3.3 Drag using conservation of energy

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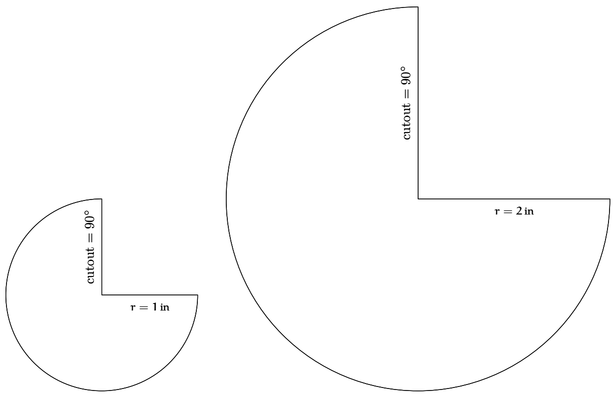

When you drop the small cone and the big cone, which one falls faster? In particular, what is the ratio of their fall speeds v_{big}/v_{small}? The large cone, having a large area, feels more drag than the small cone does. On the other hand, the large cone has a higher driving force (its weight) than the small cone has. To decide whether the extra weight or the extra drag wins requires finding how drag depends on the parameters of the situation.

However, finding the drag force is a very complicated calculation. The full calculation requires solving the Navier–Stokes equations:

$$(\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v}.$$

And the difficulty does not end with this set of second-order, coupled, nonlinear partial-differential equations. The full description of the situation includes a fourth equation, the continuity equation:

But the cross-sectional area of the cone changes...do you just use the median diameter or something to measure A?

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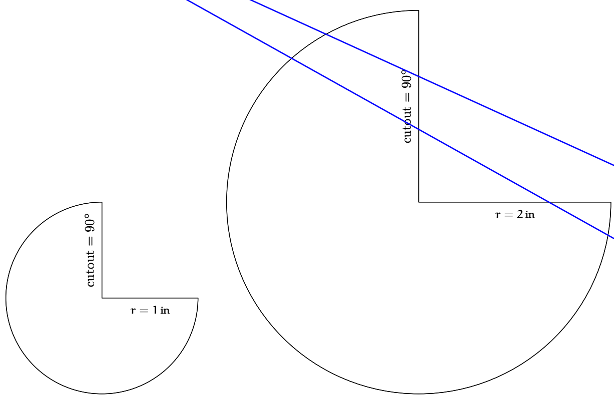

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Read Sections 3.3 (drag) and 3.4 (application to cycline) for the memo due Tues at 10pm. See you Wednesday (no lecture on Monday).

why is drag one of the most difficult subjects in classical physics?

Nice to get the explanation of this.

from the first day of class!

Yeah, this is definitely a nice throwback to the early days of class lecture.

instead of using a sub-heading, I'd use an inset box around the experiment instructions.

How much of a difference does this make? Are they just too unstable at smaller sizes?

and they are easier to cut out if they are larger

Generally easier to tape and handle. Any imperfections in cutting them out technically have a larger impact because of the percentage in size with respect to the entire shape, but this is likely minimal.

This makes it extremely unlikely that anyone is going to do this (especially if this becomes an actual textbook). It might be better to have something that can be traced (in an appendix if it takes up too much room) or just give the specifications and assume people can make a decent attempt at drawing a circle.

typo, only one of these words is needed

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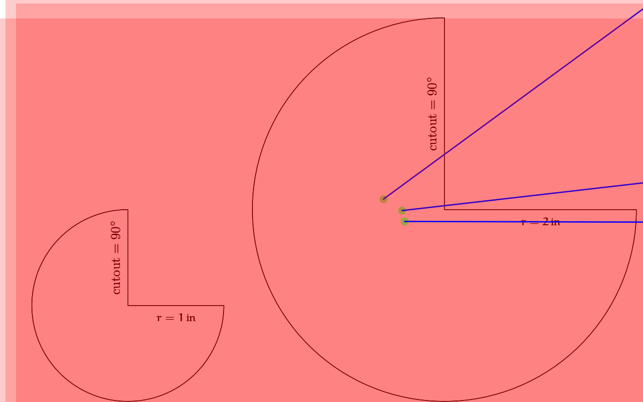

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Do the words on these templates matter? I had to zoom in pretty far to see what they said.

I think he just wanted to make clear what the radii of the two were, but I agree. He should have written that somewhere in the text instead of on the tiny picture.

or just enlarge the text?

well when you copy it it's supposed to be at 200% enlargement.

regardless, i don't know if the words would come out clearly if you were to magnify the shapes.

it might become pixelated when you enlarge it. i think it's so small because he used a much larger picture to start with and latex'd it in without realizing that the original text would get shrunk so small.

I would think that the smaller one falls more slowly. It seems like this is the case for animals.

Yea we shouldn't have to actually do the experiment because we watched it in class the 1st day. Ratio 1:1 about

Don't forget, this is a book intended to for public use, so not everyone will have seen the experiment. Though it would be nice if the images were given a separate section so I wouldn't have to worry about enlarging.

This was the experiment we did in the first lecture right?

That's right: I am delivering one of the promises made so far.

Hooray for promise fulfillment!

Haha yeah, if I recall, the ratio was roughly 1:1 right?

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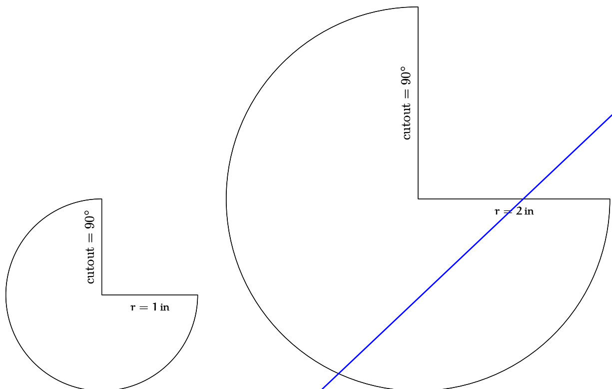

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There's never any discussion of how the cone's effective area differs from the area of the circle or the circle minus the cutout. (and what if the cutout had not been 90 degrees?)

http://math.about.com/od/formulas/ss/surfaceareavol_2.htm

Basically, area = pi*r*s+pi*r^2

True, but I think at this first pass, it's sufficient just to notice that it's larger so that we can set up the competition between increasing force and increasing drag.

Yeah–there is no mention about how the cone's effective area relates to the area of the cutout itself, but we don't need the exact relations, we just need the ratios. We know that drag is proportional to area, and we assume both cones are cut and made the same way (only difference being size). Thus, we know the ratios of radii, which we can then do math on to find the ratios of the effective areas

It might be useful to define what you consider drag to be before using it in a sentence.

Agreed. I remember that drag is the force that slows things down while falling or moving fast, but nothing more than that. Unless you are intentionally keeping it vague for now.

It might also be helpful since in the next paragraph the text refers to the "drag force" instead of simply "drag". I think an explicit definition would make this transition a bit more clear.

I feel like people have an intuitive grasp of what drag is - I've never taken a fluids class and the concept of drag being some force that makes you go slower isnt that difficult to grasp. I think it comes down to a question of intended audience and what they should know

Agree to define what drag is, people who are not course 2 or course 16 might have just heard of it but never have worked with it in formulas.

To me, this reads very awkwardly.

It would be useful if you explained how these two things play into the terminal velocity of each of the cones.

I agree with the awkward wording sentiment. Instead of "win" maybe is larger?

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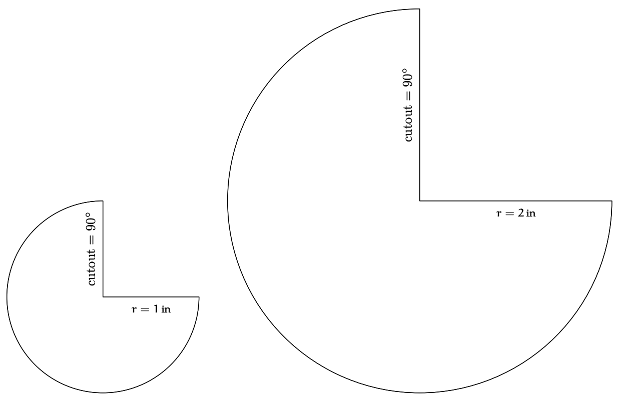

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at this point I would have stopped and done the experiment (if we hadn't already done it in class).

i think that the section would be a lot better if you referenced the outcome after this...most people aren't going to wait until the end of the section to try it out (at least the ones I know).

ie. Through this experimentation you find that the two cones fall at roughly the same speed. However, actually calculating the drag forces to prove this is a very complicated. It requires solving...

