

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

### GLOBAL COMMENTS

I think it'd be nice if you showed some dimensional analysis here

Despite being so short, this was a fairly difficult (to stomach without objecting to everything) section. I guess, we can just be glad that this class is set up such that all of our disbelief in methods and confusion can be brought to class the next day, where it can be answered in person.

why is it the square root of C?

I completely agree I don't really understand how this makes sense. Also in class we included a fact for efficiency...doesn't that matter anymore? Also are you say 's' is the approximately 1 across the plane industry or for each plane

This section was much easier to read than the last. It was very concise and that really helped me to follow it.

did this animal have a simpler estimate of fat as our example? Also wouldn't the weight/fat distribution matter?

I found this section a lot easier to read and understand than previous sections.

# 4

## Proportional reasoning

Read the introduction and section 4.1 (more on flight) for Sunday's reading memo.

I think you need to use the phrase proportional reasoning more. It still feels like we're in older chapters.

what philosophy do you refer to in this statement? It's ambiguous.

I'm not really sure what this phrase means

I also think that there's so kind of grammatical mistake here...

I like the build-up, but I agree with the above two, the end of the sentence doesn't make any sense. What are you referring to? What philosophy...symmetry?

I took it to mean "one implementation of symmetry", but I'm not sure if we've seen a different implementation of symmetry that doesn't wring out complexity.

Oh thanks for the comment. I was confused until I took philosophy to mean symmetry.

I feel it becomes clearer when distance is proportional to time is written out shortly after.

Yeah, it's not that clear but I think he means we just use symmetry once, whereas for other things, like divide and conquer, or abstraction, we might have to use them multiple times in order to actually make things clearer

in = is?

No, it should be "in". Take out the qualifiers: Symmetry wrings out complexity and reasoning IN one implementation...

no it should be is. this is a section about proportional reasoning, so therefore one would not want to "wring" it out.

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

i really hate this software. if you click anywhere it goes crazy and all these boxes pop up, it doesn't scroll like a normal pdf, and makes you do a hell of a lot of extra work.

More details would be very helpful. There were similar comments a few readings back, and Sacha worked on it. Clearly more needs to be done, and specific descriptions of what goes wrong would help that along.

For example, what scrolling doesn't work? (I use the scrollbar on the right and that seems to work.)

I think it's much improved and works fine - the main gripe i had earlier was that when you were typing and had a typo, you couldn't really click in your text box to fix it. But now that's been resolved.

"without accelerating" might be more intuitive, here.

I had to re-read this sentence to understand it, i think its talking too generally.

This paragraph seems very unclearly worded.

Could you be more specific like the example you listed right before?

This is clever, since this is basically like previously choosing an invariant. Here, we want to find two factors that are relative or proportional for reasons of comparison, simplifying two into one.

maybe "distance to time ratio"

"contributes to" might be better here.

Aren't these called extensive something?

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

What if this were re-emphasized: "However, speed, the ratio of distance to time, is invariant". Or something similar?

I like this a lot better, mentioning the invariant. Given the previous sections it makes more sense

Definitely - with no forces on it, although meaning the same thing, is somewhat harder to understand.

I like the way this would tie in the previous section since we just went through learning invariants

The beginning of this section talks about symmetry too much. This is a whole new topic of proportional reasoning, but it feels like we're still on symmetry. Judging by everyone's comments thus far, I think it confused everyone else too because all the comments are asking what this section has to do with symmetry.

Perhaps an overall description of the topic would be nice, before jumping into specifics.

Yeah, after finishing this intro I have no idea where the next section is going.

what this section has to do with symmetry: symmetry reduces complexity just like proportional reasoning

I think a lot of this confusion would be resolved by explaining the first sentence more.

I think you're trying to present something which is simple in a complex manner. Speed is useful because it relates to two other useful things, distance and time.

independent of what time? this seems contradictory.

he means its invariant in this problem—with constant speed, the velocity (the derivative) does not have a "t" term; it's constant, so it doesn't depend on time

That's a better explanation of what is meant by this- the reading as it is now is confusing because it sounds like it's saying that distance/time is independent of time which doesn't make sense

I definitely found this confusing. How can he say that velocity=distance/time, and in the same sentence say velocity is independent of time.

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

This seems contradictory with the beginning of the sentence. I understand what you're saying, but I still had to stop and think about it for awhile.

