

# 5

## Dimensions

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### 5.1 Economics: The power of multinational corporations

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In Nigeria, a relatively economically strong country, the GDP [gross domestic product] is \$99 billion. The net worth of Exxon is \$119 billion. “When multinationals have a net worth higher than the GDP of the country in which they operate, what kind of power relationship are we talking about?” asks Laura Morosini.

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### GLOBAL COMMENTS

I feel like this is just repeating already known information. If I hadn’t have known this, I would have been screwed in 8.01

When you say "this", are you referring to the reading or to particular parts of it? I don’t claim that the ideas in this section are previously unknown, only that most people don’t use them. Hence the need for the section.

Dimensionless are the best. When there are many different terms to rectify with the numerator and denominator, it gets a little tricky.

Is it still this high? I think 2006 was at the height of the peak oil scare, when oil was well over \$100/barrel...

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We're starting a new unit. Read the first two sections of this chapter for Tuesday's memo.

I hope that you don't find too much to fix in Section 5.1 because it's borrowed from *Street-Fighting Mathematics* (MIT Press, 2010), which came out in print last week. But comment freely about it (and the other section) and let the chips fall where they may.

I like how this example/topic isn't about formulas or science, it makes for a nice change of pace and adds variety to the book.

I was going to cheer also, but then I noticed that we'll be back to it in the next few sections. But it is a nice change.

I like that we're relating to something I know better.

I'm an engineer and I'm getting kind of queezy because I didn't do all that well in 14.01 :-/

Looking at the section titles, I recognize almost all of them from *Street-Fighting Mathematics*. Have they been changed at all?

## 5.1 Economics: The power of multinational corporations

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Why is this bracket here?

Probably a placeholder that got left?

Or maybe it's for a citation for the next passage

If this section was taken from another book, then the TeX could reference a citation that does not exist in the references/bibliography for this file.

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I've never had an econ class in my life, but isn't GDP yearly while net worth is the total amount the company would be worth if you sold it, not how much it makes in a year?

Yes, you are correct about both. Net worth is the value of a company (assets - liabilities) if you sold it right then.

Regarding, "I've never had an econ class in my life," maybe that is an advantage. When I gave a workshop one time and used this example to illustrate the importance of dimensions, many participants (who are faculty at leading research institutions) insisted that there was no problem!

**Does this mean Exxon in Nigeria, or the worldwide corporation? I'm not sure how these things work in specific countries.**

Worldwide. The comparison is trying to show how poor a country is in comparison to a single company (globally)

**This is a poor comparison. net worth is at a particular point in time while GDP is cumulative over an entire year. It might be better to compare Exxon's annual revenue to Nigeria's GDP**

I agree but the comparison isn't too ridiculous. It doesn't compare two similar things but, if anything, it puts into perspective the size of Exxon.

**Fun fact: Exxon Mobil is the world's second largest company by market capitalization (that's share price\*number of shares). It's also the most heavily weighted company in the Dow Jones Industrial Average, which is one of the most popular benchmarks of economic performance.**

This is really interesting...and I think it emphasizes the idea of this quote and works as a great lead-in to the rest of the section.

Which begs the question, at least from me, what is the largest company?

I don't think I've ever heard a comparison like this. I usually hear GDP's compared to annual revenue or profits, which have the correct units.

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This is an interesting way to open the chapter. It made me want to read more to see how you’d tie in this excerpt to dimensions.

Yeah I really like this - it suddenly makes me very interested.

Agreed. It’s a very general, non-technical overview that everyone can understand and ultimately relate to.

It’s also, as someone mentioned earlier, a nice change of pace and topic. Airplanes are fun and good, but sometimes you can focus too much in one topic!

I also think the choice of topic (discussing the ethics or fairness) of globalization which is a very controversial topic further attracts the reader’s interest.

I look at this somewhat differently - it seems all along we’ve been examining a few different examples. Although many were more physics/science based, we still did more than just airplanes.

This reminds me of District 9, and MNU operating in South Africa.

Yeah I am a huge fan of this introduction. It’s simple to understand and straight to the point. The non Science thing is a good point too.

This is very similar to when people make claims like Harvard is worth more than small countries because of their \$28B endowment. They don’t mention that it took 400 years to build up and that they only spend 5% of it. It drives me crazy when journalists do this.

This is a great point that gave me a different perspective. Though, at the same time, you shouldn’t compare Exxon to Harvard either. Exxon is driven so much more on a profit motive.

No matter what happens to the textbook, make sure to leave this part in!! This is the kind of stuff you always see on the news that is just nonsense. This is one parts of the class that isn’t just about approximatoin, but about general lessons for life.

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This suggests Exxon's headquarters are in Nigeria- is this true?

i doubt their headquarters are in nigeria, but i'm sure they drill from there, so they "operate" there.

I don't think this suggests anything about Exxon's HQ (which isn't mentioned at all?)

My first thought was also that Exxon's base was in Nigeria, perhaps it could be changed to something like "The net worth of Exxon, who operates in Nigeria,..."

Changing 'the country' to 'a country' should remove most of that confusion, I think.

One of different units that might not really mean anything?

Haha, you catch on quickly.

Didn't she respond to your correction of this argument saying you were right but they were keeping it the wrong way to prove their point?

Oops, you mentioned that down below.

I'm unfamiliar with this name, is she a well known critic of globalization?

Other than this quote from "Impunity for Multinationals", I don't think she's a well-known critic. A quick google only came up with the one article.

I think it would be useful to add in parenthesis after this quote who she is.

I don't think a well known, educated critic would make this argument. It is so misguided.

what do you mean by this?

I think he's just asking what is the most obvious mistake in the argument/question posed by Morosini, not in her opinion, but in the way she argues it.

Sounds right!

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I think the problem is Exxon operates in many countries. And perhaps serves more people than the population in Nigeria or serves more needs. On the topic of dimensions, I definitely wonder if the dimensions are the same since I am sure net worth and GDP are not measure in the same capacity

Right. I don’t think the comparison of GDP to net worth makes sense. And isn’t this chapter on dimensional analysis? I guess I’m cheating...

I agree as well, as GDP is the amount of money a country takes in per year while net worth is the amount a company is work after several years of worth. I wonder if the two are still comparable...

I dont think the two are comparable. I think GDP might possible comparable to net income in terms of the units but like mentioned earlier they represent two different things. That to compare net worth’s of the two you would have to include all the natural resources, inventory, other available resources, etc. that Nigeria has.

I agree with the above - if you sold Exxon it would come with a lot more than its current bank account earnings for one year. If you sold Nigeria it would come with a lot more than its GDP.

Both are measured in currency, but I don’t think the two can be compared easily.

this statement is a little confusing and not very helpful.

The field of economic reviews I think.

What field? The comparison?

I think the field of faults in the comparison

i dont even know what this means either.

maybe he’s saying that: the economic field is competitive but in this particular case, the justification is wrong.

I took this sentence to mean that there are multiple errors in Morosini’s argument/question, but we’re going to focus on the most obvious one.

It seems like he’s being sarcastic, saying there are so many errors to choose from.

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GDP is more a measure of economic activity and production than monetary flow. It's specific to what was produced in a nation with regards to goods and services rendered. Not really an important thing, I know. I'm just nitpicking.

This is a great introduction to dimensional analysis, and I like how units play such an important role in this comparison

this is a little pedantic...

But it makes the point

And is interesting :)

But adds nothing to the discussion...

It's merely discussing the terms that are involved in the statement. Nothing "pedantic" about it. In fact, we regularly ask him in our comments to define stupid stuff that we could look on our own all the time.

That's interesting. I never considered how meaningless this measurement was.

This is a really interesting comment- it definitely made me think!

I agree. I really liked the way this was worded.

I like how this points out how arbitrary are units of measurement are

an interesting way to put it...



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It doesn’t seem entirely arbitrary.

Not entirely, but think about it this way: our life has nothing to do with how long it takes the earth to go around the sun, but one year is a crucial unit in our lives.

i don’t think it’s arbitrary at all. agriculture always has an effect of the economics of the country, and in some countries, it’s a huge factor. agriculture depends upon the rotation of the sun.

I agree that a year isn’t entirely arbitrary; every civilization has had some measure of time based on the seasons or the stars, which makes sense to me. I think the 365-day year was mostly to make bookkeeping easy for everyone (why have some years have two winters and others two summers?) I agree with what the reading says, however; to me it would make more sense to have a larger sample of, say, 5 astronomical years to smooth out the data.

I’ve thought about this on occasion, and how we celebrate our birthdays and say that we were born on some day, when the construct of a year has little effect on our modern lives. I don’t think it is arbitrary in this sense though: a day as 24 hours seems not arbitrary because of our sleep cycle, it is so intrinsically tied to our bodies. If we say the measurement of day is arbitrary, then there is not much left to discuss, as anything else can be deemed arbitrary. A year’s direct influence on our body is so much more less drastic, but it does affect the changing of seasons, which directly affects agriculture, which, for much of history until now, has had a central impact on life.

I think this is kind of unnecessary.

yeah I don’t really think this is necessary...maybe the same thing could be stated in a shorter way?

