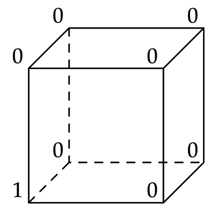
3.2 Cube solitaire



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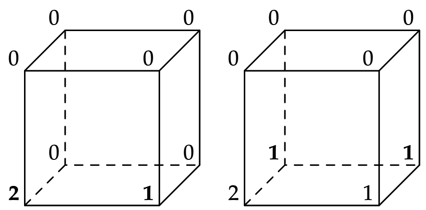

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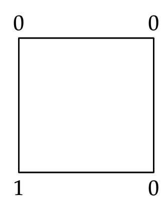

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this definition of an invariant seems different from the one in previous sections?

The explanation above definitely helped understand how each variable(side) was configured to equal one. I think we should further explain the equation in class, step by step.

I think this example is great since it draws on things we all learned in 8.01. However, I am not really sure of its connection to the cube problems. I am not sure of the comment "physic problems are also solitarie games".

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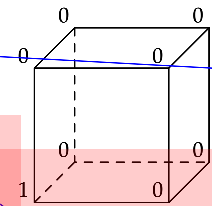

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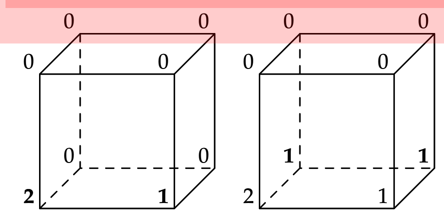

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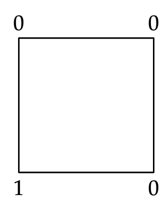

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Read Section 3.2 for the memo due Sunday at 10pm. (Note: NO lecture on Monday.)

When I think of solitaire, I think of the card game. Is this some other meaning? Or is the term more general?

I'm also very confused here - wikipedia-ing solitaire comes up with the card game as well. What does this have to do with the cube?

I'm assuming it just means it's a game that you can play by yourself....

Ya, in this sense solitaire means a one person game/challenge. You guys never played solitaire games like this (http://www.woodtoysonline.co.uk/Family%20games_files/s where you had to remove all but the last peg by jumping pegs over others to remove them? I wonder what the invariant is for those games?

As in, in the last move, all the vertices must because 3? I think I'm misunderstanding the rules...

Nevermind, didn't see the "multiples" of part.

If this really is a game called solitare, is there a way nevertheless to call it something else to avoid confusion?

a followup question to this section could be for which n's does the problem hold?

This question is great example for this section. I originally tried to do it before breaking it up into pieces. But once I saw the problem in a different light, it was obvious.

this seems really similar to the checkers board covered in dominoes. I don't think you can win this game

I still don't really get the game

I wouldn't consider this a game...even a solitary game. I think that it would be better to call it a Puzzle.

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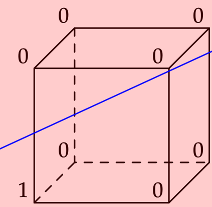

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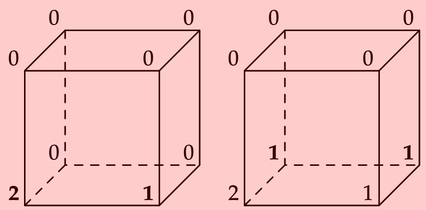

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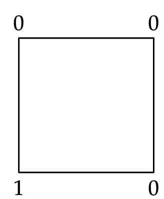

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it might be nice to have animations for this so that a person could play the game and just see how it works.

or maybe one edge highlighted by a color or bolded and then the vertices circled. i know i understood the problem without any bold/circling but it's nice to know i understood it perfectly

I agree, although it's just as easy for me to take notes physically which is what I'm doing to keep it all straight.

I like the bolded suggestion if this was to be in a textbook instead of simply online.

Bolding the line would be useful, but it's not too difficult to follow and suggesting "animations" just seems ridiculous. I mean, you chose a line and its two vertices increased in value by 1. It's pretty straight-forward.

I think that the statement was more of a 'this would be cool' than a 'i need more to understand this" ... I, personally, would enjoy playing around with an app [just for "fun"]

ps. to orig. poster, please don't highlight huge blocks of text like that...it makes it harder to pull up notes for the one lines underneath it...do something similar to what you see along the edges. Thank you.

I agree, I had to read this over several times to understand it. perhaps the above suggestion would have helped.

I feel that if you have the technology why not make animations that can easily explain a point.

I showed this to a friend who likes puzzles, he looked at it for about two minutes and then told me it was impossible, and a used a variant on this proof. (I feel kind of dumb in comparison now)

So how many moves do we get? As many as we want?

I think we get as many moves as we want, but the phrase "neither configuration wins the game" doesn't make it super clear that from these positions and after many more moves, the game can't be won.

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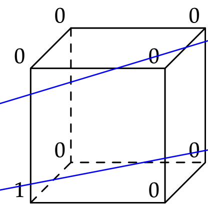

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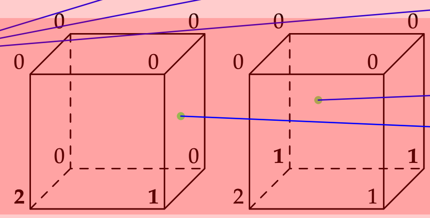

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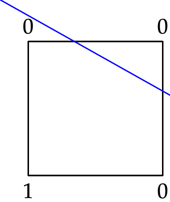

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I don't know about for others, but this is still not that clear to me... I think animation might actually help for people who are confused and are slow at understanding things.

