

4.4.2 Swimming

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Estimate each factor in turn. The first factor accounts for the relative athletic prowess of swimmers and cyclists. Let's assume that they generate equal amounts of power; then the first factor is unity. The second factor accounts for the differing density of the mediums in which each athlete moves. Roughly, water is 1000 times denser than air. So the second factor contributes a factor of 0.1 to the speed ratio. If the only factors were the first two, then the swimming world record would be about 1 m s^{-1} .

Let's compare with reality. The actual world record for a 1500-m freestyle (in a 50-m pool) is 14m34.56s set in July 2001 by Grant Hackett. That speed is 1.713 m s^{-1} , significantly higher than the prediction of 1 m s^{-1} .

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What is A in this section? Not sure what this Area is in reference too, maybe i'll refer back to the previous reading.

It's the cross-sectional area. I should add a subscript to indicate that.

How did you make that conversion?

I am really enjoying this section. It's really cool to just use a ratio with something that is known to find something that is not.

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Read the 'swimming' and 'flying' subsections for the Sunday memo. (Don't mind this spurious box. It's just there so that I have a place to anchor the comment. Otherwise I have to make a global comment, which would pull up a whole separate panel when you visit the link.)

On Friday, you briefly went over notation used in this class (ie. tilde and alpha). In future drafts, you should mention that in the earlier sections.

After reading this, I feel like it belongs earlier in the chapter. It's a clear and easy example of proportional reasoning.

you mean in relation to drag?

yes, he is saying that you can compare how the drag on a cyclist limits its maximum velocity to how the drag on a swimmer will limit their maximum velocity. All you really have to change are the cross sectional areas and densities of the fluid.

I think it would be better to reference the actual section. For example, if you say "In section ___ we predicted the world record..." it would introduce the section better and let us know where to look if we need a refresher.

I agree...I just clicked around for 10 minutes trying to find that reading for a refresher
yeah, maybe perhaps briefly say what the variables are.

didn't we already do an example for the record for running? this doesn't seem too different.

shouldn't this be the same as cycling except there's a different drag?

what is P again?

power generated by the athlete

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Wouldn't A change more in motion in swimming than cycling? Would this affect the accuracy of our analysis?

besides the obvious response that "it's close enough" i think you don't have to worry about it so much because the moving parts of your body in water are pushing through the water and generating less drag than your cross section.

Also, you are protected by the cube root – even an error of a factor of 2 turns into only 25% after taking the cube root.

Here being water

Yeah there should probably be a subscript: p_{water} .

I actually think it is intentionally ambiguous. That is, you use the density corresponding to the athlete's medium (p_{water} for swimming, p_{air} for cyclists).

p_{fluid} , then?

I think that indicating it's water makes the equation easier to conceptualize. From there, it is evident that a change in fluid requires a change in density.

what does "new" ρ and A mean?

you would need to change the density to the density of water, and you would need to change the area – when a person is on a cycle, most of their body is against the wind, when you are swimming, it is a much smaller area! (since you are horizontal (or hopefully, more on your side) most of the time)

yeah "new" is perhaps just "different" here. it's clear that we're now talking about swimming.

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This seems quicker than doing the ratio method below...or is the point of the ratio method to be able to estimate everything instead of calculate?

I think the point is 1) to estimate and 2) its easier to do the ratios without a calculator than to do the calculation

I agree, and even if the numbers were nice enough that the multiplication was simple, taking the cube root without aid would be very painful.

I think another point of using the ratio method for this problem is to teach us a new method using a familiar example- that way, we can check our answer using a different technique. I think it's cool to see how 1 problem can be approached with so many different models!

Yes, I look at this as another example of all the different ways we've calculated things like miles per gallon. Multiple approaches can lead us to the correct answer.

This is rather similar to lecture Friday, where he pointed out that using the circle-area equation and always plugging in new numbers for the radius is actually the "long" way of doing things, even though it doesn't seem like it.

I find this to be one of the more enlightening things about this class. At MIT we have so many simple formulas plugged into our heads that it always seems easier to do the math. However, much of this relies on a calculator and complete information. In this case we have neither so we need to find a simpler method.

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It may be instructive, but it is really better? To me, I don't think you're saving any time doing it one compared to another.

I agree, I would prefer to do some complex calculations in my head than to go through all this math relating the 2 values. I certainly couldn't do all this in my head.

I agree as well, simple math is easier than comparisons when you know nothing, especially if the format is something we've done previously

Maybe that's why he said instructive instead of better.

Agreeing with all of these comments - it would probably be easier to just compute the answer directly. Perhaps we'll see why this method is instructive?

it's "instructive" because it's a relatively simple example to work though & you can verify the approximation by revisiting something we've done previously

I would say simple math is only easier than comparisons when the estimations are easier. For harder, more obscure estimations, I would rather compare it to something I am familiar with.

It is easier for the reason that you don't have to explicitly find the constant of proportionality. I'm not sure why he used \sim and not proportional to, but there should be a constant of proportionality. That drops out when you take the ratio of $v_{\text{swimmer}}/v_{\text{cyclist}}$, and multiply by v_{cyclist} .

After having read the section, I have to disagree. Except maybe if I had a calculator handy, I would much rather estimate some simple ratios and multiply them than try to approximate a cube root in my head.

It's not always better, but it's an additional skill that's very useful when the equations might be messier. It's good to have more than one way to think around a problem too, and this really is a different way of imagining the problem. What if you were trying to compare the price of two buildings, one 3 levels of 900 sq ft and one 5 levels of 800 sq ft. You wouldn't calculate a per square foot cost, you would just scale the cost of one by those ratios (3/5 and 900/800) to see if it was a better deal.

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Well, for starters, it never said it's better.

Besides, whether or not it's better really isn't the point nor is it relevant. Here is just ANOTHER method to do it, which is one of the first things we're taught in this course. There isn't necessarily a best way and this happens to be a different approach that will emphasize different concepts that we should understand.

Hence, it's instructive.

