3
Symmetry and conservation

Symmetry greatly simplifies any problem to which it applies – without any cost in accuracy. A classic example is the following story about the young Carl Friedrich Gauss. The story might be merely a legend, but it is so instructive that it ought to be true. One day when Gauss was 3 years old, the story goes, his schoolteacher wanted to occupy the students for a good while. He therefore asked them to compute the sum

$$S = 1 + 2 + 3 + \cdots + 100,$$

and then sat back to enjoy a welcome break. To the teacher's surprise, Gauss returned in a few minutes claiming that the sum is 5050. Was he right? If so, how did he compute the sum so quickly?

Gauss noticed that the sum remains unchanged when the terms are added backward from highest to lowest. In other words,

$$S' = 100 + 99 + 98 + \cdots + 1$$

equals S. Then Gauss added the two sums:

$$\begin{array}{r} S = 1 + 2 + 3 + \cdots + 100 \\ +S = 100 + 99 + 98 + \cdots + 1 \\ \hline 2S = 101 + 101 + \cdots + 101. \end{array}$$

i thought this was the most interesting (and my favorite) reading so far.

i still really hate this webpage. why do we have to click on each new page, and why does it always have to adjust to the top of the page when you do that? it definitely inhibits concentration on the actual material, which you think would be the important part of the class. also, if you click anywhere a stupid little box pops up and blocks everything you're trying to read or type.

I agree that having to remember to click on each new page as you come to it is irritating. It would be nice if NB would realize that once you'd scrolled a bit down a page you were most likely reading that page, instead of the last page you clicked on.

You can always look at the pdf in your own reader and go back to nb to look at and enter comments.

> I agree about the having to click on each new page. I have been trying to convince Sacha that this is a problem. He has probably just been too busy to hear me on this point.
>
> But, I'll forward this thread to him and see if the feedback from other users is enough to convince him.

When do you know to use symmetry as a abstraction versus other methods we've learned? Is symmetry good only for simple summation problems, etc?

Symmetry is such an amazing way to solve problems. I'm course 8, and a lot of professors make a really big deal about symmetry arguments. This kind of logic has become intuitive to me and there are so many times where I can solve a problem just by looking at it now.

This works for the centers, but what about some random point in the pentagon area? and what about prior to equlibrium. Gaussian symmetry addition has very limited uses.

I really like the thermo example. It applies the message to something I understand (being course 2).

I'm confused, are the temperatures 120 degrees or are they 24 degrees at the center?

I didn't find any problems with this reading; the examples were very straightforward and easy to follow. Even non-technical people can follow along and grasp the concept.

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Read the introduction and first section (3.1) for a reading memo due Thurs at 10pm. This is the first part of a new unit on symmetry and conservation, itself part of a broader unit on "Lossless methods of discarding complexity".

Is symmetry the same thing as abstraction? Because the example below looks exactly like what we did in the last pset.

Symmetry can be considered a form of abstraction; since abstraction involves a resuable, 'modular' approach, this is in fact a similar technique.

Is this how you comment?

This is an excellent story to begin this chapter on symmetry. I think it does a good job of setting the tone for the rest of the sections in this chapter.

what is he ended up doing as a career?

He went on to become one of the world's greatest mathmaticians.

Gaussian surfaces!

Gaussian distribution - that's him!

3? I wouldn't have thought of doing this now.

I've heard this story before, but I'm pretty sure the version I've seen just said he was in school, I don't think it made any claims about his age. But it doesn't seem likely that children would have started school before age 5 or 6.

I agree. Back in the day, kids also started school later on.

3 years old definitely seems a bit of a stretch...I was probably learning how to count (if even), though considering he was a genius mathematician, it could have been 3.

Agree. Maybe the story could be fact-checked, or if nothing else the age bumped up to something like 6 so it's a bit more believable.

Yea, 3 years old is a bit disheartening.

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I disagree with this phrase. There are plenty of things out in the world that are "instructive" but cannot be true.

Yeah I found this phrasing very strange too...it's quite awkward.

I think it's rather amusing, he's trying to say that something should be true because it proves a very good point, and thus as a true story would be exemplary.

I like it. I'm with (9:44)

I'm with 9:44 also, fact and fiction are often constructed out of what we want to believe. Especially with history, so this statement was pretty amusing to me too.

3 years old!?!?!

I know–so crazy. I'm reading about this method now and i have to think about it for a sec before I'm like wow thats really clever

i drew a plan view drawing when i was 3 to show my architect mother, and she thought that was cool. i've always been better with art than numbers though.

That is very impressive...

Good example, remember that from junior high!

It seems like all the greatest scientific minds have stories like this. I wonder how many people at MIT have a story like this about themselves?

I don't think this phrase is actually needed here.

I used this in the last homework

I wish I saw this for the homework it would have been useful

I've seen this during an interview before..pretty cool how Gauss figured it out when he was so young!

The story is likely apocryphal, but the idea is clever nonetheless. Remarkable how many of these questions show up during interviews!

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$$\begin{aligned} S &= 1 + 2 + 3 + \cdots + 100 \\ +S &= 100 + 99 + 98 + \cdots + 1 \\ \hline 2S &= 101 + 101 + \cdots + 101. \end{aligned}$$

Ah yes I remember this story from elementary school.

