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$$m_{\text{air}} \sim \rho L^2 s.$$

The downward speed imparted to that mass must take away enough momentum to compensate for the downward momentum imparted by gravity. Traveling a distance s takes time s/v, in which time gravity imparts a downward momentum Mgs/v to the plane. Therefore

I have real experience with any of the physics behind flight because we don't cover it in 8.01...so this entire section is very cloudy for me.

I really like these examples in mechanical engineering, however, will we ever discuss any more comp sci/ee readings?

think this should have been mentioned earlier.

how would you modify this approach to calculate the minimum-energy speed for a car?

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A general note: it would be really awesome to have all of the week's readings posted on Sunday, so we could sit down for an hour and do them all if that would be most convenient for us. It would allow us to gain a broader sense of the topic, since when we focus really hard on one at a time I, at least, tend to lose sight of the general principles being taught.

Also, another point: sometimes I've found myself reading the memos just to post things on them (to get it checked off), rather than reading them because I want to learn the material. I don't have a solution to this problem, but it seems to act counter to the principle of not rewarding people so they actually care about learning.

Read Section 3.5 for the memo on Thursday. One topic in lecture on Friday will be a home experiment (one for everyone) to refute a common but bogus explanation of lift.

Is there a way to undo an "I agree" or "I disagree" ?

yes, click on it again...it's a "toggle" field.

A recitation teacher in my freshman year 8.01 class described flight as air taking more time to travel over a wing (due to curvature) then to flow underneath it, creating lift.

I believe this is exactly what lecture will disprove on Friday.

i don't think lecture today "disproved" the theory. it just proposed a better, more testable one.

I don't know... it seems what we did in lecture is somewhat incompatible with that explanation.

I always wondered how far they can go without landing.

I did not see any analysis of how far birds can fly in this section. If none of the further sections mention this (I think they should, becuase bird flight is incredibly fascinating) I would recommend you remove the reference to how far can birds fly, if you don't answer it.

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what are streamlines?

streamlines are kind of like electric field lines in E&M. they're lines that are always tanget to the fluid velocity.

what are streamlines?

Streamlines are the way that the air moves over and under the object, in this case, a bird. They help explain how the air moving past the bird both push the bird downward and upward.

I think this reads awkwardly. Maybe you could say "...and much more."

what are vortices? was this mentioned before in the class?

He has talked about vortices in class. it's the spinning flow of a fluid. it's when a fluid spins rapidly around a center point. think about stirring coffee or pouring water. sometimes you get little vortices in them.

missing "to", use conservation to estimate

I think it would also be helpful if we knew immediately how this relates to symmetry. Although the brief explanation of the example is great!

I really like estimation problems like these that are tied to physics. I see them naturally in physics problems a lot.

Is this actually how planes operate?

maybe tie back to real world... to minimize costs

maybe it's just me but i had to read this a few times to get it (i'm probably just slow), maybe having "minimum-energy" in quotes would help

to clarify, the following phrase confused me: minimum energy way. so i suggested that if it said "minimum energy" way, I'd understand what you were trying to say more immediately.

Invariant!

Yea! I never thought to think of flyijng as an invariant problem.

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I didn't know this was true. Thats cool.

Well, 'constant altitude' means that vertical velocity is zero, so the vertical component of momentum can't possibly change.

Hm this makes sense. But when a plane is landing, why does the pilot curl the wings in (downward)–wouldnt that cause more deflection of air downwards?

I thought that was the point? By curling wings in and deflecting air downward, the plane gets some upward lift.

The deflection of flaps and increase in camber allows the plane to gain lift at a lower airspeed.

Because the air's velocity is lower, it's momentum is lower. So this allows the plane to compensate.

Yeah, I haven't be aware of this!

I had not realized until now that that is how lift actually works in a physics forces kind of view.

I don't think that this is how lift actually works. Maybe this is an approximation for an airfoil.

Actually, it is surprisingly accurate. Some cool pictures demonstrate this:

http://people.eku.edu/ritchisong/554images/downplane5.gif

http://www.aviation-history.com/theory/lift_files/fig6.jpg

Those are awesome images.

I found this explanation helpful as well.

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This is much more satisfying of an explanation than Bernoulli's principle for why a plane stays in the air.

I don't really agree that it's a more satisfying explanation, but definitely a more creative explanation which I enjoy.

I don't think I could have come up with this explanation by myself.

we all know through physics that velocity in x is not related to y. Thus we had to come to the conclusion that although the turbines are thrusting us forward, something is keeping us from falling. That is Bernoulli.

Shouldn't this go after the 3 steps when we define the other terms?

I agree, I think that works. It doesn't matter a whole lot though... you still get the picture.

agreed... when I got to the next part, I had to look for this definition again

I also agree; the order seems a little off here.

Seems strange to introduce this in the first-person.

These steps aren't intuitive to me.

I think we can think about it like this: in order to fly we need to displace some air, but how much? We have to displace the air with a certain velocity to actually get airborne, and we want to know how much energy we need to use to do that.

So I thought this would be a very hard calculation. But I thought that it was explained very well. It especially helped me when you went through the dimensions at the end.

I think it's somewhat difficult to calculate the mass of air, because the volume is almost infinite

Why is the air deflected downward from a flat plate like the index card we used in lecture?

as the plane moves through the air, I believe it creates a vortex under and the air flows under the wing. The energy to displace the air should translate into energy which keeps the plane up.