I'm still confused about how this example as a whole illustrates symmetry. Any thoughts?

The first thing that came to mind for me was actually flattening the structure into a series of squares.

As for the image this comment is highlighting... I think it works well for understand how the game works. Most people seem pretty confused about the name "Solitaire", but after describing how the game works, and confirming the mechanics with this image, its very clear. I don't think further clarifications are needed to explain the game.

I understand how the both this game and the game of solitare are played but I don't really get why this game is called solitare.

I also think that this might not be the most effective way to begin explaining symmetry. The reading gets very dense when discussing mod and also trying to remember symmetry, and remember the solitare game. Its gets difficult trying to process, understand, and remember all of these while at the same time trying to make connection between them (i.e.. connection of how the game relates to symmetry). I just compared this to the next reading and for some reason it is much easier to read and comprehend than this. Just wanted to give some constructive feedback

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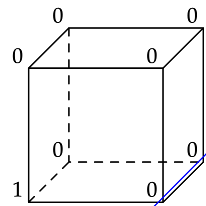

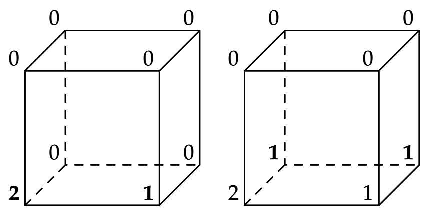

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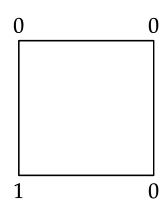

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Try to answer this question for yourself before reading onward!

This might seem like a silly request, but it's actually useful for learning. Oftentimes we glaze over things with a "Yeah, I can do that if I wanted" attitude at the expense of actually doing it and learning. Active reading = good reading.

One advantage of personal teaching, including lecturing, compared to books is that you can stop and make sure everyone tries the problem. As the next-best thing, I'll put a solitaire problem on the next problem set. (We'd do one in class if I were not away on Monday.)

Can we do one in class Wednesday anyway?

I wonder if you could include one more example where this principle of the invariant helps you prove that a final state is true, as I'm curious as to how it would resolve into a winning sequence of moves?

I accidently read someone else's comment about this not being solvable before this. Perhaps the creators of NB could make it so that the instructor's highlighted boxes could be a different color?

Yeah. Your email address shows up highlighted in red, but not until the comment is expanded, and they all start folded in so we can't tell which ones are yours without flipping through them all.

I've been trying to work out a solution but it seems hopeless! Can we go over this example or another in class and prove that this doesn't work (as I assume)?

I think, with these kinds of questions, you can usually guess that you can't win. It makes the problem more interesting, and it's fun watching people hopelessly try to find solutions.

The way I see it, is that you always have to increment two points, and there are an even number of vertices on a cube. Therefore it's impossible to get an even number of total increments (sum of the number on each of the vertices = even) when you start out with an odd count (1) and only increment with even numbers (1->3->5 etc)

Looking at this problem again it's amazing to see how now we have methods to actually attack it. When I saw this weeks ago I spent too long playing with the 3D cube picking random sides and getting nowhere, but now I have a starting place

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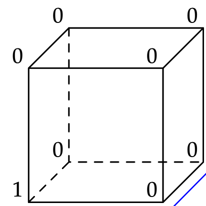

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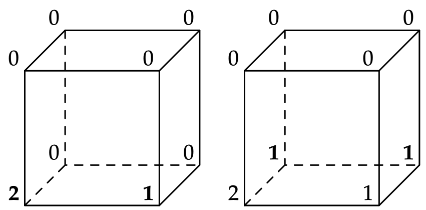

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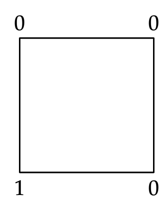

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haha. i find it funny that this was the next sentence because i stopped reading just before this sentence, to try the game, as suggested, and stopped not too long later b/c i was overwhelmed by the options possible.

haha I did the exact same thing!!!

At first glance this sounds like a quick twist to a 6.006 problem

Could we maybe code this up and run it on the computer?

how did you figure this out?

ultimately you need the corners to sum to 24 unfortunately no multiple of 2 is within one of 24 (and divisible). if the sides were going to be 9 corners then it might be possible because you could at 26 points with moves and use the initial one value corner to get a total of 27 which is divisible by 3

Shouldn't this say "there are 12^10 sequences for ten moves." The current phrasing makes it sound like there are only ten moves possible in the game.

Or even a MUCH lower number!

Agreed! Anything somewhat lower than 12^10 is still huge.

how low can you go?!

I think it would help here to discuss the difference between combinations and permutations briefly.

This seems unnecessary

i suppose this depends on whether or not you're solving something else haha

I use this approach a lot when trying to figure out combinations and permutations, just write them out systematically in all possible ways, but it gets unreasonable if there are more than 20 or 30.

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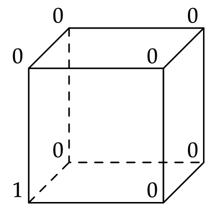

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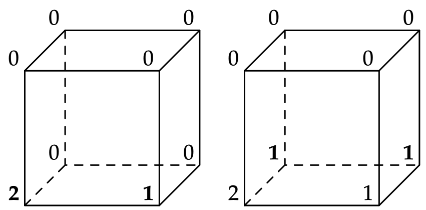

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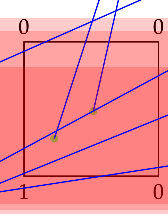

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Simplifying the problems is always a great approach to finding a solution

Although this is true, it's always easier to see and understand the simplification rather than come up with it yourself.