It confused me too..I think if the ratio distance/time wasn't there then I would have got it.

Yeah I agree - the ratio shows a clear dependence on time, and then you say there is no dependence on time (which is true) but it does throw something off.

Doesn't your proportionality statement also conflict with this?

I don't think it necessarily contradicts. We're told that there's a constant relationship between distance and time therefore speed is constant. That doesn't really contradict the proportionality statement because distance will be proportional to time according to the speed constant.

@12:09 : it's not that it actually contradicts, just that it takes rereading it a few times to realized that it's not.

It just didn't make sense at all to me...

I don't quite see why this makes it simpler than either distance or time.

On some level, this is rather silly. Time is very simple, at least at the level the average reader would take it. Perhaps a rephrasing emphasizing that we make the abstraction of velocity to simplify the relation between distance and time?

I don't really get this because it seems like distance would be proportional to 1/time or vice versa?

distance = velocity \* time, so at constant velocity the distance is proportional to the time.

it doesn't matter what fraction it is, but only that if one changes, the other makes a corresponding change. that's proportionality.

the symbol looks more lik a symmetry sign than a proportional sign

I thought it was standard notation for proportional?

do we naturally use this symmetry when we say that something is x hours away, as opposed to a distance?

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

This is an interesting way to look at velocity that does a good job of showing how we use symmetry in our everyday lives

Shouldn't this say "change for how long the object moves"?

this doesn't exactly jump out as me as symmetry.

yeah I'm a little confused about what the symmetry is

should be "moves"

Should this be: how long the object moveS?

what do you mean? How does this not change the conclusions?

I think the idea is that you are making a quick approximation and regardless of this, you will be left with a conclusion that can accurately answer your question. Not every answer we come upon will be completely accurate, but at least it will be accurate enough to make a solid conclusion.

This entire paragraph seemed to confuse what I thought was a very simple concept... the ratio of distance/time as an invariant. I'm still not sure how this is a symmetry operation.

I agree - I feel like it is trying to do with words what is intuitive for many of us. Maybe show how exponents are related on both sides of the equality? Above, distance is proportional to time. Maybe it would be easier to explain that distance is the only thing that changes because they are directly related to one another with equal powers?

There has to be a better way to word this entire section. I had to reread it twice to get the overall meaning. The idea is very simple: "Applying dimensional analysis to invariants produces useful result. Eg, knowing that speed is invariant means that you know the distance someone has traveled in a given time."

I agree that this paragraph is really abstract and complex. Thanks for the summary.

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

I feel like this sentence was written very quickly - everything before the comma and everything after the comma basically say the same thing using similar terminology.

I think this whole section might have been written speedily, there are more grammatical mistakes here than in many other sections we've read

haha is that supposed to be a joke?

I just have to say. After reading the first 14 memos, I am very impressed with the amount of varied knowledge you have. The only problem is that readers probably won't have the same amount, which makes reading this difficult. Also, I'm very glad that the entire class reads and comments. It helps me understand the examples which in turn help me understand the concepts you are trying to teach.

**What is the mean-square distance?**

I think this is just the rms of the distance

It's the average value of the square of the distance traveled over many such random walks.

It's equivalent to the variance of distance traveled for a random walk where the expected distance traveled is 0 (keeping in mind that the distance could be negative or positive, depending on direction, in a 2D example).

**why walks and diffusion? These seems like pretty random examples to use. Is there anything more general that we could use? Or one good example to follow through?**

They're both examples of gaussian (or normal) distributions. The thing about this type of distribution is that pretty much anything is gaussian.

Yeah normal distributions are extremely useful. Although not always accurate, they are great for approximations.

**I believe this is a misspelling.**

"travelled" is more common in British English, and "traveled" is more common in American English. <http://www.future-perfect.co.uk/grammartips/grammar-tip-travelled-traveled.asp>

# 4

## Proportional reasoning

4.1 Flight range versus size	68
4.2 Mountain heights	70
4.3 Animal jump heights	72
4.4 Drag	76

Symmetry wrings out excess, irrelevant complexity, and proportional reasoning in one implementation of that philosophy. If an object moves with no forces on it (or if you walk steadily), then moving for twice as long means doubling the distance traveled. Having two changing quantities contributes complexity. However, the ratio distance/time, also known as the speed, is independent of the time. It is therefore simpler than distance or time. This conclusion is perhaps the simplest example of proportional reasoning, where the proportional statement is

$$\text{distance} \propto \text{time}.$$

Using symmetry has mitigated complexity. Here the symmetry operation is 'change for how long the object move (or how long you walk)'. This operation should not change conclusions of an analysis. So, do the analysis using quantities that themselves are unchanged by this symmetry operation. One such quantity is the speed, which is why speed is such a useful quantity.