You can do these calculations even if you don't know the constants or even the exact equations, just how things (dimensionally or intuitively) relate to one another.

Or if you simply can't remember the constants off the top of your head.

i suppose the more options we see, we can choose which one makes the most sense to us, and it's beneficial in that sense.

To me (the instructor), it seemed obviously a better approach. Therefore, to answer this question I had to think quite a bit about the intuitive reasoning behind that bland statement of the "obvious".

The most fundamental reason, I think, is abstraction. The scaling argument in the text is implicitly building an abstraction: that top speeds are inversely proportional to fluid density^(-1/3) (and similarly for area). Therefore, do that computation once – use the abstraction – and then reuse it to get the similar result for swimming.

I have drawn a tree picture to show the abstraction, and I will put that into the textbook as well as show it in lecture today.

this reminds me of how, in 8.02, they'd always encourage us to keep our answers in symbolic form and then evaluate our answers based on if they made sense. (if A goes up, then B should go down. yes, it is at the bottom of the fraction...)

this requires prior knowledge of the cyclist problem, right?

Yeah, I think he assumes that we know the value of V_{cyclist}

Can we do the example of running as well, maybe really quickly in class?

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Why repeat an identical equation? You should either remove it from above or simply reference it here.

I think it helps though given the next step in the math

I think it would be good to have the equation repeated if there is a subscript on the rho term. If the first was rho_water then it could be generalized here as rho_fluid

It's in a new section, it doesn't take much space, and there isn't any reason you should expect someone to flip back when you can just print it again.

It does seem out of place, although it is helpful and I can attest that I hate it when textbooks reference "the equation on page ___ and I have to go actually search for it again.

which area does this refer to?

cross-sectional area

In the drag equation, the area always refers to the cross-section that's directed towards the oncoming flow. In this case, the swimmer's A refers to their head, shoulders, and part of the arms and legs.

Maybe a diagram would show this better.

I forget, does the cyclist's cross sectional area refer to when legs are extended or not? maybe the different is minute due to the order of proportionality

we're looking at two different areas. one is the front of the person, one is the cross-sectional. i can see how this would be confusing.

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This makes it sound like its the swimmer and cyclist's densities, not the densities of their respective environmental fluid. I know it's not consistent, but I feel like $p_{\text{water}}/p_{\text{air}}$ would be more clear and useful.

At its worst, someone could just read this formula without reading the context, and assume that it is the people's density (and reduce the ratio to 1).

I don't know, it seemed pretty obvious to me, and it did a better job of making a point of the general idea of what he is doing... Maybe density swimmer_fluid... or maybe just stating it explicitly?

I agree with the suggestion to change it to p_{water} and p_{air} , it makes it more clear and it's still obvious to which each is referring to.

It didn't seem as clear to me. However wouldn't the density of the swimmer be water and that of the cyclist be air? If so, wouldn't that be a better reference than the using swimmer and cyclist

I agree that it should be labeled as $p_{\text{water}}/p_{\text{air}}$ because I thought it was referring to the densities of the swimmer and cyclist initially.

I agree. I'll revise it to use clearer subscripts.

shouldn't swimmer be on bottom and cyclist be on top for this part or am I missing something?

OP: oh, I just noticed the negative 1/3 in the exponent, my fault.

This combines divide and conquer with proportional analysis. I like the way this is broken down.

I agree—this method of breaking things down proportionally makes problems a lot easier. In homework problems, whenever there is multiplication we can break formulas into components like this

I like how this was set up in terms of proportions. Comparing swimmers and cyclists for each element is much easier than having to calculate the v-max for both of them separately.

Do you account for the power generated by seperate muscle groups? i.e. arms vs. legs? or do you just keep power constant?

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Agreed - it helps clarify for me where different values are coming from.

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Why don't we have to consider the actual bike ever? Whatever power the cyclist is generating is converted into his velocity through the bike, whereas the swimmer is only using his body, so shouldn't the cyclist be proportionally a lot faster anyway?

Not necessarily. Bicycles increase our output efficiency at the speeds at which they're used, but a swimmer does not really need the mechanical advantage of something like a bicycle to swim efficiently. It's basically because human power output varies with the speed of the motion – that's why we have bicycles to increase our speed and levers to increase our torque. Swimming, however, seems to fall closer to the peak output for humans (although some mechanical advantage, like fins, does seem to help).

does the bicycle itself play at all into energy or power?

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Estimate each factor in turn. The first factor accounts for the relative athletic prowess of swimmers and cyclists. Let's assume that they generate **equal amounts of power**; then the first factor is unity. The second factor accounts for the differing density of the mediums in which each athlete moves. Roughly, water is 1000 times denser than air. So the second factor contributes a factor of 0.1 to the speed ratio. If the only factors were the first two, then the swimming world record would be about 1 m s^{-1} .

Let's compare with reality. The actual world record for a 1500-m freestyle (in a 50-m pool) is 14m34.56s set in July 2001 by Grant Hackett. That speed is 1.713 m s^{-1} , significantly higher than the prediction of 1 m s^{-1} .

The third factor comes to the rescue by accounting for the relative profile of a cyclist and a swimmer. A swimmer and a cyclist probably have the same width, but the swimmer's height (depth in the water) is perhaps

I know you have to assume some things to start the analysis, but I have a problem saying swimmers and cyclists generate the same power. It's plausible, but not that likely.

Cyclists produce energy principally through their legs, whereas swimmers use most of their body. I also fail to how they put out equal power.

I would be inclined to argue contrariwise. Legs are much more powerful than the arms and shoulders and cyclists are able to work with very efficient strokes. Swimmers are very limited in their range of motion with their legs. Anyhow, we are talking about differences in 20-30%, which are of little consequence here.

I agree with the previous comment - it's true that the powers generated are likely very different, but unless they're off by a factor of 10, we'll end up getting the correct order of magnitude answer in the end.

It sounds like someone who is a swimmer or cyclist got offended, to me...