I agree. I don't think I would have been comfortable reducing the 3D case to the 2D/1D case without being shown it wouldn't change the outcome.

Does that answer mean we can just simplify it and multiply our answer by the appopriate number?

i tried this! :)

yeah me too. I like how this problem goes back to the divided and conquer methodology.

I agree–perfect example of divide and conquer, and something that we could all do without a bunch of previous knowledge

how does solving for a square give us the answer for a cube?

Well, we can abstract with a decent degree of certainty that if there's no solution for a square, adding complexity will not make a solution _more_ likely. It doesn't give us an answer per se, but it lets us make some justified assumptions.

I like this idea of breaking the problem down into a smaller piece. I don't really think it's lowering the standard but just figuring out a strategy to apply to a simpler problem so you can apply it to the more difficult one.

SO does one just guess that trying this out with a square will lead to valuable information?

It could also be seen as a divide and conquer method

Makes sense

Makes sense

So I have some worries about simplifying the problem, and it's that I might skip over the key to answering the question. Ever since coming to MIT, problems aren't as straightforward. So I can't see myself really being able to use this outside of the class.

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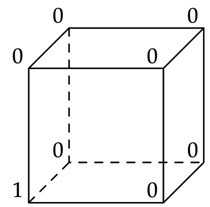

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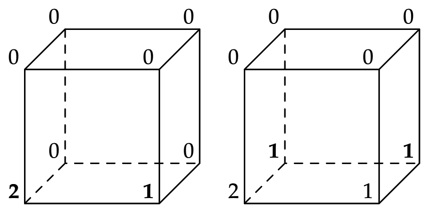

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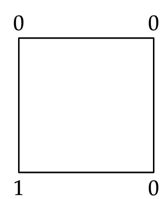

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so this doesn't count as divide and conquer?

I think this might relate to the lossy/lossless situation that Sanjoy mentioned in class in reference to something else. Lowering your standards to me means simplifying the entire problem, while divide and conquer is breaking it into smaller pieces. But I'm making this up... I could be completely wrong. I guess they both relate in that you are taking something complex and making it simpler?

Yeah, I think it has to do with both–we are lowering our standards in that if we solve this problem, we've only solved the square, not the cube–a much easier problem. BUT,it could also be seen as divide and conquer since you're breaking the cube up into its faces. So if you could find the way that each face relates to the whole, you've solved the problem using divide and conquer. that's my understanding...

great statement

author of this quote? or is this one of your own?

I think this is one of the best lines so far in this class, though I feel like it's been stated once already. I think (and I think I've stated this before) that there should be a bunch of these; sure you lose some comedic effect, but it's memorable.

A proverb to live by! It seems like all the previous units kind of build on each other. This statement kind of reflects the divide and conquer approach as well. Instead of dealing with all 12 edges, divide it into just 4 and conquer that first.

That's hilarious.

...and this is why I love the readings for this class.

I love it.

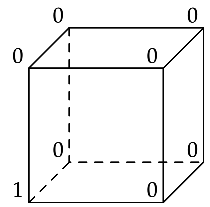
I have a friend who is known for lowering his standards when the going gets tough!

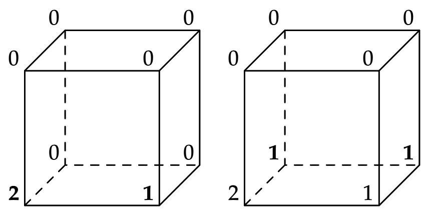
I don't really agree with the statement, there is a difference between lowering standards and simplifying things. It does sound kind of catchy, and does sort of get the point across, but not the words I would have used.

This might be good for approximating, but I don't think it's a good proverb to live by. When the going gets tough, the tough should try harder! And perhaps collaborate/ask for help.

3.2 Cube solitaire

Here is a game of solitaire that illustrates the theme of this chapter. The following cube starts in the configuration in the margin; the goal is to make all vertices be multiples of three simultaneously. The moves are all of the same form: Pick any edge and increment its two vertices by one. For example, if I pick the bottom edge of the front face, then the bottom edge of the back face, the configuration becomes the first one in this series, then the second one:



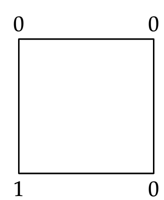


Alas, neither configuration wins the game.

Can I win the cube game? If I can win, what is a sequence of moves ends in all vertices being multiples of 3? If I cannot win, how can that negative result be proved?

Brute force – trying lots of possibilities – looks overwhelming. Each move requires choosing one of 12 edges, so there are 12^{10} sequences of ten moves. Although that number is an overestimate, because the order of the moves does not affect the final state, even a somewhat lower number would still be overwhelming. I could push this line of reasoning by figuring out how many possibilities there are, and how to list and check them if the number is not too large. But that approach is specific to this problem and unlikely to generalize to other problems.

Instead of that specific approach, make the generic observation that this problem is difficult because each move offers many choices. The problem would be simpler with fewer edges: for example, if the cube were a square. Can this square be turned into one where the four vertices are multiples of 3? This problem is not the original problem, but solving it might teach me enough to solve the cube. This hope motivates the following advice: *When the going gets tough, the tough lower their standards.*



Are you really lowering your standards? To me, you're reducing the problem into something simpler upon which you can build to solve the original problem. I would say "When the going gets tough, the tough find a new going" instead of what's written.

We are actually relaxing the constraints of the original problem. So while your recommendation makes sense, the original is just as valid.