This is a great way to get people to start thinking about dimensional analysis. if we start looking at a decade, then the GDP all of a sudden looks so much larger

And sets up why the direct comparison doesn’t work as well

I think using decades is common.. but I also wonder if that is a good idea for modern economics when things change so fast every year, if not every month. Is it OK to generalize it that much.



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I guess now it makes sense why the sentence about the year being the unit of time chosen to measure social phenomenon was in the previous paragraph...it gives a nice transition to this paragraph.

I think in the first paragraph it should be stated that GDP is a flow (which is does... good!), but additionally, I’d like to to make clear that Net Worth is an amount. I think it shouldn’t be stated much later- once it is mentioned, it becomes clear the comparison and everything flows a bit better.

steady from year to year) would be roughly \$1 trillion per decade and be reported as \$1 trillion. Now Nigeria towers over Exxon, whose puny assets are a mere one-tenth of Nigeria's GDP. To deduce the opposite conclusion, suppose the week were the unit of time for measuring GDP. Nigeria's GDP becomes \$2 billion per week, reported as \$2 billion. Now puny Nigeria stands helpless before the mighty Exxon, 50-fold larger than Nigeria.

A valid economic argument cannot reach a conclusion that depends on the astronomical phenomenon chosen to measure time. The mistake lies in comparing incomparable quantities. Net worth is an amount: It has dimensions of money and is typically measured in units of dollars. GDP, however, is a flow or rate: It has dimensions of money per time and typical units of dollars per year. (A dimension is general and independent of the system of measurement, whereas the unit is how that dimension is measured in a particular system.) Comparing net worth to GDP compares a monetary amount to a monetary flow. Because their dimensions differ, the comparison is a category mistake [] and is therefore guaranteed to generate nonsense.

**Problem 5.1 Units or dimensions?**

Are meters, kilograms, and seconds units or dimensions? What about energy, charge, power, and force?

A similarly flawed comparison is length per time (speed) versus length: "I walk  $1.5 \text{ m s}^{-1}$ —much smaller than the Empire State building in New York, which is 300 m high." It is nonsense. To produce the opposite but still nonsense conclusion, measure time in hours: "I walk 5400 m/hr—much larger than the Empire State building, which is 300 m high."

I often see comparisons of corporate and national power similar to our Nigeria–Exxon example. I once wrote to one author explaining that I sympathized with his conclusion but that his argument contained a fatal dimensional mistake. He replied that I had made an interesting point but that the numerical comparison showing the country's weakness was stronger as he had written it, so he was leaving it unchanged!

A dimensionally valid comparison would compare like with like: either Nigeria's GDP with Exxon's revenues, or Exxon's net worth with Nigeria's net worth. Because net worths of countries are not often tabulated, whereas corporate revenues are widely available, try comparing Exxon's

Wait, but that means you're comparing apples to oranges now: Nigeria's GDP for a decade, and Exxon's for a year. Wouldn't you have to compare it to the monetary flow of Exxon in the past decade...?

Ha, nevermind, didn't realize that the original quote compared GDP to net worth. That was silly.

I feel like I made the same observance. I was thinking that these two things are incomparable..but then later in the reading it explains what I was thinking.

but these can't be compared, one is over a period ten times the other

I don't get this reported business. Under what circumstances would the reported number be different than the actual number?

I think he means reported as just "1 trillion" the number without any time scale attached. Because without the time scale it's easy to lose sight of the fact that a net worth doesn't depend on a unit of time and a GDP does.

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This is an interesting point.

I missed this in my first read through of the excerpt above, and it pushes a rather strong argument against the "critics".

That is, in this case, Nigeria "makes" as much in a year than the entire value of a large corporation...

I agree. I hear some educated people make the same sort of error when they're arguing for/against certain policies. I wonder if there's some psychological block that causes us to make these sort of errors.

This intro was especially noteworthy for me because I have heard many times how a corporation has a higher net worth than a country's GDP, but never had I stopped to realize what that statement incorrectly indicates.

As for the "psychological block" you alluded to, maybe those statements are trying to say that a corporation is worth so much that it could run an entire country for a year?

Indeed, despite my economics nackground, I had not considered the arbitrariness of our definitions and the year.

I don't have an extensive background of economics outside of 14.01 and 14.02 but this example really drove home how important it is to really think about what we are comparing when we use invented forms of measurement.

I think my favorite part about this is the tone it seems to convey with towering over puny assets - it gives almost a comical view at how easy it is to play with units to change a number presented with improper dimensions

I think someone brought this out above, but at this point still, does this contrivance that Nigeria "makes" more even really have a point? Because we are still comparing GDP (a rate), to net worth at a time. Shouldn't we compare GDP to GDP, and net worth to net worth, although, measuring the "net worth" of Nigeria is a complex equation?

EDIT: oh nvm, the paragraph right below talks about this, haha.

Why are we comparing a net flow to an absolute amount? It doesn't seem like we want to compare them at a moment in time.

wouldnt Exxon's gdp change also, if we are choosing different time specs? or are you implying the 2 are measured in different ways? in that case, how is Exxon's gdp measured?

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this makes me wonder what would be an appropriate time frame (unit). in fact, is there an "appropriate" unit, people can play around with the units and therefore convey a totally different message

That's still pretty big!!!

Interesting. this is a good cautionary tale of looking beneath the words/numbers in statistics. Care always needs to be taken to reacting to these kinds of figures. I anticipate this unit will make us a much wiser consumer of information.

I agree - i really like this

You should read the book "How to Lie with Statistics" by Darrell Huff. Its about all the tricks that can be pulled with wording and numbers in statistics if you aren't careful about what you're reading.

I agree, this was a very insightful and interesting example of the concept of dimensions.

This is a wonderful argument and explanation.

It would be interesting to see how long it took Exxon to attain that net worth- i.e., did Exxon build itself up in a year or in 20 years?

Thats a good point, and what did their growth look like? similar to that of a nation?

Does this mean you can compare the GDP of Nigeria over the time span that Exxaon has been making profit and it be a valid comparrisson?

wouldn't the ratio of nigeria to exxon GDP scale relative to whatever time you are considering?

Is this comparison even valid? Is there a point to comparing between something that changes with time and something which doesn't (unless Exxon bankrupts or something that dramatically changes their worth)?

No, the whole point here is that the comparison is totally invalid because the units are different.

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A good way to put it for people who don't do a lot of work with numbers.

Adjectives are here to save the day and spice up the writing.

Alright I'm guessing we're going to end up comparing annual revenue to GDP.... I wonder if you could get the net worth of Nigeria lol assets? cash flows? Is it possible while remaining politically correct? what about human capital? Is it fair to compare citizens to employees?

I don't follow this logic- if you use the same time period, the GDP can be compared

Again, a good point. Reminds me of examples when people use lightyears as a measure of time...

This is a very interesting discussion - great way to open and motivate the topic of dimensional analysis.

Very good point. Claims like the one you mentioned above are used every day, and it's really up to the individual to pick out what is a lie and what is a twist of truth and what actually is truth.

Ever since I can remember, we have always been told to look at our units when comparing values. However, I find it extremely amusing how you show it is a very obvious error when you compare money, with money per time.

Er, this makes it sounds like the problem is the unit of time chosen (i.e., using an astronomical phenomenon), not just that the problem is comparing flow to net worth.

Maybe he is trying to point out the error in both. 1) we should probably use a time-scale that is characteristic of the phenomenon that we are trying to describe. Does anybody know what timescale this might be for GDP since the year is what is most often used in accounting? 2) One should not compare GDP and net worth because they are two different properties.

I agree with 5:58, the main point of this couple sentences is muddled by the phrase about astronomical phenomemon. It's ok that we use the year to measure GDP and we could also use something like 5-year periods to measure it. (I wouldn't really consider a decade a different "astronomical phenomenon" from a year since it relies on the same happening.) It's that we're comparing a rate to a time-independent quantity that's the problem. Mixing things that are both rates but with different time periods would be a different problem.



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Wow. I was totally fooled by this argument at first. I didn't even think to compare the units the two numbers were in. But this is a good point on how people make statistics mean whatever they want.

i didn't get this at the start, but it is the most important part. why can't the chapter start out with this?

There's a difficult tension between writing a handbook and a textbook. The handbook is for those who already know the material and want a quick memory jog. The textbook is for those learning it for the first time.

The two are quite different, because just telling people something is not a good way for them to learn and understand it. It's much better to build a structure in which the ideas come into play, and the reader forms their own understanding (with guidance).

But once they form the understanding, they are now closer to the experts, and would rather have the core ideas presented right away – but it might not have been instructive if it had been done that way.

Perhaps with the web there is a way to merge both kinds of books into one document?

This made me re-look at the quote in the beginning...i hate that people are able to get away with this kind of lie, arg.

Given that most of our examples have been scientific so far, and so few (if any) have touched on the social sciences, this is a nice change of pace.

I completely agree...its definitely relating approximation to everyday world. This was a useful example and definitely examines the usefulness of approximations.

I couldn't find where to put this comment, so I'll agree to this point (I do like economics examples), and also add that i think this first example was really useful in starting this new chapter.