Similarly, in random walks and diffusion problems, the mean-square distance traveled is proportional to the time travelled:

$$\langle x^2 \rangle \propto t.$$

So the interesting quantity is one that does not change when t changes:

Each individual sentence make sense, but I think I'm still missing the big picture.

So far, he's been talking about how in these distance/time/velocity problems, there are invariants—for the "no external forces" case, it's velocity that's invariant, and for the "random walks" and "diffusion" cases, the mean-square distance travelled is always proportional to t (meaning that the invariant is  $x^2/t$ )

Ah...thank you for that.

wouldn't  $\text{abs}(x)$  work too?

Perhaps? Is proportion in this case only linear, or are you saying only that one quantity depends on the other.

how is this related to symmetry...? seems more like using an invariant...

i agree. i dont really get all these categories and divisions and things.

what do the brackets mean? the bar means average, right?

I'm confused as well. But I think the important idea is that it doesn't always have to be linear.

This is probably a dumb question, but is it squared so going backwards is still considered as distance traveled or is there another reason?

Exactly. We square it to get something like total distance traveled and not displacement. Ordinarily I would expect to see root mean-squared and not just mean-squared though.

"Interesting quantity" means "invariant" here, yes? It would be better just to say so.

How did we come to define this as an invariant from the proportion?



interesting quantity  $\equiv \frac{\langle x^2 \rangle}{t}$ .

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

Can you define what exactly is this "interesting quantity"? I understand what you're trying to say, but it's a little strange to read.

there is no definition for this "interesting quantity" other than the fact that it's the invariant. i guess he calls it "interesting" because there's no real name for it—it's just the mean square distance travelled divided by time (since they're proportional, this is always constant)

I am not understanding this quantity of not changing with time. Can you elaborate more in lecture?

I think I'm pretty confused about this entire section, but what is this in particular meant to mean...?

I don't really understand this section either. How does the "interesting quantity" not change?

because the diffusion constant is an intrinsic property of a fluid, it doesn't vary with time or distance

I think explaining the concept with an example problem might make this material a little less confusing.

This 'interesting' quantity is not made clear; I feel like it's just being added and this part of the book is just vague and confusing.

I never thought about the diffusion constant in this context as an "interesting quantity," but now it makes sense why its important.

by that definition all constants are important, not just this one.

So it varies with each type of material?

Yes, I think that diffusion definitely varies with the type of material.

interesting quantity  $\equiv \frac{\langle x^2 \rangle}{t}$ .

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

**It might be nice to add an additional example or something. I feel like the section is easy to forget since the next section is so interesting.**

I think it's ok the way it is...I read this section as an introduction to the chapter, so it doesn't matter if stuff gets a little forgotten here since the other sections will drive the point in.

This example seemed like it was gonna be explored further, but it just ended here. Perhaps it could have been incorporated into the actual sections.

What is the diffusion constant supposed to mean/represent?

I would like a little more information on this constant and maybe a couple common values to see how they compare. I think it would be helpful and interesting even if it doesn't completely add to proportionality, and continue in the theme of everyday knowledge that this book follows

**Not sure where this is going. Need a simpler intro and focus on examples to prove your point. All your other examples are very good.**

**Maybe some introduction to this would be nice. Like "now we will look at how to apply this to the range of a plane on a full tank of gas."**

**What do you mean by range? How far a plane can go when its tank is full? On a gallon of gas? Requiring the least energy? Its somewhat ambiguous.**

I believe that when he says range, he is referring to the maximum distance a plane can travel on a full tank of fuel.

I agree, but I think it would be useful to put that in there. My guess is it is "one plane, full capacity, full tank of gas, how far can it go"

But even there there's an important question of load - an unloaded large plane may travel farther than an unloaded small plane given the larger plane's larger fuel tank. However, that larger plane carries more weight when loaded, which would presumably effect the range.