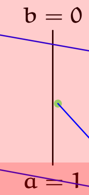
I dunno, I actually like the original quote as well. It fits the section a lot better.

Hmm this is actually very clever - you can represent all the vertices of a cube by two connected squares, so if you can solve a square, you can solve a cube (by never choosing the vertical edges).

I think he was trying to add an element of comedy into the text.

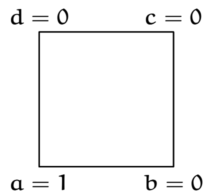
yea who cares if it's not 100% accurate of the precise situation. we get it, it's funny, and many of us like it.

The square is easier to analyze than is the cube, but standards can be lowered farther by analyzing the one-dimensional analog of a line. With one edge and two vertices, there is only one move: incrementing the top and bottom vertices. The vertices start with a difference of one, and continue with that difference. So they cannot be multiples of 3 simultaneously. In symbols: $a - b = 1$. If all vertices were multiples of 3, then $a - b$ would also be a multiple of 3. Since $a - b = 1$, it is also true that



$$a - b \equiv 1 \pmod 3,$$

where the mathematical notation $x \equiv y \pmod 3$ means that x and y have the same remainder (the same modulus) when dividing by 3. In this one-dimensional version of the game, the quantity $a - b$ is an *invariant:* It is unchanged after the only move of increasing each vertex on an edge.



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$$a - b + c - d \equiv 1 \pmod 3.$$

Therefore, it is impossible to get all vertices to be multiples of 3 simultaneously.

Comments on page 2

Are we really lowering standards? It seems like we still want correctness we are just using fewer constraints. I might prefer the term decreasing complexity? simplifying? Just the "lowering standards" has a bad connotation and there is nothing wrong with reducing the problem.

I can see how a square is helpful- isn't this simplification a little too simple to be of any help?

I get the point,but I feel like this is oversimplification. Knowing that it doesn't work in one dimension is pretty obvious, but I wouldn't consider it evidence suggesting that it does not work in 3 dimensions.

It's not supposed to show that it doesn't; it's just supposed to show how simplifications can form a relationship.

I too think that this is almost getting "too" simplistic. I can definitely understand why this example is impossible, but can't immediately see how the cube is impossible.

this is a very confusing way to say a very simple thing.

I think its ok–he says in words "they cannot be multiples of 3 simultaneously", then shows it in symbols. I understand it fine.

I think he's trying to describe with words something that is best visualized (I was able to follow along if I pictured the numbers at the vertices shifting)

I think it is kind of intuitive that they cannot both be multiples of three. If you add the same number to two numbers one apart from each other the difference between them will always be one.

I like this explanation... it does seem almost intuitive, but a bit tougher to explain. I think this explanation is very clear and convincing.

I find the explanation confusing.

But proving that the difference between a and b is always 1 doesn't apply to the whole problem since you can change a without changing b

I don't know what this is, so it'd be nice to explain it in the text.

Another form of invariance- what stays constant among variables

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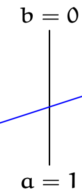

$$a - b \equiv 1 \pmod{3},$$

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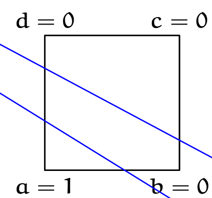


$$a - b + c - d \equiv 1 \pmod{3}.$$

Therefore, it is impossible to get all vertices to be multiples of 3 simultaneously.

Could you explain modular arithmetic in class? I have a vague idea what it does, but it might be useful to go over it in class.

I think what mod tells you is the reminder, such as 5 mod 3 is 2, and 6 mod 3 is 0

True, but I haven't seen mod's in a very long time

there's nothing else to it...

I agree, I don't see why you have to write this as 1 mod 3...how does this guarantee a multiple of 3...

This statement is saying that a-b will always equal 1 mod 3, which means that a-b will never be divisible by 3. The only way that a-b could be divisible by 3, and consequently, the only way that a and b could be divisible by 3, is if a-b equaled 0 mod 3.

The reason the modulus is used is because in this problem, we only care if the numbers are a multiple of 3- it doesn't matter exactly what multiple they are, as long as they are a multiple. In other words, we don't care if the number is 3, 6, 9, etc., as long as that number always equals 0 mod 3.

Modulus just means remainder; this is the same standard term used in programming languages like Python (which is what this is all made in).

The way this is phrased is a little confusing to me, even after I read through it a few times.

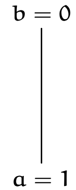
Is this really necessary? I find it kind of confusing and not really helpful to the explanation of the problem. I think you should either expand on it more or scrap it entirely.

agreed, it's slightly confusing. maybe writing: since a-b=1 and 1 is the same thing as 1(mod)3, it is also true that...

This gets confusing for me because I have no idea what mod means. I get the idea from the fact that a-b will always be 1 and there's no way that you can have multiples of 3 that equal 1 when one is subtracted from the other.

This is a nice concise explanation of mod, it conveys the idea while not going into too much unnecessary detail.

As someone that has never been exposed to "modulus" I found this part of the example very confusing... Why are we using it?



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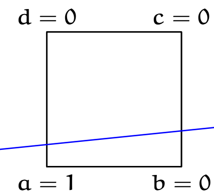

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I recall one of the previous sections mentioned invariants without this helpful following description - perhaps this section should preceded any other mention of invariants?

So for problems like these, we should look for properties of a problem that DO NOT change after every step. IS that what invariant means?

i think that's even a stretch for this problem

Yes an invariant is some property of the problem that doesn't change after every step.

so this is where the symmetry analogy comes in.

so this is where the symmetry analogy comes in.

