5.5 Buckingham Pi theorem

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$$\text{no. of independent groups} = \text{no. of quantities} - \text{no. of dimensions}.$$

Test this statement with additional examples:

Comments on page 1

Read Section 5.5 (Buckingham Pi) and 5.6 (drag) for the next memo. It's due Wednesday at 9am (since this is posted later than usual). I've also included the end-of-chapter problems, which is why this assignment seems longer than usual, but there's no need to comment on those pages.

I am not too happy with the transition from 5.4 to 5.5. Other sections seemed to have a progression, but this transition seems to me very distant and leaves the previous section hanging. It feels better to read them at very separate times than one after the other.

I thought that this entire section was very clear and a good way to explain the theorem. It may be useful to go back and show the actual theorem at the end of this section though.

I agree... i really like how he relates it to an example we recently went over as well. It greatly helps the understanding.

...step in any dimensional...

typo

insert: made

This?

Do you mean "That task is made simpler..."?

What are we referring to when we say groups

the B. Pi theorem may even be useful a little earlier – we learn this in 2.006, and so I was already thinking about it when reading the previous sections.

Is this somehow related to Buckingham Palace?

haha, unfortunately it doesn't. The theorem is named after a late 19th - early 20th century physicist.

http://en.wikipedia.org/wiki/Buckingham_%CF%80_theorem

It is important for this topic, specifically dimensional analysis. ;)

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Test this statement with additional examples:

Not familiar with this...maybe some background?

It's not clear here, but he's about to describe how to do it. It might be helpful to mention that it's the method used to form dimensionless (Pi) groups.

I agree, even a sentence here, or maybe for the intro, about what the buckingham pi theorem is, would be nice!

Oh that helped - i didn't have any clue how pi was related to the discussion

Is there a reason for it to be named Buckingham?

This is good because it lets us know why we're doing all of the example that we do later.

i don't exactly like the way this is introduced. "possible beginning" sounds a bit awk

why is this helpful to us?

It helps you figure out when you are done coming up with groups

how about rewording it as: "it is helpful to begin your theorem statement as 'the number of dimensionless groups is...' "?

does this theorem only help you decide after you pick out the relevant parameters. because if that is the case it doesn't seem like it would be that useful

It would be helpful for me to see a table like the ones before with the dimensions and meanings of each variable (theta, G, m, r, c) just so that I could see concretely that these groups are dimensionless and then be able to more easily find additional examples myself.

I think this list of dimensionless groups all stems from theta and Gm/rc2. Everything in this list is multiples of those, so a table wouldn't really do much clarification.

I agree, the paragraph may seem a bit mathematical, but having prior knowledge of what is being talked about makes it understandable.

Why is this include as well as theta? doesn't this lead room for theta^3?

I think the point here is that there are infinite possibilities with just theta alone, since it is dimensionless and thus any theta (squared, cubed...and on) would also be dimensionless

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Test this statement with additional examples:

How did we jump from light bending to hydrogen? Maybe you shouldn't mention the light bending problem first if it can't be solved

We're just going over the results we've derived before, so I think this is fine in context.

We are just briefly revisiting problems we've seen in previous sections

I like that he went back to the previous section, though i can see why the jump would be confusing. maybe open with this and then bring on the light bending?

I am not entirely confident on what is being explained here. Is it simply a motivation for why 'independent dimensionless groups' is a better statement?

It's showing that if we don't require independence, then the number of dimensionless groups is either 0 or infinity, which isn't a very useful conclusion.

Is he trying to tease us of what the theorem concludes? He explains it's possible to have infinite or zero groups.

It makes sense first read-through. I think he's just elaborating on an example we already looked at.

Yeah, the main idea is at the last sentence?

Is this saying that for ANY list, the number of possibilities for dimensionless groups is either 0 or infinity?

Yes.

It says it seems that way, it doesn't feel like thats it though.

It is true... you can always take a different multiple or exponent and have a different dimensionless group. The key here which is explained in the next paragraph is the idea of independent dimensionless groups

You are right, although we don't care about multiples...just exponentials.

I think the statement is accurate...it does seem that a dimensionless group could be formed from almost any combination of variables once you get started...but then maybe you start to think about the number of combinations possible considering the different variables involved.

This is logical though, because if there is a dimensionless group at all, you can manipulate any number of dimensions by multiplication/cancelleation of choice to maintain that.

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Test this statement with additional examples:

So if the amount of dimensionless groups is zero how can we form a relation and solve the problem?

By what you showed earlier it seems like if one group is possible then infinite dimensionless groups are possible?

Yup. Like with theta, and any combo that ends up dimensionless...since now there are no dimensions, you can divide it by any unity, or just square it or cube it or modify it in any way you want to so long as you don't add an extra diimension

I like the way that you did this. It draws attention to the fact that it is important that the groups must be independent without just saying "this is important." I think doing it this way makes it easier to understand (things are broken into pieces) and helps you remember it better

Seeing how "independent groups" is used frequently throughout both of these sections it might be helpful to explicitly explain what they are.

I think it just means that the group isn't a multiple of a group you already came up with or the addition of two groups...

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Test this statement with additional examples:

in that they have no overlapping variables?

I think so

I think they can have overlapping variables...I think this means that one dimensionless group is not a function of the other. For example, theta and Gm/rcˆ2 would be independent, but theta and thetaˆ2 would not be.

Nope - actually you guys are right, no overlapping variables. I realize that that's exactly the same thing I said (not functions of each other) so I basically contradicted myself...

Here independent means that you can't construct one group from some combination of the other groups.

In any case, it should probably be explicit in the reading.

you can't construct one from a combination of the others is the correct definition of "independent." It says nothing about each variable on its own, because on their own they are likely not dimensionless.

i don't understand what 10:37 means. It seems just that one can only use a certain variable once in a group.