When we consider power generated we should look at the environment in which each person is racing. The cyclist only has to deal with air resistance while the swimmer has to push against water. So it's true that the cyclist is using primarily his legs (which are stronger) and the swimmer is using his arms, the factors probably even out enough that we can assume both generate about the same amount of power.

Both athletes are using most of the muscles in their bodies. Have you seen a cyclist with a weak upper body? The fact is they are both working their body's to a maximum, which is why we can make this assumption about power.

i fail to see what you're getting at here. how the power is generated doesnt matter, only quantity. i agree that we're making some allowances here, and glossing over any discrepancies.

Being that these are two specimens of simnilar physical fitness of the same species, it's reasonable to say they generate the same power within an APPROXIMATE range.

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Isn't the answer 1/10 of the speed of a cyclist that was calculated for as the average speed over a whole hour? If you are trying to calculate the maximum almost-instantaneous swimming speed you should not compare it to a cycling speed of such different time difference. And if I happen to be confused then you should be more explicit about what speed ratio you are using, more specifically what speed for the bicycle you are using.

I commented on this in the cycling reading, but I was bothered then when we didn't worry about time. We seemed to be calculating a maximum, but then we arbitrarily assumed this power could be applied for an hour. I'm not really sure what we're looking for with the swimming speed, instantaneous or over some period of time.

why would you say this when clearly there is a third factor? of course any formula is going to be incorrect if you're leaving things out.

I don't think it's to point out that we need another factor, instead it helps us understand how the final factor is going to affect our answer. Without this third factor, we are well off.

Here I had to go look up our results from a few readings ago to see how you arrived at this. Would it be possible to repeat somewhere in here that we had originally estimated $v_{\text{cyclist}} = 10 \text{ m/s}$?

Good point, I'd like to see that too.

I agree. also, what swimming world record? because for the cyclist, the speed we found was the speed over 1 hour..

I think review like this is good for an online textbook where we read different sites/chapters days apart, but not necessary for a published book where it's possible to flip through pages

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They swim much faster in a 50m race than a mile race! I think the 50m record is like 20.91s, making the speed 2.39m/s.

Sprinting is much more entertaining but I think it might be the distance in a bike race and the mile swim might make the type of muscles used more comparable. (powerful fast twitch vs. endurance)

Right, we need to keep in mind our comparison. Can't compare distance/endurance with sprinting.

Besides, it makes little sense to compare burst of speed competitions to a more settled rhythm.

It's be interesting to compare the records for cycling in a sprint to a sprint swimming. Though it's a relatively long sprint (compared to 50m which I can't find anything on cycling for), the record for 200m cycling is 11 seconds. That's 18 m/s! Which is still significantly higher than the 2.39 m/s mentioned earlier for swimming.

It's ok that we used the distance swimming race here for our comparison because the speed we got was still much faster than what we expected.

It took me a few seconds longer than it should have to read this. Could you maybe put a space in so it's more clear? I was expecting a speed since that is what we are actually interested in.

also, maybe write out 'min' or 'minute', since m is used as meters everywhere else. Or just convert the number to seconds for us.

If we are talking about the world-record fastest swimmer, then why would you take a 1,500m race, instead of a 100m race?

The argument was made in another comment thread that the cyclist power was based on endurance, so he uses an endurance swim race as well.

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I believe this is the only long-course Olympic record that predates 2008 (and Wikipedia backs this up).

Interesting...thought this would have been broken more recently with all the fuss about how improvements in swimming technology have made athletes faster.

This season the NCAA banded most of the really high tech suits but you would think that the high tech suits would have really helped especially in a distance event

Improvements in swimming technology? What makes a high tech suit...?

I was curious, so I looked into this as well. Apparently, high tech suits usually cover more of the body, and because they are "corset-like," enable those with stocky and muscled bodies to be as streamlined as a long and lean one, and a soft abdomen as effective as six-pack abs!

Where does this discrepancy come from?

why are we looking at how close a third of our process is? doesn't it make more sense to wait and compare the final solution anyway?

it may give us a better idea of where our estimations should lie if we didn't have any clue about the area. but then we would need the answer in the first place. yea this seems strange.

how should we know how close our prediction has to be? usually within a power of ten is supposed to be good enough, which this is

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When I think about a swimmer traveling 1 meter in one second versus almost 2 meters in one second, I can see how 1.7 m/s is a lot higher than 1 m/s. However, given that this class is always working on the scale of powers of 10, my first reaction was that these two velocities were very similar (almost equal to 1). Why is it that such a small difference matters here? Is it because the numbers are so small?

I agree with the above, its large given what we know about swimmers, but given that we just nonchalantly drop factors of two everywhere, I don't see how we can come to this conclusively.

This is only off by about 70%. In class we definitely would have said that this is a very good estimate. We drop coefficients of 2 all of the time. So why is this 1.7 coefficient suddenly significant?

I believe this is "significantly higher" due to the low order of magnitude.

The difference between 30m/s and 30.713m/s is, relatively, a lot smaller than the difference between 1m/s and 1.713m/s.

In general, there is no silver bullet for what is close enough and what isn't. We have to make that decision based on the context.

I agree with most of the statements above... while it is a much smaller order of magnitude, so the percent difference is much larger, I'm a bit skeptical given how many other assumptions we're making that we wouldn't have just gotten some other closer speed by accident.

maybe stating it as miles per hour would be helpful. Then it is more apparent that the speed is almost doubled. 2.2 mph to 3.8 mph.

Maybe briefly mentioning mph just to emphasize the difference would be nice but then again it is another conversion that will lost more accuracy and is much less convenient than the conventional units we used here

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I think a factor here is that while a person is cycling, the wind / drag will ALWAYS be against the athlete. No amount of power will be able to significantly change the volume of air around the athlete, nor it's direction (only perhaps the small volume behind the cyclist, but that doesn't help him/her).