**i understand that this makes the approximation a lot easier, but it doesn't seem like it's true that a fighter plane has the same shape as a passenger plane like the 747, and I feel like this will have a huge impact on the resulting conclusions**

$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2} Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2} M g}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2} M g} = \frac{\beta\varepsilon}{C^{1/2} g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

**Didn't know this equation.**

Actually, I think you might.

If you notice the " ", it suggests that rather than being an explicit equation, it's just a statement of proportionality.

Here, it is just saying that the Energy of a plane to fly a distance is merely related (somehow or another) to a constant  $C$ , its Mass, the gravitational constant, and the distance traveled.

This seems very reasonable and I would bet that if you were asked what factors does the energy required to fly a certain distance depend on, you would come up with these factors. After all, a heavier object will require more energy. Traveling longer will also require more energy. And likewise, stronger gravity will require more energy to overcome it.

In this way, you don't have to think of this expression as an "equation" of the intimidating sort. Just think of it as an approximate relation that summarizes the various factors which influence energy.

**It might help to explain where this comes from...just have a sentence or so.**

I think this was covered in the previous section

**It would be nice if some of the variables were explicitly stated - you mention  $C$  is the drag coefficient later, but it probably should be mentioned here.**

I agree. Even if they've been mentioned in previous readings, it would be nice if we could be reminded again.

**It might be nice to have  $C$  defined. (just realized it was defined lower down, maybe just define it when you introduce it?)**

I think it was defined in an earlier section, if you assume continuity in the book across its sections.

Even with that assumption  $C$  can stand for more than one thing in many science/engineering examples. While the drag coefficient makes the most sense, I'd like to see it defined again.

$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

**what do these variables mean?**

I am also confused. But I think 'M' is mass and 'g' is the gravitational constant?  
yeah, this equation als gave me pause, maybe mention the coefficients, like C, closer to the beginning perhaps?

**maybe go over what each coefficients mean? It is not completely obvious from the equation**

**What is s in this equation?**

Look at the two words before the section you highlighted.  
s is distance in this equation

**does s refer to distance here? or seconds? It seems like a random letter to choose.**

distance. distance is often denoted 's'.

Perhaps, but I am also used to distance as "d" and "s" as seconds...

I think you're confusing the variable with its units. 'd' and 's' are variables that have units of distance (meters, for example). Time is often 't' or 'T', and that has units of seconds, or 's'. 's' as a variable almost never represents time.

It does state that "s" is distance right above, but I agree that it tends to get confusing especially when the "s" is at the end.

He's used 's' to denote distance in previous sections.

**is this always constant? I feel like planes would have different size tanks**

**How do plane manufacturers determine beta? I'm surprised that they have not made planes that could fly between any two points on Earth.**

Beta is dependent on factors like airspeed, lift-to-drag ratio, and specific fuel consumption.

$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

I remember being confused in the last section about what beta was since it wasn't clearly defined. Maybe this definition should be moved up there.

I agree. This paragraph cleared up a lot of confusion with the last section.

This was a nice clarification. I wasn't sure if I had it before but now it's pretty straightforward.

Great succinct explanation.

this is a really nice description of the equations and summary of the units

Definitely the best explaining paragraph in the section thus far.

I agree. In the last chapter I was very confused about beta and epsilon. This paragraph explains them much better. I also like that it's concise.

Thanks for explaining this so quickly!

I think  $C$  is more appropriate here.

what does the drag coefficient depend on again?

It depends on the both the geometry and the material.

[http://en.wikipedia.org/wiki/Drag\\_coefficient](http://en.wikipedia.org/wiki/Drag_coefficient)

can you define this above?

wow this is pretty interesting...so theoretically if a bird had a lot of energy it could fly the same distance as a plane

isn't that true of anything in the air then? this seems like a silly statement.

$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

Well surely a Cessna can't fly the same distance as a Boeing 747... I feel like the assumptions made in this problem were too big, and thus you end up with a result like all planes fly the same distance. Until I see a Cessna and a 747 flying the same distance myself, I won't believe it.

This is from the wikipedia page on drag coefficient:

0.021 F-4 Phantom II (subsonic) 0.022 Learjet 24 0.024 Boeing 787 [16] 0.027 Cessna 172/182 0.027 Cessna 310 0.031 Boeing 747 0.044 F-4 Phantom II (supersonic) 0.048 F-104 Starfighter 0.095 X-15 (Not confirmed)

so you're right - the assumption that the drag coefficient is the same is not 100% correct, but they don't actually differ that much, so all planes can roughly fly the same distance.