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this is also represented as "angle of attack"

Angle of attack is only part of this calculation, which has to have some other parameters like wing length in there.

As in how much air, or the angle of deflection?

Good point. I think this is referring to finding m.air, so it's what quantity.

obviously this is divide and conquer- where does symmetry tie in?

This is a very interesting way to think about this problem

Every step leads to the next one. I would have liked to see a tree here. Steps make it seem like we do 3 things when we really do 2 and combine them to find the third.

is this a function of the speed at which the airplane is traveling?

and also, is it a function of height?

I think that makes sense. I'm not too familiar with this type of a problem, but i imagine maintaining a particular angle of deflection will maintain the same lift, regardless of height.

It'd be cool to eventually see a tree for this- it would show yet again another tie between what we have done and what we are doing now

I totally agree. This reading has a lot of math and it would definitely be helpful seeing a tree to help tie it all in.

Divide and conquer hard at work!!

Why is kinetic energy imparted to the air matter? Doesn't the flight just depend on how much recoil there is?

Why is the area L^2 and not L*width of wing?

Why L^2 and not just the area of the wing's foot print?

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Typo?

Why this area [is] L^2...

I understand the first L, and I understand s, but not the other L. If we looked at it like a box: L = width s = length L = Height? I don't understand the intuition

yeah i'm confused about where the L^2 is coming from too.

I think, based on the paragraph, that we are looking at how far the air is disturbed, rather than just where exactly the wings pass. As such, the air is disturbed in roughly a square, where on side of the square is the wings themselves (length L) and the other part is the region right behind where the plane just flew, also a length L back, leading the disturbed air area to be L^2

Oh, if that's what it's supposed to mean, that explanation made a lot more sense to me than what is in the actual text. Maybe explaining it in the text like that would be helpful?

I agree that that explanation made more sense. What I'm still confused about is why the width of the wing is not taken into consideration.

Seconded.... I was thinking about this for like 5 minutes and searching through all these comment boxes just to find a thread discussing this.

by this do you mean that as a plane is traveling there is basically a zone in front and behind of the plane where the surrounding air that is traveling with it acts as part of the wing?

This is very hard to follow; I feel like this should be expanded and made clearer.

Although the overall picture is different, this is the same area calculation that is used in using fluid dynamics to calculate lift force.

what's with all the random boxes. i hate this program...

i didn't think we were considering cross-sectional area at all. isn't the area under the wings we would look at instead?

Yeah, I think what he's saying in this sentence is that we are choosing this L^2 over the cross sectional area but stating that the reason for doing it this way is "subtle"

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So the cross sectional area has no effect on air disturbance?

It seems so; this makes sense as the plane will be moving so the area over which the wing can deflect air is roughly a square.

Is this because vertical momentum is conserved?

I am not very clear about this, is it supposed to be an approximation?

Yes! That's why he used a tilde ( ) - it means "approximately"

This sentence confused me, as its a bit wordy and I'm not familiar with the physics of flight. Is this saying that the only important factor is the length of the wings, and not their width at all?

Yeah, this seems a little confusing–how does area not matter? why does it only depend on length? wouldn't a plane with wide wings deflect more air?

Maybe it only depends on where the most disruption is ocurring, which would be the leading edge of the wing. And thus the dominant contribution goes as L^2 instead of being proportional to the cross sectional area of the wing?

how do you know that? did you learn it from physics or general knowledge

Why? This doesn't really seem like an explanation. More just like another fact you happened to know.

I agree that this seems like a fairly obscure fact to know. I'd imagine it's derived from computing velocity*arbitrary unit of time, which probably gives a distance traveled of about L. However, regardless of how this was arrived at, it should be shown.

Good point. A beautiful picture of the air disturbance is given in the Wikipedia "Vortex" article. Equally good, the picture was made by a US government agency (NASA), so it has no copyright, which means I can use it in the textbook.

are we talking characteristic length for the equation here? then again, i don't know why its l^2 rather than 2l...

I'm not sure about the L, but I know it can't be 2L since we need an area (units of distance^2)

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This has to be a vertical distance right? I guess the passage of the plan disturbs a rectangular prism of air that has dimensions s x L x L?

Also, I suppose most wings have a similar shape where the longitudinal length is some fraction (a) of the span, L, making the total area some fraction of the total area: aLˆ2.

Surely their area along with their span must play some role. Otherwise airplanes would have thinner wings, right? Someone who knows more please explain.

With rho being the density of air?

of the fluid through which the vehicle travels, which in this is case, is air.

Didn't you just say this above?

Yes, but I think that it is rephrased slightly, making a momentum to momentum comparison.

While it's clearly stated what 's' means in this context, when we start throwing around speeds and masses, it's pretty natural to expect 's' to mean seconds. I think using 'd' for distance would make this much easier to follow.

Not a big deal or anything, but you don't actually define M (though it's pretty obvious of course).

I'm slightly confused as to how you got this so an explanation would be nice. Instantaneous momentum is given by Mv, but Mgs/v seems to be F*t?

So he has Mgs/v –> meaning gs/v has to equal a velocity if you want this to be momentum. g is m/sˆ2 and s/v is s, giving you m/s – so it all works out in the end. Also, the units of momentum are N-s, so F*t also works (as it should)!