In contrast, swimmers are in a pool with a limited amount of water. Compared to the volume of a swimmer, it's not much. If you consider all the swimmers in each lane, the water:swimmer ratio is even less. In swimming, the athletes actually cause the water to move one direction while they go, so for the length of a pool, they're actually aided by the tides/waves they create. This could be in part responsible for this disparity.

Cyclists can also be aided by a tailwind. If the athlete happens to be cycling in a windy area with a very strong tail wind, the wind could also aid them.

a profile figure might be nice for this

This isn't actually the swimmer's height is it? I thought we were measuring the cross sectional area going against the water?

That's what he's saying. The swimmers height is defined by his depth in the water, I'm sure this assumes that the swimmer isn't completely submerged which is usually true in a swimming race.

I think this makes sense—I mean either way, whether or not he is fully submerged or not, the cross sectional area is significantly less: in the swimming case, you're looking at basically a head's height and shoulders' width, whereas for the cyclist you're looking at shoulder's width over a much longer height...

haha unless you've seen a butterfly swimmers shoulders.

The order in which you made this analysis was a bit off-putting. It felt like you might have somehow made the numbers work, instead of allowing them to naturally fall out correctly.

one-sixth that of a crouched cyclist. So the third factor contributes $6^{1/3}$ to the predicted speed, making it 1.8 m s^{-1} .

This prediction is close to the actual record, closer to reality than one might expect given the approximations in the physics, the values, and the arithmetic. However, the accuracy is a result of the form of the estimate, that the maximum speed is proportional to the cube root of the athlete's power and the inverse cube root of the cross-sectional area. Errors in either the power or area get compressed by the cube root. For example, the estimate of 500 W might easily be in error by a factor of 2 in either direction. The resulting error in the maximum speed is $2^{1/3}$ or 1.25, an error of only 25%. The cross-sectional area of a swimmer might be in error by a factor of 2 as well, and this mistake would contribute only a 25% error to the maximum speed. [With luck, the two errors would cancel!]

4.4.3 Flying

In the next example, I scale the drag formula to estimate the fuel efficiency of a jumbo jet. Rather than estimating the actual fuel consumption, which would produce a large, meaningless number, it is more instructive to estimate the relative fuel efficiency of a plane and a car.

Assume that jet fuel goes mostly to fighting drag. This assumption is not quite right, so at the end I'll discuss it and other troubles in the analysis. The next step is to assume that the drag force for a plane is given by the same formula as for a car:

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Then the ratio of energy consumed in travelling a distance d is

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Estimate each factor in turn. The first factor accounts for the lower air density at a plane's cruising altitude. At 10 km, the density is roughly one-third of the sea-level density, so the first factor contributes $1/3$. The second factor accounts for the faster speed of a plane. Perhaps $v_{\text{plane}} \sim 600 \text{ mph}$ and $v_{\text{car}} \sim 60 \text{ mph}$, so the second factor contributes a factor of 100. The third factor accounts for the greater cross-sectional area of the plane. As a reasonable estimate

it just seems to me that making the guess of one-sixth after the fact that the first estimate was too high seems a bit contrived?

I agree - it seems like you just picked a number that would make it close.

yeah i agree...is there a way to make this approximation more accurately so that it's not just a somewhat random number?

I think it feels contrived because it is so close. But that is the power of good guesses. After all, the world obeys the laws of physics so why shouldn't we get a close answer, if we follow the physics relations as closely as we deem possible. I just laid on the floor to measure my height and it does come out to be about $1/6$ th of how high my bike is.

I find how close you get to real values to be quite remarkable given how many assumptions you make.

I think that it was deceptive of you to compare your estimate with the speed of swimmers in a 1,500m race, since they are pacing themselves. This estimate is most likely not very accurate.

I think that it's well within the error of the reasoning we've used to get this far. I don't think it's too much of a problem.

i agree. we have to consider the timescale over which these things are happening. the 50m sprint is likely what we're looking for here.

I think it was unclear but not unfair. First, I reorganized the sections badly, and somehow dropped the analysis of cycling (so the "previous section" points to nowhere).

Second, that analysis, which you had no way to know, compared the prediction to the record for long-distance cycling. In particular, the record in question is the "one-hour" record: How far can one cycle in one hour? That is a fair comparison with a long-distance swimming record.

For cycling, the record one-hour distance is about 50 km, for a speed of 13 or 14 m/s, which I took as reasonable agreement with the prediction of 10 m/s.

never would have thought of that

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Cool! This is very close to Michael Phelps record for the 200m freestyle- 1:43.86 which comes out to be 1.9 m/s

Dang he's fast...

does this mean swimmers have come close to "maxing" out the record?

They can swim much faster than 1.8m/s, the number from above is for a mile – and you can imagine that one would swim much slower over the course of a mile – the shortest race these days is 50m (for a long course pool), and I think I mentioned above that is about 2.39m/s.

but they have come close to the max _for long distance_ swimming.

There isn't a hard upper bound. Reducing their drag coefficient, changing swimming style, increasing potential power output, etc., could all increase maximum speed. This is just an estimate.

this is why swimmers are relying on technology, mainly new body suits to make them swim faster

Definitely true, the new suits they use significantly reduces drag for the swimmers in the water. This is still an impressively close calculation!

This isn't a maximum or even an estimate of one. The number relies directly on the athlete's power output, so better training may allow them to increase that number and exceed our estimate.

To me it kind of seems way too convenient that the numbers worked out the way that they did.

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But we still needed to know the maximum velocity of the cyclist which we approximated and worked out using arithmetic. So I guess I would note that one downfall is that you'll have to use raw approximations without scaling for something, and then use that as the base for subsequent approximations.

I think this is just another example of proportional reasoning and how scaling can save us time

I'm not sure that the scaling was too much faster than plugging in new numbers would've been - we remember P , know the new ρ and can guess the new A ...things we have to do anyways to create the scaling factor. Is there some error propagation benefit to scaling?

One reason that it's beneficial to scale is to easily compare your estimate for one number with that of a number you already know. It makes it easier to see if any of your estimates are unrealistic.