But no one would actually use this formula. We would estimate based on the surface area!

So given this is a continuous book, the N-S equations have been shown 2-3 times by now, and I think it's unnecessary to show it this many times.

I disagree, I think you need to show the equation to illustrate how complicated it is and why approximations are useful. I think it's good to remind the reader what it looks like every time it is mentioned, even if you don't go into great detail about it every time.

Also, is it really necessary to go directly to navier stokes to solve for drag? Can't you use newton's second law and the coefficients of drag that have been approximated for various shapes to solve for the drag force?

I think the answer is yes, you can approximate this using drag coefficients and Newton's Laws, however the point is that this is still an approximation for the drag force, and not actually a calculation of the drag force itself.

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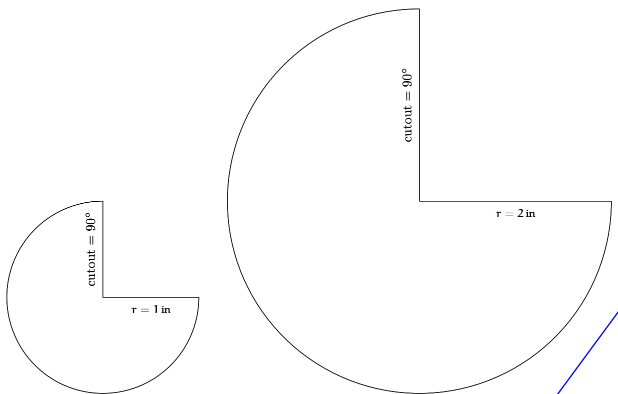

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I really like your examples that relate to equations or principles I've used in my other Course 2 classes – it helps clarify a lot of things.

Aren't the Navier-Stokes equations unsolvable (at least at the present moment), thereby necessitating estimation or numerical methods?

Not quite. You can solve them for certain very simple cases, but I think they are only solvable in 3 or 4 situations.

Or rather, they're solvable when you can cancel out enough of the terms (which is true for certain simple cases)

Is this the same equation mentioned earlier as an example of super complicated things easier investigated via abstraction techniques?

The very same one.

What are the assumptions for the cases that you can solve N-S for again. We learned it in 2.005 but that was so long ago?

nice tie to earlier notes

Although I know the equation, it would be cool if you defined the variables so non-engineers knew what you were writing about

Yeah I agree..I've never seen this equation and I was a little confused. There is no need to explain where the equation comes from or how it derives, but perhaps just stating what each variable represents would be helpful

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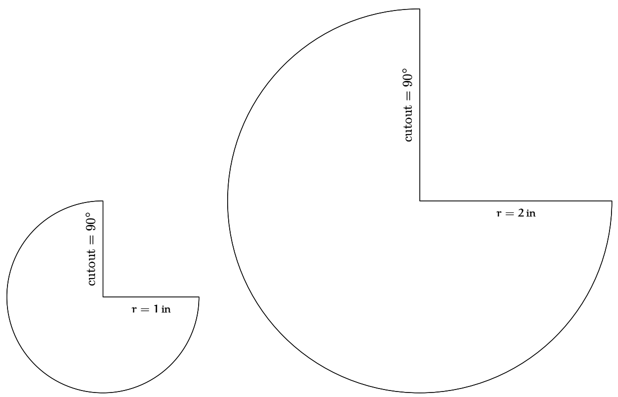

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That sort of equation just sounds terrifying. I have rarely seen such a string of scary math words strung together like that.

Now you know why all us course 2 kids love 2.006 so much.. (I actually think its material is really cool, just really hard, too)

I'm not familiar with how to solve any of the properties of that equation... yay for abstraction and estimation!

I'm sure it's intentional ;-). And just imagine what that reads like to someone who isn't used to the sea of jargon we're already floating in at MIT.

I agree i like this sentence - makes you really want to find a way to get away from that equation!

I also think that the paragraph could have ended here. It gets kind of into the realm of irrelivnt after this.

I don't think there is a need to "drag" this out. If the point was to show that the calculation is really complex, I think even just putting only the "full description" in one go, instead of separating the equations, would do it.

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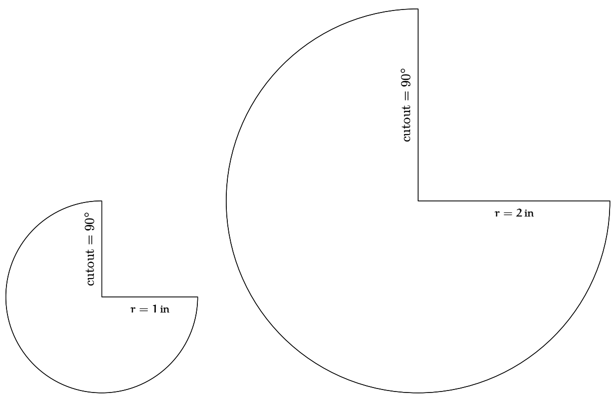

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What does this question mean / why is it relevant? Sorry if this is elementary but I haven't dealt with drag before. I guess it just serves to prove that solving this problem via math is quite difficult and through this mathematically analysis it is very apparent.

I think that was the point. We don't want to solve for drag using these equations, we would rather solve it using some handy estimation tool! That being said, the continuity equation is not terribly scary.

The continuity equation is useful in solving the Navier-Stokes equation as it establishes quantities that are conserved – in this case, mass (you don't lose mass!). But I agree...the point is that it is very complex (and sometimes impossible) to solve N-S.

Agreed - I think this examples is included just to demonstrate how hard the task would otherwise be.

I am familiar with these equations, but I'm not sure what the point of showing the continuity equation is...

The point of these complicated equations isn't to actually use them to solve the problems, or even for us to learn them to use in the future. The whole purpose of this class is learning ways to approximate things without using these crazy math/physics/whatever equations.

$$\nabla \cdot \mathbf{v} = 0.$$

One imposes boundary conditions, which include the motion of the object and the requirement that no fluid enters the object – and solves for the pressure p and the velocity gradient at the surface of the object. Integrating the pressure force and the shear force gives the drag force.

In short, solving the equations analytically is difficult. I could spend hundreds of pages describing the mathematics to solve them. Even then, solutions are known only in a few circumstances, for example a sphere or a cylinder moving slowly in a viscous fluid or a sphere moving at any speed in an zero-viscosity fluid. But an inviscid fluid – what Feynman calls 'dry water' [9, Chapter II-40] – is particularly irrelevant to real life since viscosity is the reason for drag, so an inviscid solution predicts zero drag! Conservation of energy, supplemented with skillful lying, is a simple and quick alternative.

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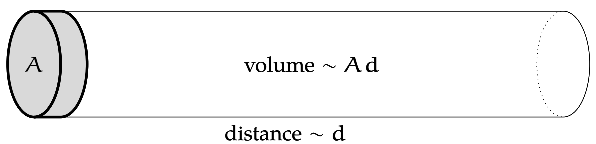

The drag force is the energy consumed per distance. The energy is consumed by imparting kinetic energy to the fluid, which viscosity eventually removes from the fluid. The kinetic energy is mass times velocity squared. The mass disturbed is $\rho A d$, where ρ is the fluid density (here, the air density). The velocity imparted to the fluid is roughly the velocity of the disturbance, which is v. So the kinetic energy imparted to the fluid is $\rho A v^2 d$, making the drag force

$$F \sim \rho A v^2.$$

The analysis has a divide-and-conquer tree:

This pagebreak confused me - took a minute to discover that this _was_ the continuity equation

Same here...dont know if there's an easy way to correct this formatting so that the equation is close to the text that refers to it rather than hanging on the next page

I agree too. Especially if I were reading in a textbook, it would be confusing. I think when all relevant materials are in one page, it is the easiest to follow what's going on.

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Fluid? I thought we were dropping it in air. Can you explain why we are considering a fluid instead of considering air, or how they are related, or something along these lines?

Navier-stokes is effectively the fluid dynamics equation. It models how fluid moves and behaves in different situations (stresses, pressures, velocities, sizes of tubes), and air can be treated as a fluid (and is) when considering drag. I think a handy analogy, for this paragraph at least, would be to charge/current and maxwell's equations

It's really easy to understand air as a fluid with just a very very low density.