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**Financial analysts often use income data for several years to project a company's lifetime net worth. Perhaps this could be used on countries. This method is called the discounted cash flow analysis.**

I don't think it's possible to calculate a realistic net worth of a country using a DCF. DCF itself for a company is very theoretical and sensitive to assumptions (about discount rate, projected cash flows, etc.), and I could only imagine this sensitivity increasing if looking at a country.

I agree. We are able to come up with projected cash flows from the current situation of a company and a market but it would be very difficult to look at the change in rate of a country. Being the second derivative implicitly makes it more sensitive.

**It would be interesting here to find a comparable dimension, to show just how flawed it is (like, figure out the monetary flow rate of Exxon)**

Or maybe the net worth of Nigeria. That seems like it would be a huge value, counting all sorts of infrastructure.

These ideas are intriguing... I would have never thought of this comparison without being enlightened by these facts.

**Excellent way of transitioning from on opening introduction to the topic of this chapter!**

I agree, I'm enjoying this example a lot.

That was a great example to lead into this paragraph. By showing the problems with comparing things with different units you show why it is important to take care when comparing data.

Yep, I also agree. Interesting, clear, and gives me reason to care about the topic at hand.

**This sentence is a good example of the difference between units and dimensions.**

I think this sentence should be in BOLD and called out instead of hidden in a parenthetical, since it's an important and maybe subtle point.

**I think this should say "A dimension is a general..."**

Whoops, never mind. My correction is completely off.

lol



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can you look at a quantity and know instantly that it is either a dimension or a unit or it is also a factor of context?

You can tell right away. If you can sensibly write a number before it, then it's a unit; otherwise it's a dimension. For example, "5 meters" makes sense but "5 length" makes no sense.

this comment would be good for a side bar.

Good clarifying statement. I think people interchange these in everyday speech, so this sentence helps clear up some possible misunderstanding.

I also like how you define the topic very early in the reading, even as you do it in an example. yeah, this is a great point. You simply can't compare a flow or rate with an amount. This is done a lot, and if you're not careful, you might not be getting the facts straight.

While I'm glad that you point this out early on, I wish you would have expanded on it more. I'm not sure I entirely understand what you mean.

Rereading the above example of money versus dollars, I now understand what you mean.

i'm confused between a dimension and a unit.

I'm not sure but I think a dimension is a general characteristic like time, distance, etc. A unit is a measurement system for dimensions (i.e. inches, meters, cubits, etc, for measuring distance)

might be good to italicize dimension and unit here

So, what would be a more proper comparison? Annual Revenue of Exxon vs. GDP of Nigeria? I think a revenue comparison is more apt than a profits comparison

I actually learned this in one of my Economics class and it is really cool that this concept is mentioned in this class as well.

Never mind, I see

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so this is equivalent to comparing velocity and distance?

Never mind, the very next paragraph answers my question.

Answered previous question. I see where this paper is going now

Why is this bracket here?

This is also the second time in this reading that this has happened?

Maybe there is supposed to be some reference to another section/appendix/something? It seems odd that there are multiple instances of hanging brackets...

My fault. I did not include the citation in the bibliography database, so the reference pointed nowhere. I'll fix that and the other spots (though I had done it before, but I must have missed a few).

What is a "category mistake" besides something that is "guaranteed to generate nonsense"

they belong to different categories, so it doesn't make sense to compare them. ie: apple and orange are 2 different things, and it doesn't really make sense to compare them

why category though? it's just wrong dimensions. hence, dimensional analysis.

I think i like dimension better than category as well

This clause seems unnecessary, why can't you just leave it at "Because their dimensions differ, this is guaranteed to generate nonsense"?

...or powerful propaganda for uneducated people

its true, a lot of platform statements are based on incomparable quantities or biased statistics

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**i don't consider this nonsense. though the time is in some ways "arbitrary," it's still important to compare with that value of a "year."**

But as we saw, it makes a big difference what unit of time we choose. If both sides of the comparison don't use the same unit of time then our answer doesn't make any sense.

I think about the amount of money something will cost me in comparison to my annual (or daily or monthly) salary all the time and it's not nonsensical. Similarly, house prices are compared to annual income, assets to annual income, PE ratios... It's not nonsense if we have a sense of what we're comparing. If we have a sense of a year, and a sense of the size of companies then it can be a useful comparison. It doesn't mean that the magnitudes have to be comparable (though that's probably the rhetorical point of the comparison and the danger of misused numbers).

**I loved this example as the start of this section- it's really intriguing.**

**however, one intimates the other.**

**This is a nice problem to start off with to get acquainted with what we're doing next.**

It's a bit sad that this actually took me a minute!

**These are units**

**This question would be more effective as "indicate which of the following are units and which are dimensions: ..." The way you have it is rather leading.**

**These, and energy, are dimensions.**

These are dimensions which are composed of the units: meters, kg, and seconds.

**This paragraph comes out of nowhere...it's unnecessary...and i think that it's actually distracting to the discussion on dimensions because it's just thrown into the middle of a section on economics (which it has nothing to do with.)**

**it sounds so stupid when you say it like this**

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**I really appreciate this comment because people say things like this all the time. I am quite positive no one really things about the importance of dimensions.**

I was just going to comment that I've never heard someone say something like this. The GDP/net worth example is a good one and seems realistic, but I don't think I've heard anyone compare speed and distance incorrectly.

Another thing that gets me- people measuring distance in units of time... As in, "the grocery store is 10 minutes away." No, the grocery store is half a mile away and it will take 10 minutes to walk there. This happens ALL THE TIME

I feel like when someone says the grocery store is 10mins away the "...by X method of travel" part is implied, but I agree people mix up units a lot in general conversation although I haven't quite heard one like the example given here.

I think the grocery store thing might be a better example (despite what is obviously implied) because I've never heard anyone say anything like the example quotes about the Empire State Building. Or if I have heard something it is different enough that I read those quotes and was rather confused by how anyone could let that pass...

In the grocery story case, I feel it's because people don't actually care how far away it is in units of distance but rather how long it will take them to get there.

I agree about never hearing the example here before. Perhaps it would be more useful to put an example that would be more relevant/related to students like the grocery store example given above.

I have also never heard this anything like this before. It seems so unreasonable, I think that anyone intelligent enough to use Meters, would not make any mistake like that.

I agree that nobody says the speed versus length comparison. That's why I chose it: because it's so obviously bogus that no one would say it. Yet the equally bogus GDP versus net-worth comparison is made so often. Somehow I should make it clearer why I chose the speed versus length comparison.



steady from year to year) would be roughly \$1 trillion per decade and be reported as \$1 trillion. Now Nigeria towers over Exxon, whose puny assets are a mere one-tenth of Nigeria's GDP. To deduce the opposite conclusion, suppose the week were the unit of time for measuring GDP. Nigeria's GDP becomes \$2 billion per week, reported as \$2 billion. Now puny Nigeria stands helpless before the mighty Exxon, 50-fold larger than Nigeria.

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I was surprised that the speed vs length example was even in here; i read it and said, "There is no way anyone would actually say that. It would be ridiculous." The grocery store one would probably not be as good as an example because I feel it's semantics. I know, personally, I say "It's 5 minutes away", which is just me rephrasing "It will take us about 5 minutes to get there."

The ridiculousness of it is exactly why it is in there. Nobody would say such a thing about speed vs length, but the equivalent comparison about GDP versus net worth is made all the time, and I want people to see, by analogy, how ridiculous that one is.

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Even though it's based on certain implicitly agreed upon conventions, this immediately makes me think of how people say that places are "2 hours north" or that they're "ten minutes away from you." I realize that it's followed by an unspoken "at a constant speed near the highway limit" or "at my current walking pace," but it's still something that sticks out in my mind.

I've never heard something like this without time also mentioned. Relating velocity to distance through time.

I think this is a good example to show the flaw in the earlier comparison. I know I'm not too familiar with GDP and net worth, etc. so this example really solidifies the issue.

steady from year to year) would be roughly \$1 trillion per decade and be reported as \$1 trillion. Now Nigeria towers over Exxon, whose puny assets are a mere one-tenth of Nigeria's GDP. To deduce the opposite conclusion, suppose the week were the unit of time for measuring GDP. Nigeria's GDP becomes \$2 billion per week, reported as \$2 billion. Now puny Nigeria stands helpless before the mighty Exxon, 50-fold larger than Nigeria.

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**why did you write the same example (essentially) twice?**

I guess he try to pinpoint that if you use different units,  $\text{ms}^{-1}$  and  $\text{m/hr}$  you would get totally different conclusions, once is smaller than the height of the Empire State building while the other is much larger than the height. Yet the speed is same but just expressed in different units

(OP) this makes a lot of sense actually, thanks

It's the same structurally, but in the Exxon/Nigeria form it is easy to not see the problem, whereas in the speed versus length form it is much easier to see. So, I was trying to make the first example as obviously problematic as the second one.