It's definitely amazing how taking a step back and looking at a problem in a new way can help so much. I often find myself trying to forget things I know when I look at this type of problem to catch some form of symmetry that would help me out

I really love this story and I actually use the idea all the time to solve problems. In fact, I rarely bother to remember what the formula is for the sum of sequential numbers, sum of sequential even or odd numbers, etc. because I know that I can always derive it again from this trick.

Exactly. And then sit and feel proud about myself for deriving it until I remember how young Gauss was when he figured it out, and the fact that I can only derive it because I know the trick.

edit: "sum *was* 5050"

I've seen this example before but never thought of it as a "symmetry" problem. It's obvious now, and this example will help me find future solutions.

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I should have thought of that... But it would have taken me a while, and I'm pretty sure I couldn't have done it at age 3.

This is pretty awesome - in fact, I've never seen this before. Outsmarted by a 3 year old...

Why was he going to school at the age of THREE? Did any one else besides me not know addition at the age of three?

I love how we all went wild over this. It's always nice to have the reading sprinkled with interesting anecdotes.

I've also never heard this anecdote before or even knew this trick, it's really interesting and I'm glad it's in the book! Even though it does make me feel like I pale in comparison to a child...

I remember learning this method years back, but I'm pretty sure it took a while for me to figure it out.

About being outsmarted by a 3-year old – that's my daily life these days. As other parents of toddlers will probably agree, any time I manage to win an argument with my daughter (who is 2.7), and my winning happens rarely, I cheer to myself, "Yeah, I outsmarted a 2 year old!"

Dude that's wicked smart. I would not know what to do if my kid was that much a brainiac.

I don't know if you have more examples of symmetry... but this section is short and this example is pretty awesome, maybe insert another example?

Additional examples would be nice, though this I think is suitably sufficient for its purposes. I feel like more examples would require additional content beyond just that, which is pretty time-intensive to do.

So I haven't done this problem in a while, and didn't remember the formula, but from the reading it's easy to see the formula is n(n+1)/2

I have always best understood this problem graphically with two triangles combining to be one rectangle, does anyone else look at it that way?

I do

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o wow ingenious (especially for a 3-year-old boy)

Yeah i always new the formula on how to get the sum, but I never knew/always forgot how it was derived. Good example!

This is a wonderful first example of symmetry.

the way i've heard this story told is that he realized that if he took pairs that summed to 100 (0 and 100, 1 and 99, 2 and 98, 3 and 97, etc), he'd have 50 pairs of 100 and a 50 left over.

This is the way I would go about doing the problem but it is pretty much the same thing. Also it would be fifty pairs that add to 101 not 100.

This is a neat impressive trick that can be useful for summations . I wonder if the story is actually true..

This version is also correct - it's 100 pairs of 101 that add up to 2S.

50 pairs of 100 + the left-over 50 as mentioned above is also correct.

yeah that's what i heard too, but same difference.

I hope that we see an example of a more complicated calculation using this method. Doing it with a simple sum is pretty intuitive, but I think right now I'd be hard-pressed to use this for anything else.

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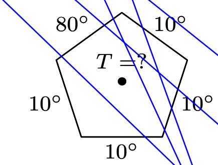
In this form, 2S is easy to compute because it contains 100 copies of 101. So $2S = 100 \times 101$, and $S = 50 \times 101 = 5050$.

Gauss tremendously simplified the problem by finding a symmetry: a transformation that preserved essential features of the problem. The idea of symmetry is an abstraction, and fluency in its use comes with practice.

3.1 Heat flow

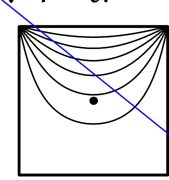


As the first example, imagine a uniform metal sheet, perhaps aluminum foil, cut into the shape of a regular pentagon. Attach heat sources and sinks to the edges in order to hold the edges at the temperatures marked on the figure. After waiting long enough, the temperature distribution in the pentagon stops changing ('comes to equilibrium'). Once the temperature equilibrates, what is the temperature at the center of the pentagon?

A brute-force analytic solution is difficult. Heat flow is described by the following second-order partial differential equation:

$$\kappa \nabla^2 T = \frac{\partial T}{\partial t},$$

where T is the temperature as a function of position and time, and κ is a constant known as the thermal diffusivity. Eventually the time derivatives approach zero (the temperature eventually settles down), so the right side eventually becomes zero. The equation then simplifies to $\kappa \nabla^2 T = 0$.



Alas, even this simpler time-independent equation has easy solutions only for a few simple boundaries. Even these solutions do not seem that simple. For example, on a square sheet with edges held at 10°, 10°, 10°, and 80° (the north edge), the temperature distribution is highly nonintuitive (the figure shows contour lines spaced every 10°). For a pentagon, even for a regular pentagon, the full temperature distribution is still less intuitive.

Symmetry, however, makes the solution flow: Rotating the pentagon about its center does not change the temperature at the center. Nature, in the person of the heat equation, does not care in what direction our coordinate system points. Stated mathematically, the Laplacian operator

Is this class taught in such a way that the previous concepts build upon the upcoming ones? It seems we used D/C to explain abstractions and are now using abstractions as a stepping stone for symmetry

> I have tried to design it that way, and am happy that it is showing results. One reason I put abstraction early on is that everything else in the class is an abstraction.
>
> In a couple chapters from now, we'll use symmetry to understand dimensional analysis. Then the limits of dimensional analysis will lead us to extreme-cases reasoning.