I actually thought the previous memo was a good introduction to invariants, and then this memo followed up quite nicely with a longer example.

really like the progression from line to square to cube. Helped me understand the problem a lot better.

This is just a nick-picky thing (or just a lazy thing), but this paragraph is not hard to follow; however, I didn't really read it and refer to the diagram - I just assumed I knew which edges were being chosen...another small graphic, or even color could help with this.

meh you don't really need to know.

I know what you mean... I didn't feel like slowing down and mentally going through the proof. However, when I got to the end of the 3 pages, I wasn't sure why the argument was proved so I decided to actually examine the proof. And, it did make sense. But I feel like there is a psychological aversion to proofs, especially more mathematically daunting ones which make people avoid reading paragraphs like these. The one difference between the examples in this reading versus the ones in the previous reading is that the previous reading had it organized and spaced out better. The paragraphs here are long and have equations/symbols mixed right in. It would be much easier to read a proof, if the equations were spaced out on separate lines with less text in between new lines.

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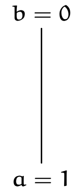

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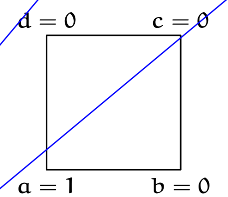


$$a - b + c - d \equiv 1 \pmod 3.$$

Therefore, it is impossible to get all vertices to be multiples of 3 simultaneously.

I definitely get what this is saying, and can picture how the different combinations may or may not change a-b or d-c, and I like how it connects with the invariant we were discussing with the one dimensional line. I just kind of have this strange uneasiness with some of the phrasing. I can't put my finger on it, but since I understand, I guess it might not be such a big problem.

I was struck by this same feeling. I read the sequence a few times trying to pinpoint it with no luck. I dont know if diagrams or equations would help or if its just something that takes a few reads to swallow

Wow. That was pretty eye opening. Can we get a little clearer of an explanation in class? I think I understand it, but I also think I am confused enough to not be able to explain it to someone else.

Woah, that's really neat, but I don't think I would have ever figured that out on my own! I guess before I read ahead I'll try to apply it to the cube...

I think a good way to think about this is that every edge touches either a or c, and every edge touches either b or d (but never both). So each edge adds 1 to the value of (a+c) and to (b+d). Hence (a+c)-(b+d) is invariant.

it'd be helpful to see all 3 invariants written out separately on the left or something so we can see more clearly how they come together to get the concluding eqn.

Actually the explanation you just gave is really a great one. I think its much clearer than the one given in the text.

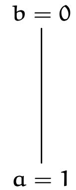
This is extremely helpful. Taking advantage of the fact that every edge touches a or c or b or d but not both makes the problem solvable.

Very neat. I was wondering how the overly drawn out explanation of the line earlier would apply. Very cool.

This is where solving the problem becomes clear. I was a bit shaky before this.

I agree. wow. this is pretty crazy. that (a-b)-(d-c) stays constant. this makes solving the problem so much clearer

I don't really see how this problem demonstrates symmetry. It seems more like the point is invariants, and while often invariants and symmetry are linked, I don't think that link is very clear in this example.



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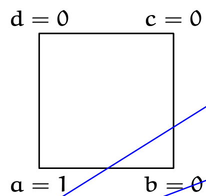

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$$a - b + c - d \equiv 1 \pmod 3.$$

Therefore, it is impossible to get all vertices to be multiples of 3 simultaneously.

I'm confused with 1(mod3) notation. Is the "1" showing that the remainder has a difference of 1?

I'm also a little confused with this, maybe it's because I don't entirely understand mod()

This means that the answer to (a-b)+(c-d) =1 when modded by 3. For example, this means that the only possible answers to (a-b)+(c-d) are 1, 4, 7...

Thanks for the explanation I would have had the same question.

But a-b+c-d = 1, exactly and always, not just 1 (mod 3)...

We only care about it being 1 (mod 3) because our goal is 0 (mod 3) != 1 (mod 3).

Would an explanation involving linear combinations help? Sorry, after taking 18.06 this math seems very applicable to spaces and linear combinations.

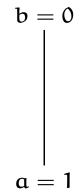
Even though we've found this, it was obviously much more difficult to find than the 1-d case- was there no way of generalizing to make this easier for a cube?

This was much easier for me to solve by taking (a+b+c+d) = 1 mod 2. Any move you make increases two vertices by 1, so the above statement is always true. Unfortunately, the solution has the sum equal to 0 mod 2.

However, using mod3 is helpful in this case because we are trying to make all edges equal to a multiple of 3... I see how the way you did it is equivalent, but for someone without much knowledge of mods it is useful to see the original problem statement in the equation

we don't know that a solution has to have a+b+c+d = 0 (mod 2). What if a possible solution were a=b=c=3, d=6? That sum is 1 (mod 2), and it's not obvious from a (mod 2) argument that it's an impossible configuration

That wasn't as enlightening/clarifying for me as it seems to have been for other people.



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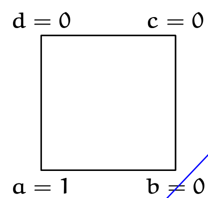

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$$a - b + c - d \equiv 1 \pmod 3.$$

Therefore, it is impossible to get all vertices to be multiples of 3 simultaneously.

I don't quite follow the implications of this therefore. Why does this expression prove that solving the problem for the square is impossible?

This took me a while, and this isn't the most rigorous answer, but if all the numbers were multiples of 3, then a-b+c-d would also be a multiple of 3 (could be positive or negative or 0). The fact that it's offset by this remainder of 1 means that at least one of the numbers isn't a multiple of 3.