**I feel though you cannot compare Nigeria and Exxon, sure Exxon might operate in Nigeria, but it has plenty of offices in Houston and London that generate themselves millions of dollars in sales, of which Nigeria has no access to in these regions. So I'm not sure weather arguing for either case is correct**

I don't think he's arguing for either side, rather pointing out the error in comparison.

I think the comparison is just to show that a company has a higher production than an entire country. But here he's just pointing out the error.

**What author?**

**What!!!!?? that's so ridiculous! Why is that okay? Does the strength of a argument suddenly compensate for the accuracy?**

I feel like the accuracy should largely determine the strength of the argument.

It compensates if your reader is uneducated or easily manipulated...

This is completely repulsive. Making a mistake is one thing, but keeping in a mistake because it might fool a few people into believing your point is horrendous. It's akin to making up experimental data to suit your hypothesis. Awful.

This is why the only articles I read now are science ones. I hate journalists.

steady from year to year) would be roughly \$1 trillion per decade and be reported as \$1 trillion. Now Nigeria towers over Exxon, whose puny assets are a mere one-tenth of Nigeria's GDP. To deduce the opposite conclusion, suppose the week were the unit of time for measuring GDP. Nigeria's GDP becomes \$2 billion per week, reported as \$2 billion. Now puny Nigeria stands helpless before the mighty Exxon, 50-fold larger than Nigeria.

A valid economic argument cannot reach a conclusion that depends on the astronomical phenomenon chosen to measure time. The mistake lies in comparing incomparable quantities. Net worth is an amount: It has dimensions of money and is typically measured in units of dollars. GDP, however, is a flow or rate: It has dimensions of money per time and typical units of dollars per year. (A dimension is general and independent of the system of measurement, whereas the unit is how that dimension is measured in a particular system.) Comparing net worth to GDP compares a monetary amount to a monetary flow. Because their dimensions differ, the comparison is a category mistake [] and is therefore guaranteed to generate nonsense.

**Problem 5.1 Units or dimensions?**

Are meters, kilograms, and seconds units or dimensions? What about energy, charge, power, and force?

A similarly flawed comparison is length per time (speed) versus length: "I walk  $1.5 \text{ m s}^{-1}$ —much smaller than the Empire State building in New York, which is 300 m high." It is nonsense. To produce the opposite but still nonsense conclusion, measure time in hours: "I walk 5400 m/hr—much larger than the Empire State building, which is 300 m high."

I often see comparisons of corporate and national power similar to our Nigeria–Exxon example. I once wrote to one author explaining that I sympathized with his conclusion but that his argument contained a fatal dimensional mistake. He replied that I had made an interesting point but that the numerical comparison showing the country's weakness was stronger as he had written it, so he was leaving it unchanged!

A dimensionally valid comparison would compare like with like: either Nigeria's GDP with Exxon's revenues, or Exxon's net worth with Nigeria's net worth. Because net worths of countries are not often tabulated, whereas corporate revenues are widely available, try comparing Exxon's

**Hmmm interesting. Now that you've exposed the Nigeria-Exxon article, perhaps the readers of your book can write letters en masse!**

Only if Sanjoy is willing to name names...

Hardly sur[rising, this is what everyone does when posing an argument. Sad, but true. I too would love to see this article though.

It's entirely true people manipulate/present information in a certain way all the time to drive a point home. The interesting this about this example is that I don't think most people would even realize the error in the comparison to begin with.

Yeah, this example reminds me of how polling can be manipulated so that the poller's hypothesis can be confirmed. This is especially apparent in political polls when a specific party leader will conduct polls and claim that "80% of Americans agree with X,Y, and Z" but doesn't mention that the only Americans polled were members of his party.

**wow. I really want to be surprised by this comment, but I just cant.**

**Sound Logic is not as powerful as clever rhetoric if you are trying to sway the masses**

**I really don't find this surprising. He doesn't really sound like he's from a technical background, which isn't surprising since he's making politically motivated comparisons.**

Agreed -politicians only have to say things the majority of the voters will believe.... unfortunately most of them aren't talking dimension.

**I thought so too. I wonder what he numbers are like. Probably a lot weaker since the author didn't use them in the first place.**

I also thought this. I guessed and commented on these comparisons earlier in the reading although I didn't know they would be explained later.



annual revenues with Nigeria's GDP. By 2006, Exxon had become Exxon Mobil with annual revenues of roughly \$350 billion—almost twice Nigeria's 2006 GDP of \$200 billion. **This valid comparison is stronger than the flawed one, so retaining the flawed comparison was not even expedient!**

That compared quantities must have identical dimensions is a necessary condition for making valid comparisons, but it is not sufficient. A costly illustration is the 1999 Mars Climate Orbiter (MCO), which crashed into the surface of Mars rather than slipping into orbit around it. The cause, according to the Mishap Investigation Board (MIB), was a mismatch between English and metric units [ , p. 6]:

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Make sure to mind your dimensions and units.

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**Did you make the author aware of this?**

**And oil companies say they aren't ripping us off..**

**In what year was it 99 billion?**

The quote that says 99 billion comes from an article that was published in 2002, if that's any help.

**this seems like a silly statement to me: one is wrong, the other is right. of course it's better.**

**wow, I'm sure that author would be glad to hear that!**

And I'm sure he also feels silly for using a flawed comparison instead of a valid one when both confirm his hypothesis!

It shows the power and how easy it is to manipulate the written word. Hard facts always wins out in the end.

**when you wrote the author did you give him these numbers? or just say that the argument contained serious mistakes?**

**wow I'm really surprised. I thought the author was making the "mistake" on purpose. I guess there are other flaws than time when comparing assets and annual cash flows. But really what is a country and what is a company? Maybe there are even more errors in making this comparison than in the units.**

True, it may in fact be that Exxon Mobil has more political might than Nigeria

I agree but wasn't that the point of the statement in the beginning? I wonder why the author didn't just put this comparison originally.

**You should send this to the author!**

or just resubmit the paper with the correct calculations. seeing as the author didn't seem to want to do it.

**Did you explain that to the author?**

He probably didn't listen even if it was tried. people are stubborn

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Is this referring to the ignorant author referenced above? Because it seems like s/he was a totally unrelated anecdote to our nigeria example, but if they are one in the same I'd like to know as a reader.

I think after this you need to briefly explain what this - now valid - comparison means (tying up the "what kind of power relationship" quote).

This sentence sounds awkward. It took me a couple reads to understand what you meant. suggestion: The condition that compared quantities must have identical dimensions is necessary but not sufficient"

On this point, I would think there's a better statistic than revenue to compare to GDP.

I understand your point, that we need to mind our units, but it doesn't mean that we can't compare quantities with the same dimension but different units. For instance 1 meter &gt; 3 ft, etc.

Earlier in this sentence, he says, "the compared quantities must have identical dimensions", so I think that covers the case where you're discussing length with both meters and feet.

Yes, but my point is its possible to make valid comparison without matching units (and therefore, that matching dimensions is sufficient). It's certainly easier with matching units, but not necessary to have them.

saying that 1m/3ft is not ignoring units! in trying to figure out which one is larger...you do the conversion in your head - you are just talking about unit conversions. And people should still be careful with this - remember the Martian lander than landed below the surface of the planet..

hahahaha i should have read just one sentence later...

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**Several similar cases have come up in recent history. Why don't people learn to keep units consistent? Learn to include units when giving calculations to others!!**

Part of the problem also stems from having a globalized world without a common unit system. Same thing with language.

The situation is not great in America, but it's even worse in England (and maybe the rest of Europe). There, because the metric system is pretty much standard, all physics is done completely in the metric system and with SI units (meters, kilograms, seconds). So, teachers in high school tell students not to include units in the intermediate stages of a calculation but only put them in at the end.

This unwise advice used to drive me crazy, and I shared the Mars Climate Orbiter story very often with the students (one of the students had told me about it when he saw it reported in the newspaper).

In America, perhaps because feet and slugs and kilograms and meters are used in engineering, people are more careful to write units.

**Was this the example that was mentioned in a couple weeks ago? I always like when examples in lecture correspond to the readings!**

That was the example I mentioned in lecture.

**I think this is an overused example...but maybe it's just because we go to MIT that we've heard it so often?**

even so...its a good thing to keep in mind, so we never forget, just like the Tacoma-Narrows bridge (I see that video in every class too)

I have my doubts about whether this one was true. Don't they use simulations to test whether the program works? How could a gross mismatch in units slip through the simulations.

yeah I've also heard it said that they had to "type" the numbers into the program. who does that? they would have been read in automatically

I've never heard this before, so I'm glad for the example.

yeah I had never heard of this example either so it was really interesting and surprising when I read this

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**There's a Mishap Investigation Board? What types of incidents do they investigate besides NASA?**

That's a great name. It's an ad hoc thing that NASA does when things go wrong and a common military term, but we could probably use more investigations of mishaps.

**I guess these brackets are for citations that haven't been filled in yet?**

Good observation.

They are that. Though I thought I had found all those spots and had augmented the bibliography table – evidently not. That reference should be to, "Mars Climate Orbiter Mishap Investigation Board", Phase I Report, NASA (November 1999).