Maybe it would be useful to put the actual formula for the sum after we have been given the symmetry argument, to really show the power of representing symmetry into non trivial formulas

I find this example easy to understand.

that is a interesting way to do this, its the kinda thing that can be useful in many different types of situations

I've actually never heard of this trick before, thats awesome! But it only works for finite sets (obviously, I suppose) which means it's only applicable sometimes. Still very cool.

Well not exactly... I used this for computing the sum of 1 + r + r^2 ... more or less... sorta a combination of abstraction and symmetry. You can multiply this sum by r and you have the same sum - 1. If you add them together you will surely find your answer.

I remember learning about this in HS when we learned about series

This is really useful. I feel like we used this a lot in 2.001.

I prefer a similar trick that I find slightly more intuitive... There are 100 numbers, with average 101/2. So their sum is 100*101/2, which results in the same math, but is faster for me to apply in other occasions.

Before I read the explanation, I was reasoning it this way: 1+99=100, 2+98=100... all the way to 49+51=100, and 4900+100+50=5050.

I was imagining some kind of symmetry around the "50" axis.

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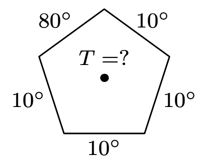

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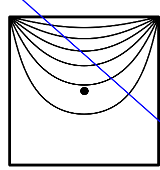

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Instructive, interesting example (though I'm not sure the story is that he was 3...it's very hard to believe).

For some reason I have a hard time realizing when to use symmetry. I suppose I should check that first every time, it's always pretty simple when it works.

He also says that it takes practice to get good at realizing when to use it.

I feel like symmetry and recursion are similar in this manner, the only wy to get good at them is to see a bunch of examples

Definitely. It seems more of a manner of thinking than anything out, just as in programming sometimes people understand recursive intuitively while others struggle to apply it - it depends on how you think through problems.

i feel like anything can be an abstraction at this point. any problem solving tool is an abstraction because it is separate from a specific problem? this makes abstraction rather....abstract. i think it's more useful to focus on the tools.

As you point out, all the problem-solving tools are themselves abstractions. Every word is an abstraction. Abstrations are everywhere! One purpose of this unit is to help everyone see how prevalent abstractions are and to start noticing them everywhere – as the first step toward using them and then making new ones.

At first when I read this I wondered why you didn't tell us how generally applicable abstraction really is. Now, I'm thinking we just wouldn't have understood without all of the examples.

I think of symmetry as a feature of the problem, not as the transformation that exploits that symmetry.

When an old physics teacher explained the idea of symmetry being a feature of the problem rather than something to try to force upon a problem it really cleared it up for me - I think its a good way to think about it that helps things make sense.

I like how symmetry is put into the category of abstraction here. I had previously thought of it as its own thing, but it really is just reusing information over lines of symmetry.

So what differentiates symmetry from abstraction? is it just repetition?

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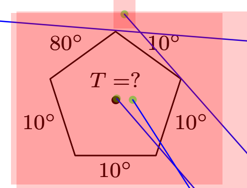

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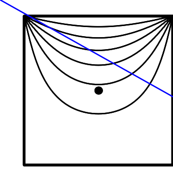

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I haven't ever dealt with heat flow problems before...not sure if this is a course 2 familiarity, but the symmetry approach makes it seem almost too easy. What a great example.

I totally agree with you...I finally get it!

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I agree- the examples in this section are excellent.

Heat flow implies that we might be calculating d/dt of something. Maybe a better title is "Temperature gradients" or something like that.

I agree. The title Heat Flow implies that there will be some transfer of heat, whereas you're really just interested in temperature differences and gradients.

If there's a temperature difference, there will be heat flow. The temperature distribution is just at equilibrium.

Oh, I just realized it's the temperature not the angles... Maybe that could be clarified as well. It is obvious when I read through the material but not when I glance at it.

This is a regular problem in 18.04...we use complex numbers symmetry together with regular physics equations

I am confused about how angles are notated here...

This is a regular pentagon, so each angle should have 360/5 degrees. The notation is actually the temperature at which the edges should be held at.

what are heat sources and sinks? A meche term?

In real life would there by anyway to keep the points between where the 80degrees and 10 degrees meet from changing each others temperature?

Could this even be easily approximated (divide and conquer style)? Or is it too complicated to easily break down like that?

I was wondering a similar thing - how do we decided to approximate something vs trying to find a clever way to find and exact answer?

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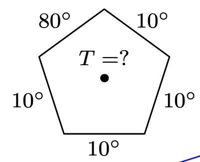

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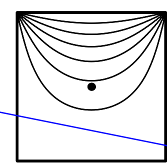

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can you explain what exactly brute-force is? I've also heard of this term, but I don't really know what it means.

Brute-force methods are those approaches where you know you'll get the right (usually exact) answer, but that will take more time/effort to complete. They usually involve either using a difficult analytical solution or some repetitive process that could be automated or collapsed into fewer steps.

The complexities involved in actually mapping the calculations and heat flow by brute force is nearly impossible. The equations would just be too complex.

The brute-force method is suggested below with the differential equation and explanation.

An explanation of what this is would be helpful.

The next sentence explains the variables and the sentence after explains the equation.

If I never took Thermodynamics, I would be a little confused here.