I actually really like this bit of unit analysis, thanks whoever posted it!

$$m_{air} v_{down} \sim \frac{Mgs}{v}$$

so

$$v_{down} \sim \frac{Mgs}{v m_{air}} \sim \frac{Mgs}{\rho v L^2 s} = \frac{Mg}{\rho v L^2}.$$

The distance s divides out, which is a good sign: The downward velocity of the air should not depend on an arbitrarily chosen distance!

The kinetic energy required to send that much air downwards is $m_{air} v_{down}^2$. That energy factors into $(m_{air} v_{down}) v_{down}$, so

$$E_{lift} \sim \underbrace{m_{air} v_{down}}_{Mgs/v} v_{down} \sim \frac{Mgs}{v} \underbrace{\frac{Mg}{\rho v L^2}}_{v_{down}} = \frac{(Mg)^2}{\rho v^2 L^2} s.$$

Check the dimensions: The numerator is a squared force since Mg is a force, and the denominator is a force, so the expression is a force times the distance s. So the result is an energy.

Interestingly, the energy to produce lift decreases with increasing speed. Here is a scaling argument to make that result plausible. Imagine doubling the speed of the plane. The fast plane makes the journey in one-half the time of the original plane. Gravity has only one-half the time to pull the plane down, so the plane needs only one-half the recoil to stay aloft. Since the same mass of air is being deflected downward but with half the total recoil (momentum), the necessary downward velocity is a factor of 2 lower for the fast plane than for the slow plane. This factor of 2 in speed lowers the energy by a factor of 4, in accordance with the v^{-2} in E_{lift}.

3.5.2 Optimization including drag

The energy required to fly includes the energy to generate lift and to fight drag. I'll add the lift and drag energies, and choose the speed that minimizes the sum.

The energy to fight drag is the drag force times the distance. The drag force is usually written as

$$F_{drag} \sim \rho v^2 A,$$

where A is the cross-sectional area. The missing dimensionless constant is $c_d/2$:

Although I understand where this came from it may have been helpful to have a description beforehand, maybe even a visual, of how you planned to use the information you were looking for.

this is very clear

Why does the twiddle become an equal sign at the end of this?

It's nice to see this work out mathematically because the intuition at the beginning is that d should not affect the air velocity

Yeah I like this little proof here.

I agree, it's similar to when you calculate things like Force/Length but need to pick an arbitrary length in the beginning of the problem only to cancel it out later.

Why isn't this 1/2mv^2?

Maybe he doesn't think the 1/2 is significant for this approximation. I think in another section he left out a factor of 1/2 for a drag equation.

I think this has to do with the approximation. This came up in the last reading as well.

Other people hinted at this, but I think there should be a twiddle in front of this equation.

is this "s" seconds?

Never mind. I see we are now looking at total energy from the flight

It's distance

Though that brings up a good point - when I first read the comment I thought, "of course it's seconds! That's the SI unit." s as a distance variable is a little confusing when numbers are involved - maybe change it to d?

I agree. We used d for distance in the last memo. It's good to be consistent.

No, it's distance. Kind of confusing notation, right?

$$m_{air} v_{down} \sim \frac{Mgs}{v}$$

so

$$v_{down} \sim \frac{Mgs}{v m_{air}} \sim \frac{Mgs}{\rho v L^2 s} = \frac{Mg}{\rho v L^2}.$$

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I thought kinetic energy was 1/2 * mv^2, or are we just ignoring the 1/2 for reasons of approximation.

I think we are ignoring that factor of a half for approximation purposes.

I had the same thought, and he seems to do it consistently, so yes.

We're ignoring the 1/2 not because it's a small factor, but because it's a constant factor at all. The ' ' in the equation indicates we are just trying to find a proportional relationship between energy and the other variables so that we can infer the effects of altering one of those other variables (like v, m, etc.)

This is something I have come to use a lot when solving problems with various units. Its a very good sanity check.

I agree, especially in these types of problems. If you can see that the dimensions work out the problem ends up becoming simple.

I don't understand the wording of this sentence- are we saying that it's a squared force because Mg (a force) is squared or is it because Mg is a force that it must be squared?

I believe it's your first answer, since we're just trying to put some meaning to these terms.

I like that we checked the dimensions without having to revert all the way back to mass, length, and time. I tend to do that, and clearly getting to force * distance is the quick and easy way!

This is true, but not obvious at first glance. A quick note about dimensional analysis might help.

It's equivalent to the dimensions of the drag force we saw earlier, when you consider [L^2] = [A] (both meters^2). [rho*v^2*L^2]=[rho*v^2*A]=[Force]

I like this note.... dimensional analysis is always a helpful tool... especially in approximations.

s is easily confused with seconds. A final draft might want to use a more recognizable variable, like 'd' or 'x'

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so

$$v_{down} \sim \frac{Mgs}{v m_{air}} \sim \frac{Mgs}{\rho v L^2 s} = \frac{Mg}{\rho v L^2}.$$

The distance s divides out, which is a good sign: The downward velocity of the air should not depend on an arbitrarily chosen distance!

The kinetic energy required to send that much air downwards is $m_{air} v_{down}^2$. That energy factors into $(m_{air} v_{down}) v_{down}$, so

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what is the reasoning behind this? I thought it was the opposite

The equation shows v^2 in the denominator, meaning that - mathematically - its increase results in a decrease for E.

Is that interesting? Wouldn't this be logical?

I thought it was interesting and I don't know very much about planes, so I didn't think it was that intuitive.