I really want to see an answer to this. I know a single-engine two seater is not going to be making it across the ocean anytime soon, so how can they possibly go the same distance?

Different fuel tank sizes?

The point about Cessna's is a good one and shows me that my statement was too hasty. Rather, 747's and other long-distance planes, which fill a similar "ecological" niche, have about the same shape and drag coefficient and range.

But there are several other niches: (1) medium-distance commuter flights (e.g. Boston to Washington, DC) or (2) short-distance commuter (e.g. Boston to Bar Harbor, Maine or Nantucket). For those kinds of journeys, a 747 is totally unsuitable. The leading planes in each of those niches has a very different design than a 747 (and a very different range).

I agree...I really want to know what order of magnitude this is working on? just about anything can appear to be equal if you're looking at it with a large enough scale.

$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

Wow I didn't know you could just cancel things out like that.

isn't everything proportional to 1? I don't get the point of this line.

It's just a way of saying that  $s$  is constant.  $x \propto y$  means that  $x = (\text{some constant}) * y$ , so  $x \propto 1$  means that  $x = \text{constant}$ .

thanks! the explanation helped.

I don't agree with the fact that the fuel fraction is equivalent. Bigger planes carry more fuel so will be able to fly further. They may weigh more but I'd have to see data to believe it.

I'm assuming this is for planes of similar engine types? i.e. prop engines vs. turbines?

I believe that's a correct assumption as we're only looking to compare planes based on size, not the efficiency of their engines. By assuming they are geometrically similar, we also assume they are similar in engine.

Only all planes that are the exact same size and mass...which is pretty obvious already. what does this show??

....if they have the same drag coefficient, fuel capacity and fuel efficiency

....if they have the same drag coefficient, fuel capacity and fuel efficiency

....if they have proportional fuel capacities ( $\beta$  was a fuel fraction).

Upon initial glance, it looked surprising, but reading all the assumptions made, the result is pretty obvious. Keeping  $\beta$  and  $C$  constant, means that even though there are planes of different masses, heavier ones will naturally carry more fuel.

$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

this is cool - but does engine efficiency play no part?

You can account for the efficiency in the energy density of the fuel?

I was really surprised that this is how this worked out!

But I feel like this doesn't take into account that some planes hold a LOT more people...

Yeah, I don't know... My intuition is fighting against the math here.

But I feel like the weight would still be proportional to the plane's volume. A bigger plane would hold more people than a smaller plane, but all big planes probably have similar mass.

Indeed, this does seem quite counter-intuitive. But after reviewing the Breguet range equation, this conclusion seems sound.

My intuition against this is really different than the other comments in this thread. I have a problem with all planes traveling the same distance because of the implications that would have on the airline industry. If it really takes the same amount of energy to move any sized plane, then flights would not be scheduled as they are.

I think that each airport will focus its flight schedule to maximize profit. For example you won't find any airports in Nebraska that will fly you all over the USA.

I followed, but am still confused. This, by experience, just doesn't seem very logical.

I agree, why do they use small shuttle planes to take people from new york to boston when they could just as easily use a big one?

it may be that smaller commuter flights run at more times carrying fewer people making them more efficient for the airline to run small planes rather than large empty ones. Also the larger planes may have a larger fuel fraction.

How true is that in reality?

I don't know if this is a curiosity equation or not, but I don't believe a true answer is the purpose of the statement.



$$\text{interesting quantity} \equiv \frac{\langle x^2 \rangle}{t}.$$

This quantity is so important that it is given a name – the diffusion constant – and is tabulated in handbooks of material properties.

#### 4.1 Flight range versus size

How does the range depend on the size of the plane? Assume that all planes are geometrically similar (have the same shape) and therefore differ only in size.