Is this constant specific to the type of material/thickness?

it's a material property

From wikipedia, thermal diffusivity is "the thermal conductivity divided by the volumetric heat capacity", so it's variable depending on the object.

This bit of information seems irrelevant to the question at hand. (Not that it's not a useful thing to know, just it's unclear why it's here.)

I think it's still relevant in that it explains why the dT/dt component of the equation drops out. If the text had initially introduced the heat flow equation in the time-independent/stable form then I would agree.

I think what Sanjoy wants to do is start with the original equation, and show how it simplifies to this easier-to-deal-with form where the right side =0.

redundant (you already used the word eventually in this sentence)

The exact meaning of this diagram seems a little unclear, and it isn't really referenced directly in the text. Maybe label it more clearly? And explain the contour lines?

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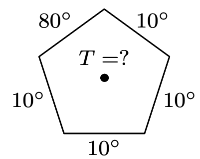
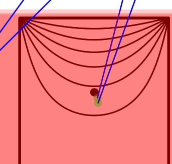


As the first example, imagine a uniform metal sheet, perhaps aluminum foil, cut into the shape of a regular pentagon. Attach heat sources and sinks to the edges in order to hold the edges at the temperatures marked on the figure. After waiting long enough, the temperature distribution in the pentagon stops changing ('comes to equilibrium'). Once the temperature equilibrates, what is the temperature at the center of the pentagon?

A brute-force analytic solution is difficult. Heat flow is described by the following second-order partial differential equation:

$$\kappa \nabla^2 T = \frac{\partial T}{\partial t},$$

where T is the temperature as a function of position and time, and κ is a constant known as the thermal diffusivity. Eventually the time derivatives approach zero (the temperature eventually settles down), so the right side eventually becomes zero. The equation then simplifies to $\kappa \nabla^2 T = 0$.



Alas, even this simpler time-independent equation has easy solutions only for a few simple boundaries. Even these solutions do not seem that simple. For example, on a square sheet with edges held at 10°, 10°, 10°, and 80° (the north edge), the temperature distribution is highly nonintuitive (the figure shows contour lines spaced every 10°). For a pentagon, even for a regular pentagon, the full temperature distribution is still less intuitive.

Symmetry, however, makes the solution flow: Rotating the pentagon about its center does not change the temperature at the center. Nature, in the person of the heat equation, does not care in what direction our coordinate system points. Stated mathematically, the Laplacian operator

not the best example.

I disagree. I think the square is a good example because it is a simpler symmetrical shape that we can compare with.

I also think the drawing is good, but maybe a little more, like a directional, or some labels would be better. If you miss something in the reading, then you will completely skip the picture, and it has a profound effect.

I think labels here might help if someone were to quickly go back to look at it again. I always find figure captions very helpful for reviewing materials later.

i'm not really sure how this all makes sense

just refer to the diagram

I think it's fairly straightforward, since the figure is right next to it.

It sort of makes sense... there shouldn't be a gradient on the bottom of the diagram since the edges are the same, and you expect to see a larger gradient on top. I doubt I would have gotten the exact shape right, but seeing the shape, it makes sense

The idea behind the gradient makes sense, but I think the exact shape of the contours in the diagram is quite non-intuitive.

I find the shape to be somewhat intuitive, maybe not the spacing. I wouldn't imagine it being in any shape other then an arc.

I too agree that the diagram makes sense, but given the edge temperatures and asked to find the stable temp at a particular point, my answer would be very inaccurate. Therefore I would tend to agree that the distribution is highly non-intuitive.

I agree. It's intuitive that there is a gradient, but as far as the shape of it, I would have no idea. Plus, as the shape gets more sides, I'd expect that the gradient would get harder and harder to visualize

I would think that it turns out to be something like the above picture but with closely spaced lines towards the edges that are 10 degrees. the far 10 degree lines would have spacing farther apart than the ones next to the 80 degrees edge

In this form, 2S is easy to compute because it contains 100 copies of 101. So $2S = 100 \times 101$, and $S = 50 \times 101 = 5050$.

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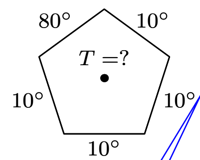

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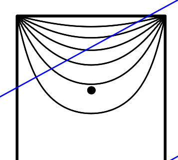

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Is this supposed to be a pun? If so, well done!

Is there a way to rotate something about the center and have the center change temperatures? I can't really think of anything.

Agreed - how would rotation every change the temperature at the center, even if the shape is not symmetric?

Well, if the shape weren't symmetric, the temperature at the center would change based on which heat sources were where. For instance, if one point of our pentagon were longer, then putting the hottest source there versus at a closer point would definitely change the answer.

wha?

blink what person?

I think this is not good word choice. "Nature, as exemplified by the heat equation..." or something similar to that

or "represented by"

I agree. "Person" is both distracting and confusing.

typo for persona?

This statement seems (for lack of a better word) obvious. I can't think of any systems in which the temperature would change in the center due to the directions of coordinate systems?

I am experiencing a bit of a disconnect. Even though nature doesn't care in what direction our coordinate system points, why do we have to consider the 5 directions that our pentagon can point. Why can't we just consider 1 orientation, add up the temperatures, and divide by 5?

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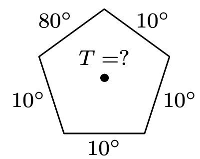

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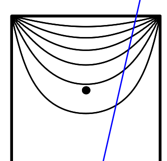

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What exactly is the definition of this symbol?