I think its both logical (since it makes sense given how we think about planes) but it isn't intuitive.

So that's why you need to speed up in order to fly (and also the reason why I can't fly)

Birds don't fly that fast though so how do they generate all that lift? By flapping their wings?

their mass is smaller? Just a guess... I don't know much about flight/birds

their mass is smaller? Just a guess... I don't know much about flight/birds

is it right to think of this because youre flying faster so you're generating more lift per time step?

This seems to make sense however, as mentioned earlier it is hard for me to wrap my head around it. Gravity acts instantaneously so how can we say gravity has less time to pull the plane down. Is this one of those things where you have to blur your vision?

I had never thought of this before, it is an interesting way to think about the problem

Yea, I wouldn't have thought to think about the amount of lift needed this way. Intuitively it makes sense that a faster plane needs less lift, but actually calculating how much less is a different story.

is the E a function of time? I didn't think they were

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I guess in theory this does make sense, but it's hard to wrap my head around this intuitively.

I agree. Seeing it written down on paper makes sense, but it's hard to picture it intuitively.

I disagree, I found the paragraph explanation very descriptive and exlpanatory. I think it is natural that this does not make intuitive sense, hence, the explanation.

I like this explanation. It is just like a projectile from 8.01 - except with a jet engine, but gravity still has the same effect.

This paragraph has been the best one so far. I think this was very informative and explained how lift is inverse to speed well.

That's really cool, I've never heard of that before. Maybe we could go into this a bit more- how fast would you have to go to not need any energy to produce lift??

The wording of this seems a little awkward. Maybe it would be better to go ahead and show the equations with the wording

a picture of all the forces acting on a plane would be nice here.

Agreed!

I agree but i think its important not to make any types of pictures too specific. I feel that in this class being able to extract the ideas from specific problems and apply them to any type of problem is crucial.

i thought "drag" would be in the "x" direction, while "lift" would be in the "y" direction, perpendicular to it. if this was the case. the magnitude of the vector sum of the two will not be the same as a scalar addition.

It seems like you can get around this issue by using "C" and the wingspan area instead of Cd and the cross sectional area.

Well, since the plane is moving in the 'x' direction, wouldn't lift have to have some effect on that as well? There is also airspeed velocity....what is generated by the engines

Drag and lift forces are vectors, but the energy used to overcome those forces is a scalar. So we can add Energy spent on lift and energy spent on drag to find total energy consumed.

that would be true if we were talking about summing forces, but not energy. Energy has no direction.

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Are we supposed to know all these formulas? Maybe you can make a file on the class site with all the formulas you've used so far in the book.

This was in the last chapter. I think he expects us to remember things from chapter to chapter

I think it would be nice to have a continuously updated list of different formulas that we have seen. Even one at the very end would be nice for reviewing for the final.

Why isn't this just written with the equation? Why is it mentioned afterward?

I agree. I'm not sure why you bothered to tell us that Fdrag is proportional to things and then immediately after tell us the exact formula for the force. It seems kind of redundant.

It's the same thing we do in class: figure out the relationships between the variables of a problem, then attempt to arrange them into a (semi-)formal equation.

yea same thing with the missing half in the kinetic energy equation. A usefull representation of how variables relate to each other is the more important then an exact answer.

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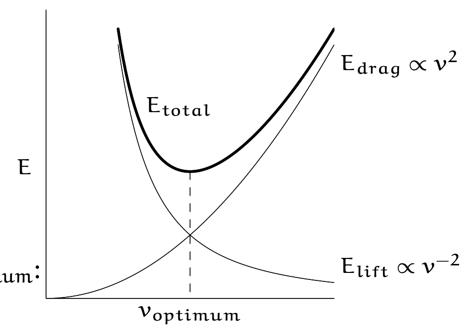

Comments on page 3

Where could we get something like this?

Why do we get hung up on including the constant coefficient C when we just nix the 1/2 from the KE equation so quickly?

According to wikipedia, a streamlined body has a drag coefficient of 0.04. This is a much more significant factor than a factor of 1/2 (that is, it changes the answer by a factor of ten).

Also, the drag coefficient mentioned above is c_d. Since C is much smaller even than that, C is yet more important.

One reason is that the 1/2 is the same for every object, whereas the C (drag coefficient) might vary a lot. And if want to understand the effect of shape on flight, then we need to keep the C around.

is the drag coefficient dependent on the cross-sectional area or L^2?

oh, hah, nevermind, answered in the next paragraph.

I'm still not sure what determines the drag coefficient and why it would need the additional coefficient of 1/2.

I think it has more to do with convention than need... I remember we discussed this in 2.006 but I forget where it came from.

Obviously the coefficient of drag could be expressed double the normal values in order to remove the 1/2, I'm pretty sure they didn't originally have the coefficient of drag and found that there was a factor of 1/2 different than just rho*v^2*A. Eventually they realized different geometries affect that coefficient, left the 1/2, and have been defining c_d in this way ever since. (A lot of geometries have a c_d of 1.)

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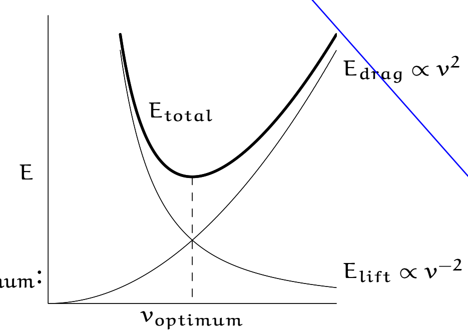

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Does the capital C represent the 1/2*c_d? And if so, why do we consider the 1/2 here and not for kinetic energy?