Ahhh thank you for that short explanation, I wasn't sure how we got to the therefore either

Yea, he's stating the invariant, which is that a-b+c-d is 1(mod 3). Invariant means this statement will always hold no matter what move you make in the game, so that's why after stating it once he can say therefore, etc.

So the earlier guess was right, and this makes sense

Yeah, I am definitely surprised by this answer since it didn't agree with my initial gut, but this makes sense now why this is true.

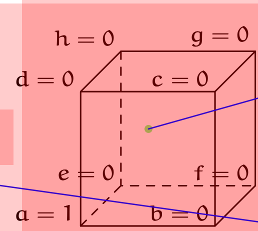
I thought of this another way originally. The sum of all vertices increases by 2 each time, and for all 8 to equal 3 the total sum must be an even number (24). But 1+ even numbers is always odd, so at no point can't it sum to 24.

Is it possible for other numbers?

but does this hold true for the cube? the cube is a fundamentally different problem

my way seems a lot simpler than this. Is it correct?

The original three-dimensional solitaire game is also unlikely to be winnable. The correct invariant shows this impossibility. The quantity $a - b + c - d + f - g + h - e$ generalizes the invariant for the square, and it is preserved by all 12 moves. So



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Comments on page 3

Are there confusing varieties that are solvable with only one solution?

I think this is really cool, but I still don't think I could ID and solve one of these on my own (the physics problem below excluded, because that one made sense beforehand)

So this is where I got stuck really fast – I wasn't sure how to generalize it, and what still held true, since I was so confused with all the vertices. If there a good way to essentially put together the problem again once it's been broken down?

Try to find the sets of vertices that every edge contains exactly 1 of, for example <a, c, f, h>. The other set is <b, d, e, g>. This is basically how the previous cases were formed, although it wasn't stated explicitly. Now, each edge you add will add 1 to exactly one element of each set... and the invariance can be proved from there.

Adding this explicit explanation to the text would be great. I subconsciously did this, but hadn't figured out why. You can see the pattern in how the letters alternate between +/-, but this could depend on how you assign the letters to the vertices.

yeah, having a more detailed explanation about how we got to this equation would be very helpful...while I understood how to get the equations for the line and the cube, I kind of got confused in this last part, which is the most important to solve the problem!

This is really cool, but what is your general approach for finding invariants? In general, I think I would have a tough time computing what the invariant in a problem would be.

how did you choose which direction the vertices were chosen? Or does it not matter, so long as they lie on two different squares?

See below for a good explanation (tied to the box that starts around "this impossibility").

Yeah, how were the edges chosen? The front face has horizontals chosen and the back face has verticals chosen. Is this crucial?

I would like to be walked through this in class.

I think I would actually have tried this first and probably subconsciously done the divide and conquer, but I really like how it's being fleshed out here.

How did you prove this invariant is true for the cube?

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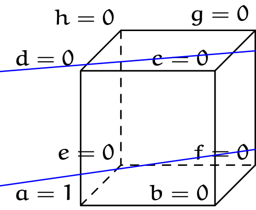

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i find the problem to be spatially ambiguous in some respects and have a hard time visualizing all the moves.

I am still a little confused on this equation. Can you go over it in lecture?

The previous example where we "lowered our standards" really helped me to understand this more complex problem. In this way, I can relate this problem to my understanding of symmetry as well as divide and conquer.

I was right (referring to the question posed in the beginning)!

haha, me too! For once I wasn't outdone by some clever shortcut!

I'm kind of disappointed. I was hoping for a really cool shortcut to solve this problem.

Same here, because my intuition told me there wasn't a solution I was ready to be proved wrong by a simplification I never would have seen.

I never would have thought to look at this problem this way, but it is incredibly useful now that I see it.

Me too–I've never actually thought of this as a problem solving strategy. see what stays constant, so you know what can change relative to what. applies to much more than this problem.

Just an FYI... I believe both 6.042 and 6.005 use the term invariant and I remember there being some confusion between the two as they used slightly different definitions. So it might be helpful to formally define the term earlier as I believe some students have never heard the term before, and other students have heard too many definitions.

I agree that a formal definition would be helpful. "quantities that remain unchanged" seems like it might be misleading when your invariant is a relationship between systems that can't easily be explained with a numerical constant.

I think this concept should be put at the beginning of the section so the reader kind of knows what to look for when thinking about the problem.

I agree. It would be very helpful in understanding what you meant in the first page when you mention invariants if we were given a definition earlier in the section.

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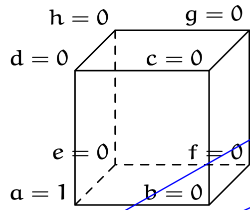

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I like this problem because it explains the notion of invariants in a different context than explained in lecture. Now I see their usefulness.

great quote

Forgive my ignorance, but I'm not sure how these are "solitaire" games.

Yes, how exactly are you defining solitaire? Isn't any problem solved by oneself a solitaire game?

...unless you collaborate

lolz

I think "solitaire games" refer to a subset of games in which the result of the game depends only on your actions and not on another's.

solitaire games are all games with some invariant?

solitaire in that only one person plays, right, not like this models a card game?

puzzles! not games...

I assume you mean an unpowered roller coaster *car*?

or maglev!

I'm pretty sure I've seen this before, and spent a good long time trying to find a way to lift the roller coaster above its starting point! The only thing I think I could come to was to violate conservation of mass...and dump people out

yeah right away this sounds fishy.. im pretty sure physics doesnt work this way

Is this even possible? (doesn't conservation of energy say it's not, even with lots of loops and curves?)