In fact, if you go to a high enough pressure for a given fluid, the transition between gas and liquid (both fluids) is non-existent. i.e., by changing the temperature you might change from one to the other, but there is no defined boundary (or even a difference) between the two.

I still think it would be worth it to mention that by fluid you actually mean air here.

The term "fluid" can refer to any liquid or gas that has the ability to take the shape of it's surroundings. Since the N-S equations deals with pressure and density, then we don't have to necessarily use air.

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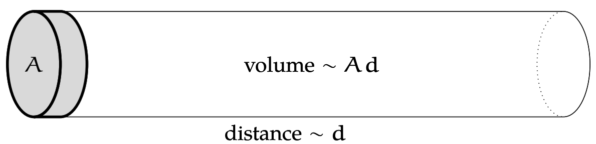

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On first reading, I tried to go back and think through the equation, b.c. I've never had to use the navier stokes eq. But I realized that wasn't the point; the point is that it is difficult.

Yeah, that whole paragraph was kind of in one ear and out the other for me. It looks like the typical difficult textbook sentence, which is difficult to understand/absorb.

I think people can gather from looking at the equation that it is very complicated and nobody wants to ever have to solve it. Maybe this paragraph could be shorter so that it doesn't distract people from its purpose, which is to show that a quick estimation is much more enjoyable than cranking out the navier-stokes equation.

I agree that as someone who has never encountered the Navier Stokes equation before this paragraph is mostly just clutter. All I really needed was the sentence, "In short, solving the equations analytically is zero". But if you had seen Navier Stokes before it might serve as a useful review of the difficulties inherent in solving the equations.

That's an understatement.

You should–it might win you a million dollars.

why?

yeah I've heard that fluid drag is extremely complex and it is very difficult to model. Why is that? How accurate are current models?

I think those equations are only valid for "ideal" conditions, but in the real world, there are so many external factors we might not be aware of, or this equation might not be able to account for, so this makes the math very complicated and it's kinda hard to set the boundary conditions as well

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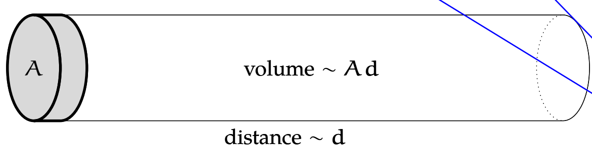

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Does this refer to solutions for the Navier-Stokes equation? Or to solving the two equations together?

Solving them together. Navier-Stokes is just part of the system of equations that defines the physical system.

If there are only limited solutions, do MEs typically just use numerical methods to solve them?

Also, how did people design bridges before computers. For example, the Golden Gate Bridge in the SF Bay Area was designed in the 1930s, pre computers. Despite this, it is still designed to survive the high winds/etc. How did they manage to do this with these complex equations

define please! we can make a good guess of what it means using context, but it'd be nice to see this word defined specifically.

I'm also unfamiliar with this term. I'm also eager to know what 'dry water' is and how it got this name.

I'm going to guess, but water with no drag and viscosity?

yeah...inviscid means no viscosity, which helps simplify some problems when you can consider boundary conditions separate.

I think it's pretty clear what this means - coming right after discussion of zero-viscosity fluids, we should be able to infer the meaning of inviscid.

Yeah, I don't think it needs to be defined because you can guess what it means after reading the sentence.

What's this citation referencing?

interesting name

why does this matter?

why does an inviscid solution predicts 0 drag?

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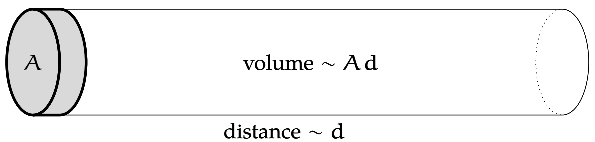

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$$F \sim \rho A v^2.$$

The analysis has a divide-and-conquer tree:

Can you really call it lying if it arrives a correct answer?

I'm not sure where the skillful lying comes into play.

I believe that the skillful lying is in pretending that the object is shaped a certain way in order to get the physics to work the way that you want them to.

I like this term; I think it has a nice ring to it, and adds comedy to truth. Another memorable tidbit to keep the reading interesting.

The word 'lying' seems a little uncomfortable when relating it to an alternative approach to finding the answer.

I agree about how "lying seems a little uncomfortable". It suggests that we're doing something wrong...

I'm pretty sure that's just personal moral attachments to the idea of "lying" and not anything implied by the text.

maybe "moral fudging"?

i might go with skillful fudging, or educated fudging.

$$\nabla \cdot \mathbf{v} = 0.$$

One imposes boundary conditions, which include the motion of the object and the requirement that no fluid enters the object – and solves for the pressure p and the velocity gradient at the surface of the object. Integrating the pressure force and the shear force gives the drag force.

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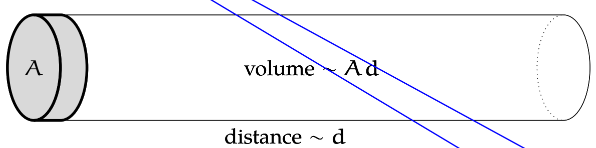

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I don't really see how this involves symmetry. Unless your conception of symmetry is different than mine. It seems more like just simplification, I think that whatever link to symmetry that you see should be more explicitly explained.

I haven't finished reading, but I think the point it to see the invariant point (conservation of energy), and then use symmetry operations. (the roller-coaster example!).

I agree, I don't see how this is supposed to be "symmetry". All of the "symmetry" problems and readings we've had so far seem like arbitrary simplifications. Could we possibly have a more concrete definition of symmetry, and how it is applied?

I disagree that the simplifications were arbitrary - a lot of times, simplification can be done by looking at how to turn the problem you have into a problem that is similar but easier (ovals to circles, for example). It's all a matter of looking at a problem and thinking "oh, this would be sooo much easier if it looked like that other thing I know how to solve."

I think sometimes I get too caught up in the predetermined definition of "symmetry" in my head where it is linked to a visual image being symmetric about an axis. Calling it "invariant" clicks better for me.

I agree–if you think of it as "invariant", it makes a lot more sense–energy is conserved so if we lose some in one place, we gain it somewhere else. It's not symmetry in that geometric sense of a square having two symmetrical sides each with 2 sides at 90degrees; it's more like if we change one thing we know what happens to other things (once we find what stays constant or invariant).

Technically speaking, Noether's theorem says that time invariance will correspond to some conserved quantity: energy.

can we always use skillful lying, just curious

typo

typo here, remove one 'analysis'

$$\nabla \cdot \mathbf{v} = 0.$$

One imposes boundary conditions, which include the motion of the object and the requirement that no fluid enters the object – and solves for the pressure p and the velocity gradient at the surface of the object. Integrating the pressure force and the shear force gives the drag force.

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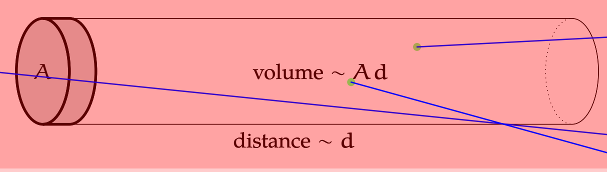

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Besides the double typo, I'm not sure "The analysis imagines" makes any sense. Perhaps you mean something more like "For our analysis, imagine"?

Just a different way of saying something similar, I believe.

Is the double repetition a typo or is it intentional? Also, it is impossible to spell check your comments on NB. Normally on firefox, when typing in forms, a red squibble shows up underneath the word and you can right click to get a spelling suggestion. This does not work on NB though since right clicking opens up another comment box. Can you fix this?

I'm assuming this is a generalized case then? Since with the example of the cone, the velocity isn't constant. Or are we assuming it is?

Diagrams are always helpful. Having a picture which shows drag force in terms of Energy per distance greatly improves my understanding over the material.

I believe this example is being used to explain the drag force and therefore the problem is more simple to understand with a constant speed.

shouldn't it be faster because it is actually a cone rather than a cylinder?

My guess would also be that a cone is faster, but I think the basic concept is better taught using a cylinder as an example.

An arrow or something to show the path would be helpful. In this diagram I'm not sure if the puck is moving left to right or has already moved right to left.

By this figure if seems like the large cone would fall more slowly. I guess this also depends on how the surface area and mass scale.

Also you could separate the puck and the cylinder. That would clarify instantly that this diagram is an object trying to move through fluid, where as right now glancing at it, it just seems like a cap of a cylinder.

what do you mean by energy consumed?