Right, but it also means that if you are going to set them equal, you cant have the same variable on both sides (they would cancel) so if X is used in XR/L, then it can't also be used in XYG because setting them equal would make the Xs cancel and no longer be dimensionless (I chose the letters at random, so they don't mean anything)

I thought that this little lead in didn't read as smoothly as you had intended. Perhaps changing the order of the sentences and ending with "The number of ... is" would be better? It just seemed kind of clunky.

i think that because this is more familiar to you, you can just <know> which are convenient choices. this is much more difficult for someone like me, who is just starting out.

So what would be a different set of independent groups we could create? I'm only seeing the obvious one that we chose.

I agree... I don't see how there is room for improvement or change...

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Test this statement with additional examples:

I'm a little confused by this statement. If there are only 2 independent groups, how can the two choices be "convenient?" If they are the only ones, they are the only ones (unless I'm misunderstanding something here).

I think he might be saying that we could have chosen the two groups differently (a different set of two dimensionless groups), but that our choice of having one group as theta was convenient for what we were trying to accomplish in the problem.

I agree. Also, doesn't theta have immediate dimensions?

I thought in class he has said that theta is dimensionsless

I believe we're using theta in radians not degrees, in which case it is dimensionless.

Yeah, theta is dimensionless. it has units, but it's still dimensionless. there is a difference between units and dimensions. he went over this in class.

Until he went over this in class I was a little confused - they might've mentioned it in other classes but it definitely didnt stick.

In general, angles are dimensionless. Think of all those times you've approximated sin(theta) as theta. sin definitely doesn't have dimensions, so entiehr does theta

So I like the idea of dimensional analysis, but it seems like a pain to go through the units yourself trying to cancel them all out. How does this theorem make this easier?

I really like how you refer back to some problems we saw in the previous section. Also its helpful that you put them in a list instead of paragraph form because it makes its easy to read through them and remember what the results from before were.

What does this size correspond to physically?

This is the Bohr radius between the center and the single orbiting electron.

So is it just h that makes the difference?

not exactly happy with the transition between 2 and 3. i think it's an excellent example but i think there could be some sort of transition so we know where you are going would be helpful. i found myself rereading 2 a few times, had i known that we'd find out in 3, i'd understand why you did 2 better without getting stuck in between

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Test this statement with additional examples:

Maybe list the independent group here, since the groups were listed above for light bending.

what was h again?

I believe it represents the atomic number because we are using a0 as the size. In this case it would just be 1.

would be nice to clarify what that group is just for the record.

is it always 3 or is it the number of dimensions. there are others besides length, mass, and time. temperature is one for example

# of dimensions. If there was something with length, time, mass, and temperature, you would subtract 4 for example

I think you probably made this comment before reading the next page, but it shows that your thinking is right on track!

wow this is so cool, but is it always true?

I have a vague memory of being taught that once is an incident, twice is a coincidence and thrice is a pattern. Is this true?

I think you can have a pattern if you give the rule ahead of time. If we were just empirically observing things, we couldn't say anything about 1 or 2 observations, but we might be able to figure out a pattern with 3 or more observations (which speaks to your point). However, since we have the rule ahead of time, we can call it a pattern right off the bat.

I think that's more an adage than a hard rule. 3 examples are often used to show something, but as a rhetorical device, not a scientific one.

where did this number come from?

This 3 just came from looking at the three examples he just listed...they all have 3 more variables than independent groups.

It almost seems too easy, though - usually a pattern is more complicated...

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Test this statement with additional examples:

what are quantities? variables?

What else could they be? We're experimenting with dimensionless groups.

I prefer the # to no. notation here but thats just personal preference.

agreed when I first saw this I thought you were saying no to negate previous statement.

I thought we had endless possibilities with the bending of light quantities? Following this, we get 2 possibilities (5quantities -3). im confused

i concur to this being distressing. are there no counter examples at all?

Even more distressing, I think, because you start off with negative 3 pi groups.

I'm confused by this post. When are we multiplying something by pi?

I believe this post is referring to it as a Pi group, not 3*pi groups. Meaning, before we do any analysis, we start with negative 3 independent groups (or pi groups).

Yeah, that is in fact what I meant. I guess it's not stated in the text, but these independent groups are called pi groups, hence the Buckingham Pi Theorem. The use of pi as the representative symbol for the dimensionless groups comes from Buckingham's use of the symbol when he was first writing about them in 1914.

If I recall correctly, it had something to do with taking products (sort of like how you use sigma for summation, you use pi for adding up multiplications). Nothing to do with 3.14

Hmm. I think a line about how Pi is used to describe the groups would be well placed in this introduction. I think it makes the idea of the Buckingham Pi theorem a little easier to follow.

First, I thought you meant the 3 listed above.

well, he does. as a rule, but not the three in the list

...so basically dimensional analysis is pattern matching. Please correct me if I'm wrong.

It is pretty clear when I read the rest of the sentence, but I think it is a little distracting.

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Test this statement with additional examples:

so this is the number of basic dimensions that make up the quantities we are examining?

I believe so. I can't quickly think of anything that doesn't involve some combination of these 3 dimensions.

there are others like candela (like luminous intensity), temperature, etc. I think there are a total of 7 basic dimensions in SI however, these are the most common ones and the only ones relevant to our current analysis.

Isn't it common to have something that only involves two of these quantities though?

I also don't understand how the other dimensions such as charge are not factored into this theorem. Are they somehow not as fundamental as Length, Mass, and Time? Or are they included in a larger more comprehensive theorem?

Sanjay, you should specify whether this state meant is only for this example or for all examples.