I think this example doesn't really seem that much better with scaling because it doesn't allow us to eliminate anything - we just had to do the whole thing out basically. Maybe taking an example where there were constants in the formula or larger similarities - say between the areas - that would allow us to eliminate numbers by using the scaling trick.

I just realized, in writing another response, that I forgot to include the cycling calculation when I reorganized the book draft. Sorry about that.

The main results from the ghost section are the prediction of 10 m/s and the world record of about 13.5 m/s (for the maximum speed over a one-hour stretch).

So the purpose of kind of re-doing this example was to understand the source of estimation's accuracy?

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This is a very good point. We could hung up on details such as whether or not the cyclist and the swimmer actually exert the same amount of power. But the debatable issue decreases in significance after cube rooting.

The opposite is also true though, and I feel like he glosses over the fact. If something goes like a factor squared or cubed, minor errors become greatly exaggerated.

Yeah, this is true, actually. while in this case the cube roots make the errors smaller in the final answer, there are cases where things are squared, and error propagates much larger.

This part does clear up my question.

Correct me if I'm wrong, but we never made an estimate for power produced, simply assumed they were equivalent values (as it should be, if it's the same muscles being used). Perhaps you meant about the densities of water?

I think this might be referring to the equation for velocity that we didn't bother using; even if we didn't estimate the power directly, the cube root compresses the P term. It probably didn't mean the densities because the values for densities of water/air are more or less correct, and the only things we're estimating are A and P.

This does make it seem like we estimated the power (but even then this doesn't specify whose power this is referring to). We should either do the estimate earlier or nix this specific value.

I think we made the estimates of power earlier in the cycling example—remember the stair climbing/time calculation? You're right, we don't have to think about them again here since they're not changing, but if we were to compare people capable of different power outputs (due to age, fitness, etc.) we would need to do more estimation for the ratio above.

It does say "for example" right before, giving the impression that it is all hypothetical. I guess it could be fixed by writing "an example" instead of the "the example."

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ah yeah, I need to start thinking about this more – its a bit confusing! So you do the factor you might be off ("n") to whatever power it is, so $n^{1/3}$?

Try it with some sample numbers—say you had an estimate of the volume of a cube and wanted to find its radius, any errors you made in the volume would be scaled down by the cube root. If the true volume were 1000 (length = 10) and you guessed 1100 (10% off, a factor of 1.1) you would get 10.3 (only 3% off, since $1.03 =$ the cube root of the error factor of 1.1). Of course, there are cases that magnify the error, for instance, going from radius to volume, the reverse of my example. Lesson: be more careful with your original estimate if your error is scaled up, or at least be aware that it is scaled up.

This may be a bit philosophical, but what is a "mistake" in estimation? In other words, at what point do we say a number/answer is "wrong?"

I think "mistake" is a bit harsh here, maybe misjudgement

Yeah agreed, "mistake" is not the right word to use here.

Whether or not it was a mistake to use the word mistake or not, I think that the point here is pretty clear.

I think if something isn't exact it's always 'wrong'. There are just varying degrees of wrongness. For example. This was wrong by 25%. I think a good way of judging if something is "too wrong" to be a good estimate is if it is outside of an order of magnitude estimation.

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this seems to happen in most cases lol (at least so far)

If you estimate enough numbers, it is probable that some will be overestimates and some underestimates. I'm not sure whether its really "luck"

That's a good point, I never looked at the canceling that way.

It would be nice to see a more rigorous explanation of how things always seem to cancel, because right now it seems they just do. It doesn't have to be a mathematical proof or anything, just a general "well, we overestimate on things in both the numerator and denominator and it works out well" would be fine.

I agree, I don't remember talking about error analysis much in the readings... this is cool!

It is cool. However, do you think he plans to over estimate and underetimate in every case, or is it luck?

I don't usually get that lucky in my estimation mistakes.

me either. are you picking problems where errors cancel each other out on purpose? if you had two errors that contributed in one direction by 25% each...that would make your answer pretty inaccurate. I think a 25% error rate is already pretty bad...but I guess we are usually only looking at orders of magnitude

I think he just consciously makes a point to remember when a value has been overestimated, and thus will underestimate another to compensate.

I definitely do that. If you know one of your estimates is an overestimate, why not consciously look for an underestimate elsewhere to balance the two?

Is there some way for us to intentionally increase the chances that our errors will cancel?

Yeah, what if our errors compounded into more egregious ones?

I think that's why we always have to check our answers. If we're dealing with something where you can't check your answer, assume the worst with your errors and have wide confidence intervals.

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not really understanding this error analysis..

it's just saying to utilize the powers of the ratios here in order to reduce order of magnitudes. i'm not sure its something you could account for at the beginning though.

I think it's more about understanding how close your approximations really have to be. We are given a bit of leeway in this problem because we are taking the cube root so it is not so important to be perfectly accurate.

It would help to say this - I saw in another comment a reply about how a factor of 2 becomes only 25%. Including that here would no doubt clarify matters.

I feel pretty familiar with drag now.

what does it mean to "scale" the drag formula? i thought it was just ready to receive values of various magnitudes, depending on what we were working on.... what scaling is needed?

agreed, I think the drag coefficient takes care of this.

This is probably dumb, but I don't really know the difference between a jumbo jet and a 747 off the top of my head. Obviously I can look it up but its not something I know.

As far as I know they're different names for the same thing.

I don't think the difference is significant. Basically the message is: a really freaking big plane!

Speaking in terms of sets, a 747 is a subset of jumbo planes, probably the most well-known one :)

I wonder why this would be seen as large and meaningless. couldnt we work backwards in the future and using the fuel efficiency of jumbo jet to find that of a plane and car

I'm sure we could, but it's much easier for people to understand if we find the fuel efficiency of a jet in terms of that of a car—something we are all familiar with.

Agreed. The point about it being large is that we should look at things that we come across in our every day lives.

is this different that a 747?