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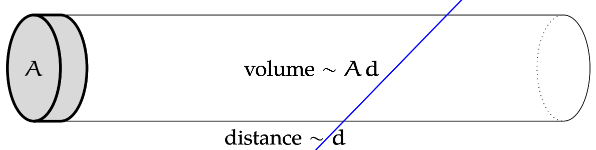

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Is this supposed to be obvious? If so, it's not obvious to me. How do you arrive at this?

W=F*dx, or F=W/dx. The force, F, is the drag force, and W is the work done, or energy consumed. 'dx' is your unit distance.

For me this would have been helpful as part of an explicit definition of drag towards the start of the section.

I think it would be useful to have some other equations here to refresh those of us not well-read in the fields of physics.

More and more demands seem to be answered if the reader only continues reading before he asks them. How about reading an extra page and then asking for definitions and equations?

I think it be a poor design for a text to introduce a new and unfamiliar topic, only to define it pages later. It makes sense for people to request definitions and equations for drag, after now reading 2 pages about it and potentially being lost and confused. The non-course 6 people were confused and demanded explanations about UNIX, and now the non-course 2 people are confused and would like explanations about drag, and I think everyone was completely right to do so.

Really? I found this sentence to have the purpose of defining the drag force as energy consumed over distance. That's what it is.

How does viscosity remove KE?

I'm not sure about the exact details, but I can give a general explanation. Viscosity is the measure of the resistance of a fluid. Simply in one word, it is "thickness." So the thicker the fluid is, the more that the kinetic energy of the object decreases as it is traveling through the fluid.

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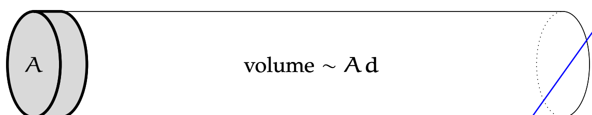

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Kinetic energy is usually 1/2 times this quantity. Is this just for approximations sake?

Good question, I was wondering about this too. I assume it's just to make approximations easier since it really only affects the answer by a magnitude of 1 (half of 100 (10ˆ2) is 50 (5*10ˆ1))

It's because we don't care about constants, regardless of how large they are (in fact, I believe there are other constants missing, like the drag coefficient, which is geometry-dependent). What's important is how the force, below, scales with these constants. So really we have KE M*vˆ2, M rho*A*d, so KE rho*A*vˆ2*d

That's why we're writing these with instead of =, since it's rough estimation.

No, I believe stands for "goes like" or "is proportional to" or "scales like" (take your pick). It is different, and in many ways more technical than "approximately equal to", which would be the 'double squiggle'.

The reason is used, is because several constant factors are omitted. Since we only care about how two velocities relate to one another, they are ok to leave out. If you wanted to find out what the actual terminal velocity was, however, you'd have to include them (and determine their values, of course).

I think this would have been easier to understand written out as equations rather than into a paragraph. It is hard to follow math in word form.

Yeah: the shows proportionality, since we're not including constants here. Remember, in the problem we're looking at how the two cones will fall; we don't care about their absolute velocities or drag, we only care about their velocities relative to each other. So we don't need to worry about constants (which will cancel out when we find the ratios anyways)

What is mass disturbed?

The mass of the volume displaced by the object (density*volume=mass)

$$\nabla \cdot \mathbf{v} = 0.$$

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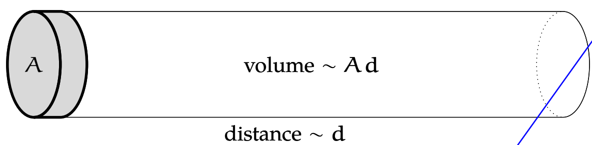

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I am continually impressed that all this analysis takes is a little concentrated thinking and application of easy 8.01 principles. These are such good exercises!

This is amazing. I admit, I always wondered where the v^2 term in drag force came from.

I agree with the above. This short paragraph is a great breakdown, and this type of explanation makes something very complex understandable for a student in high school physics.

Although I agree this is a clever, simple way to solve for the drag force, I don't think it's so useful in practice. The point of calculating drag forces (the "real" way) is to predict what final velocities will be or what forces on necessary to account for the lost energy. It's rare that you care about the drag force if you already know parameters like final velocity or energy lost.

Is the energy lost due to the fluid or the interaction between the fluid and the member it is traveling through?

This is mass disturbed per unit time right?

This is an amazing simplification of the previous set of equations

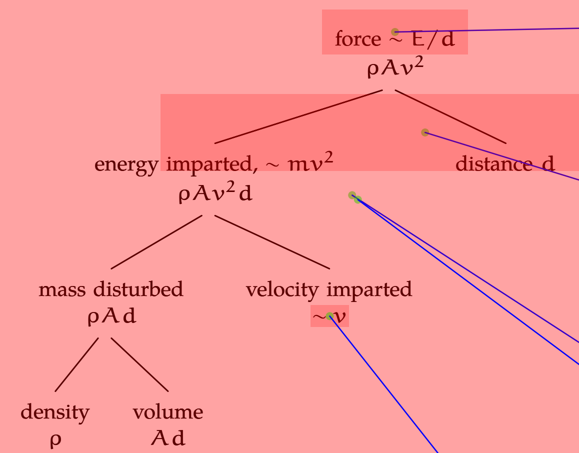
should probably define what these are, even if they seem obvious. the density threw me off, I wasn't sure which density you were referring too (fluid vs air?)

I really like how this book builds upon techniques from previous chapters. It's nice to be able to relate something new and unfamiliar (symmetry) to something we already understand (divide and conquer).

Agreed, I like the tie in with earlier sections. Although, to be honest, I'm having a tough time distinguishing between symmetry and D&C in some cases. What are the main differences?

Yeah I agree with both points. It is really nice to see that things we learn in the beginning of the class don't prove to be useless by the end of the class. I am also a little unclear on what symmetry is.

i can't describe how happy i was to see this...the above wouldn't have made nearly as much sense otherwise



The result that $F_{drag} \sim \rho v^2 A$ is enough to predict the result of the cone experiment. The cones reach terminal velocity quickly (see Problem 8.6), so the relevant quantity in finding the fall time is the terminal velocity. From the drag-force formula, the terminal velocity is

$$v \sim \sqrt{\frac{F_{drag}}{\rho A}}.$$

The cross-sectional areas are easy to measure with a ruler, and the ratio between the small- and large-cone terminal velocities is even easier. The experiment is set up to make the drag force easy to measure: Since the cones fall at their respective terminal velocities, the drag force equals the weight. So

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To test this prediction, I stood on a table and dropped the two cones. The fall lasted about two seconds, and they landed within 0.1 s of one another. So, the approximate conservation-of-energy analysis gains in plausibility (all the inaccuracies are hidden within the changing drag coefficient).

Comments on page 3

It would be nice if F_drag were at the very top of this tree (above force)

Building up a tree like this came earlier in my thought process about this problem, so could it go in earlier? Or at least the root could and we could start building branches as the reasoning progressed?

The "force" mentioned is the same as the "F_drag" in this problem. The idea is that we used energy and distance to determine the force, if we didn't know the drag force equation (which explains why we simplified the problem by removing certain coefficients).

Why didn't you use the 1 and -1 powers along the lines here?

Maybe he hasn't had time to edit this since he introduced those?

Also if he's using the tree generating code, maybe said code does not (yet?) have support for powers.

Vaguely related (but not entirely), the other day a friend showed me some tree-generation software called graphviz for those of us who want to make nice trees but don't know how to program (www.graphviz.org)

Like the use of earlier notes on diagrams

This does make everything so much easier

thank you!

I really like how the tree diagram is shown after the explanation of how we got the approximation for the force of drag. It makes a lot of sense and makes stuff more clear.

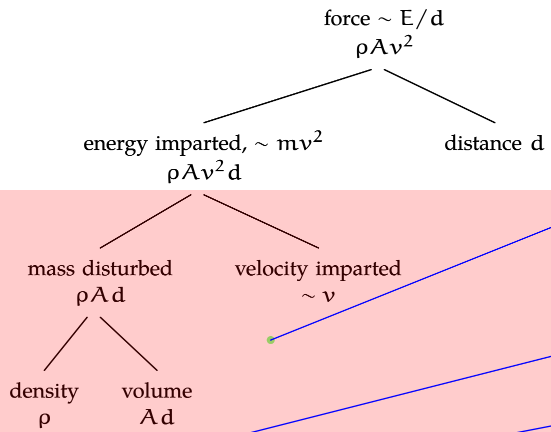
I think for all of your diagrams, figure captions would help a lot.