I've seen the Buckingham Pi theorem before (2.006), so perhaps I'm not a fair judge, but this all seems clear and well written to me.

So is the number always 3 or does it change with the number of dimensions? What if we added charge? You might go into this below, since I haven't gotten there yet, but maybe it should be explicit up here if it is always or isn't always =3. (kind of like a few...)

It changes with the number of dimensions. I think it's fairly explicit here. In the example, we are only dealing with length, mass, and time. Perhaps if we added angle or some other dimension we would see a change to the pattern.

i've never seen this before it's really cool

Although it's a review for me from 2.006, it's presented a little differently here which helps me understand the material even better.

5.5 Buckingham Pi theorem

The second step is in a dimensional analysis is to make dimensionless groups. That task is simpler by knowing in advance how many groups to look for. The Buckingham Pi theorem provides that number. I derive it with a series of examples.

Here is a possible beginning of the theorem statement: *The number of dimensionless groups is....* Try it on the light-bending example. How many groups can the variables θ, G, m, r, and c produce? The possibilities include θ, θ^2, Gm/rc^2, $\theta Gm/rc^2$, and so on. The possibilities are infinite! Now apply the theorem statement to estimating the size of hydrogen, before including quantum mechanics in the list of variables. That list is a_0 (the size), $e^2/4\pi\epsilon_0$, and m_e. That list produces no dimensionless groups. So it seems that the number of groups would be zero – if no groups are possible – or infinity, if even one group is possible.

Here is an improved theorem statement taking account of the redundancy: *The number of* ***independent*** *dimensionless groups is....* To complete the statement, try a few examples:

1. Bending of light. The five quantities θ, G, m, r, and c produce two independent groups. A convenient choice for the two groups is θ and Gm/rc^2, but any other independent set is equally valid, even if not as intuitive.
2. Size of hydrogen without quantum mechanics. The three quantities a_0 (the size), $e^2/4\pi\epsilon_0$, and m_e produce zero groups.
3. Size of hydrogen with quantum mechanics. The four quantities a_0 (the size), $e^2/4\pi\epsilon_0$, m_e, and $\hbar$ produce one independent group.

These examples fit a simple pattern:

$$\text{no. of independent groups} = \text{no. of quantities} - 3.$$

The 3 is a bit distressing because it is a magic number with no explanation. It is also the number of basic dimensions: length, mass, and time. So perhaps the statement is

$$\text{no. of independent groups} = \text{no. of quantities} - \text{no. of dimensions}.$$

Test this statement with additional examples:

How did we determine this was the number of dimensions?

I think this just refers to the basic dimensions in the sentence above (length, mass, time).

It seems this number would be the number of dimensions included in the quantities used...which like you said is length mass and time in the previous examples

I think that, at this point, it is simply a hypothesis based on the observation that there are (almost) always 3 basic dimensions that apply to most problems: length, mass, and time.

Does this have anything to do with 3D objects and the equation relating faces, edges, and vertices?

that doesn't feel right, but i'm not sure

Are we still using the minimum amount of quantities required in order to form a dimensionless group? (like in class we added c in order to get a dimensionless group). Without defining this it seems like there would be a lot of variability

is the number of dimensions the only thing that this can represent? It seems like that was just a guess.

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The theorem is safe until we try to derive Newton's second law. The force F depends on mass m and acceleration a. Those three quantities contain three dimensions – mass, length, and time. Three minus three is zero, so the proposed theorem predicts zero independent dimensionless groups. Whereas $F = ma$ tells me that F/ma is a dimensionless group.

This problem can be fixed by adding one word to the statement. Look at the dimensions of F, m, and a. All the dimensions – M or MLT^{-2} or LT^{-2} – can be constructed from only *two* dimensions: M and LT^{-2}. The key idea is that the original set of three dimensions are not independent, whereas the pair M and LT^{-2} are independent. So:

Var	Dim	What
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That statement is the Buckingham Pi theorem [3].

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$$F_{drag} \sim \rho v^2 A, \tag{5.1}$$

where ρ is the density of the fluid (e.g. air or water), v is the speed of the cone, and A is its cross-sectional area. What can dimensional analysis tell us about this problem?

Comments on page 2

The auto switching to the current page is a very nice upgrade

oh, now I see what you mean by quantities.

I knew what "quantities" meant from the start, but I can understand why someone could be confused. Perhaps it would be more clear if the variables were more frequently introduced as quantities throughout the chapter.

Since T and x_0 are explained in one word each, it might be more coherent to do the same for k- spring constant and m-mass

i think he assumed that we would know what those meant while T and X_0 can be ambiguous for time and distance. But this probably isn't a very good assumption for readers

Time and distance could refer to different measurements in this setup, but mass and spring constant are extremely straightforward. It's a perfectly valid assumption.

so when designing dimensional analysis problems should we decide the quantities based on our intuition?

These are the variables used in a spring mass system relating to the dimensions described above. I don't think it's intuition as much as it is prior knowledge.

I have been having trouble figuring out what parameters I can use to determine the dimensionless group. How do u choose these parameters? There seems to be some a priori knowledge about the dimensionless group that allows us to find the parameters, but trhat knowledge doesnt seem evident.

maybe you also wanna say that there are three dimensions (to be absolutely clear)

well nevermind my earlier comment about being absolutely clear by including dimensions. This does a good job of clearing things up.

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So would a different way to state the theorem be: # of variables - # of dimensions that make up those variables? In this case, 3-2=1.

I think that's exactly how we defined it above.

that helped clear things up for me a bit...thanks!

I was a bit skeptical at first, since 3 coincidentally happened to be the number of dimensions, but it's really cool that this actually works.

agreed

I like the way you phrased this

Always causing trouble. But I liked this example and clear derivation!

have you considered rewriting this as like 3-3=0.