The equation relating C and cd would be helpful to see here before C is substituted in. Is it C = (1/2)*cd*A/L^2 ?

Yes, I am confused why we introduced the 1/2 in the first equation but then have eliminated it here. It almost seems like the 1/2 was made up, and I don't know why.

Read the lines directly above in the reading. He explains it there.

is the drag that we are measuring the skin friction drag that occurs as air flows over the wing or the drag due to the air directly hitting and pushing against the front of the wing (momentum transfer). shouldn't both drags be considered?

I'm pretty sure the drag due to the air hitting the wing directly is quite low. The drag on the top of the wing has a larger effect on the bird because of the anatomy of birds and their ability to cut through the air well.

I don't understand the difference between using c_d and C

c_d is the coefficient in relation to the cross sectional area of an object, while C is a "modified" coefficient to use with the squared length of the wingspan. As mentioned, C is much smaller than c_d.

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c_d=1/2*C

you are assuming that the drag generated by flow over the wing is much greater than the cross-sectional area to the point where the latter can be neglected?

Is this really just for most flying objects, or does it apply to all flying objects?

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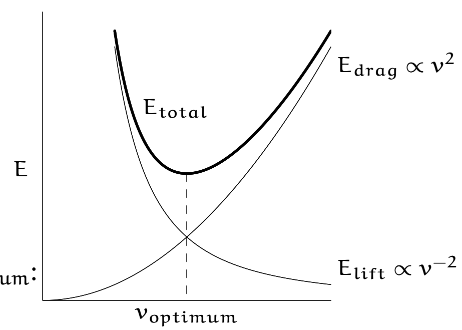

I'm a little concerned about the usage of ' ' combined with the '+' sign in the right hand side of the equation. Unless you introduce a factor of C.Lift as a coefficient for the first term in the r.h.s., I think you're covering up the fact that here the constant C is also taking into account this lift coefficient.

The reason is that E CL*E.Lift + C*E.drag is not the same as E E.Lift + C*E.drag when you have already defined C.

I'm finding myself getting lost in all this math. It's not that I can't understand it, but it's so densely packed in here, that it's hard to concentrate. I'm not sure if you could get away with removing part of it, but it might help with flow and comprehension.

I think it's a necessary evil. You can't really explain things like lift and drag without resorting to mathematics.

We do need math, but the operations and substitutions could be presented more clearly or explained better. There are some quick leaps in here.

I too would like to see some clearly proofs, as well as an image of the plane. I do like the sequence of thinking however, moving from lift to drag and then combining the two.

I think a really well done diagram of the plane would be really helpful.

I too would like to see a diagram of the plane. Perhaps a force diagram showing what parts of the equation look like on the plane.

I agree, we have seen a lot of equations so far..and I'm kind of confused about which way (vertically or horizontally) the forces are acting on the plane.

A diagram would also be helpful in the beginning of the problem in terms of defining L and s.

Yes, I do think math is unavoidable at this here point now too. However, perhaps we could include a picture, that would make people feel that it is easier to understand, like when we talked about how pictures are more easily digested!

I agree I would definitely like to see a labeled force diagram of the plane here.

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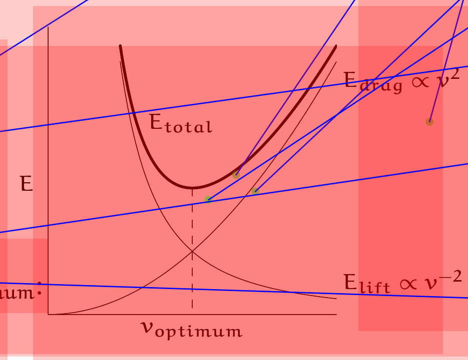

so this means that the energy doesn't depend on the height at all? or is this the energy for a specific height

I think that this is ignoring the differences in the height, and therefore the density of the air, since it was previously estimated to be about 1 earlier. I would guess that this could make a big difference in the numbers. I think this would be a great point for the text to discuss.

I really like this graph- it really helps

So is this why birds fly in flocks, and why airliners are considering grouping up planes in fleets for trans continental and oceanic flights?

for birds, yes...I didn't know that they group planes for really long flights, but it does make sense.

I like this diagram. Because there are so many terms, this visualizes and simplifies the problem.

Good graph! This clarifies a lot, in terms of visualizing the shapes of the curves.

I agree very much as well. This graph really put things into perpective.

This is really cool to look at. I just always assumed the low speeds at takeoff and landing were mostly due to safety!

so are commercial planes designed so that this optimal cruising speed is just under mach 1?

I'm a little confused on how going under mach 1 effects the energy. What are you getting at?

Shouldn't we be calculating how fast birds actually fly, rather than the optimum speed?

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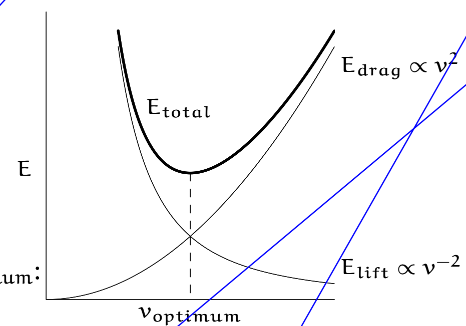

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This is really interesting! I've always wondering what determined a plane's cruising speed.