**Another good example of metric-imperial confusion:**

[http://en.wikipedia.org/wiki/Gimli\\_Glider](http://en.wikipedia.org/wiki/Gimli_Glider)

I find it amazing that such simple mistakes are made on multi-million (or billion) dollar projects. Obviously, none of us will make these mistakes after taking this class...

It takes 2 to make these mistakes...always check not just your work but others

That page has double value. I was thinking about using it as part of a discussion on gliding. Life is short and we didn't get to it, but I'll mention it in the book, perhaps in both places.

**How embarrassing.**

This happened with some plane company as well. This kind of stuff should never happen.

Just shows that the little mistakes we make on psets and stuff can be made by people in real life too, and with much more serious repercussions.

Yeah I've heard about this before. Ever since reading this I've made it a habit to include dimensions whenever necessary.

That guy's life was ruined



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**Why doesn't the world just pick one and stick with it? Wasn't there a country somewhere that drove on the English side of the road forever and ever until like a decade ago one day the government said "switch!" (with a clever marketing campaign and many signs and patrolmen) and then they did? I might be making this up.**

Wouldn't this cause even more of said problems? We're more or less in too deep in this situation and I think generally there aren't enough advocates or good reasons that outweigh the difficulties in the US to make this drastic change.

Possibly for a similar reason that everyone uses a different currency...

I think that was Sweden in 1967. The rest of Europe (except England, which was isolated by the Channel) used right-hand drive, so it made sense to do.

As for why left-hand drive started out, I heard various explanations while I lived in England. The most convincing, even if it isn't true, is that it originated on the original "highways" – roads for people to walk on. There, people walked on the left side and carried a sword (to defend against robbers or "highwaymen" as they were also called). The sword was normally on the left side, so that a right-handed person could reach across and pull it from the scabbard, and then would have the sword ready to protect their right side. Therefore, walk on the left to shield yourself from the rest of the road.

This choice, allegedly, was adopted when cars came to drive on the highways.

**I love the examples used to reinforce this lesson!**

**Could this sentence be fleshed out to "Make sure you mind your dimensions and units...." and something about the cost of the MCO?**

I think most people would have an appreciation for how much the MCO accident must have cost, so repeating it here might appear redundant. However, I sorta expected "and units" to be emphasized somehow (italics, bold, etc)

I like the succinctness of this sentence because it gets the point across well.

I agree, it acts almost like a concluding sentence. Correct dimensions and units are both necessary in approximation problems.

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Annual budgets of cities vs. endowment values for universities. This is a constant source of town-gown tension and newspapers don't help the situation by trumpeting endowment values. Budgets would be a more relevant comparison.

A common reoccurring example is the recycling issue. I always think if the recycling numbers are true when compared to the numbers that produces the bottles themselves. Or the numbers of reusable aluminum cans, the production and transportation of them might as well overpower the recycling and production of the plastic ones.

"now with 20% less plastic!"

Miles per gallon. When comparing the miles per gallon between a porsche and a pontiac, I've seen them say that they both can get 25 mpg. They fail to mention, however, that they were using premium gas for the porsche and regular unleaded for the pontiac, which I'm sure makes a big difference to someone who is looking to find the most fuelcost-efficient vehicle...

though i agree that this is an example of deceit, but i don't think the problem here is with units.

plus, it might as well be accurate. afterall, the person with the porsche is probably going to be the one that buys premium gas and the one with the pontiac will likely stick with regular.

This might not be relevant, but I heard that when they calculated the iron in spinach for the old Popeye cartoon they overestimated by a factor of 10.

Hehe

Did they overestimate the additional strength of his biceps due to the iron?

I find that this section was a really nice read and helpful to understand what we are going to be learning about.

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In 2.006, one of the early topics was dimensional analysis and it was all about forming dimensionless groups. I thought it was really cool how we could understand literally none of the physics about a system but using the dimensions of its critical components formulate important dimensionless ratios to solve the problem. I'm looking forward to the angle this class takes with this topic.

Actually, I'm beginning to notice that a lot of my engineering classes actually use a lot of the topics in this class. In my power electronics class they mentioned symmetry a bunch of times. And I just realized that in every one of my classes, a dimensionless constant is always used somewhere.

Isn't this another name for scaling.

You mean when comparing two numbers of the same dimensions?

This is a really useful section, and this is a very useful tool. I think that this section should possibly be expanded with a few more short examples; I believe that learning this well can get anyone through any type of estimation; find the groups and find an answer. (It's also a way to find equations; you have these knowns and you need length: so we need to end up with a group that equates to  $L$ ).

It actually took me some rereading to realize that this was referring to the Exxon example, since 'oil' wasn't specifically mentioned in that context (I was trying to think back to some estimation we had done on oil imports..). Maybe just stick with 'Nigeria' or 'Exxon example'?

Same here. Thanks for this comment!

Yeah I would consider rewording this.

It might be worth to at least mention, if not go into detail, the Buckingham Pi Theorem. For those who are interested in it here is the wikipedia page: [http://en.wikipedia.org/wiki/Buckingham\\_](http://en.wikipedia.org/wiki/Buckingham_)

Dimensionless ratio means all the units cancel right?

So it is more efficient to use ratios of quantities than absolute quantities? that may be help for for quick calculations but isn't that less practical?



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Well, you mean for estimation purposes, right? Not all the time...

I mean all the time! The universe doesn't care about our system of units (e.g. whether we measure speed in meters/second or furlongs/fortnight), so our descriptions of the universe shouldn't care about the system of units either. And the best way to ensure that our description doesn't care about the system of units is to use dimensionless quantities, for they are invariant to a change of units.

I was starting to see this pattern in proportional reasoning, and I'm glad we're looking at it now

I guess this is true actually—if you think about it, if someone told you a plane travels 500mph, you wouldn't instantly be like oh 500mph i know exactly how fast that is. You'd think to yourself, ok a car goes around 65 mph on a highway, then think about the ratio of speeds to get an idea of how fast that plane goes.

I think this is a great explanation! I think the paragraph could benefit from a narrative comparison like this.

I'm really glad that someone in class the other day asked about dimensions and proportional reasoning and why they don't always match in that case... I think you should include something about that in the reading (it also helped clarify the and proportional symbols)

Yeah, this was a huge help. And the quick example presented a couple of comments up helps to make this link between proportional reasoning and dimensionless ratios.

Yeah, because without the comment stated above with the example I would have been kinda confused as to why you don't care about dimensions. So by dimensionless quantities we are talking about dimensionless ratios? Not just dimensionless things in general like comparing degrees to a constant.

On that note, I saw the class notes from one of the other classes taught by Sanjoy (Streetfighting Mathematics maybe?) and there was a good little table explaining the difference between  $\neq$  and  $=$  and stuff. I think that would be good to include somewhere.

That's a nice explanation.

annual revenues with Nigeria's GDP. By 2006, Exxon had become Exxon Mobil with annual revenues of roughly \$350 billion—almost twice Nigeria's 2006 GDP of \$200 billion. This valid comparison is stronger than the flawed one, so retaining the flawed comparison was not even expedient!

That compared quantities must have identical dimensions is a necessary condition for making valid comparisons, but it is not sufficient. A costly illustration is the 1999 Mars Climate Orbiter (MCO), which crashed into the surface of Mars rather than slipping into orbit around it. The cause, according to the Mishap Investigation Board (MIB), was a mismatch between English and metric units [ , p. 6]:

The MCO MIB has determined that the root cause for the loss of the MCO spacecraft was the failure to use metric units in the coding of a ground software file, Small Forces, used in trajectory models. Specifically, thruster performance data in English units instead of metric units was used in the software application code titled SM\_FORCES (small forces). A file called Angular Momentum Desaturation (AMD) contained the output data from the SM\_FORCES software. The data in the AMD file was required to be in metric units per existing software interface documentation, and the trajectory modelers assumed the data was provided in metric units per the requirements.

Make sure to mind your dimensions and units.

#### Problem 5.2 Finding bad comparisons

Look for everyday comparisons—for example, on the news, in the newspaper, or on the Internet—that are dimensionally faulty.

## 5.2 Dimensionless groups

Dimensionless ratios are useful. For example, in the oil example, the ratio of the two quantities has dimensions; in that case, the dimensions of the ratio are time (or one over time). If the authors of the article had used a dimensionless ratio, they might have made a valid comparison.

This section explains why dimensionless ratios are the only quantities that you need to think about; in other words, that there is no need to think about quantities with dimensions.

To see why, take a concrete example: computing the energy  $E$  to produce lift as a function of distance traveled  $s$ , plane speed  $v$ , air density  $\rho$ , wingspan  $L$ , plane mass  $m$ , and strength of gravity  $g$ . Any true statement about these variables looks like

this seems like an extreme (not to mention counter-intuitive) statement

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i agree. often units can help guide you towards the right answer and are easier to deal with than dimensionless ones.

Its about getting things from one place to another using only unit. It wont always work, but I can only imagine that its very useful

What a fascinating statement

It would be nice to apply the same analysis for SHM to this problem and get a crude expression for the energy.

dont really understand what a "true statement" do you mean an equation that is valid

I think 'Any true statement' is hyperbolic. Do you mean "We could write a true statement about these variables that looks like..."

$$\triangle_{\text{mess}} + \square_{\text{mess}} = \circ_{\text{mess}}$$

where the various messes mean 'a horrible combination of E, s, v, ρ, L, and m.