"However" and maybe add "so there is at least one." to the end of the sentence.

The word is "dimensions" right?

Why are these in a different color? or is it just my laptop?

they're the same color on my laptop

Great table!

well explained. It would seem to me now that we should always look to use as many independent, as opposed to duplicate, pairs in any approximation. Should we only worry about independents here, or look for them in other situations as well?

This is a great explanation and really helpful in understanding this theorem.

Agreed. this section has been pretty good about explaining things as questions arise

Is it correct to call LT-2 a dimension? I am beginning to become confused about the difference between Dimensions, units, and just regular quantities. What exactly is LT-2?

I'm confused why LT^-2 can be used as a single dimension. Is it because both L and T only appear as LT^-2 and no where else?

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I like this part! very clear. i think people could easily mistaken that as 3 independent groups instead of 2

Yeah I would have definitely used 3, but this makes more sense now that I see it.

This is very clearly explained.

super clear...gets the point right across

I also found this sentence helpful in clarifying the independence point raised before.

I really liked this part as well. I feel that the example was clear and familiar enough for me to understand the theorem.

I'm confused. Why can we say that LT^-2 can be treated as a single dimension? I would first think that L and T themselves are dimensions. Again, the big question I've been getting reading this is the definition of independent.

This is a really excellent derivation of an interesting result!

I still feel like this was pulled out of a hat, yes we've gone from 3 to the number of independent dimensions, but is there some way to motivate why the Buckingham Pi theorem is true, without just examining a number of examples?

I agree. I don't really feel like this is at all a proof, even if we are being incredibly liberal in our definition of proof. I mean, you could have speculated that the 3 means anything, say "a few". Of course we would have found out a few examples later that this doesn't work, but how can you better motivate that the number 3 really means the number of dimensions?

Are there some other analogues to this sort of equation? I feel like it's sort of like in geometry where the number of vertices and edges are similarly related by a fixed difference. Or solving sets of linear equations, where you need n independent equations to solve for n variables.

yeah, I feel like we've used very specific cases to narrow things down to the theorem, and now we're using it to generalize everything, which seems a bit wrong...

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Ok, this is a nice and interesting formula. But I'd really like an explanation of why it's useful. It feels like it's thrown in here, and it doesn't feel connected with the earlier readings on dimensional analysis.

I think it's pretty obvious why this might be useful and I think it will be shown in the next section. I'm curious why this formula wasn't brought up earlier in the chapter.

I felt like this was what we have been trying to derive for the past few paragraphs. Also, the result of this gives us the number of dimensionless groups, the centerpiece of dimensional analysis.

This is great that this is emphasized here, read in the next section. You'll quickly realize how useful this is.

This is an interesting distinction. I would have thought to make three groups, but obviously the fact that two are related makes that incorrect. How would one determine the connections without knowing the formulas, though?

does the "independence" here has anything to do with notions of independence such as in probability or linear algebra?

I think it just means it appears alone as a dimension of a quantity. If a dimension pair appears along but the two dimensions creating the pair do not then it is an independent dimension pair.

I think the idependent needs to be defined right off. I keep flip-flopping my definition of independent from no overlapping variables to non creation out of each dimension. But actually I think I am just lost.

It's interesting that you mentioned that because I was thinking the same when I read through this (regarding references to probability and linear algebra and determining independence). At first, when I read that there are either 0 solutions or infinite solutions, I made the comparison that in linear algebra, if there is one solution to $Ax = 0$ or $Ax = b$, then there are infinite solutions since we can have linear combinations of that solution. With probability, this reference came to mind when reading this sentence exactly, looking at independent dimensions, particularly pairwise independence (and also dependence correspondingly). Where mass and force (and acceleration and force, respectively) are independent, mass, acceleration and force are pairwise dependent. At least that was my intuition.

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where is the [3] referencing to?

There are references missing thoughout the reading–I think Sanjoy said they'll be added in a future edit.

Is this called the Pi theorem because of the 3 basic dimensions?

Yeah what does Pi have to do with it? seems like if it were actually Pi because of "3" is just a slapped on name.

I personally would have liked to see this theorem at the beginning, and then the process of explaining why. Also, is there a more formal proof? Examples just don't seem as concrete

agreed. I'd like to know what I'm trying to convince myself of through use of the examples.

No offense to this commenter, but I think getting away from what you're talking about is exactly what this class is all about. To us, the proof doesn't matter if we know it works.

I actually liked the way the text derived the theorem with all these examples, but I do think it would be helpful to have a little bit of background after we arrive at the theorem.

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I thought this was a great way to explain the above conclusion, but I was under the impression that this was not the actual conclusion of the Buckingham Pi theorem, just a useful bit of pre-information.

I thought the actual Buckingham Pi theorem was pi1=f(pi2, ..., pi_k), or equivalently F(pi1, ..., pi_k)=0.

In fact, going back through the readings a bit, it seems like we use this fact (and continue to in the next section) without actually stating it explicitly as any sort of theorem. (I could have missed it, though.) I think fully fleshing out the Buckingham Pi theorem might make it a more readily available tool.

Where did the theorem get its name from? I also agree that a more fleshed out definition would be useful.

Edgar Buckingham, US physicist. He published papers on the subject in 1914-15.

http://en.wikipedia.org/wiki/Edgar_Buckingham

and

http://en.wikipedia.org/wiki/Buckingham_π_theorem#Original_sources

This is a great link that clears up a lot of questions. Thanks.

I like how the Buckingham Pi Theorem was mentioned and slowly built up before directly saying its formulaic meaning.

Maybe include a sentence now on what this theorem can be used for, or something like that...?