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I love that this plane fuel consumption problem keeps coming up, and that we've had the opportunity to solve it different ways each time. More than any single example, I feel like this one has been a good exercise for each method.

Yes definitely agree! Having one common thread like this through different estimation methods really highlights strengths and weaknesses between the different types.

I have noticed that this is a very common way of finding solutions, but wouldn't you want to use an independent way just as a check for your answers?

why is it meaningless? we've been coming up with a lot of large numbers.

I like this explanation - it makes scaling seem much more reasonable because it produces an answer that is more easy to think about for the average person.

This is a profound statement. It's interesting to consider the amount of fuel required to bring rockets to orbit vs. having them coast in space where no atmosphere exists and no extra boosts are needed

i never knew that drag was so important in all these applications, the amount of places you can do a drag analysis is amazing

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This is helpful so I know that the actual complexity of the problem will be explain later, so I don't have to ask now.

Agreed, this is definitely an improvement from some of the previous articles.

Sorry to disagree here, but I think stating this takes away from the strength of your methods and approximations. Discuss limitations and further refinements after you have produced an initial prediction. Simply stating that you are making this assumption should suffice for any doubters until you get to this discussion at the end of the section.

Disagree – if you just look at comments from earlier sections, a lot of people got caught up early on when he made some assumption people found hard to believe (i.e., cubic mountains). So it helps to know that it'll be explained – because at the very first half of the sentence I was already skeptical that most of the jet fuel goes to fight drag.

I agree that it's useful to point out there are further complexities not yet covered, but I also agree that it shouldn't sound like the estimate is doomed from the beginning. Perhaps saying that "the assumption is not quite right - but still allows us to arrive at a reasonable estimate" would be helpful?

I agree with the fourth post as well. A comment like that really brings more perspective into the problem as you are doing it. In a way it makes me feel somewhat more confident, even though the "assumption is not quite right"

I totally agree - as soon as i read this I thought - oh that's nice it gets rid of all the comments regarding shortcomings that are eventually explained anyway!

as opposed to having a drag coefficient that goes towards geometry?

I think I've finally got this one memorized!

For some reason I keep forgetting all of these.

i think they'll become easier once we hit the dimensional analysis chapter! :)

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when did we switch from fuel efficiency to fuel consumption?

probably because

fuel efficiency = energy consumed in a distance d

fuel efficiency = E/d

since we compare the same distance for both plane and car, the ratio of their respective fuel efficiencies is simply the ratio of their energy consumed.

a little confused on what this term means..

Why "up-high", but not "down-low"?

Because then you'd have to fit in a too-slow

This is to refer to the density of the air at a higher altitude (I think). The 'down low' is rho low underneath, which is roughly the density of air under normal conditions.

Shouldn't you be accounting for the amount of people each can hold? A car that holds 5 people can not be compared to a plane that holds 300.

Sorry I noticed your discussion on the passengers after the first note was written.

you could always delete your note, if you want... I think nb still saves a "hidden" deleted note which the admin can see.

(one reason for deleting a note you don't want anymore is that it cleans up the clutter, since everyone draws boxes in the same place)

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much better than 's'

So much for using 'r' for 'range' instead...

Not that this matters, but perhaps it should be $d_{\text{plane}}/d_{\text{car}}$.

i thought 'x' would have been better than 'r' anyway...

x sometimes becomes a problem when you are also using it as the multiplication sign

Clearly the appropriate choice of variable to represent distance is a contentious issue... Deep breaths guys, it's really not a big deal.

You're right, you get the picture whether its d , s , r , x or whatever... as long as it's defined. I agree with above though, that it might be better to use the plane and car subscript with the distance variable.

And d_{plane} and d_{car} are the same thing. There is no reason to put any more detail than what's already up there.

I'd still like to have seen the intervening equation $E = F_{\text{drag}} \cdot d$ since we both added that and took the car/plane ratio in one step.

Based on the analysis in the preceding section, can we assume that the squared factor would amplify any errors in estimations of velocity?

Yes, it would. but in this case, this value is probably the easiest to estimate—the relative speeds looks much easier to gauge (for me) than relative cross-sectional areas or air densities at various elevations...so even though error would amplify, it probably shouldn't be too big a deal.

But what about holding a plane up in the air?

Never mind, it's mentioned later on.

I like how you break everything down into manageable estimates. This makes the problem much less difficult, and it really proves your points in earlier memo's about using abstraction to your advantage.

I agree...these examples that make use of the previous techniques we've learned in conjunction with the new one we're studying really help bring things together

one-sixth that of a crouched cyclist. So the third factor contributes $6^{1/3}$ to the predicted speed, making it 1.8 m s^{-1} .

This prediction is close to the actual record, closer to reality than one might expect given the approximations in the physics, the values, and the arithmetic. However, the accuracy is a result of the form of the estimate, that the maximum speed is proportional to the cube root of the athlete's power and the inverse cube root of the cross-sectional area. Errors in either the power or area get compressed by the cube root. For example, the estimate of 500 W might easily be in error by a factor of 2 in either direction. The resulting error in the maximum speed is $2^{1/3}$ or 1.25, an error of only 25%. The cross-sectional area of a swimmer might be in error by a factor of 2 as well, and this mistake would contribute only a 25% error to the maximum speed. [With luck, the two errors would cancel!]

4.4.3 Flying

In the next example, I scale the drag formula to estimate the fuel efficiency of a jumbo jet. Rather than estimating the actual fuel consumption, which would produce a large, meaningless number, it is more instructive to estimate the relative fuel efficiency of a plane and a car.