Since the energy required to fly a distance  $s$  is  $E \sim C^{1/2}Mgs$ , a tank of fuel gives a range of

$$s \sim \frac{E_{\text{tank}}}{C^{1/2}Mg}.$$

Let  $\beta$  be the fuel fraction: the fraction of the plane's mass taken up by fuel. Then  $M\beta$  is the fuel mass, and  $M\beta\varepsilon$  is the energy contained in the fuel, where  $\varepsilon$  is the energy density (energy per mass) of the fuel. With that notation,  $E_{\text{tank}} \sim M\beta\varepsilon$  and

$$s \sim \frac{M\beta\varepsilon}{C^{1/2}Mg} = \frac{\beta\varepsilon}{C^{1/2}g}.$$

Since all planes, at least in this analysis, have the same shape, their modified drag coefficient  $C$  is also the same. And all planes face the same gravitational field strength  $g$ . So the denominator is the same for all planes. The numerator contains  $\beta$  and  $\varepsilon$ . Both parameters are the same for all planes. So the numerator is the same for all planes. Therefore

$$s \propto 1.$$

All planes can fly the same distance!

Even more surprising is to apply this reasoning to migrating birds. Here is the ratio of ranges:

$$\frac{s_{\text{plane}}}{s_{\text{bird}}} \sim \frac{\beta_{\text{plane}}}{\beta_{\text{bird}}} \frac{\varepsilon_{\text{plane}}}{\varepsilon_{\text{bird}}} \left( \frac{C_{\text{plane}}}{C_{\text{bird}}} \right)^{-1/2}.$$

Take the factors in turn. First, the fuel fraction  $\beta_{\text{plane}}$  is perhaps 0.3 or 0.4. The fuel fraction  $\beta_{\text{bird}}$  is probably similar: A well-fed bird having

Following your notes, this makes sense, but I guess I still have trouble how you can just simply everything like this.

This is a very easily grasped concept for symmetry.

Fuel really makes up 40% of a plane's weight? That seems high when you factor in engines, people, and luggage....

I also agree that the 30-40% percent of airplane mass is due to fuel seems rather high. I would have guessed 25% myself. But then again I would have guessed 20% for the fat percentage on a bird, so I guess it works out. However, if I had guessed 40% for plane but 20% for bird, then my calculation would approximate that a plane has the same maximum travel range as a bird. How do we avoid these errors, or are they inevitable?

25% and 30% are well within the error of our estimation.

Also, fuel is really heavy. Think about lugging one of those little tanks of gasoline. Now think about how long it takes to fuel a plane and they're probably using a pump that can provide a lot more fuel per second than a standard gas pump. if you really think about it, 35% is total reasonable.

no it's actually totally reasonable. the fuel is a very large portion, which is in part why it doesn't really matter how many people are on board and why they didn't charge for luggage before.

I agree. Further detail on how you arrived at this estimate would be useful.

If you calculate it out: A 747 has a max fuel capacity of about 45000 gallons and @ 6lbs/gal for Jet A the fuel weighs 270,000lbs. A fully loaded aircraft is about 950,000 so the fuel is about 30% of the aircraft

I don't see how fuel fraction of the bird and the plane would be similar??? What does a fuel fraction mean precisely?

I'm a little confused about this too. Is this the overall efficiency of converting the stored energy from the fuel to kinetic energy?

Confused again on how these ratios are known. Should we just look these up or is this common knowledge?

Well, one could certainly look them up, but I suspect that divide-and-conquer would allow you to get a pretty good estimate of beta for a plane.

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\mathcal{E}_{\text{plane}}/\mathcal{E}_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

so the thing is, you can't use up all of a bird's fat in one go like you can with fuel without bird health serious issues. i'm guessing this is where we just blur our vision a bit but it's still a fundamental difference. if we wanted to be more precise, how would this affect our calculations?

Not the pigeons in Boston during winter...those things are OBESE.

But I doubt the pigeons in Boston are migratory birds...

how does this compare to other animals?

My question has to do with how they maintain their flight ability given they've put on weight? If people get fat, they can't run as far as they might like to.

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\epsilon_{\text{plane}}/\epsilon_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

**How did you come up with those numbers? I would have guessed slightly less.**

I agree I would also have guessed significantly less, but I don't have any other animals to compare these values to which would probably help.

I know for humans, normal is around 15-25%. I have no basis for applying the same to birds though.

I agree that this sounds extremely high...how would we have gotten to this otherwise?

i dont think it matters how he got there, its just trying to prove a point.

Well birds' bones are pretty small and they most use their wings for moving so I wouldn't think they have much muscle. Compared to humans 30-40% sounds about right.

So, I was kind of skeptical about this percentage (mostly just off the guess of "Well, the chicken I eat is certainly not that fatty!"), but googling around I found this book excerpt: <http://tinyurl.com/ya7td8v> It seems to depend on the bird, but 30 to 40% is in range.