I agree, why does this theorem help us? I saw how having dimensionless groups helped us solve problems, but how does knowing the number of independent groups via the Buckingham Pi theorem help us in this course?

Is it because it helps us check if we forgot any of the dimensionless groups?

I feel like normally you'd precede or follow this with the history of the theorem.

Yes, who was this Buckingham, and why Pi?

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This is all clear until maybe the last paragraph; the 'independent' dimensions confuses me, and although I still need to read on, it is definitely something that should be further explained before going to a new subsection. I'm pretty lost at the whole idea of dimensions being combined and then being independent; I feel like this then simplifies more, or is being redundant.

This was quite clear, although I do agree with people that it should've been introduced at the beginning of the explanations of deriving dimensionless groups.

It could be interesting to lack back at the end of the class to all the different methods we analyzed drag problems into one single lecture

I really like that there are a few examples that are used throughout the readings. It means we already have a familiarization with the example and can develop a better intuition.

Definitely. I also didn't quite grasp the entire background the first time through, so reinforcement also aids in my general understanding.

I like that you go back to old examples. Drag is a popular guest.

Its also really nice seeing the same problem tackled in different ways coming up with the same conclusion.

I almost want a list at the end for all the different ways we solved for drag...they are getting a bit confused in my head!

What problem? The equation?

I believe the problem is to estimate drag.

The problem is the cone experiment- I think we are supposed to discount what we already know and try to solve it using dimensional analysis and hopefully come up with the same equation that we previously found.

I think it might be more clear if it is specified as the falling cone experiment.

The strategy is to find the quantities that affect F_{drag}, find their dimensions, and then find dimensionless groups.

▶ *On what quantities does the drag depend, and what are their dimensions?*

v	speed of the cone	LT^{-1}
r	size of the cone	L
ρ	density of air	ML^{-3}
ν	viscosity of air	L^2T^{-1}

The drag force depends on four quantities: two parameters of the cone and two parameters of the fluid (air). Any dimensionless form can be built from dimensionless groups: from dimensionless products of the variables. Because any equation describing the world can be written in a dimensionless form, and any dimensionless form can be written using dimensionless groups, any equation describing the world can be written using dimensionless groups.

Problem 5.3 Kepler's third law
Use dimensional analysis to derive Kepler's third law connecting the orbital period of a planet to its orbital radius (for a circular orbit).

▶ *What dimensionless groups can be constructed for the drag problem?*

According to the Buckingham Pi theorem, the five quantities and three independent dimensions give rise to two independent dimensionless groups. One dimensionless group could be $F/\rho v^2 r^2$. A second group could be rv/ν. Any other dimensionless group can be constructed from these two groups (Problem 5.4), so the problem is indeed described by two independent dimensionless groups. The most general dimensionless statement is then

$$\text{one group} = f(\text{second group}), \tag{5.2}$$

where f is a still-unknown (but dimensionless) function.

▶ *Which dimensionless group belongs on the left side?*

The goal is to synthesize a formula for F, and F appears only in the first group $F/\rho v^2 r^2$. With that constraint in mind, place the first group on the left side rather than wrapping it in the still-mysterious function f. With this choice, the most general statement about drag force is

Echoing the previous comment at the end of the previous page, I'm not sure why we are doing this dimensional analysis. It might be useful to give some motivation at the beginning.

He's already told us why in the previous sections when we did these examples. This is attempting to be a more math specific section.

These dimension charts are very helpful for quick glances

I like the consistency of the charts!

Force is in the other chart but not here. I immediately said in my head '4-3 = 1' but realized when I read on that it was 2. You have force in the Newton chart but not here, and I think for consistency, it should be in both.

If we didn't think of density at first, could we get to this in a roundabout way, like mass of the air & distance traveled & size of the cone?

this isn't included in the equation above? how does it fit in?

Also, this is slightly misleading because this is not the typical viscosity people denote with mu, but rather the kinematic viscosity which is the aforementioned mu/rho.

i don't really know much about fluids...what is viscosity again?

I don't understand this sentence.

Yeah I didn't understand it at first. This is a really roundabout way of stating something I think is obvious.

I do think this sentence is redundant/worded awkwardly...but it's saying that by multiplying some combination of variables, you get dimensionless products that form dimensionless groups.

Sometimes, it's confusing to put v and nu in the same place. Depending on the font, they can be almost identical...

This paragraph is kind of silly i think. It goes off on making big general statements without supporting any of them. Then it draws a conclusion from those loaded statements.

I feel like it's not that they CAN be, isn't it that they SHOULD be?

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Is it possible to give an example for this sentence? The use of the word "dimensionless" is confusing me since it it referred to 3 times. An example will clarify your point.

What is the difference between forms and groups?

I am also confused by this, But my guess would be that dimensionless form just means that it has no dimensions, while a dimensionless group is an instance of something with dimensionless form.

This is kind of cool. I never considered this before. I'm still not entirely convinced it's true, but I like this statement because it got me thinking.

I'm not convinced it's true either. It just came out of nowhere.

Actually, it has to be true. Think about it:

If I have a statment, x = y, then x and y must have the same dimensions. So let's define a constant with those dimensions called C. Thus, x/C=y/C, but x/C and y/C are dimensionless groups, so we've expressed an arbitrary relation using only dimensionless groups.

its like finding the invarient all over again

This makes a lot more sense after the comment in lecture that the world doesn't care about units/dimensions, it cares about relations.

Interesting - and I agree that the comment was quite clarifying. Dimensions, as we define them, are just relations to some known physical quantity - take, for example, the kilogram, an arbitrary block of substance. We're simply defining the relation of something else to this arbitrary baseline.