Definitely! This course really does help answer some of the mysteries of life...

I always kind of assumed that planes fly close to as fast as they safely can.

The follow up question that immediately occurred to me...what determines their cruising altitude? Because the density changes as you move up or down...

I also always thought planes flew as fast as the safely could, it's interesting to see that they are maximizing their profit and could fly faster but it is not to their advantage.

Same for cars, like when there was a big gas crisis in the 70's people were told to stay at 55mph on the highway because that was the most efficient at the time.

55 is still the most efficient for most cars...i'm not sure about hybrids, but the rest are pretty well set there.

This is really interesting. Does this mean that different size planes travel at different speeds in order to minimize their energy consumption? I always thought that all airplane flights traveled at a roughly uniform speed. Does anyone have any insight or thoughts on this?

Yes! Similarly, different-sized birds have different cruising speeds. But for planes there is an additional constraint, which is that they follow one another on a flight path. So, it's not efficient to have planes with different cruising speeds. The 737, which otherwise would have had a different cruising speed than the 747, was redesigned so that it has the same cruising speed as a 747.

This is an interesting point, especially because it's not intuitive to me. I would think that going slower would consume less energy but now it makes sense why it would actually consume more.

you could also think about it in terms of a car, you use a lot more gas running around the neighborhood at 30-40 than you do racing down the highway at 55 mph...then again, that 30-40 uses about the same as 70-80.

nevermind, if we're doing planes.

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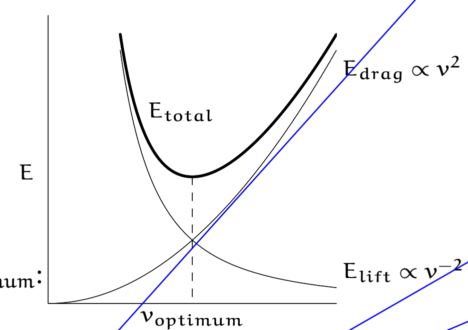
$$E \sim \underbrace{\frac{(Mg)^2}{\rho v^2 L^2} s}_{E_{lift}} + \underbrace{C \rho v^2 L^2 s}_{E_{drag}}.$$



A sketch of the total energy versus velocity shows interesting features. At low speeds, lift is the dominant consumer because of its v^{-2} dependence. At high speeds, drag is the dominant consumer because of its v^2 dependence. In between these extremes is an optimum speed $v_{optimum}$: the speed that minimizes the energy consumption for a fixed journey distance s. Going faster or slower than the optimum speed means consuming more energy. That extra consumption cannot always be avoided. A plane is designed so that its cruising speed is its minimum-energy speed. So at takeoff and landing, when its speed is much less than the minimum-energy speed, a plane requires a lot of power to stay aloft, which is one reason that the engines are so loud at takeoff and landing (another reason is probably that the engine noise reflects off the ground and back to the plane).

Does this sort of logic to find v_opt apply to non-flying objects like cars too? I've heard it's more efficient to drive at some lower speed, but I was never sure what the forces were behind that.

I believe that's generally based on fuel efficiency of the engine. I've heard the most efficient spot is in top gear but with a low tachometer reading. It may have to be fine tuned to take air drag into account, though.

The drag term still holds (in some form) but the lift part doesn't, since a car doesn't need to sustain a certain velocity to stay up. It depends on the engine as mentioned, and lower rpm are more efficient, but the most efficient speed could theoretically be very slow, where drag was near zero. Then other operational definitions of efficiency might be needed, e.g., I need to get to work in less time than a day.

Is that always true? Is it true for supersonic speed?

I think that this is true for everything, except the planes designed to break supersonic speeds...they don't really care how much energy those take...

I don't think any plane actually 'cruises' at super sonic. It's like cruise control in a car I believe; and super sonic is like going 100 on the freeway.

Is this just saying that a plane is designed such that it is most efficient at its cruising speed...?

yes. If it wasn't then that would be very poor engineering. It's a good point that I never thought about.

Wouldn't takeoff be different from landing? Before landing, isn't the plane resisting less of gravity because it is heading towards the ground?

Interesting, that makes a lot of sense.

yeah really cool to learn that!

I never even thought about these two reasons. I always compared it to starting a car engine...but then the plane's engines are loud for such an extended period of time!

I never thought about this before, but it makes a lot of sense.

Aha! So at lower speeds, to compensate, the use of flaps increases the effective area of the wing. This shifts L^2 higher to compensate for a lower v^2.

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Comments on page 4

nice, I like this assumption!

convenient.

The graph really helps in showing this is true.

Wait, isn't that a completely false statement? shouldn't their derivatives (with respect to v) be equal in magnitude and opposite in sign to minimize the sum of the energies?

Say, for example, that drag energy happened to be constant with velocity. Then there's no reason why you would want lift energy to equal drag energy, when you could just keep reducing it further.

Update: So I did the math, and coincidentally you actually get this exact same relation, but not for the reason stated in the text. Taking the derivatives of C1/v^2 and C2*v^2 to be equal and opposite you get:

2*C1/v^3 = 2*C2*v, which simplifies back to C1/v^2 = C2*v^2.

But this is only a coincidence because the velocity on the left side of the equation is raised to the opposite power as the velocity on the right side. I still believe that it is misleading to state that the energies will be equal without noting this fact.