I think this diagram is self-explanatory. It actually helps a lot as it is...

I don't think a tree diagram needs any more figure captions than the info that's already on it. From here it's clear what breaks down into what

I agree. I get 80% from reading the above paragraph but the tree diagram cements it for me.

this is "approx" v right? what's the need of the "approx" if we're approximating anyway



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All of this could have been done with the powers along the lines instead of writing out the equations. It's nice to see them in writing, but I think you should be consistent throughout the book.

I also like the power method for it's simplicity and ease of reading. It would be nice to have the equations after though.

This is just like the coefficient of drag approach that I was talking about earlier. However, this is a more rudimentary and straightforward approach and is easier to derive/remember.

It might be better if the form of this was consistent with the rest of the document, so that it reads pAv^2.

Agreed.

So can I assume that the p=1 if it's air? In which case it all comes down to the Av^2?

This still seems like a lot of work for two cones that will hit the ground at nearly the same time

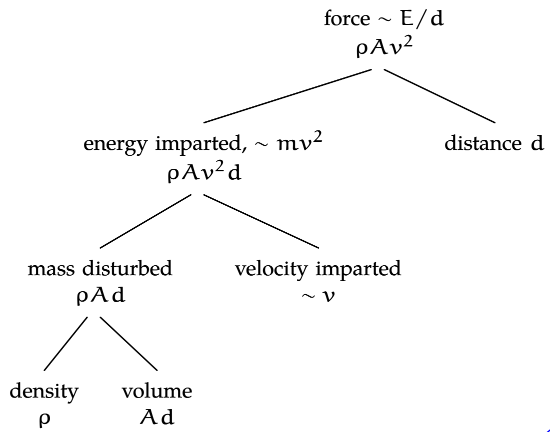
It seems that because the result is simple. But the method is a lot less work than solving the Navier-Stokes equations to get the same result.

Also, lots of other results are plausible: for example that the bigger cone is twice as fast. For highly viscous flow (e.g. big and small cones in honey), it turns out that the two objects do not hit at the same time. That analysis may end up on a problem set...

That would be a cool follow-up.

I don't remember the distribution, but the first day of class not everyone got the right answer. This isn't an entirely intuitive answer.

Where is this problem? I couldn't find it.



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This makes sense intuitively (it is not difficult to imagine the cones not accelerating too much after a second or two), but is there another way, perhaps more rigorous, to come to this conclusion?

You can estimate a drag coefficient, find a more precise equation for v, and get an estimate of terminal velocity. From there, you can take a stab at figuring out its acceleration (which will depend on the cone radius), and then estimate how long it will take to reach terminal velocity.

Is there an abstraction here? As in some time constant that can represent how quickly something goes from rest to terminal velocity?

I think if you just think about the quantities involved it falls out pretty naturally. Drag should be proportional to cross sectional area and the weight is proportional to the mass. So for paper cones which have a "large" cross sectional area and a "small" mass you would expect the forces to balance out pretty quickly.

I think it might be useful to first put newton's second law and equate the forces before you skip straight to the terminal velocity equation. Just so that people know where this terminal velocity term actually comes from

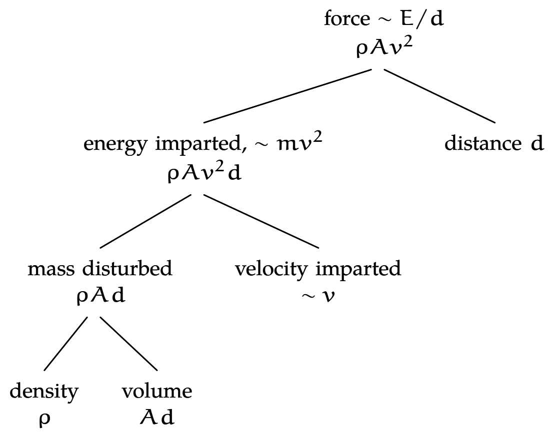
Is this always the _terminal_ velocity? Above when we found the relation of Fdrag to pAv^2, I didn't think we had specified that.

The definition of terminal velocity is defined when these quantities are equal, or when the drag forces are equal to the opposing forces whether it be cycling or falling. This was intuitive to me, but it may help to define the terminal velocity for those that haven't seen a problem like this before.

I think you may have missed 2:55's point. The equation that immediately follows this use of "terminal velocity" is just a rearrangement of the drag equation (F_drag pAv^2). The velocity the drag equation uses is just the velocity of the object with respect to the fluid.

Terminal velocity occurs once we've replaced Fdrag with Weight (since at that point there's no acceleration and thus no change in velocity).

So I think this use of the phrase "terminal velocity" is mistaken.



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I feel like a good portion of readers wont actually try this experiment... i think it would help to have a quick sentence concluding the findings so I'm not as uncertain?

He does summarize his findings a little later in the reading.

Good idea for an introduction to the subject!

This is an interesting thought but why not make this inference in the beginning of the problem.

Why W instead of mg? I feel like the concept that acceleration is 0 is left out here when that is what tells us the drag and weight forces cancel. Free body diagram?

I wouldn't ponder too much over asking why W instead of mg, but I would defnitely agree that a free-body diagram would be extremely useful here.

I just think "work" when I see W (as someone used it in earlier comments), instead of Fw or mg.

so it turns out is does depend on the ratio between surface area and weight

It might be useful here to write down in equation form that the weight and cross-sectional area are proportional to each other by a factor of 1, which is why the terminal velocity is independent of area.

Agreed.

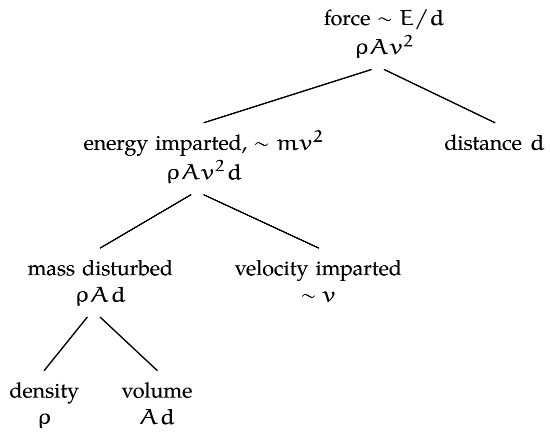
wait....how are they proportional by a factor of 1? Maybe im just tired but I dont see this.

I don't get this transition. I understand that each cone's weight is proportional to its area by why does this mean that mean the velocity is independent of area?

So are we saying that for this case (same shape & same material), if you triple the area you also triple the weight? Thus the velocity will be the same no matter which 3 variables we change.

So are we saying that for this case (same shape & same material), if you triple the area you also triple the weight? Thus the velocity will be the same no matter which 3 variables we change.

I'm happy that we saw this demonstration in class! It makes the example so much easier to relate to!



The result that $F_{drag} \sim \rho v^2 A$ is enough to predict the result of the cone experiment. The cones reach terminal velocity quickly (see Problem 8.6), so the relevant quantity in finding the fall time is the terminal velocity. From the drag-force formula, the terminal velocity is

$$v \sim \sqrt{\frac{F_{drag}}{\rho A}}.$$

The cross-sectional areas are easy to measure with a ruler, and the ratio between the small- and large-cone terminal velocities is even easier. The experiment is set up to make the drag force easy to measure: Since the cones fall at their respective terminal velocities, the drag force equals the weight. So

$$v \sim \sqrt{\frac{W}{\rho A}}.$$

Each cone's weight is proportional to its cross-sectional area, because they are geometrically similar and made out of the same piece of paper. So the terminal velocity v is independent of the area A: so the small and large cones should fall at the same speed.

To test this prediction, I stood on a table and dropped the two cones. The fall lasted about two seconds, and they landed within 0.1 s of one another. So, the approximate conservation-of-energy analysis gains in plausibility (all the inaccuracies are hidden within the changing drag coefficient).

I found the demonstration in class helpful.

but this might not be so accurate though because your arms might be at different heights, but then again, this is an approximation class

When you performed the experiment in class, it actually looked like the cone fell at different speeds, but I guess it's difficult to be precise.

Does this mean that drag doesn't have as much effect as we thought it would?.. or that weight and drag scale similarly?

It would be helpful to see the numbers from the expirement plugged into the equations to obtain this result.