Wow. That's a pretty powerful statement. If dimensionless groups are so powerful, why is it that I've never really heard them mentioned till this class???

Yeah.. Bold statement.

The strategy is to find the quantities that affect F_{drag}, find their dimensions, and then find dimensionless groups.

▶ *On what quantities does the drag depend, and what are their dimensions?*

The drag force depends on four quantities: two parameters of the cone and two parameters of the fluid (air). Any dimensionless form can be built from dimensionless groups: from dimensionless products of the variables. Because any equation describing the world can be written in a dimensionless form, and any dimensionless form can be written using dimensionless groups, any equation describing the world can be written using dimensionless groups.

v	speed of the cone	LT^{-1}
r	size of the cone	L
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ν	viscosity of air	L^2T^{-1}

Problem 5.3 Kepler's third law
Use dimensional analysis to derive Kepler's third law connecting the orbital period of a planet to its orbital radius (for a circular orbit).

▶ *What dimensionless groups can be constructed for the drag problem?*

According to the Buckingham Pi theorem, the five quantities and three independent dimensions give rise to two independent dimensionless groups. One dimensionless group could be $F/\rho v^2 r^2$. A second group could be rv/ν. Any other dimensionless group can be constructed from these two groups (Problem 5.4), so the problem is indeed described by two independent dimensionless groups. The most general dimensionless statement is then

$$\text{one group} = f(\text{second group}), \tag{5.2}$$

where f is a still-unknown (but dimensionless) function.

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Maybe you should spell out what kepler's third law is, so we know if we got it or not.

I agree! It doesn't require any explanation, just a basic equation would do.

So I was going to tell you Kepler's third, but I forgot the exponents on T and R, which I can never remember. This problem actually turned out to be really instructive. Find dimensionless groups between G, M (of star), R, and T, and figure out how T and R are related based on the powers of T and R in the dimensionless group.

I feel like this problem might be better placed if you moved it to the end of your example about drag. Here it interrupts the flow, but at the end it would give the reader an opportunity to try and apply what they had just learned in the drag example to a different problem.

I agree, the placement of Problems has been erratic before too. Putting them at the ends of sections is a good idea. One or two at the end of each section is still better than 10 at the end of the chapter, where it's easier to ignore them and harder to see how to apply what you've just read about.

I also felt that when my mind was on drag forces and the cone experiment and suddenly this example problem was thrown in, it kind of threw off my mental picture.

when this is stated so briefly and succinctly, it always throws me for a loop a little. could you provide more steps and help guide us through it?

I don't remember having problems embedded in the text in previous sections (Maybe I haven't seen one in a while). I think its pretty neat though and encourage them to be included in other chapters!

The strategy is to find the quantities that affect F_{drag}, find their dimensions, and then find dimensionless groups.

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It might be helpful to state that the fifth is the force itself.

I agree. I just glanced at the table and saw 4 things listed and then see the number 5 here. Maybe the force should be listed in the table too.

I don't know if it's necessary. Earlier readings mentioned that you need to consider all quantities, so the reader should remember to include force. This is presuming he/she is reading the chapter as a whole, instead of in the fragmented form they we have.

Counting on the reader to remember force seems like it's causing confusion. I think it should be included in the table.

agreed

so theoretically you can construct an infinite number of dimensionless group but the B.Pi theorem tells you only 2 of them are valid, so how do you choose the correct two from a pool of infinite groups

There are no "correct" 2. Any two independent groups will work... But it's usually easier to pick the simplest ones.

I think the B. Pi Thm says there are only 2 "independent" groups in this case, but there are infinitely many groups of 2 that would work.

Yeah, he's talking about independent groups, where they can't be divided down any further and still be dimensionless (any multiplied combination of the two is still dimensionless but not independent).

Why does F/p(vr)^2 count? This is just putting the drag force on the same side as it's equation.

This example solidified it for me.

How come these are so different than the analysis above F pv^2*A I thought that they would match more closely. do they provide similar results to each other?

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can variables be in more than 1 dimensionless group? I always thought that they could not be repeated

Yes. They are independent because one group does not "contain" another group. You could not, for example, 'factor out' the first group from the second group.

But both groups may contain the same variable.

The v and nu are nearly indistinguishable.

This threw me off as well.

Like that you introduced this at the end of class Monday!

I'm confused by this. Do you mean that any new group is a function of the two already defined? If you really do mean group 1 is a function of group 2, doesn't this mean that they're not independent?

Yeah, this statement is pretty unclear to me as well. It seems contradictory to what we already read about independent groups.

But isn't it true that any dimensionless group is a function of another dimensionless group? I thought before it discussed independent dimensions, not independent dimensionless groups.

This is a fundamental point of dimensional analysis, any non-dimensional group can be described as a function of the remaining non-dimensional groups.

The tilda versus the proportionality sign i think...

how is this so different than the proportionality stuff we did with drag before? just that the units match? but how to they really differ from each other.

this statement would have been more useful earlier in the chapter...actually, this whole section would be more useful earlier in the chapter because it walks you through the method really well

I like this break down of the key points to the problem

I like that they're posed as questions, so the reader can work the following step and check it piece by piece as they go.

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I was wondering that! How do you know which to define as a function of the other?

I lost what we were looking for in the long paragraph about dimensionlessness. It would be a good idea to restate the question.

I disagree, I think this does a good job of not restating what has been said before but instead moving through the problem using the information that was just described.

Well, but only because you set up the dimensionless groups like that. So I guess you have to have in mind what you're going to be looking for when you set up the groups?

i really liked this paragraph. it asked a question i was wondering and then answered it well.

$$\frac{F}{\rho v^2 r^2} = f\left(\frac{rv}{\nu}\right). \quad (5.3)$$

The physics of the (steady-state) drag force on the cone is all contained in the dimensionless function f. However, dimensional analysis cannot tell us anything about that function. To make progress requires incorporating new knowledge. It can come from an experiment such as dropping the small and large cones, from wind-tunnel tests at various speeds, and even from physical reasoning.