This operator is from 18.03 I believe- from Wikipedia- The Laplace operator is a second order differential operator in the n-dimensional Euclidean space, defined as the divergence of the gradient.

The Laplace operator is div(grad).

It took me a little while to make the connection between "Laplacian operator" and Laplace...maybe I just need a little more sleep...

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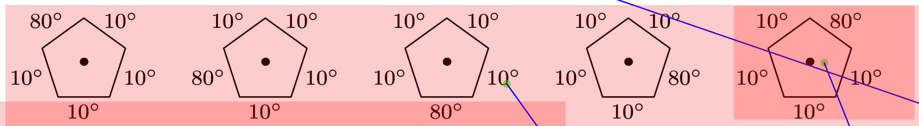

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can't you just say it doesn't have an inherent coordinate system rather than making us think about spinning pentagons?

I think you need the pentagons in there to show what the symmetries are. A scalene triangle has no symmetries but the Laplacian is still independent of the coordinate system.

Other than doing this rotation part, couldn't you have simply "weighted it"? Known that the temperature was going to have 4 parts influence from 10 and 1 influence from 80, and added to 120 and divided by 5 for this reason?

this is what i would have naturally thought to do as well. is this a correct line of logic?

I think so – both methods seem to be using the same idea, but I think the one expressed in the text is more intuitive. Your mileage may vary, though.

The idea that the temperature moves in the way the pictures show seems pretty intuitive to me. It would be nice to see a comparison between this and the way Gauss was able to flip the numbers backwards then add them up.

It's the same concept though, averaging over different perturbations of the same problem to figure out what one answer would be from multiple solutions

To me, it is clear that the pentagon is rotationally symmetric, more clear than the operator being invariant. Perhaps you could say "By inspection, the pentagon is rotationally equivalent. This fits with the definition of the operator, which is also defined as such"

I think the visual presentation here is good. I remember looking at the tree drawings and thinking they look messy and drawn in paint, but this looks very professional

But they were drawn with his "tree" language!

it would be great to have a diagram with the line spacing to illustrate the temperature distribution.

This example was explained VERY well and made perfect sense. It was clear, concise, and easy to understand.

I dont understand this part

I wish I had known this trick when I took 18.303. We had to do things like this the long way...

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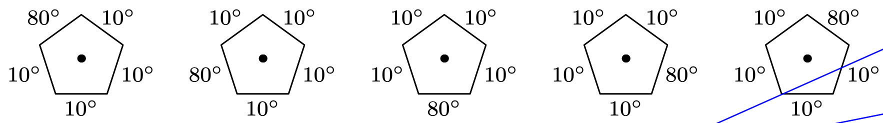

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I would have just guessed that since the dot is in the middle, that each edge contributes equally. I forgot if you said it in class, but is this any different from stacking them?

How do we know that temperature addition obeys linearity? I would have approached the problem by averaging the temperatures of the edges, which I know works – e.g. mixing hot and cold water – and that does give the same answer.

On second thought, since the differential equation is linear, I suppose both approaches should be the same, though I still don't like adding temperatures because that is not as rooted in a physical phenomenon.

Here's is where an abstraction is useful. Blur your vision a bit and don't look too closely at what you are adding. The method works because of a general property of the differential equation (linearity), independently of what the particular quantity being differentiated means.

Alternatively, think of *averaging* the temperatures rather than adding them. Averaging temperatures is a physically legitimate operation.

I think it is fairly obvious to me that it would obey linearity, just from experience. Also, averaging is a linear operation, so aren't you also assuming temperatures add linearly?

This still seems counter-intuitive. If I have a 2 sheets of metal at 100deg and stick them together, their temperature doesn't become 200deg...

More explanation regarding how this is legitimate would be fantastic...

I agree with this comment; it does seem very counterintuitive since temperatures don't magically add when you stack them together.

I am also very confused as to why using symmetry enabled us to stack the pentagons as a way of simplifications..

I thought when materials of different temperatures come together, they would reach a certain equilibrium. I don't understand why we can add up all teh temperatures

I'm just struggling to conceptually understand why we can add all the rotated pentagons to come to the right answer. Maybe I just don't understand this Guassian principle.

I think this brings up a good question. In general, when do we know that we can assume a problem to be symmetrical?

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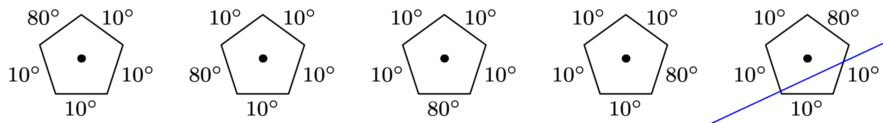

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i'm not following this logic

temperature is summable???

This is really clever, actually. But I don't think I'd be able (even after seeing this example) to be able to apply this idea–can we use it any time when we have things evenly spaced from a certain point? and in this example–even if the points weren't evenly spaced, could we still use the same method? how would we take the average for the temperature "at the edge" if the edges were different distances from the center?

You couldn't. This only works (I think) because of symmetry. Once the object in question is no longer symmetric, this trick no longer applies

Is it possible to pick a different point to find the temperature at using symmetry? Or is it generally always at the center?

Can you find another point where the figure is symmetric?