As horrible as that true statement is, it permits the following rewriting: Divide each term by the first one (the triangle). Then

$$\frac{\triangle_{\text{mess}}}{\triangle_{\text{mess}}} + \frac{\square_{\text{mess}}}{\triangle_{\text{mess}}} = \frac{\circ_{\text{mess}}}{\triangle_{\text{mess}}}$$

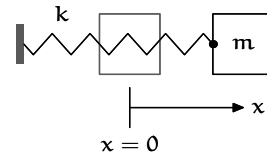
The first ratio is 1, which has no dimensions. Without knowing the individual messes, we don't know the second ratio, but it has no dimensions because it is being added to the first ratio. Similarly, the third ratio, which is on the right side, also has no dimensions.

So the rewritten expression is dimensionless. Nothing in the rewriting depended on the particular form of the true statement, except that each term has the same dimensions.

Therefore, any true statement can be rewritten in dimensionless form.

Dimensionless forms are made from dimensionless ratios, so all you need are dimensionless ratios, and you can do all your thinking with them. Here is a familiar example to show how this change simplifies your thinking. This example uses familiar physics so that you can concentrate on the new idea of dimensionless ratios.

The problem is to find the period of an oscillating spring-mass system given an initial displacement  $x_0$ , then allowed to oscillate freely. The relevant variables that determine the period T are mass m, spring constant k, and amplitude  $x_0$ . Those three variables completely describe the system, so any true statement about period needs only those variables.



**haha this is exactly how physics works in my head**

This is how most of my class work looks like.

**I don't know if you are missing a quote at the end or this is just a typo**

mm I read it as "prime a" but I guess that doesn't make sense... lol

**How is this useful?**

**I'm not sure using shapes here is the best way to communicate your point. I understand what you're saying, but I can also see how other readers (perhaps non-MIT students) might be confused. I would consider reworking this.**

Or, you could keep the shapes and just make note that same shapes imply that the two "messes" have identical dimensions.

I actually think this is really clever and makes the point clear.

I also like the pictures, it reminds me of high school when we learned about how to convert dimensions by "canceling them out." The point was that it didn't matter what you were multiplying (3 squares divided by 7 circles) as long as the top equaled the bottom.

I had to look at it twice to get it, but I really like visual examples and this one make sense to me

The shapes serve as a good visual, but I think a quick, basic example should be included as well (much more simpler than the spring example).

I think it was a great example, especially for the visual learners like myself that did not benefit as much with all of the math equations and essays from previous chapters. Also, it requires no context or problem set-up.

**So what kind of dimensionless constant could you use for comparing the company and the country?**

**This explanation is still a little fuzzy to me... I don't really understand how we can go from 3 "messes" to 1 dimensionless statement.**

**How do we know this is true?**

$$\triangle_{\text{mess}} + \square_{\text{mess}} = \circ_{\text{mess}},$$

where the various messes mean 'a horrible combination of E, s, v,  $\rho$ , L, and m.

As horrible as that true statement is, it permits the following rewriting: Divide each term by the first one (the triangle). Then

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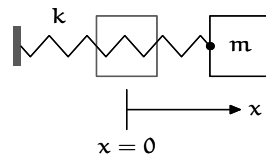
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This is pretty neat. I never thought about it like that before.

they should teach this reasoning in high school, dimension theory is somewhat avoided completely

This is really neat—so then any statement can be written as dimensionless? by just dividing by one of the terms?

Yes, any statement can be rewritten as dimensionless. And yes, just by dividing by one of the terms since all the dimensions match to begin with, dividing everything by the same term will still keep all the dimensions the same for each term and leave it dimensionless.

This is a clever trick and definitely worth remembering.

It's also worthwhile to point out that this works because, by design, everything had the same dimensions from the very beginning. Which could be another potentially useful fact.

–Edit: And reading another two lines down I see that this was brought up. :P

I agree, really cool way to think about problems!

I actually don't see how adding different dimensionless ratios can give any valid information, even if the dimensions work out

I don't understand how adding this ratio to a dimensionless one is proof that it is dimensionless...?

I believe the point is that because the first one has no dimensions, adding the second one and getting a third requires that the final two have no dimensions. It's just a rule of math.

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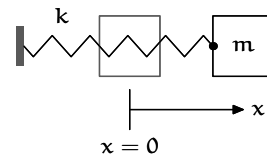
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Interesting point. I hadn't realize this until reading this section.

Yep this is really cool.

Hmm, interesting. Can someone give me a quick simple example?

I think the easiest way to think about it is that a lot of true statements that we assert can be expressed as equalities. When you take a true statement, such as  $F_{\text{drag}} = F_{\text{gravity}}$ , the dimensions on both sides of the equation are already the same. The left and right side of the equation are in the units of Newtons (dimension would be force). At this point any division of the two sides that preserves the equality also preserves the same dimensions (its like an invariant of equalities). If you simply divided left by right you would have two dimensionless sides that contain the exact same information as the initial equation (force/force = 1).

Why is this true?

Should this be "any true relation among physical quantities"? I don't see how to write the sentence "Mr. X's favorite dimension is the length dimension" in dimensionless form.

And why the restriction to true statements? Surely any false statement can also be rewritten in dimensionless form. Sentences like "the ship's velocity is equal to its mass" are not false, they're meaningless.

You're right that I should add the restriction "among physical quantities". (Now I had better check whether I said it correctly in *\_Street-Fighting Mathematics\_*.)

That's a good point about false versus meaningless statements. I never thought about it that way before, and I think I'll revise the claim to, "any meaningful relation among physical quantities." Although sometimes it's hard to tell the difference between false and meaningless (as the Exxon/Nigeria quote shows).

We've always been taught that units needs to match in an equation. Time can't equal speed. I run into the problem of needing to make things dimensionless a lot in coding models.



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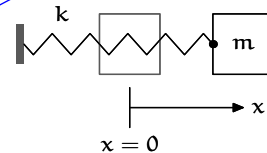
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Ok so after the first section, which showed exactly how important the dimensions are, why are we being convinced to forget about them? What's the advantage?

You won't be forgetting them. You'll use them – to figure out how to write statements in dimensionless form. The dimensions will then tell you the possible structures of that dimensionless statement.

If you are trying to make something dimensionless, is it reasonable to divide the quantity by another quantity with dimensions that you already know the quantity of? I.e. I am trying to make mass dimensionless and I divide it by the mass of a ball that I know.

If you do that, you are never going to get any answers....if you just divide by the mass of your ball for example, you will just get 1...

It's good to make quantities dimensionless, but you want to use a relevant mass as the standard of comparison (i.e. as the denominator. So, if you are comparing various sports, it might be useful to list all the ball masses in terms of one of them.

But wouldn't their dimensions effectively become the ball? I.e. a basketball's volume is 2.5 footballs (that was a guess).

I understand from physics why these variables completely describe the system, but how would I know this a priori?

One way is to think, "How can I describe the system completely, and what information is required for that description?" Here, the spring differential equation and initial conditions completely describes the system because its solution tells you the entire (past and) future history of the spring.

To write the differential equation, you need to know k and m. To write the initial conditions, you need to know  $x_0$  (the initial velocity is zero by assumption). So k, m, and  $x_0$  are all you need to put into the system. And the period T is what you get out of the system.

I think here you don't have to know it, you're being told. A deeper explanation of \_why\_ is probably beyond the scope here.

Since any true statement can be written in dimensionless form, the next step is to find all dimensionless forms that can be constructed from  $T$ ,  $m$ ,  $k$ , and  $x_0$ . A table of dimensions is helpful. The only tricky entry is the dimensions of a spring constant. Since the force from the spring is  $F = kx$ , where  $x$  is the displacement, the dimensions of a spring constant are the dimensions of force divided by the dimensions of  $x$ . It is convenient to have a notation for the concept of 'the dimensions of'. In that notation,

$$[k] = \frac{[F]}{[x]},$$

where [quantity] means the dimensions of the quantity. Since  $[F] = MLT^{-2}$  and  $[x] = L$ ,

$$[k] = MT^{-2},$$

which is the entry in the table.

These quantities combine into many – infinitely many – dimensionless combinations or groups:

$$\frac{kT^2}{m}, \frac{m}{kT^2}, \left(\frac{kT^2}{m}\right)^{25}, \pi \frac{m}{kT^2}, \dots$$

The groups are redundant. You can construct them from only one group. In fancy terms, all the dimensionless groups are formed from one *independent* dimensionless group. What combination to use for that one group is up to you, but you need only one group. I like  $kT^2/m$ .

So any true statement about the period can be written just using  $kT^2/m$ . That requirement limits the possible statements to

$$\frac{kT^2}{m} = C,$$

where  $C$  is a dimensionless constant. This form has two important consequences:

1. The amplitude  $x_0$  does not affect the period. This independence is also known as simple harmonic motion.

Var	Dim	What
T	T	period
m	M	mass
k	MT <sup>-2</sup>	spring constant
x <sub>0</sub>	L	amplitude

There was a long interlude, so now I don't remember what we are doing and what the "next step" is exactly. Maybe clarify just a little in this sentence. The reader shouldn't have to flip back a page to understand your thought process.