I agree that this is a misleading way to state this. I wouldn't have been able to get there on my own with this explanation.

Yeah, I read this part of the text and did a total double take. He should really use the explanation provided by 12:20pm.

So once he explained it in lecture it made sense. Maybe that explanation should be made clear here....

Can someone reiterate that? I wasn't able to make lecture. Thanks.

This is a really good point. It makes it so much simpler and really understandable.

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what? This seems nonsensical to me...missing words/typo?

"Simplifying the sum" = simplifying the sum that gives the total energy, written on the previous page.

Since E E-drag + E-lift, and these two are equal at min energy speed, E 2E-drag E-drag.

(I found this difficult to follow at first too.)

Why not just say we're looking at just 1/2 the total energy? Also, it would help to make explicit that we are substituting into the earlier equation for Edrag with this new relation to get Edrag in terms of Mg instead of p,v,L. "simplifying" seems like a fuzzy, relative term and it's used three times in quick succession here.

I also am pretty confused by what this sentence is trying to convey.

I think if you said "Instead of simplyfying the sum from the total energy equation you can simplify just the drag term..."

That may read better. I think the way that it is now is very confusing.

It seems strange to include C through all of this, given that it's unitless. Mg * s is an energy, correct? So would it be a twiddle here?

I like that we always analyze our results like this to ensure that they make intuitive sense.

what?

these are good sanity checks to verify that our equation makes sense.

I agree. In a lot of the problems, along with making sure dimensions match up, its important to understand that as the various variables change in value, the ultimate answer will change the way we expect.

WHy are we using these variables? where do they come in?

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By this, we still mean approximations, right?

Yes. He is now assigning "real" values to the symbols from before. Of course, the "real" values are just approximations like everything else.

This is one of my favorite parts of this course, we can see how approximations really do work in the real world where nature doesn't care about being "exact" It gives us a real place to start when we look at complexity instead of wasting out time trying to figure out every detail about a problem

Range of what?

yeah, a definition of what "explicit range" is would be nice here.

I think here by range he means the distance that can be traveled by a 747 without stopping, but I agree that a definition here would be nice.

What exactly does the fuel fraction describe in this case? I originally thought it was a measure of efficiency, but we treat engine efficiency separately later in the paragraph.

I'm also wondering this. What is it a fraction of?

I feel like beta should be explicitly defined. 'energy density' seems to me to be a common enough term, and C was previously defined.

Fuel fraction is the fraction of the plane's mass due to the fuel. This quantity coupled with the energy density and efficiency relates the amount of energy we can carry to the mass of the plane.

The issue is that there are two masses that are important – the mass of the plane affects the lift energy and the mass of the fuel affects the total energy available. The fact that these masses end up being on opposite sides of the fraction (so we only have to know their ratio, not their values) is not necessarily intuitive, so this abstraction to a fuel fraction is not obvious.

I'm confused as to what you need this estimates for, and where you got them.

interesting- but is that really that accurate? Shouldn't we use a car/gallons of oil instead?

hahha that's a good idea!

yea I agree it makes everything flow and make sense

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I don't understand how butter can be related to jet fuel. The process where the fuel is converted to energy could be entirely different for foods than that of combustible liquids for all that I know.

I agree, I understand that fat is energy dense, but how do we know it is anywhere near as energy dense as jet fuel?

This seems like an abrupt transition, maybe explain the use/relevance of Beta and Epsilon....

I agree - I literally went 'woah!' as I read that first sentence. Where did all those variables suddenly come from?

I agree. Also it wasn't immediately clear to me that "an explicit range" meant we were going to calculate the maximum distance the plane could travel. Based on the previous section, I thought we were going to calculate a "range" of energy values used by the plane during flight.

Agreed! Where does beta and energy density play in? And typical engine efficiency? This went from making sense to making very little.

B*Eps = Energy/M or how much energy is our plane carrying per total mass B = fuel mass / M Eps = Energy from fuel / fuel mass All our energy is from fuel, so we can substitute B*Eps in for E/M in our formula for s.

The variables didn't bother me, as they were defined, but at the beginning I was also kindof confused what "explicit range" was exactly going to entail

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I'm not sure I understand how exactly we go from the energy density of butter to the energy density of a plane?

I think the text is showing that we can estimate the energy density of jet fuel by calculating the energy density of a high-fat (high energy) substance like butter.

The Wiki article on Energy Density has a table that shows that Jet Fuel has almost the same energy density as biodiesel (vegetable oil) so I think the assumption the text makes is valid.

I think this needs much more justification in the text.

Right, it's energy density of any compact fuel source, and long-chain hydrocarbons (fatty acids in butter, alkanes in jet fuel), are pretty similar in energy density from C-C bonds. Butter does have about 20% water content (since it's an emulsion), so shortening or olive oil would also work.

I think it needs a transition sentence like the ones used above... something along the lines of "we can estimate the energy density of fuel based on that of butter"

Ok this explains it. Haha I was so lost. Questions like this require some research and can't be simply estimated. However, I think the way you attack this question is very interesting.

Why do the 'big calories' in butter help us in this calculation?

They help give a frame of reference for estimating the quantity of energy density.

This section seems kind of abrupt. Some introduction into what we are trying to calculate or accomplish here would be helpful.

Awkwardly phrased

"needs to be converted," or change from passive voice

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how is it appropriate to look up this number, since this is an estimation class?