Exactly – that's why energy is a convenient invariant in physics.

I still think the idea of throwing people off the coaster after you've gained speed is a pretty solid one.

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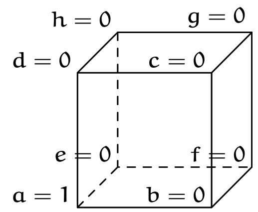

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The rollercoaster example really helps make the point. it's something that we all can understand, so it shows how applicable this principle of looking for invariants really is.

Using GIR-level physics is great for explaining these concepts, especially right after an example that may have lost some of the less engineering-inclined.

this sounds awkward

agreed

Overall clear, easy to understand, and makes a good point. No real comments here.

This is a fresh look at a concept i've known since high school. Looking at different properties in physics or engineering can simplify a lot of problem... or at least make them more understandable.

this is also found in LTI systems, and optical systems

The inclusion of invariants that we've all seen before seems to be an effective way to tie it all in together. Its helped a lot whenever that has happened in the book so far

This would have clarified how to solve the cube problem if I had seen this first

I agree, the first time an invariant was introduced it would have been nice to see a type of invariant like this that everyone knows.

I can kind of understand the connection between this example and what we are learning, but not entirely. Could you explain more in lecture?

yeah that makes more sense

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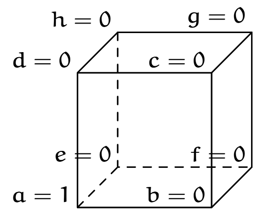

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I feel like this example is a little too obvious. I'd be surprised if there was anyone in the class that didn't know this right off the bat. I feel like a more difficult real-life example would be more appropriate here.

I don't know - while the answer was obvious, it is kind of nice to have a problem that I know the answer for sure. It is actually great, because it really gets you to think about and understand the concept of the invariant point, because it points out that you've been doing it all along!

I agree. All the examples up to this point have been kind of non-trivial. We know how to solve this problem, but that allows us to pay attention to the invariant part as opposed to solving the problem itself. This whole section in general I feel really helped solidify what was meant by "When there is change, look for what does no change"

I feel like energy is also a sort of abstraction, it hides all of the more complicated underneath it and allows us to greatly simplify most real world problems.

Agreed - it's nice to see something from old 8.01 for some familiarity.

I think this is a great example because it's simple, clear, and easy to understand. An additional problem that is more difficult would be fine too, but as a supplement, not a substitute.

My comment would be to reorder examples. It's really hard to follow math in text and I think the first section needs to be hashed out more. Seeing this example first as a "warm-up" might make me more prepared to understand the first bit.

I would see what it is like reordered; I also think that the energy example is easier, but I just spent some time looking at the cube example. It may be better reordered to get the brain around the concept early.

Actually, I like the ordering – I struggled a lot with the first one and then came back to this and it helped really solidify the concept seeing something familiar. If it'd been ordered hte other way around I don't think I would have gotten that same "It makes sense!" moment, since I'd be too lost in the details...

I agree with that. To me, I only really get the topic in this class if I couldn't solve it using my old methods. I struggle to get everything out of this example because I want to revert back to what I already know.

I think that putting the physics example first would have helped to hit home the point that one way of solving difficult problems is to find invariants.

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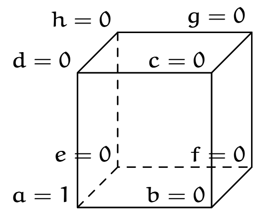

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yeah I agree that having the more simple example first would be helpful in understanding the invariant method...also I disagree that this example is too obvious, while it is simple to understand, I haven't seen 8.01 stuff since high school so it wasn't that obvious!

I agree that this example would've been a nicer introduction to the invariant, and the cube being more an application than an introduction.

Seeing something familiar gives me faith in the approach. After finishing the reading, the last problem forced me to go back and make more sense of the cube problem because I knew there was truth in the approach.

I think it would be nice to have an example that was this simple and short toward the beginning of the section to introduce the idea of an invariant and make the section easier to follow.

While I am unsure as to whether or not the examples should be rearranged, I really feel that this example solidifies this section.

It's nice to see something I know but I feel like I've been told energy is an invariant since high school. This doesn't provide practice with finding invariants; although it does show it's relevance. I would like another "harder" example in this chapter.

This is exactly what I thought of doing after I read the problem. Sounds like I'm getting the hang of this!

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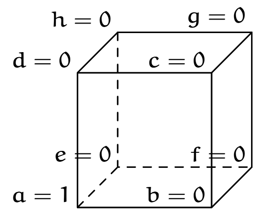

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Now what if the car split in half? Then you could get each half of it above the starting height (at different times) without violating the conservation of energy. I believe you would add entropy though...

Really? Would you be able to do that?

split it in half or raise each half above the starting height? If splitting the train in half took no energy (perhaps you could just uncouple a car), then it would not violate the law of conservation of energy to have one rise and the other fall. Suppose two cars were strung together with a cable, and the cable was looped over a pulley. One car could fall and the other rise, and since friction is negligible, the process would be reversible.

I feel like the issue with this is that you have to put energy into the system to separate the cars. Without friction, if you split both the cars in half both halves will continue to travel in the same direction at the same speed together. The only way to separate them is to use some device that pushes them apart, and that would require adding energy to the system.

I agree. Moving this forward would really help to hit home the point of the section. Which is that looking for invariant quantities can simplify problem solving tremendously.