Assume that jet fuel goes mostly to fighting drag. This assumption is not quite right, so at the end I'll discuss it and other troubles in the analysis. The next step is to assume that the drag force for a plane is given by the same formula as for a car:

$$F_{\text{drag}} \sim \rho v^2 A.$$

Then the ratio of energy consumed in travelling a distance d is

$$\frac{E_{\text{plane}}}{E_{\text{car}}} \sim \frac{\rho_{\text{up-high}}}{\rho_{\text{low}}} \times \left(\frac{v_{\text{plane}}}{v_{\text{car}}} \right)^2 \times \frac{A_{\text{plane}}}{A_{\text{car}}} \times \frac{d}{d}.$$

Estimate each factor in turn. The first factor accounts for the lower air density at a plane's cruising altitude. At 10 km, the density is roughly one-third of the sea-level density, so the first factor contributes $1/3$. The second factor accounts for the faster speed of a plane. Perhaps $v_{\text{plane}} \sim 600 \text{ mph}$ and $v_{\text{car}} \sim 60 \text{ mph}$, so the second factor contributes a factor of 100. The third factor accounts for the greater cross-sectional area of the plane. As a reasonable estimate

This would have been helpful information for the last pset (the last problem on the pset required this information).

I think this was brought up in lecture before pset #4 was due, but not sure when exactly.

The Monday before, I think. It was still something you could estimate, though, if only with a gut feeling.

I think it would be useful to put the $e^{-(h/H)}$ formula that you showed in class into the text. I thought that was pretty handy.

You noted in the previous section about how an error of a factor of 2 in power estimation only came out to a factor of 1.25 in our result. It might be good to show the dark side, too, and note that any error here gets squared (or doubled, depending on how you think about it... 10% error goes to 21%...).

I think our 600 and 60 numbers are relatively accurate, so this may not apply here, but at least noting it somewhere would be useful and honest.

Yeah I agree, as I read it I was already asking myself what would happen if the errors didn't cancel out but only compounded on one another, so a "dark side" problem would be useful.

$$A_{\text{plane}} \sim 6 \text{ m} \times 6 \text{ m} = 36 \text{ m}^2,$$

whereas

$$A_{\text{car}} \sim 2 \text{ m} \times 1.5 \text{ m} = 3 \text{ m}^2,$$

so the third factor contributes a factor of 12. The fourth factor contributes unity, since we are analyzing the plane and car making the same trip (New York to Los Angeles, say).

The result of the four factors is

$$\frac{E_{\text{plane}}}{E_{\text{car}}} \sim \frac{1}{3} \times 100 \times 12 \sim 400.$$

A plane looks incredibly inefficient. But I neglected the number of people. A jumbo jet takes carries 400 people; a typical car, at least in California, carries one person. So the plane and car come out equal!

This analysis leaves out many effects. First, jet fuel is used to generate lift as well as to fight drag. However, as a later analysis will show, the energy consumed in generating lift is comparable to the energy consumed in fighting drag. Second, a plane is more streamlined than a car. Therefore the missing constant in the drag force $F_{\text{drag}} \sim \rho v^2 A$ is smaller for a plane than for a car. our crude analysis of drag has not included this effect. Fortunately this error compensates, or perhaps overcompensates, for the error in neglecting lift.

Is this an average area? the nose is more narrow than the middle part of the plane, where the carry on baggage also contributes area.

When you are computing drag you only care about the frontal cross sectional area (The entire area you can see from the front) so a tapered nose doesn't affect the area, only the coefficient of drag which we left out of the F_{drag} equation back in reading 12 or so

why do we even need this? (it's just equal to 1, I don't see why we need it)

the fourth factor is the ratios of the distances, right? since energy = force * distance, you need the distance term...

I think it's important so we compare on the more familiar basis of energy, but it was an unstated leap from F to E above.

I was wondering this as well.

I think its just for consistency, so we keep track of what we are calculating.

I think it's good to see any unities done out. I'd rather have them there so it's understandable where it came from.

Is it unity though? A plane can go straight across the US while a car has to follow roads which add about 400 miles to the trip (2600 mi flying vs 3000 driving). It's a difference of about 15%, is that small enough that we can just discount it?

By saying that it contributes unity, this example is assuming that the plane and car travel the same distance.

How much would this actually effect the overall result?

Whenever these planes hypothetically fly to Los Angeles it makes me homesick.

hahaha, me too

when comparing energy efficiency, it is important to include the inefficiency of indirect freeways, and the requirement to stop to refuel, sleep, go to the bathroom. Also traffic, and elevation gain and loss play a role. I would guess flying cross country is far more efficient than driving.

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I really liked this example in class.

Yeah! I also like how he waits until the end to account for passengers, it makes you think after the first calculation why it's so off from what you'd expect.

I think the entire idea of looking at a problem and how each factor contributes to the answer is amazing. Even though we could do this stuff with the simple equation, it is much easier to relate it to something we understand, like cars.

...but not a bus

But there are a lot of other perks that make flying much more appealing to many people like the time factor. Are the factor of time and perhaps the specialization of pilots the main factors in price difference of travel by the two modes (since you've said fuel efficiency is pretty similar as well)?

Another thing to think about is that planes are going to make their trip to California with or without you, full or not. By choosing to drive instead of taking a plane you are in a way being more wasteful by inaction as well as what you use driving.

Haha, very true!

Even in MA. I noticed this the other day when I was stuck in traffic during rush hour.

I thought this was funny, but wondered why he chose California when it hasn't been mentioned anywhere else? I am from there, so I can at least admit that it IS quite fuel inefficient by and large...

Sanjoy got degrees from Caltech and Stanford—that would be a non-negligible # of years in California.

<http://web.mit.edu/tll/about-tll/mahajan.html>

typo

does one or the other provide a more "green" environmentally friendly option? or are they about the same?

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Found some cool data. As of 2001, this number was 1.57 for cars across the US (and higher for SUVs/Vans, lower for trucks and motorcycles)

http://www1.eere.energy.gov/vehiclesandfuels/facts/2003/fcvt_fotw257.html

Again, I still think the number would be even higher for the average trip to California.

But are they in fact really equal? I read below that the 2 errors in estimating scaling ratios for the plane basically compensate for each other, but in general are the relative efficiencies about equal? I would have guessed differently.

This seems reasonable, but I would like to see the facts here to compare. (Or I could just look it up again.)