You can make any number of arguments about two points having the same temperature, but without the full symmetry of the central point, you can't figure out easily what that temperature is...

This is so cool! I love seeing Gauss's method applied in a way I never would have thought of.

gauss' method? not really...it's symmetry which i'm sure was not invented by him.

This is very clever, clearly written, and unites the idea being presented here to the introduction about Gauss.

I thought the edges are also stacked on top of each other? maybe I am thinking of a wrong picture

They do..all of the edges sum up to 120deg.

I am so confused as to how this is able to work- I understand the math, but temperature??

This wins the award for my favorite problem we've done so far. Definitely elicited the biggest "I didn't see that coming" reaction yet.

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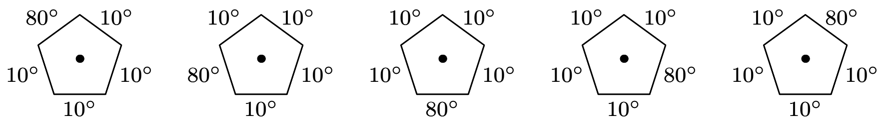

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Can you just add temperatures like that?

I was wondering the same thing...at one level it sounds like it should be good but it still seems too easy to be true.

I agree especially when the problem says the edges are held constant with sinks and sources. Wouldn't the sinks just eat up the 80 degrees?

I guessed this would be the solution. But it just seems so simple that I didn't want to trust the answer.

Well, didn't the professor say that part of this class was learning to trust our gut?

This only works because we're talking about the system at equilibrium, right?

okay, so it makes sense intuitively to divide by 5 – but what I don't understand is if the temperature is 120 throughout, why does having the 5 centers aligning mean you need to divide by 5, shouldn't it just be the same throughout?

The way I see it is if you stack the 5 pentagons on top of each other then the sums of the temperatures at a point on the combined pentagon will be 120 degrees. Since the center of each pentagon has the same temperature (it doesn't change with the orientation of the heat sources) the center of each pentagon must be at a temperature of 1/5 of 120 degrees.

Hope that helps.

That does make sense, but I thought the temperature distribution was uniform and at equilibrium?

noooo the temperature is no longer CHANGING but that does not mean its uniform. (See the above temperature gradient plot for the square). What this says, is that the ONLY point on these 5 overlaid pentagons where each perturbation would have exactly the same temperature is in the middle, where we're looking, so you can in fact average every possible rotation and divide by the number of rotations. It would not work if it was not the center, or if we had assumed the temperature gradient was still changing

all right, that makes more sense.....the reason why I assumed it was uniform was because in the previous paragraph it says "so the temperature throughout the sheet is 120"...that should be changed so it is not so confusing.

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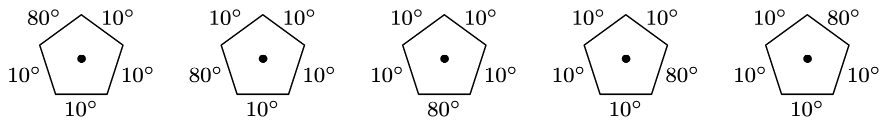

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I would have guessed that even though I don't know much about thermo

so this is a simple add and distribute heat problem since each side is equidistant from the center.

this is the kind of stuff I was hoping to learn in this class. I hope I can start thinking like this.

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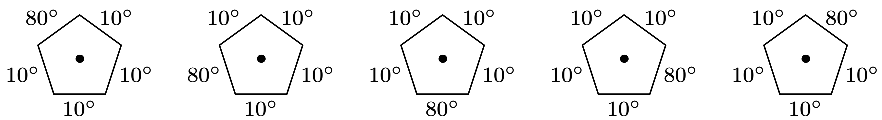

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I'm not at all convinced that this is the correct answer. Honestly, it felt like we were arbitrarily stacking pentagons and adding their temperatures for no particular reason. I understand that temperature doesn't change under rotation, but how does that imply that we can stack them and add their temperatures?

Is this the same as just finding the average of the pentagon's sides temperatures?

yeah that would've been an easier explanation

Agreed...averaging the temperatures of the edges is the same argument, and much easier to follow. I think he was just looking for a symmetry example that was easy to grasp, but perhaps this is not the best one...

I agree as well - this explanation had me thinking "Wow! _IF_ that is true, that's a cool method of solving it" with a big emphasis on the if. I wasn't convinced as to its validity.

Um, given that it was already stated the temperature at each side is set (forever) at 10 or 80 degrees, this would not work to find the temperature at a side (that is already known). it means that instead of solving complicated equations and looking at gradients...at ONE point (the symmetry point, the center) we can average over 5 possible configurations which would all give the same center value. NOW (since all 5 have been averaged) the whole pentagon is at one temperature, including the center. So then you divide out by the number of times you had to rotate it to get back to the center temp for ONE pentagon and have the answer

It ends in the same calculation, but the symmetry explains why it works to average the side temperatures. The symmetry argument also shows what the necessary ingredient is: a linear differential equation. If heat flow had been described by a nonlinear differential equation, then it would still seem plausible to average the side temperatures but it would no longer be correct.

I think that one point that might help is that the edges are heat sources. If you put all the sources at one point (or edge) then the resulting temperature output will be the sum of the individual temperatures.