Not so. The energy required to split them needn't be wasted if it comes from something conservative like a spring. As long as at the end of the day both cars have v=0 and the height of their center of mass is at hstart, you can rearrange their parts any way you want without expending any net energy (assuming no friction etc.)

I understand how identifying invariants is critical to making a symmetry argument, but I'm still a little confused how the cube is an example of a symmetry problem? Is it simply because we were able to generalize from the 1D/2D cases? How exactly is this different from abstraction or divide and conquer?

The symmetry doesn't come from the reduction of dimensions, but rather from the fact that no matter which move you make, there is some quantity (the invariant) which is preserved. This is just like saying that no matter how many 90 degree rotations you apply to a square, its shape is the same. The reduction to 1D just helped us discover the invariant behind the symmetry.

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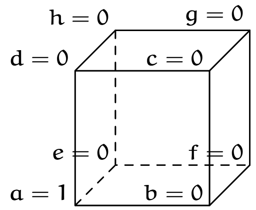

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Maybe this is a good time to tie in Noether's theorem. There's probably not enough space to prove it, but it sure shows how powerful symmetries are in physics. (Noether's theorem says that for each symmetry there is a corresponding conserved quantity. Time-, translational-, and rotational-invariance correspond to energy, linear momentum, and angular momentum conservation laws.)

umm i dont think thats necessary. we got the point.

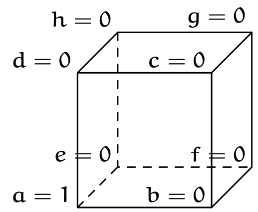
I had never heard of Noether's, and it sounds applicable, but maybe at too high a level for right here. It could be recommended for deeper scope.

I think stating this, or something similar to this, in the introduction would help clarify the purpose of this section. As it is it just seems like a long example with it's relevance not known until the end.

Agreed. I found that reading the section a second time was easier, since I constantly referred to this statement.

similar idea to recursion–looking for patterns inside patterns, ie patterns that stay constant throughout the large pattern

great summary.



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I like paragraphs like this, it summarizes everything so concisely yet easy to understand

Agreed

Yeah they're really helpful.

Is it intentional irony that the moral didn't change?

It's appropriate, not ironic.

I also liked how the ending paragraph ties together multiple units (symmetry and abstraction)

So truly, each subsequent section does build on each other.

I agree, I was getting confused as to how this was going to help us make estimations. this section was more about solving this logic puzzle.

I concur; in fact, repeating exactly what was taught in the previous section with another example that specifically stresses the point via invariants has hit the nail on the head so to speak.

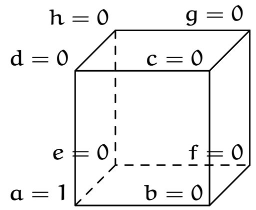
I agree with all... I was sorta wondering how symmetry would be brought up... or simply how this applies to what we've been looking at. Everything is brought together nicely in the paragraph though. Very cool.

This paragraph is great; The bringing together of our estimations tools as building blocks really helps me to see how things can be applied. These things are not just tools we can do down a check-list for, they can be combined!

On a side note to symmetry, there are also cool theorems in group theory that say for example that there are only 17 different ways to make wallpaper, and only 7 ways to make a strip of horizontal wallpaper, all of this proved by symmetry arguments. But definitely beyond the scope of this section, maybe a cool one liner somewhere.

I think the last paragraph of the readings is always a very helpful summary.

I'm confused- I thought this chapter was about symmetry, why is the conclusion talking about only abstraction. To be honest I'm not clear on what is meant by symmetry.



The original three-dimensional solitaire game is also unlikely to be winnable. The correct invariant shows this impossibility. The quantity $a - b + c - d + f - g + h - e$ generalizes the invariant for the square, and it is preserved by all 12 moves. So

$$a - b + c - d + f - g + h - e \equiv 1 \pmod 3$$

forever. Therefore, all vertices cannot be made multiples of 3 simultaneously.

Invariants – quantities that remain unchanged – are a powerful tool for solving problems. Physics problems are also solitaire games, and invariants (conserved quantities) are essential in physics. Here is an example: In a frictionless world, design a roller-coaster track so that an unpowered roller coaster, starting from rest, rises above its starting height. Perhaps a clever combination of loops and curves could make it happen.

The rules of the physics game are that the roller coaster's position is determined by Newton's second law of motion $F = ma$, where the forces on the roller coaster are its weight and the contact force from the track. In choosing the shape of the track, you affect the contact force on the roller coaster, and thereby its acceleration, velocity, and position. There are an infinity of possible tracks, and we do not want to analyze each one to find the forces and acceleration.

An invariant – energy – vastly simplifies the analysis. No matter what tricks the track does, the kinetic plus potential energy

$$\frac{1}{2}mv^2 + mgh$$

is constant. The roller coaster starts with $v = 0$ and height h_{start}; it can never rise above that height without violating the constancy of the energy. The invariant – the conserved quantity – solves the problem in one step, avoiding an endless analysis of an infinity of possible paths.

The moral of this section is the same as the moral of the previous section: *When there is change, look for what does not change.* That unchanging quantity is a new abstraction (Chapter 2). Finding invariants is a way to develop powerful abstractions.

I think this reading as a whole was the best so far. Interesting and easy to understand throughout. There wasn't a point where I got stuck because of wording or complexity, and there were some great quotes to keep me reading.

I also really enjoyed this reading. The presentation of a complex, non obvious example followed by an obvious example really highlighted the points well.

I really dont like puzzles...this was a good way to look at solving this one...but i still don't like puzzles.