What would be interesting would be to predict this time difference. It should be integral(Vsmall(t)-Vbig(t))/Vterminal. Basically we'd have to estimate the area between the velocity curves.

just as we saw in the experiment...yippie! :)

how are the inaccuracies hidden? I understand that we significantly simplified the problem and will therefore have errors, perhaps these errors should be explained in more detail?

so the drag coefficient is not constant, but we assume it is constant?

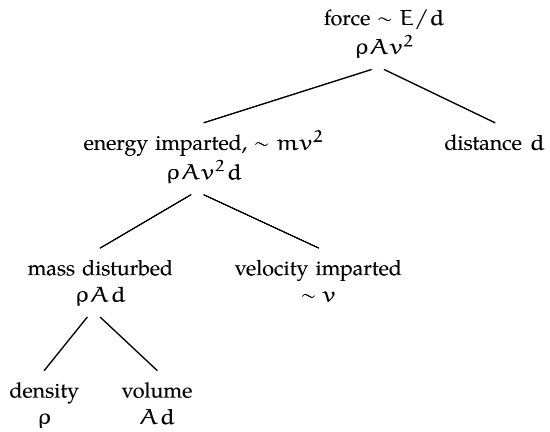
Are they hidden or we just don't consider them? Are we deeming the inaccuracies insignificant, or are they really not there...?

> I think he means hidden away as in we used abstraction to make the problem a much simpler one–remember all those crazy equations in the beginning

Again, the idea of abstraction comes into play as well here. We have abstracted away all the nasty math into one variable - drag coefficient.

> That abstraction was amazing for understanding - the drag coefficient in a small equation is much easier to understand than a whole mess of math that I never want to see again.

These three pages are really good, minus the one typo. I think the explanation is really clean and very easy to follow. If there is ever a question on how a subject should be brought up, this would be a great template.



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wait, what did this have to do with symmetry again?

When you did this in class, I figured they'd fall around the same time without equations or math – just figuring that the increased surface area and weight canceled each other out. Does that count as an estimation method?

I had the same intuition- I think that method of estimating is more like the "gut feeling" that Sanjoy has talked about. This method is better for estimation because it gives us concrete evidence for our estimation as opposed to a general feeling.

This example is one where thinking about it a little bit often leads to the right answer, as does thinking about it a lot (e.g. the analysis in the text).

But thinking about it a moderate amount can lead to puzzlement. For example, should the drag be proportional to area or to radius? For very slow or viscous flows (e.g. a fog droplet), as you will learn when we do extreme-cases reasoning, the drag is proportional to radius.

Therefore, maybe the experiment to introduce drag is four small cones versus one small one, which I'll do on Wednesday in lecture.

I think this gut decision is hard to make - without knowledge of what ratios and exponentials are present in the equations, we cannot determine whether weight, surface area, or neither dominates the situation.

3.4 Cycling

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To estimate the speed, make a model of where the energy goes. It goes into rolling resistance, into friction in the chain and gears, and into drag. At low speeds, the rolling resistance and chain friction are probably important. But the importance of drag rises rapidly with speed, so at high-enough speeds, drag is the dominant consumer of energy.

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It indeed rises rapidly with velocity, supporting the initial assumption that drag is the important effect at world-record speeds.

Setting $P_{drag} = P_{athlete}$ gives

$$v_{max} \sim \left(\frac{P_{athlete}}{\rho A}\right)^{1/3}$$

To estimate how much power an athlete can supply, I ran up one flight of stairs leading from the MIT Infinite Corridor. The Infinite Corridor, being an old building, has spacious high ceilings, so the vertical climb is perhaps $h \sim 4$ m (a typical house is 3 m per storey). Leaping up the stairs as fast as I could, I needed $t \sim 5$ s for the climb. My mass is 60 kg, so my power output was

Is this supposed to be due Thursday?

Tuesday (with Sec 3.3). For Thursday you'll learn about flight.

Cool!

This phrasing seems a bit awkward - the meaning is clear, but it just bothers me a bit.

non sequitur

I had to read it a few times before I understood this sentence.

Are we going to talk about fleas in later sections, or is this just to speak to the ubiquity of this content?

Why fleas?? haha

These results? As in, the results of this section?

Yeah, this doesn't quite make sense in the context.

speed skating or skiing would be more relevant right now.

Maybe true but winter olympics only come around every 4 years...

but he's hoping to write a textbook! hopefully it will be read by many people for many olympics to come :)

Simply due to the Olympics?

I think that, in general, more people are able to relate to swimming over speed skating or skiing...

I think the first comment was a just a joke. Swimming is a perfectly relevant example.

I would say more people have direct experience swimming than speed skating or skiing... especially since we all have to pass the swim test :)

SWIMMING FTW

100 MPH?

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is there a technical term for this? i feel like there is... i just don't know it off the top of my head...

Well, the techinical definition isn't so important. Really, we just need to make it clear so that if we compare answers or methods, we don't have conflicts based on having calculated multiple scenarios. This is a good point - I would have certainly missed it.

Armchair proportional reasoning? I'm unfamiliar with this term.

I think this means just sit around and think about it (as though you were in a comfy armchair) rather than look it up

Yeah, I think this is an awkward term and could be rephrased better.

The first thing that came to my mind when I read this was that we should approximate a cyclist as having the cross sectional area of an armchair...

Yeah, same. I tried to think of ways approximating an armchair would actually be helpful.

This makes me think that you are referring to the instantaneous speed record, which is quite different from the one hour average speed you used in the example, it should be clear before hand what exactly you are trying to approximate.

I agree. When I first read this I was trying to think of "whats the fastest anyone has ever pedaled a bicycle? were they on a hill? etc."

"Using a regular bicycle" is a very dangerous term. A mountain bike's max speed will differ from that of a road bike, which will differ from that of an expensive road bike, which will differ from that of a time trial bike, which will differ from that of a triathlon bike....

These differences are also quite considerable, even just between types and models of road bike. Indeed, even the helmet used can effect the speed by a decent amount.

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Being a racing cyclist, I cannot agree more with your choice of example and the analysis here. Air drag is so incredibly annoying. So much of competitive cycling involves trying to find the tiniest changes in equipment and rider form to reduce air drag. It can get ridiculous pretty fast, in fact there are even men who shave their legs becuase they think it will cause less air drag and make them go faster.

Figuring out whether shaving their legs would cause less air drag would be an interesting problem.

Listing all the ways energy can get dissipated in riding a bicycle is a good way to see the problem. Could we also be able to do a free body diagram while performing a force balance order to find the drag force? (since acceleration=0 at terminal velocity)

In the D-Lab wheelchair design class, we do the opposite calculations...when determining the power output of a person, you ignore drag completely because in a wheelchair you never travel fast enough for drag to matter more than 10%. Interesting to see the other end of the spectrum here.

What about motorized wheelchairs? Doesn't drag come into play? Also, what about windy days? The cross section of someone on a wheelchair seems to be bigger.

Is this necessary? Couldn't we just estimate it based on other types of vehicles and based on power?

Power comes into the analysis later on. We could use other vehicles, but it might take longer time since we need to compare the fuel used by the vehicle and the energy/strength of a person.

I like that in the previous section we saw the equation for drag, so we are familiar here with the fact that drag and velocity are directly proportional.

I agree, using the same concepts twice is really useful for understanding the estimations and not getting hooked up on the equations.

Yeah I agree too. Seeing as I am not very familiar with the concept of drag, it was very helpful to have seen the simple cone example before reading this section.

But it's not a linear proportion (see below). This is why driving your car at 65 mph burns nonlinearly more gas than at 55.

So do you just not think about friction in this case?

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I feel like the (from my feelings when riding a bike) rolling resistance should be rather important...even at high speeds.

This is a really cool way to solve the problem

Ah, perhaps I could have used this for the last problem set asking about the force of wind on a cyclist!

Indeed, and you still can use it. That's why I put the problem on HW 03.

This does seem useful for the current problem set - I had a tough time figuring out how to approach that one.

Do the mathematics for calculating drag have any relation to calculating draft...like drafting off of a car or a cyclist etc.??

I believe drafting is a much more similar mathematical calculation but relies on the same principal. The person that is drafting is at an advantage because they don't feel the drag.

We can see from this section that drag is the main force of resistance to any moving object, so if we can eliminate it by drafting we save a surprisingly (or not) huge amount of energy

It's spelled story in the U.S. and storey in Britain. I'm not sure which is the principal audience...

Interesting way to approximate this.

Haha, agreed. I would have definitely used some other sort of method, but actual, physical experimentation is also nice! I sometimes forget about that as we delve further into thought experiments in this class.