I'm not sure this holds here.... I think this just gives the average temperature...not necessarily the temperature in the the center of a particular pentagon

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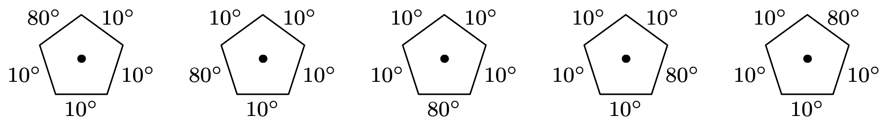

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I need a little more help with how the heat transfer stuff is the same as the gauss's sum

This symmetry method seems very useful, but I feel like it would be extremely hard to intuitively devise- or find- a symmetric way to solve a problem, especially a hard problem.

I agree... I would have never thought of this approach... I think with more practice with symmetry problems this will become more intuitive....hopefully.

Yeah I agree...I think it will become a lot more clear how to use this method if we go over some more simpler examples in class.

I agree. I understand it now that it's worked out, but I'm not sure I could think to do it this way given another problem.

but you didn't actually solve for the temperature curves, so you wouldn't have used that anyway. and your statement that it was "non-intuitive" doesn't apply when you don't solve for the curves either. we only looked at one extremely convenient point.

It's more that he took the only possible point. This wouldn't work for elsewhere in the pentagon, but for this point (symmetric) it does work, and that's the beauty of it. The Gauss example is clearer, but only applies to a very limited number of cases, whereas this could be applied to many, but only to solve for one point.

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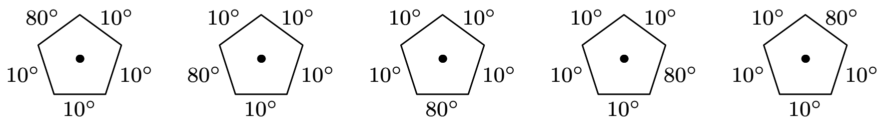

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I find it interesting that both problems are actually solvable in other ways. Is this always the case? Is there ever a case where using symmetry or some derivative of symmetry is the ONLY way to solve a problem?

I doubt it, but I do feel that symmetry makes things a heck of a lot easier.

I agree. I can't think of a situation in which symmetry allows you to solve a problem that is otherwise impossible (assuming infinite computational resources, of course).

This may not actually answer your question, but symmetry is extremely important in mineralogy and crystallography. The space groups and all those groups related were at least originally solved used symmetry - and mineral structures can also be solved using symmetry. I'm not really certain of more modern, computational methods, so there may be ways to do it without symmetry now - but I just don't know.

I don't think so. I think it's just a shortcut for problems that are normally more complex.

I agree, just as in Gauss's problem, it is possible to solve the problem with computational methods but we are learning quick computations.

I'm sure there are always multiple ways to solve a problem, but it looks like whenever symmetry is possible, it is the easiest–the trick is just being able to identify the symmetry

what, precisely, is a "symmetry operation" in most concise words? the act of.....?

only after some time was I able to understand the solution. Its not immediately apparent

This paragraph is very good. It sums up the lessons from these two problems. Sometimes these examples can get caught up in the details of the physics/engineering that the approximation lessons is lost. These types of paragraphs are great.

not necessary.

"rotated on each of its sides" would fit more with your goal of making it appear simple.

I disagree. It's precise than saying "one rotation" or something like that.

*more precise

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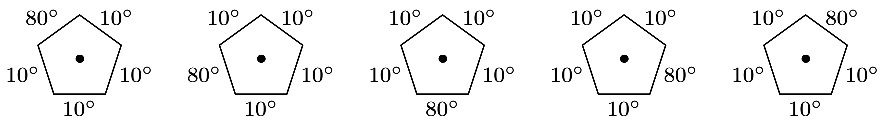

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I really like this explanation and how it ties back to the Gauss story - flipping the pentagon is just like flipping the number sequence - kind of gives new meaning to lines of symmetry for me.

I would like to see more examples. I feel like symmetry is best grasped through lots of practice and lots of exposure to different ways of applying symmetry arguments to different types of problems.

This is analogous to controls in a scientific experiment.

Possibly putting this before the examples would be helpful

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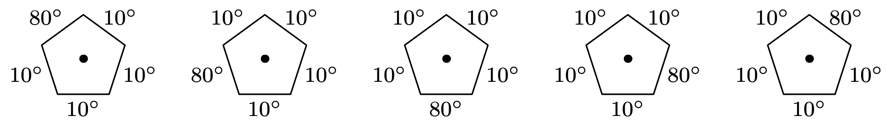

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This is a good summary and it answers my question from earlier. Are you going to add practice problems to this section like you did in previous sections? I think they would be very helpful since this is a concept that requires a lot of practice to master.

While this is a good summary of the section, it would have been nice to see the example problem broken down in to a summary like form, in order to have more clear insights as to the thought process of using symmetry to solve a problem.

I'm a little confused here - what exactly "did not change" in this example? The temperature at the center when we rotated the shape? I'm also still failing to see how symmetry makes the problem easier when we could have just averaged the temperatures of the edges (although that also uses a symmetry argument - perhaps use that instead?).

Perhaps this concept of invariants should be a section in and of itself? It would greatly simplify certain classes of problems.

I agree that the idea of the invariant can be discussed further. I hope to see it expanded in other sections, or else I suggest the concept of the invariant be expanded. It makes sense, but I'd like to see more examples.