Didn't we figure this out in class on wednesday?

I believe we did. I'm wondering why we did it in lecture before seeing it in the reading. It seems backwards from how we've done everything else. Perhaps he forgot he also used this example in the reading?

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I would think this number would be higher, especially with the focus on environmental responsibility and carpooling.

I agree, I think that a more accurate approximation would be that a typical car carries 1-2 people, so I'd use 1.5 people. This subsequently causes the fuel efficiency of a plane and the fuel efficiency of car to be unequal, which I think seems more plausible.

in lecture he said something about actually sitting and watching cars ... it seemed to be a good thing to go with from that.

from my experience watching cars, i would actually guess 1.1 or 1.2 ... yes, the typically carry 1-2 people, however they carry 1 _a lot_ more than they carry 2 (or more).

Yeah but how many people drive to california alone? It's usually a road trip of some sorts. I've never heard of anyone driving out west with fewer than 2 people in the car.

Well, the distance is just an arbitrary distance that scales to unity in the ratio anyhow, so the fact it's a road trip is irrelevant. Generally for every day use of a car, people are driving to and from work and it does tend to be about one person to a car...

The average certainly can't be one person per car because there aren't any cars with less than one person, so I agree the estimate should be higher. Is there a reason you talk about the typical car instead of the average number of people per car?

well, with the point being that a jumbo jet capacity is 400, does 1 or 2 or 3 people really make that much of a difference? and I agree with the 6th reply that the fact that its a road trip is irrelevant also.

Agreed. Just remember, it's an approximation class!

That is a really interesting result. However, I would think that a plane would be more efficient given the fact that they are strongly preferred for long distance travel.

This answer really surprised me too—but I guess it still makes sense that planes would be preferred—plane trips are shorter, and it's less of a hassle when you can sleep and don't have to drive. But also, this estimate is fuel efficiency based on drag alone. There are many other ways planes and cars waste energy right? do things like AC play a significant role?

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is it the same kind of fuel that is used by both the car and the plane? I'm assuming one is more cheaper than the other...

that's wicked cool

When you say it is comparable, do you mean that it is about the same amount of energy? Because wouldn't that mean that the total energy is almost double?

That's how we treated it in earlier parts: $E_{\text{drag}} + E_{\text{lift}}$ and total E is the sum.

Does this mean that we should therefore divide the ratio of $E_{\text{plane}} / E_{\text{car}}$ by two since only half the energy is consumed by fighting drag?

I think we multiply it by two, because the ratio is comparing total energies, and we only considered the energy of drag, when infact the total energy a plane expends is twice that.

A curious thought I had after reading this was "what is the fuel efficiency of a flying car?"

Probably really poor. Cars are designed not to fly unfortunately, so it would take a lot of energy to make a car fly, if it's even possible at all.

Cars are designed to create downward force.... if you could fly it people would start to take off at high speeds.... and that wouldn't be so good.

Actually that's why drag racers are shaped the way that they are....so that when traveling down the track they don't take off and lose control.

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So since a plane is more streamline and this effect dominates the other errors it would be more efficient than a car? I had kind of understood that planes were much more efficient than cars when you take into account the # of people on the plane. I was on a flight in the last couple of years where the pilot said we were using about \$20 in fuel per person. That's only about 7 gallons of gas or about 200 miles on the highway

interesting, thanks!

I feel like a plane is much more efficient than a car, if you factor in how much quicker and farther they go, along with how much more they transport.

To Sat 6:23: I agree that a plane has other advantages such as speed or maximum distance, but I don't think these can be considered as components of the fuel efficiency that we're estimating here. I wonder though what kind of expression we could use to include some of these factors, at least to consider something like: Energy Consumed per Person vs Velocity.

I understand this is a class about estimation, and this information doesn't seem important. But out of curiosity, how are we able to determine when certain details (such as comparison of energy and drag) are negligible.

I wouldn't think the fact that the planes move quicker would mean they are more fuel efficient.

I agree, also just as a matter of the way to the two move. All things considered, the drag of th eplane from the air is going to be less than that of the air AND friction from ground on the car to start, not ot mention the fact that planes are more aerodynamic in design.

Needs capitalization.

I like "error" here better than "mistake" earlier. You can make errors in estimation, but I feel like a mistake is hard to come by since most of us are just guessing anyway!

I think this recap is really good to go over the parts we neglected in the calculation.

I agree that the recap is good, it does come across as a whole bunch of stuff going back and forth on sources of error and their mitigation.

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I think knowing when to compensate for error is part of the art in the art of approximation. I think i tend to neglect a lot of things when estimating, but I guess the trick is to know when a value is rounded by too much or a guess was too wild.

I agree, I think a lot of times the hardest part of these problems is realizing when two errors will cancel each other out, is there any good way to recognize this?

And even if things don't cancel out, you have valuable info if you know your answer is a little big or small

This is a good question! I feel like if I were estimating and got an answer, I wouldn't be able to tell if I made any errors (or where I made them). It would just be luck that they canceled out.

I guess my question would be...how much can I overcompensate? What's reasonable? a factor of 10? and if i overestimate should i just assume that something else must be underestimated? or that I can neglect another variable?

I think it is just a fun fact of estimating. It's pretty easy to change the estimate later by finding values or getting better estimates, but I don't think it's that hard to realize something is above or below an estimate (like mph of a car at 50 is an underestimate or something).

Still, how are we to tell if our overall estimation is over or under? Are we to tell somehow based on the overall direction of our estimation?

I wonder how the friction of the wheels on their chassis adds to the energy loss equation along with drag.

I think you should add to the reading what we talked about in class about how planes are not a direct substitute for cars. Like you said, people from the east coast wouldn't go to California as much if they had to drive. Therefore, there's something to be said about plane's having a larger affect on the environment due to their generation of sometimes unnecessary travel.

I feel like a lot of these estimations are still based on numbers I wouldn't know off the top of my head, which also help guide whether you round up or down to cancel errors