If you don't have a dynamometer, this is a good substitute.

My physics teacher in high school used this example a lot. He talked about how much power was exerted running up all the stairs in the green building (he went to MIT).

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Brilliant!

This is really cool...as is the explanation of the estimation for olympic athletes in comparison

I agree–this way of finding out power output is so easy that we (I do at least) overlook it–just think of real life examples, or better yet, do them!

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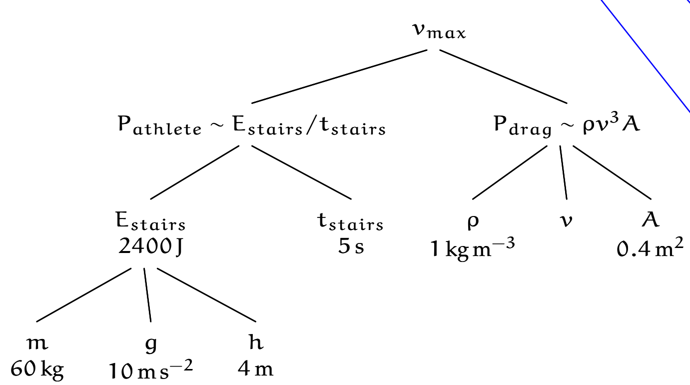

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The cube root might suggest using a calculator. However, massaging the numbers simplifies the arithmetic enough to do it mentally. If only the power were 400 W or, instead, if the area were 0.5 m! Therefore, in the words of Captain Jean-Luc Picard, 'make it so'. The cube root becomes easy:

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Can we use these methods during the final? I think that'd be wicked funny.

To me, this seems way too specific for one person. Some people would do the same thing and go twice as fast, or twice as slow. I realize factors of two aren't that important in this class, but when trying to determine a world record speed, a factor of two seems too large to even be worth approximating.

But, importantly, Power is raised to the power of (1/3) in the above equation. So even a factor of 2 becomes just a factor of 1.26.

If it were, on the other hand, raised to the power of 3, then that factor of 2 would be amplified and could end up being about a factor of 8 in our final result.

it might be useful to compare this number to a light bulb or some other familiar source of power to get an idea of how useful we are as sources.

This wasn't really clear in the introduction.. I thought you just meant world record top speed.

I agree, a bit more definition in the beginning would help.

Yes, this can be stated here also (for the estimation effect), but needs to be in the intro to cycling.

What do you mean by endurance record...? As in the power output the athlete has on average?

I thought that was a joke, as we just needed a ballpark number to compare.

this is a little hard to follow

cycling and running up the stairs deliver power in different ways. is it prudent to use the same numbers?

I like this estimate

Me too.

I think it's fine but thinking about cyclists, I'm sure they're power output is almost 1.5x that of normal humans.

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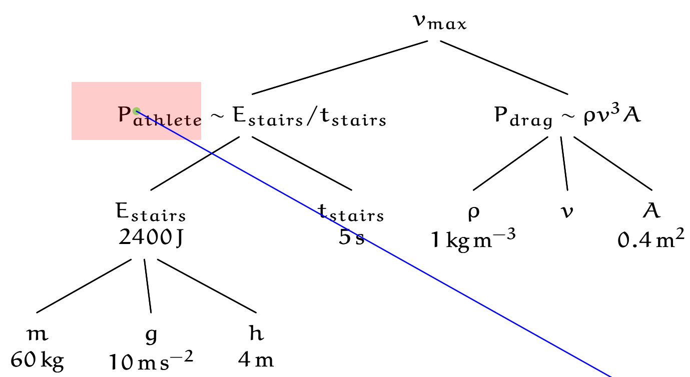

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interesting. why not 0.3 (few/10) or 0.5 (one-half)?

seeing the later calc, i understand why 0.4 makes things easier. is that why you choose it?

I'd like to know this as well - how would you have the foresight to estimate something as .4 instead of .few?

I also wonder. Plus, he later changed 500 to a 400 to match the 0.4, he could've started with a 0.5 and gotten the same thing.

I've always estimated the body to be 1m^2. But in this case the cyclist is crouching. Attention to detail got me on this one for the practice test.

This tree confuses me a little bit; it's different from the others we were using. Is a tree the best analysis tool here?

I think it would be helpful to see a tree before the analysis. It can have the values or not, but it helps me to understand why you are making certain calculations.

I agree. I also think a few of the previous problems we've seen in the reading for this course could use a bit of a roadmap (either in the form of a tree or written description) which could help indicate the assumptions/facts we need to focus on as we read through the example.

I like the tree analysis. With all the different components that combine to calculate the top speed, its nice to lay them out in a tree. It helps me reiterate my process and double check the work before diving into calculations.

Perhaps at the beginning, we could have a simple tree with vmax, Pathlete, and Pdrag. Then, when we start estimating Pathlete, we could draw that part of the tree, and then, once we've created all the subtrees, we can draw them together as one final tree.

This would have the advantage of giving us a general outline from the start without presenting confusing information (having 'Estairs' on the tree before it's mentioned in the text, for example, would just be confusing).

I'm not a fan of this font.

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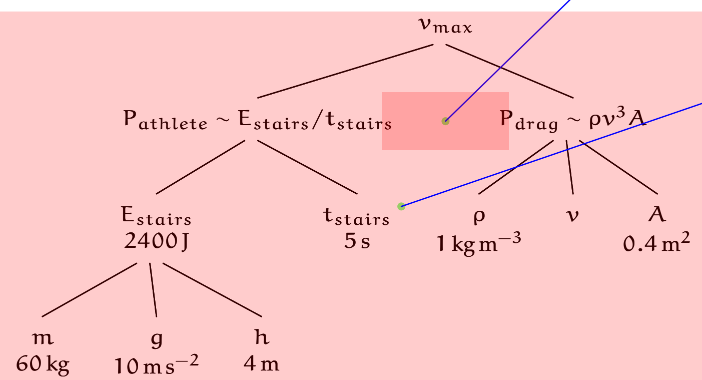

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In class I think we used a dotted line to note that these values should be equal- it'd be nice to see that here.

We were also using it for redundancy, but these values are not redundant, they just come together to give us vmax.

Yes, I think overloading the dotted line notation might be confusing, but some sort of indication could be helpful.

yeah, in class a dotted line showed redundancy, but I dont think this is a case of redundancy...we're finding vmax based on Pdrag and Pathlete...

But based on the notation we learned this diagram says they should be multiplied, which is wrong. An improvement needs to be made to the diagram system that better explains the relationships between the nodes.

This tree is very helpful in explaining the analysis.

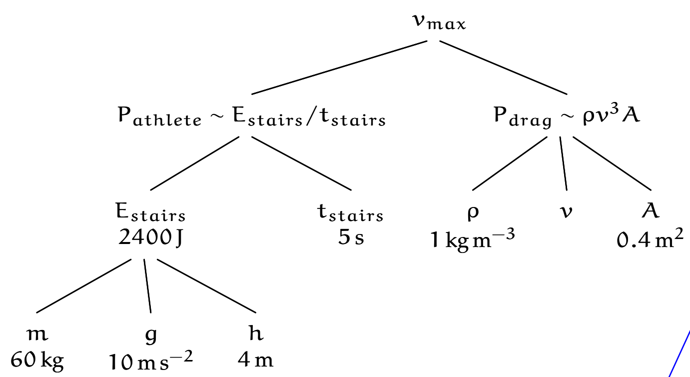
so in HW, we need to specify the range of error (or deviation), so if you just massage the numbers here, how do you account for it when you calculate your range of error

$$P_{author} \sim \frac{\text{potential energy supplied}}{\text{time to deliver it}} = \frac{mgh}{t} \sim \frac{60\,\text{kg} \times 10\,\text{m}\,\text{s}^{-2} \times 4\,\text{m}}{5\,\text{s}} \sim 500\,\text{W}.$$

$P_{athlete}$ should be higher than this peak power since most authors are not Olympic athletes. Fortunately I'd like to predict the endurance record. An Olympic athlete's long-term power might well be comparable to my peak power. So I use $P_{athlete} = 500\,\text{W}$.

The remaining item is the cyclist's cross-sectional area A. Divide the area into width and height. The width is a body width, perhaps $0.4\,\text{m}$. A racing cyclist crouches, so the height is maybe $1\,\text{m}$ rather than a full $2\,\text{m}$. So $A \sim 0.4\,\text{m}^2$.