I don't know how useful this is to you guys, but I believe it. I've had a pet bird and all they do is eat.

I also would have guessed less for both the plane and the bird, but then my ratio would still be 1 so I think it's more important to consider that they are both about the same than the exact numbers.

Well when compared to chickens, which spend a significant amount of time eating and even less time flying (I wouldn't say that they are 30% fat). Plus fat is not very dense so that means that makes it even less likely to be 30% of their weight even if there is a lot of it.

"...for humans, normal is around 15-25%"? no way, 25% seems to be a bit too much; a quarter of the body is fat? 10-15% seems more reasonable.

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\epsilon_{\text{plane}}/\epsilon_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

Also, 15-25% is totally reasonable for humans. For athletic competitive men, it's 5-10. Less than that is unhealthy, but body builders, wrestlers, rowers, etc do it for competition day. For women, it's higher due to various different bodily needs; in fact, if body fat is too low (less than 15%?), women will not menstruate.

With today's obesity epidemic, about 30% of Americans are over 35% body fat (or maybe that's 40?). Another 30% exceed 25%. Kinda eye opening, no?

That sounds surprisingly high to me also, especially since they have to be flighted, I'd think it's lower than that. Birds have SUPER low density bones and very muscular wings / diaphragm so it's hard to think fat % is so high. Perhaps our force-fed chickens would be 40%.... (sorry, just watched Food Inc.)

**I like the thing you said in class that the density of gasoline is the same of fat/calories. Very useful fact**

**This is always very interesting to me. I guess this is part of the reason fat is so hard to get rid of, it's got as much energy as jet fuel lol**

haha wow, did not know this

How do we know this?

I think we can reason it because fat and jet fuel are both consumed by oxidizing long hydrocarbon chains, so it makes sense that they would release similar amounts of energy.

**I find it difficult to assume that animal metabolism is about as efficient as a plane engine. My gut really doesn't have a good feel for this.**

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\epsilon_{\text{plane}}/\epsilon_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is **not** a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

**how would we know this, if you hadn't just told us?**

Well we know that people are about 25% efficient as well that unless we are using an incredibly fine tuned machine 25% efficiency is pretty standard and a valid approximation

It is interesting analysis and reasoning, but as mentioned earlier, I would have trouble coming up with these numbers and approximations on my own without use of an outside resource, which would have made my resulting conclusion different.

I don't really see how to factor this 25% in... the epsilon is a measure of "energy density" —> does energy density take into account engine efficiency, i just assumed it was the amount of energy in gasoline...or fat?

I think the 25% efficiency is just another check to confirm that comparing birds to planes is sane. Also, it could be factored into the energy density: if the bird had twice the efficiency, but half the fat percentage, then the effective  $\epsilon_{\text{plane}}$  over  $\epsilon_{\text{bird}}$  would still be one.

Also, all of these ratios seem valid to within a factor of two, so it seems pretty reasonable to hope that the extra little factors will roughly cancel (as they did for the engine efficiency and drag coefficient terms in lecture on Friday).

**True, but still I suppose it works for this**

**if you're admitting that it's not a great approximation, why are we allowed to make it?**

because this class is about approximation—since we're really stuck here, we have to make some sort of assumption to move on in our estimation

I agree. I believe it was made simply to allow us to move forward.

Completing the calculation gives us the advantage in that we then have an answer to work with (or refine, if necessary).

**Like the comparison in calculations between birds and planes**

**I think this part really amuses me! very interesting!!!**

I agree! The explanation as well as the end result were really interesting!!!

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\epsilon_{\text{plane}}/\epsilon_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

**what makes non migrating birds different?**

If they do not migrate, then they most likely wouldn't have as long of a maximum range since they are not accustomed to it. Also, it would be hard to try and predict for the number since the birds never test their maximum range.

**this seems like a stretch to me**

**What about other planes and other birds? Does it really scale across models/species?**

**this is a very surprising, yet cool, result.**

I really want to believe all of this should always add up, but I can't help but think part of it is coincidence.

maybe other examples can be shown to show more proportionality reasoning

...Well in reality, it doesn't add up. Lots of assumptions were made that were very liberal. Most birds probably can't fly the same range as a Godwit. Most jets probably don't have the same range as a 747. That being said, I think this comparison is a really neat trick.

I agree that it's surprising. Aren't planes modeled after birds anyway?