Probably. However, you can reasonably assume that something aerodynamic has a small drag coefficient, so an estimate would work too here (I would have guessed a drag coefficient of 0.01-0.1, which contains the actual value).

I understand that an aerodynamic object would have a small drag coefficient, but is there another way to arrive at this estimate if we don't have any intuition about what qualifies as a "small" coefficient? Or is it just something we should memorize?

I find it a little weird that we just looked up a number for the drag coefficient... but estimated the fuel's energy density via butter. I feel like if you're going to look up values like the drag coefficient, why not look up other numbers to get a more accurate answer anyway?

Sometimes you have to use easily available data.

It would be nice to mention that it's "measured using the wing area [in a different definition]:".

I agree that it's weird that we just have the coefficient of drag. Especially since we didn't use it for the problems in the previous section.

Alas, this formula is a third convention for drag coefficients, depending on whether the drag is referenced to the cross-sectional area A, wing area A_{wing}, or squared wingspan L^2.

It is easy to convert between the definitions. Just equate the standard definition

$$F_{drag} = \frac{1}{2} C' A_{wing} \rho v^2.$$

to our definition

$$F_{drag} = C L^2 \rho v^2$$

to get

$$C = \frac{1}{2} \frac{A_{wing}}{L^2} C' = \frac{1}{2} \frac{l}{L} C',$$

since $A_{wing} = Ll$ where l is the wing width. For a 747, $l \sim 10\,\text{m}$ and $L \sim 60\,\text{m}$, so $C \sim 1/600$.

Combine the values to find the range:

$$s \sim \frac{\beta \mathcal{E}}{C^{1/2} g} \sim \frac{0.4 \times 10^7\,\text{J}\,\text{kg}^{-1}}{(1/600)^{1/2} \times 10\,\text{m}\,\text{s}^{-2}} \sim 10^7\,\text{m} = 10^4\,\text{km}.$$

The maximum range of a 747-400 is 13,450 km, so the approximate analysis of the range is unreasonably accurate.

when would you know when to use which?

They're all interconvertible, just defined differently.

Yeah, in many of these problems there are various ways to go about getting the correct answer. Just like many physics problems, it all depends on how you define your directions.

I think there are too many random variables being introduced here. It makes it hard to follow/ keep track of what they all mean.

I agree, if I hadn't seen this material before I'd be very confused.

Why would you make wing width "l"? Why not a different variable like "w"?

but before we were using L^2... when do we use L^2 and when L*l?

Why did we know to multiply these?

I tried to address this on the previous page in a comment on Beta and Epsilon, but B*Eps = Energy/M in our equation rearranged to solve for s.

I am kinda lost here, how did you get this equation?

On just one stick of butter?! wow! (just kidding... but this is where having defined beta would add clarification to what this result means, I think.)

clarified butter or clarified beta?

I would be kind of interested to know what sort of distances most planes actually fly, as compared with their theoretical maximum range.

if im not mistaken, planes are required by law to carry atleast 20-25% extra fuel in case of emergencies and/or the need to divert to another airport. this is a good way to approximated the distances most planes would fly at based on their max range, as the range of planes various by their design points.

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Maybe I've forgotten after reading 5 pages, but I don't remember at the beginning anything about trying to find the max range of the plane. The whole section seemed more like an exercise in math than trying to make a specific point or reach a certain conclusion. Maybe in the opening paragraph include a sentence about what specific values you want to calculate- which necessitates the calculations of lift and drag.

That's unfortunate. It took some paying attention, but I thought the section was pretty instructive.

I also think this should be thrown in at the beginning as a "goal" so we know why we care about lift/drag in the first place. I think it would give the chapter more of a direction.

Well, the very first sentence is: "how far can birds and planes fly?", which gives the whole section a goal. It should perhaps be emphasized a bit better, since it's so easily overlooked.

I just thought there were too many equations throughout the way that got me a little lost.

I agree, it's easy to get lost in all of the equations. It would be helpful if we could have a short recap of the key equations and processes just so we can put everything together.

I'll admit that I lost interest in the particulars of these calculations. What does this all have to do with symmetry again? I think it'd be a perfect place to draw things back to the topic of the chapter.

I agree, reading this section seemed like just reading some interesting information with lots of equations. It would be nice to know why I should pay attention to the details in all the equations given.

I think I'm a little lost after a once through read on this one; Somehow, the beginning and the end didn't work for me; it hasn't come together.

Can we more explicitly relate this back to the energies to generate lift and fight drag? That would tie this into the previous sections better.

How can it be unreasonably accurate? I guess all the estimation errors just happened to cancel out

why is it 'unreasonably' accurate?

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I don't really like the word 'unreasonably' at the end. To me, it carries the idea that we shouldn't be this close, but maybe we're just that good at making approximations (or more likely, our errors canceled).

Yeah I was sort of confused about what made it unreasonable...

Yes, aren't you expecting to be accurate? If you were not accurate, you probably would not have put it in the book.

was it a typo? Maybe use the word reasonably?

I'm still a little iffy on this as well, but maybe he just means that we are much closer than would be reasonably expected. I am often impressed with how the errors in this book cancel out to our advantage. Is this from careful choice of approximations or just plain luck?

why is it 'unreasonably' accurate?

I'm not quite sure what lesson this section was supposed to teach. I was able to follow and see how to calculate the energies needed for flight and gain a better understanding of the phenomena, however, I am not quite sure what the take home message is here?