Before reexamining the results of the cone experiment in dimensionless form, let's name the two dimensionless groups. The first one, $F/\rho v^2 r^2$, is traditionally written in a slightly different form:

$$\frac{F}{\frac{1}{2}\rho v^2 A}, \quad (5.4)$$

where A is the cross-sectional area of the cone. The 1/2 is an arbitrary choice, but it is the usual choice: It is convenient and is reminiscent of the 1/2 in the kinetic energy formula $mv^2/2$. Written in that way, the first dimensionless group is called the drag coefficient and is abbreviated c_d. The second group, rv/ν, is called the Reynolds number. It is traditionally written as

$$\frac{vL}{\nu}, \quad (5.5)$$

where L is the diameter of the object.

The conclusion of the dimensional analysis is then

$$\text{drag coefficient} = f(\text{Reynolds number}). \quad (5.6)$$

Now let's see how the cone experiment fits into this dimensionless framework. The experimental data was that the small and large cones fell at the same speed – roughly $1\,\text{m}\,\text{s}^{-1}$. The conclusion is that the drag force is proportional to the cross-sectional area A. Because the drag coefficient is proportional to F/A, which is the same for the small and large cones, the small and large cones have the same drag coefficient.

Their Reynolds numbers, however, are not the same. For the small cone, the diameter is $2\,\text{in} \times 0.75$ (why?), which is roughly 4 cm. The Reynolds number is

Oh wow. Didn't see this coming. So for this, we haven't actually looked up the equation for Drag force right? We just analyze the components and turn them into dimensionless parts. What happens if you need three dimensionless groups though and cannot set them equal to each other?

Why must this only work in steady state?

does this function only indicate that these two dimensionless groups can both be used to calculate the drag coefficient based on some factor? I didn't see a solution for this function below

I find it interesting how we come to the two important parameters, Cd and Re, from dimensional analysis. I would have never thought to get to them this way!

> How do we come up with these though? Did you just know these similar formulas, or did you look them up?
>
> > Yeah...I can't see myself coming up with these on my own...even knowing common formulas such as kinetic energy.
> >
> > > The drag equation may not be intuitive but something like kinetic energy is something all people should know, much less an MIT student.
> > >
> > > > Yeah, but just because you know it, doesn't mean your intuition will tell you "this is a dimensionless group".

why? just for further insight, or will this come up again later?

Was there any reason to change the function as it was?

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Even at this stage in the term I don't feel comfortable taking those leaps of faith.

I was a little confused by this too, sometimes I see the connection but am not sure when I can make the bridge like this

yeah i dont see the connection either. just a random 1/2 in an equation doesn't exactly set off any alarms.

I think his point is that the random constants dont really matter that much and like mentioned, probably are only derived accurately from testing. However, since he already knows the answer, he goes ahead and "guesses correctly"

I thought constants weren't important in dimensional analysis, they are?

The point is not that you have to do it this way, but as he says "traditionally" it is done this way. You will still get a correct answer if you don't but past research and convention suggests to use this form.

yeah, this seems like a leap for me too, but i guess you could maybe see the connection. Force and kinetic energy are related, obviously, and kinetic energy is (mv^2)/2. the term we have in the denominator is (rho*v^2)/2 *A, which is kinda similar i guess..

It is a huge leap, but I think the connection here between the rather random number 1/2 and kinetic energy is cool.

I'm confused by this statement because in past readings we have dropped the 1/2 in the kinetic energy formula (and in others also I believe) because it was irrelevant to the final answer. So here, I am a little confused why we are adding it if it is just an irrelevant number?

Perhaps reword to "Written that way"

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Can we perhaps get a short explanation of what the Reynolds number is? I feel like this comes out of nowhere, and it's never fully explained.

very much agree

its a dimensionless number that compares inertial forces to viscous forces in a fluid flow. this is probably more obvious to course 2s.

Agree as well, especially since it is referenced a few times in the following paragraphs

it is used to determine when a fluid becomes turbulent.

If I recall, there is a really cool video online somewhere with different reynolds number values and what happens to perturbed fluids

you should mention that we abbreviate it as Re

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i guess im more used to the rho*v*D/mu form

Note that the v on the bottom is the kinematic viscosity which is equal to mu/rho, so these to terms are the same.

This is just an unfortunate property of this font, I guess, but these nu's and v's look ridiculously similar. Is there an option for another style nu, or can it be in bold or italics throughout, or some other distinguishing feature?

I agree, I actually thought this was rv/v = r until a few lines ago.

same here! maybe you can capitlize one of the variables so as to not confuse people

I was pretty confused by this too...I wondered why they didnt cancel out and had to stare really closely at the font.

Definitely not intuitive, I would have guessed they were.

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Is this a result of the 1/4 of the circle you cut out before folding into the cone?

i believe so. i think he's trying to illustrate the fact that we don't have to recalculate from the first formula, since we can scale.

what is this getting at? the dimensions are fairly straight forward...

we are comparing the two cones knowing that they fall at the same speed. he is trying to show how you get to this point, i believe, so the numbers must be brought into it.

$$\text{Re} \sim \frac{1\,\text{m}\,\text{s}^{-1} \times 0.04\,\text{m}}{1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1}}, \tag{5.7}$$

where $1\,\text{m}\,\text{s}^{-1}$ is the fall speed and $1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1}$ is the kinematic viscosity of air. Numerically, $\text{Re}_{\text{small}} \sim 2000$. For the large cone, the fall speed and viscosity are the same as for the small cone, but the diameter is twice as large, so $\text{Re}_{\text{large}} \sim 4000$. The result of the cone experiment is, in dimensionless form, that the drag coefficient is independent of Reynolds number – at least, for Reynolds numbers between 2000 and 4000.