I agree. This is awesome.

This result is quite surprising to me. I can see where it is coming from, but still I am quite surprised.

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\epsilon_{\text{plane}}/\epsilon_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

**Does this mean that the godwit was 30 to 40% lighter when it arrived?? It seems implausible that an animal would lose a third of its body weight in one trip... but maybe..**

That actually would not surprise me as much. Animals implicitly do things that have been done for years and years, and losing a lot of body weight is one of them. Think of penguins raising their young, or polar bears.

It also took the bird way longer than the plane. He didn't need to drink any water that whole time?

this is a good point, how do we account for the difference in velocities? also, it doesn't seem like all birds could make this long of a trip...

The difference in velocities comes from a difference in size. A bird is most energy-efficient at a lower speed, but the net distance ends up being the same (because we assumed beta, C, and epsilon were the same for each object).

I believe birds do generally stop to eat, drink, and sleep during migration. So the bird has probably not lost all of the weight it invested into the flight, since it would have gained some back during the trip.

Wouldn't this not be a nonstop flight then?

**does anyone know the time that the godwit took to make this flight?**

For those intrigued: <http://news.nationalgeographic.com/news/2007/09/070913-longest-flight.html>

**The maximum range of a Cessna is something like 1400km. That's pretty far off...**

That may be because a 747 has jet engines whereas a cessna is just a prop plane.

fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\epsilon_{\text{plane}}/\epsilon_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

**A well fed airplane:**

<http://www.greenpacks.org/wp-content/uploads/2008/10/record-breaker-a-bar-tailed-godwit.jpg>

that is definitely not what I expected for a bird that flies so far. I was expecting something much more lanky. The godwit must make a lot of stops along the way. This is probably still a good comparison because the godwit probably doesn't eat anything while it's floating around the open ocean.

He specifically says it's the longest nonstop flight by a bird.

True. It is still a hard fact to swallow but is pretty good at supporting the approximations and calculations done so far in the reading.

It sleeps while flying!

**It might be useful to include a brief tie back to how this interesting result used symmetry...i.e what irrelevant complexities were thrown out**

I was wondering about that also. Was symmetry used when you took the ratio of different measurements?

**Is this relevant? Shouldn't it be just as likely that this wasn't true, especially since the amount of weight of fuel by each could be different?**

**How do these birds eat? Do they make any stops on the way?**

Well, I guess they burn off fat but I guess the better questions is whether they need to drink water.

**but is this only true for the extreme cases? are there are birds that can fly a comparable distance?**

**Great fun fact!!!**

Now this point is a strong one. It really strikes home the point of proportionality. The plane example was a bit reliant on assumptions, but relating it to birds and using fuel proportionality method, made the point very clear.

**Never would have guessed this.**



fed all summer is perhaps 30 or 40% fat. So  $\beta_{\text{plane}}/\beta_{\text{bird}} \sim 1$ . Second, jet fuel energy density is similar to fat's energy density, and plane engines and animal metabolism are comparably efficient (about 25%). So  $\mathcal{E}_{\text{plane}}/\mathcal{E}_{\text{bird}} \sim 1$ . Finally, a bird has a similar shape to a plane – it is not a great approximation, but it has the virtue of simplicity. So  $C_{\text{bird}}/C_{\text{plane}} \sim 1$ .

Therefore, planes and well-fed, migrating birds should have the same maximum range! Let's check. The longest known nonstop flight by an animal is 11,570 km, made by a bar-tailed godwit from Alaska to New Zealand (tracked by satellite). The maximum range for a 747-400 is 13,450 km, only slightly longer than the godwit's range.

**This is really cool!**

That is cool.... I think this is a cool intro to proportionality and a nice example to follow it.

**stupid boxes. stop popping up all the time!**

I understand that it is frustrating. Please describe what actions precede the boxes popping up, so that the user-interface bug(s) can be found and fixed.

Hi, I would love to be able to help with this box problem, but in order for me to fix the bug, I need to understand where it comes from. If it happens next time, could you please let me what you were typing or clicking when the boxes started to pop up. Also, a screenshot would be great, ideally. Best, Sacha &lt;sacha@mit.edu&gt;

Is there anyway to access boxes that have been overlaid by multiple boxes?

yes, if you clicking multiple times on the box you're trying to select, it will eventually come to the foreground and become selected.