These two capital T's are confusing. At first, I had gone through the pages thinking they were the same.

Only later did I realize one was serif and the other sans serif. Perhaps, it would be clearer and more consistent if you used a small "t\_p" for the variable time or just something to visually distinguish the two.

agreed

It's hard because the period is usually denoted T and if you're abbreviating dimensions, it seems most straightforward to have them as capital letters. An unfortunate coincidence here, you're right.

This makes me wish you could call my 7th grade algebra teacher and get points back for 7th grade me for not writing units down in my answer.

haha, my 5th grade math teacher would always ask us if the units were aardvarks when we didn't write any down. Although we did just demonstrate the first part of this reading that dimensions are very important!

If your answers were supposed to have units, I don't think Sanjoy would be on your side.

Besides, wouldn't your answers be different numbers if you wrote them in dimensionless ratios?

But I do agree with your point.

**Making a little table like this is very useful when performing dimensionless analysis**

it is helpful for analysis, but i still dont see the benefit to doing extra work just to find dimensionless quantities.

Well this looks like it could have helped the people building the Mars spacecraft...I think the benefit in working with dimensionless units is that they are more helpful when working with more people.



Since any true statement can be written in dimensionless form, the next step is to find all dimensionless forms that can be constructed from  $T$ ,  $m$ ,  $k$ , and  $x_0$ . A table of dimensions is helpful. The only tricky entry is the dimensions of a spring constant. Since the force from the spring is  $F = kx$ , where  $x$  is the displacement, the dimensions of a spring constant are the dimensions of force divided by the dimensions of  $x$ . It is convenient to have a notation for the concept of 'the dimensions of'. In that notation,

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unless  $F$  has a unique coordinate system, or it's made clear that we are only discussing magnitudes, I feel like the minus sign shouldn't be omitted from  $F = -kx$  (a system with  $F = kx$  would have no periodic oscillation given  $k > 0$ )

I don't think the sign matters because we are looking at the dimensions of the formula not the final answer.

May be good to explicitly say that the notation is brackets around the symbol for the quantity.

I really like this way of determining the units.

This is only a bit confused because of the odd use of brackets earlier!

I think they were in a different context

Since any true statement can be written in dimensionless form, the next step is to find all dimensionless forms that can be constructed from  $T$ ,  $m$ ,  $k$ , and  $x_0$ . A table of dimensions is helpful. The only tricky entry is the dimensions of a spring constant. Since the force from the spring is  $F = kx$ , where  $x$  is the displacement, the dimensions of a spring constant are the dimensions of force divided by the dimensions of  $x$ . It is convenient to have a notation for the concept of 'the dimensions of'. In that notation,

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Var	Dim	What
$T$	$T$	period
$m$	$M$	mass
$k$	$\text{MT}^{-2}$	spring constant
$x_0$	$L$	amplitude

**I did not know this relation, where did it come from?**

$F = ma$  where  $m$  has dimensions of  $M$  and  $a$  has dimensions of  $L/(T^2)$ . (Think about common units for acceleration:  $m/(s^2)$ .)

maybe you could explicitly define  $M$ ,  $L$ , and  $T$  as mass, length, and time somewhere? Especially to distinguish  $M$  from  $m$ ... (Mass in general from a specific mass from meters... I think we need more letters

This got pretty confusing too...why do so many things start with  $m$ ?

He does define them in that box at the top of the page.

That's true. But in terms of clarity, I am a proponent of the easier it is to decipher, the better it is in terms of benefiting the reader.

In this case, perhaps being more literal by writing out  $[F]=[\text{Mass}][\text{Length}][\text{Time}]^{-2}$  would help since that is the most basic starting premise.

Then, moving to substitute  $[\text{time}]=[\text{Period}]$  with the justification that the magnitudes are meaningless.

Lastly, specifying the relations for  $[x]=?$ ,  $[k]=?$  and how they match the parts of  $[F]$ .

Then say that you are choosing the  $[k]=$  relation to form a dimensionless combination.

Another thing to consider is whether it might be more appropriate to use "proportional" signs instead of equal signs in  $[F]=$ ,  $[x]=$ , and  $[k]=$ . This way, the  $C$  is not so unexpected, when it shows up.

This comment and several others have got me thinking about adopting the following policy in the book: No abbreviations! Edwin Taylor, who wrote the best special-relativity textbook (*Spacetime Physics*) ever written, told me that he is doing that for the revised edition of his general-relativity textbook (*Exploring Black Holes*).

The idea intrigued me as soon as I heard it, and the more comments I read, the more convinced I am that it would be a good idea. In short, why create roadblocks to understanding, even small ones?

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very well explained, thank you.

though i agree that this is most certainly correct, i find that, in practical problem solving, it is usually far more helpful to say things like  $[x]=kg$ , and  $[k]=kg*s^{-2}$ . this helps prevent problems such as that of the mco.

I also find myself referring more often to the units being used rather than general dimensions like Mass or Time. Are there any disadvantages to taking this approach instead?

Scale could be a problem, if you're thinking in milligrams about an elephant. As in the previous unit, it's generally better to think "if this system had more mass, then  $x$  would happen" instead of "if  $x$  had more kilograms." Force is mass times distance over time squared, not a kilo times a meter over squared seconds (that's a Newton, not force, and if you picked other units, you might be thinking about some unusual unit no one uses) Try just using the particulars of the units to remind you which dimensions you need (oh, a Newton has kg in it, so that's mass...) and then cast aside the particular units.

I really like how you were able to make dimensional analysis systematic. However, sometimes it's hard to pick the powers just by inspection - sometimes you can make combinations involving things to the 2/3 power and what not. It'd be nice to learn a systematic way to do that part too

You might want to be more specific...'The quantities reliant to this problem are, however, redundant: you can construct them all from just one group.'

So then when you find these, what do you do next? you just rewrite the equation in terms of this "dimensionless" unit?

I'm confused by how  $T$  went from something that seemed like a dependent variable of the 3 things that completely defined the system (above) to now being on the same footing as those variables when we look for dimensionless combinations. I knew beforehand that  $x_0$  didn't affect the period, but why is THIS the outcome of our finding that it's not in the dimensionless constant? (devil's advocate: why not say that  $x_0$  doesn't affect the mass?) You sort of cheat to get there by not including it in the list, since  $kxT^2/mx$  is also a valid ratio. There could be a comment about combining things in the simplest way possible.

Since any true statement can be written in dimensionless form, the next step is to find all dimensionless forms that can be constructed from  $T$ ,  $m$ ,  $k$ , and  $x_0$ . A table of dimensions is helpful. The only tricky entry is the dimensions of a spring constant. Since the force from the spring is  $F = kx$ , where  $x$  is the displacement, the dimensions of a spring constant are the dimensions of force divided by the dimensions of  $x$ . It is convenient to have a notation for the concept of 'the dimensions of'. In that notation,

$$[k] = \frac{[F]}{[x]},$$

where [quantity] means the dimensions of the quantity. Since  $[F] = MLT^{-2}$  and  $[x] = L$ ,

$$[k] = MT^{-2},$$

which is the entry in the table.

These quantities combine into many – infinitely many – dimensionless combinations or groups:

$$\frac{kT^2}{m}, \frac{m}{kT^2}, \left(\frac{kT^2}{m}\right)^{25}, \pi \frac{m}{kT^2}, \dots$$

The groups are redundant. You can construct them from only one group. In fancy terms, all the dimensionless groups are formed from one *independent* dimensionless group. What combination to use for that one group is up to you, but you need only one group. I like  $kT^2/m$ .

So any true statement about the period can be written just using  $kT^2/m$ . That requirement limits the possible statements to

$$\frac{kT^2}{m} = C,$$

where  $C$  is a dimensionless constant. This form has two important consequences:

1. The amplitude  $x_0$  does not affect the period. This independence is also known as simple harmonic motion.

Var	Dim	What
$T$	$T$	period
$m$	$M$	mass
$k$	$MT^{-2}$	spring constant
$x_0$	$L$	amplitude

in 2.006 I thought there was a slightly more formulaic way to find what the dimensionless groups were- any suggestions?

I like this way of thinking - looking forward to this unit.

I agree, this is some pretty eye opening stuff.

So these dimensionless constants also help us see more simplicity in the problem?

Is making something dimensionless also kind of like abstracting?

Yes! The dimensionless quantities are themselves abstractions. For example, the famous Reynolds number, a combination of speed, size, and (kinematic) viscosity, is dimensionless – and it hides all the details that we don't care (like the particular speed or size or viscosity). All we care about (in some situations) is the Reynolds number itself.

I think this second sentence should read 'This independence is a property of simple harmonic motion' for two reasons:

- Simple harmonic oscillation is usually defined by the system obeying Hooke's law ( $F = -kx$ ) (and not usually by the result that the amplitude is constant)
- Technically, simple harmonic motion is not the only way to have this independence (i.e., the independence of amplitude and period can exist for other types of systems).