I think that the part that didn't change was that the Laplacian operator doesn't change with pentagon orientation so by adding multiple pentagons to each other all sides are now 120* which makes the problem simple. Same with the Gauss problem, adding the numbers doesn't change if you add them up or down, so you can add both ways together and divide by 2.

I really like how you gave a strategy for examining problems, and think it is a great way to wrap up this section and go into more detail about symmetry.

I agree, previously we would often have examples explaining a particular concept but it was sometimes difficult to extract the actual lesson from the example. I think this summary does a good job of reinforcing the concept and also leaving the reader with a practical application point that one can remember.

What I am learning from this thread (and many others) is that it is very helpful to reap the fruits of each example. In other words, after doing an example, figure out what are the transferable lessons (and maybe do another example with those lessons explicitly). I'll try to do that more as I revise the book.

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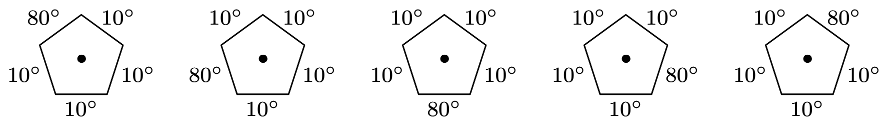

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I'm really excited to see how this book is going to end up!

This sentence cleared it up for me. But I don't like how I have to wait until the end of the article to get to the moral of the section. The path that you take us through could be much simpler.

Likewise...the mention of invariants brought back other examples of invariants that have been used for problem solving and opened up a lot of other ideas of symmetry applied - it'd be nice if this was brought up earlier so that we could have other applications in mind too

Actually, I'm not sure this statement would have made sense to me if I hadn't seen the example first.

I agree, it's easy to understand what he's talking about after seeing the example, but I don't know if would be this clear in the beginning.

I do understand this summary here, but what are some other examples of problems where symmetry can be used? and what is unchanged/changed? it looks like addition problems and anything involving shapes (as long as there is constant distance from a certain point/center) can be solved with symmetry, but what else?

Additionally, I feel like if I had seen this problem I wouldn't have tried solving it symmetrically this way because I would have convinced myself its more complicated than that. How can you tell when it really is that simple? (and when something looks like you can use symmetry but shouldn't)

Good Summary!

Will we always solve for these unchanged quantities by adding up the symmetrical results and dividing by the total number of symmetries (as we did in both of these examples)?

I feel like that there's some abstraction here... so probably not. I guess I think the point more is to look for symmetry and see how you can use it in general, not that there is always this specific formula for symmetrical things.

These examples were both easy to follow.

Try not to highlight the entire paragraph with a comment box. It makes it difficult to isolate the comment boxes underneath it.

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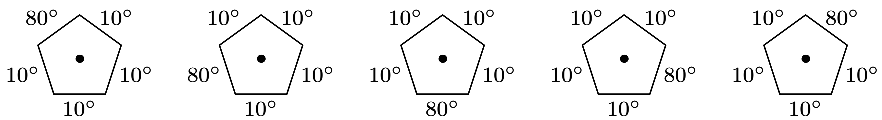

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why won't these stupid boxes stop popping up???

Which boxes? It sounds like an NB bug. Can you send me and Sacha <sacha@MIT.EDU> an email describing specifically what happens?

Me 2. when I'm writing a reply and I click with the mouse. Another "options" box pops up.

I've found the way to avoid this is to use the arrows to move around while you're typing instead of clicking. If the box pops up, you can click in another part of your answer to move it around so you can see your text or click the Save button.

Yeah this happens to me all the time too...I think its a bug in the NB user interface.

3:47 ... yes, that is a way to get around it, but it's still annoying ... and I'd really like to be able to highlight something I'm writing easier.

to be more specific, this box pops up when we're typing a reply and then click inside the box to, perhaps, go to a different part of our text to readjust wording. the cursor still goes back but another box pops up to give us the option to reply to it. quite annoying.

Ah, that must be it. The 'reply' dialog box somehow does not shadow the comments underneath it. So a click in the reply box is interpreted by NB as a click on the comment underneath it, so NB then offers another reply box...

I never noticed it because I never use the mouse unless I have to, so I always used emacs editing keys (ctrl-P to move up, ctrl-N to move down, etc.) to edit comments.

But I agree, it is an NB interface bug. I suspect it is easy to fix. I'll point Sacha to this thread.

Hi everyone, Thanks for the feedback... I'll try to fix that ASAP.

OK... so it looks like it's related to the "options" menu: While this gets fixed, a workaround is to bring the context menu not by clicking on the "options" link but by RIGHT-CLICKING anywhere on the note (mac users: read control+click)...

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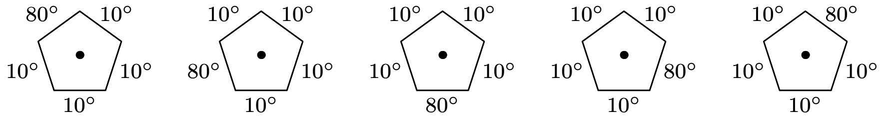

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It should be fixed now. Be sure to reload the page using Shift+Reload to that Firefox doesn't use its cached copy of the code, and give it a try

Thanks for letting me know, and have a good w-e.

I like the diagrams in this reading, the previous ones, and the next one (went ahead). They are very helpful and very simple; it makes understanding the readings easier, and also gives some nice supplement.