Here is the tree that represents this analysis:



Now combine the estimates to find the maximum speed. Putting in numbers gives

$$v_{max} \sim \left(\frac{P_{athlete}}{\rho A}\right)^{1/3} \sim \left(\frac{500\,\text{W}}{1\,\text{kg}\,\text{m}^{-3} \times 0.4\,\text{m}^2}\right)^{1/3}.$$

The cube root might suggest using a calculator. However, massaging the numbers simplifies the arithmetic enough to do it mentally. If only the power were 400 W or, instead, if the area were 0.5 m! Therefore, in the words of Captain Jean-Luc Picard, 'make it so'. The cube root becomes easy:

$$v_{max} \sim\sim \left(\frac{400\,\text{W}}{1\,\text{kg}\,\text{m}^{-3} \times 0.4\,\text{m}^2}\right)^{1/3} \sim (1000\,\text{m}^3\,\text{s}^{-3})^{1/3} = 10\,\text{m}\,\text{s}^{-1}.$$

Is it really worth it to do it mentally? It's true that you get the answer a bit faster, but you might be sacrificing accuracy unnecessarily. I'm not quite sure how I feel about it, but it's something I thought of while reading the section.

I guess it depends on how much you care about accuracy. Since he's willing to fix the equation to suit his needs he must not care so much about being really close to the correct answer as he is of getting an answer that is on the order of the answer and that is reasonably close.

Is this really a question of whether or not to do it mentally? The answer in an Approximation class will always be to do it mentally. Since the beginning of class, we've been learning methods about how to make calculations simpler and easier to do 'mentally.'

Thats true, it does work reasonally well. On the other hand, I don't think I'd be confident enough to try this approach. If the number we were calculating is orders of magnitude higher, thats a different question. When calculating top speed, a few m/s off stands out, so I think I'd need some more practice with this.

I actually think we lose a lot these days because of the reliance on calculators. Yes, it does reduce some error, but I find my mental math to be greatly inferior to my mental math when I was in 4th grade...

nice, he is my favorite captain.

I agree that the Star Trek reference is awesome, but if this is a book for a general audience as opposed to just MIT, this may be lost on some people.

Was lost on me, but it could be intended for an MIT audience, or at least people who would be interested enough to look something like that up.

While the specific reference is lost on me, I like the point he's making with it. 'Make it so' is certainly useful.

Agreed. Maybe you may want to include a footnote defining who he is.

using the few method!

$$P_{\text{author}} \sim \frac{\text{potential energy supplied}}{\text{time to deliver it}}$$
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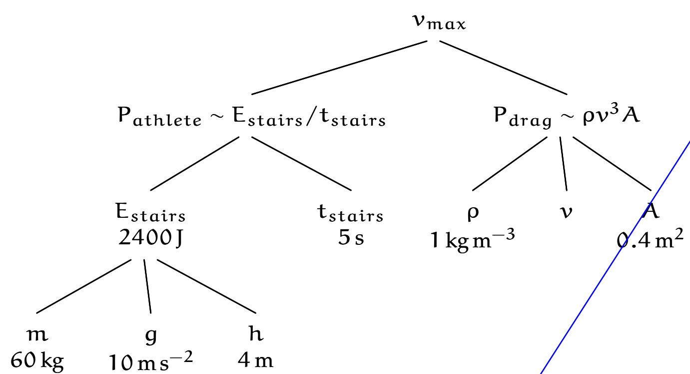

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I'd use the 0.5 m^2 instead of 400W... (1) 5s are "nicer" numbers for people to deal with in general [even though it doesn't matter at all] (2) you've already admitted to the distinct possibility that the power is an underestimate & now you want to make it smaller. (3) I'm more comfortable with the idea of the cyclist being a bit larger than with him/her using less power.

So the world record should be, if this analysis has any correct physics in it, around $10\,\mathrm{m\,s^{-1}}$ or 22 mph.

The world one-hour record – where the contestant cycles as far as possible in one hour – is 49.7 km or 30.9 mi. The estimate based on drag is reasonable!

I still have to do this conversion the long way (*3600/1600) every time. My intuition for speeds in m/s beyond a small scale just isn't very good yet.

I used to have that problem until I starting remembering a few different numbers in meters per second. for example, if you remember how fast people walk, run, a few common speed limits (30mph and 60 mph), and the speed of sound (those are the ones I know), you gain an intuition really easily by comparison

Although I think this is a powerful estimation tool and an interesting application, I'm not convinced of it's practicality. In athletics, the difference between 22 miles and 30.9 miles is astronomical.

that's way cool

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Plus you lowered the power guess, which is part of the lower estimate error.

I was going to say the same thing if we had left the power at 500W the answer would be even closer

Barely though. This only raises it to about 10.7 m/s, or 24 mph.

I still have problems with this... if you did all the same calculations with a runner instead of a biker, you're really calculating how fast the person is capable of sprinting exerting his/her maximum power. Assuming you get a value similar to how fast a person can actually sprint, that value does not come close to representing how far someone could run in one hour since humans can barely sprint over a minute a max speed.

I would argue though that most of a sprinters energy isn't lost to drag. Also, his power estimate was for an athlete's "endurance" power production.

Actually, drag is very important in sprinting. Thats why records have to be wind dated below a certain level to count. However, the running example probably wouldnt work because people tend to run on tracks where the wind would mostly cancel out. Therefore the biking example seems to make more sense.

The wind definitely doesn't cancel out (speaking from many years of track and field at a highly competitive level). The wind against you imposes a force, changing your form and slowing you down. The wind behind you, while it makes it somewhat easier to run, again changes your form and will thus tire you out. This is a similar effect to how running downhill for a good portion of a cross country course can be detrimental if one hasn't trained for it.

You're probably right about that. I would argue that the wind definitely does speed you up when its against your back, but probably not enough to offset the wind blowing against you on the opposite side of the track. As for running downhill, people generally do run much faster which is why there are restrictions on the net change in elevation in road races.

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It definitely does make you run faster in terms of instantaneous velocity, but you'll be more tired afterwords than you would be running a similar distance on level ground due to increased muscle involvement in slowing you down, as well as spending significantly more time with your feet on the ground (since you'll be striking more in front of your body than underneath your hips), thereby making your form less bio-mechanically efficient

Yes I agree that your muscles would be more sore from running inefficiently. But my point is that despite this your speed would be improved by the wind at your back or gravity pulling you down a hill. In that sense, it will "cancel" with the opposing force.

So if you aren't satisfied with the explanation of wind "canceling out" on a track, a better explanation would be that when running slower speeds the wind speed is less negligible (since v is in the equation for drag). That's why biking makes a better example.

But even Lance can't maintain 500W for more than 30 minutes, so there is some other source of error here. Is coasting helping more than we think, or is drag not as high as we think?

I don't really think drafting is why we messed up just because I think the record is a one person event. If what you meant by coasting is stopping the pedal motion, I think our calculation takes that into account because we're averaging power. Since stopping would slow you down and you'd lose ground to that air resistance, you'd have to pedal harder afterwards and your average power would be more or less the same.

That's an interesting question about sprinting, but the power delivered in sprinting is pretty different. When you cycle you really pushing the pedals down just as you are pushing down on the ground when you run up stairs. When you run forward, in some sense you are just falling forward, but I think there is some power lost due to that up and down motion.

Good to see we have a bunch of runners in the class. Thanks for the info guys! As for the question, I think the output power is high but the explanation is crisp and clean.

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side bar this!

this is still quite a bit larger than our estimate, especially considering we know some pretty strict bonds on what it could be, even without all the math

This is a great method for figuring this out. I definitely understand drag more. Can we apply this to viscosity.

Did you even talk about fleas? You said you would at the beginning of the section

Agreed. The fleas sounded intriguing. Maybe that was just a statement indicating how general this technique is? It certainly sounded like an example with fleas was coming.

Haha, I noticed that too. I was rather interested to see what you had to say.

It might be useful to succinctly restate a general way of calculating drag (to conclude the section)

I agree. For me, it would also be helpful to have a 1 or 2 line reminder of the estimations/simplifications we made to arrive at that final drag force equation.

Thats not a bad idea. I really like some of the closing paragraphs in previous sections. They really concluded the processes well. Even though the application of symmetry was pretty apparent, I don't think the word "symmetry" was ever used in this section.

I agree, a concluding paragraph would be nice. I can see that I will be using some of the information in this reading to complete the homework, but my thoughts on this section are a little scattered and unorganized. It would be nice to tie everything together at the end.