I think you should point out the usefulness of this tool in its ability to determine that it is simple harmonic motion. Sure, it's nice to use dimensionless analysis, but it is also good to be able to omit dimensions from a problem to understand what is causing changes in the system.

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The requirement that dimensions be valid has simplified the analysis of the spring-mass system. Without using dimensions, the problem would be to find (or measure) the three-variable function  $f$  that connects  $m$ ,  $k$ , and  $x_0$  to the period:

$$T = f(m, k, x_0).$$

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Here  $h()$  means a function of no variables. Why no variables? Because the right side contains all the other quantities on which  $kT^2/m$  could depend. However, dimensional analysis says that the variables appear only through the combination  $kT^2/m$ , which is already on the left side. So no variables remain to be put on the right side; hence  $h$  is a function of zero variables. The only function of zero variables is a constant, so  $kT^2/m = C$ .

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**Do you mean it is independent of both of them, or that it only depends on their ratio and not on them individually?**

It only depends on the ratio and not each of them individually. Sort of like a lot of the examples we have been doing this past unit.

Why not also say independent of  $T$ ? The useful outcome of this that I'm most used to is the formula  $T = \sqrt{m/k}$ , and it might be nice to bring this up as a result.



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I'm missing something.

Right now, I am thinking that  $C$  is constant for a specific set of  $k$ ,  $m$ ,  $T$ . Why would a different set of  $k$ ,  $m$ ,  $T$  necessarily have the same value of  $C$ ?

How does that follow from above?

Ok, I think I get it now.

You started from the equality:

$$[F] = MLT^{-2}$$

and matched the different dimensions with their variables.

Then rearranging the variables to one side, you'll be left with a unitless unknown factor,  $C$ , that accounts for not worrying about exact relations when focusing only on the dimensions.

Since the force relation is an equality, every set of  $k$ ,  $m$ ,  $T$  must obey that and thus,  $C$  will be constant.

There are 2 things that impeded my understanding initially:

1) no visual representation of the mapping/substitution when matching the dimensions of  $k$  and  $m$  into the dimensions of  $F$ .

2) using period instead of time within the dimension of  $[F]$  itself. I thought it was a nonspecific time at first and also the two similar looking letters for the variable and the unit helped hide it all the more. This is rather a sneaky/subtle maneuver, because on first thought, I wonder whether it is appropriate to use Period in the units for Force instead of a generic "time,  $t$ ". But then, I realize it won't matter, since the magnitude of the period,  $T$ , will be absorbed/offset by the extremely accommodating unknown constant,  $C$ .

This is really cool and challenged my original preconceived ideas, especially the inflexible way I was taught of adhering to classical representations of these relations. The dimensional analysis approach eliminates the inflexibility of exact relations by extracting useful ideas in the form of a group of familiar dimensions and shifting all the trivial exact magnitudes onto a constant,  $C$ .

shiny! very useful to know...thanks

Why is  $C$  independent of  $k$  and  $m$ ? Aren't they in the equation for it?

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This whole statement is confusing. Aren't  $k$  and  $m$  in the constant definition?

So using this type of dimensional analysis actually gives insight to problems—I never would've thought so! Before I always thought dimensional analysis was just for checking your answers.

Indeed it is simpler. I wish I would have had this insight when I was solving physics problems earlier in my academic career.

I guess it might be easier to figure out the dimensions of a problem especially physics before you tangle the problem...It would definitely make it easier to think more clearly about the equations I might want to use.

I'm not really sure that defining  $h$  as a function of no variables instead of just stating that it should be a constant really adds anything to the discussion.

I think you may have mixed up left and right, or else I am very confused. If you were referring to the right side of the equation  $T = f(m, k, x_0)$ , then it might make more sense. This whole section could be much clearer.

are you saying that  $h$  is a function of 0 variables because all the variables are already being used on the left side? so if you didn't use one of the variables would the right side be a function of that? and both sides are always dimensionless correct?

variables is not a concrete term, a variable is something that you don't know, just because it has no variables in this case doesn't mean it always is variableless

Right side of which equation?

It is confusing as it is worded. It means the equation presented immediately above, the right side of which is  $h()$ . If the right side were to have variables, they would have dimensions  $k$ ,  $T$ , and  $m$  (no other dimensions are relevant here). However, we know by dimensional analysis that they would have to take the form  $kT^2/m$ , which is already on the left side. Thus it can be said that  $h()$  depends on nothing.

The fact that he says " $h()$  depends on no variables", and then says: "the right side contains all other quantities" seems contradictory, as worded.

I'm not sure about this either.

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"Why no variables?" would make a good side bar ... you only need to read it if you don't understand why  $h()$  is a function of no variables. [i read it thinking...yeah, yeah, come on, next]

I'm still a little confused. So I understand the example, but I don't see what situations dimensionless groups would be useful in. Since dimensionless groups can represent (what is seems like) many different things, how does it help us in finding answers to our calculations?

An easy useful dimensionless number is the Reynolds number, and depending on the number, you know if the flow is turbulent or laminar. Also, to find the heat transfer coefficient as a function of known properties, you use the Nusselt number. In this case, the point is if you know  $C$ , and you know  $T$  and  $k$ , you can solve for  $m$  no problem.

I understand how you made things dimensionless but not what the solution of the problem. Does this mean that the period is a constant?

It might be worth it to mention that  $C$  turns out to be  $4\pi^2$  and say as an exercise prove this through the physics.

what era was he most prominent? using measurements such as bookcases and libraries seems to date him.

Roughly, the whole of the 1900s. He was a brilliant physicist and statistician. Alas, his approach to statistics, the Bayesian approach (pioneered by Laplace), got overshadowed by the biologists' approach (p-values, confidence intervals), which is now called "orthodox statistics."

But the winds are turning toward Bayesian statistics, and I'll try to show the fundamental ideas in the unit on "probabilistic reasoning" (part of the next group of methods on how to discard information).

To prior to the last 10 years? People still have all these items in physical form, and we still use these concepts as metaphors all over the place (web pages, DNA libraries...).

what is meant a table of functions of one variable? is it a table of every function that that variable is in?

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I ran across this quote when I was in high school, and estimated that he wasn't too far off!

That's a cool quote, it really helps add to the argument here about the power of dimensionless quantities. I'm glad its in the text.

How would this work with equations whose forms depend on dimensions? For example, Maxwell's Eq look different for CGS units and SI units.

I think they look different for different definitions of units, not that the forms depend on dimensions per se.

Er, wait, I'm confused. Are we trying to find dimensionless invariants, or are we trying to use dimensions...?

That phrasing needs improvement. I meant, use dimensions to find dimensionless quantities. And these quantities are invariants in that they are independent of the system of units (e.g. if you change from meters to furlongs, you don't change the Reynolds number).

This explanation makes a lot of sense. The next paragraph also ends up clearing a lot of the question up.

sounds like a concept in group thoery

This is a really nice paragraph tying together a lot of what we've learned and really makes me think about how I should be approaching problems.

I think what becomes key when using dimensionless analysis is that we solve for an INVARIANT.

This definitely clarifies matters in terms of how this ties in with the previous sections.

I agree, this paragraph really ties past units together nicely! before I was confused as to how we were supposed to know what invariants to look for, now I realize that looking at units is a really great way to define them. However, some invariants that depend on numbers alone (the cube game, for example) would still be difficult to find...

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Are we going to develop a method for looking for dimensionless groups? We did in 2.006 and I'm curious how this class will do it.

We will, but you won't be surprised to know that I hardly ever use or teach the formal, linear-algebra method (where you solve simultaneous equations). Instead we'll find them by educated guessing.

"is a recipe"

Before reading this section, I never really thought much about dimensional analysis beyond being something that I could use to check my answers. Now, I'll try to use them to find answers or simplify problems before I kill myself trying to solve something harder than it should be.

Yeah! My thoughts exactly. This makes dimensional analysis much more useful than I initially thought.

I completely agree! It definitely help to see how all the items of this class are important and in some how related. Initially when looking at the different forms of approximations we have used I assumed after this class I would only use divide and conquer. However I am starting to see how everything is related and see the significance of most of the approximation techniques we have used so far.

This somewhat occurred to me back when we did abstraction, and I'm glad to see my hunch was correct

Any quick examples?

this is very much in line with proportional reasoning, "look for what does not change"

yeah this is a cool way to build on that approach

Yeah it really shows why we did proportional reasoning before this section. I like it.



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**Could we get an example of how we would use this to actually solve a problem?**

I'm guessing those will be forthcoming, but the process as I remember from 2.006 is to get your dimensionless groups and then go to the lab and run experiments to see how they are related to each other. Once you have the relationship you can extrapolate to find a lot of other stuff.

"All good things come to those who wait." The upcoming sections, plus the lecture examples, will I hope answer your question.

**I particularly enjoyed this chapter and am looking forward to the rest of the units. I used dimensional analysis a lot in a high school physics competition when time was running out and I just starting guessing answers!**

And it's also saved me from writing a wrong answer when I realize my units don't match...