This conclusion is valid for diverse shapes. The most extensive data on drag coefficient versus Reynolds number is for a sphere. That data is plotted logarithmically below (from *Fluid-dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance* by Sighard F. Hoerner):

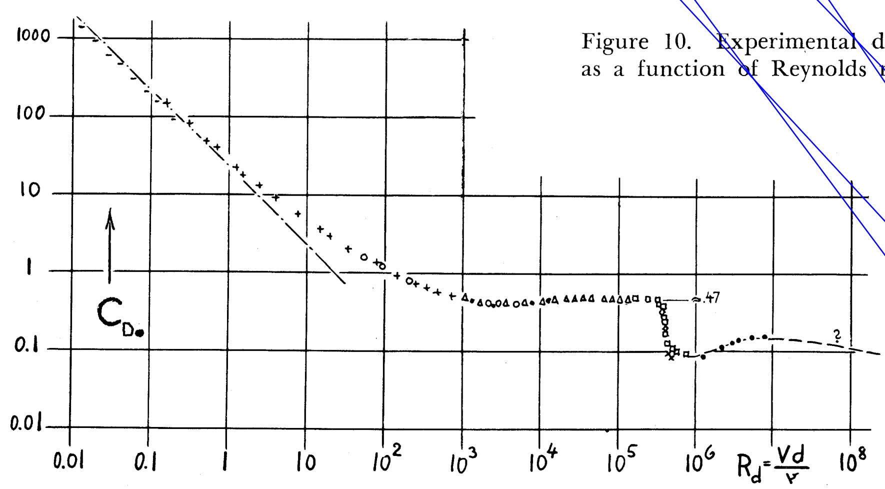

Figure 10. Experimental d as a function of Reynolds

Just like the cones, the sphere's drag coefficient is almost constant in the Reynolds number range 2000 to 4000. This full graph has interesting features. First, toward the low-Reynolds-number end, the drag coefficient increases. Second, for high Reynolds numbers, the drag coefficient stays roughly constant until $\text{Re} \sim 10^6$, where it rapidly drops by almost a factor of 5. The behavior at low Reynolds number will be explained in the chapter on easy (extreme) cases (Chapter 6). The drop in the drag coefficient, which relates to why golf balls have dimples, will be explained in the chapter on lumping (Chapter 8).

I really liked this. Now I understand the Reynold's number's used in 2.005 and a way to get them.

So does this invalidate the conclusion above that the drag coefficient can be expressed as a function of Reynolds number? Or does it simply mean that there is no easily defined function (as shown by the graph below)?

It's not that we can't express it as a function, just that, over this range, the function is a constant.

isn't the drag coefficient a function of geometry? and the cones have the same geometry so this makes sense.

I think he's trying to show that he proved this point without knowing that the drag coefficient is a function of geometry.

but didn't we just say that drag coefficient = f(reynolds number)? if it's independent, then is this not true?

Yeah this paragraph here just lost me...

yeah i'm confused

It is neat to see this insight come from an application which in the beginning of this class I would not have been able to create.

however, this indicates the switch from laminar to turbulent flow, which is important.

This is only necessarily true if we assume Cd(Re) is a monotonic function...

it'd kinda be cool to have a graph (if it exists) that has not only sphere info but also cone info (and more shapes if avail) superimposed on the same graph as a comparison

This is a good explanation, but I feel like it could have been split like the last reading into sub-sub sections (groups, reasoning, numbers) to give a bit more clarity and order.

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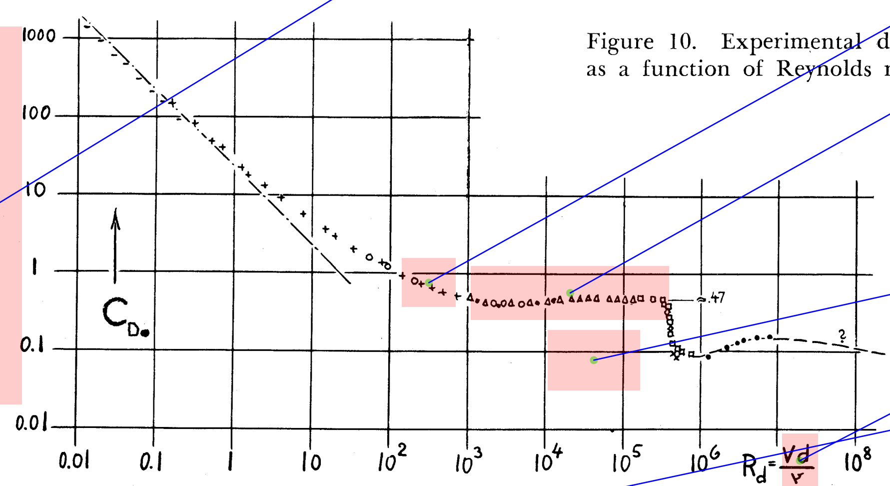

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I'd like to see these axes labeled

I agree, but the text does point out what is being shown and from the previous reading it should be pretty self explanatory. On another note, for a text book being published, it might be worth it to try to run these tests or find a more clear figure, this seems like it was almost hand done. Overall, the graph was extremely helpful in understand the significance of similar reynold's numbers.

Well, if you read the text, it's not that hard to figure out.

I agree! I do like that he adds real data to give insight into all this approximations.

Maybe draw a little box here to show where we are on the graph? Could be helpful to direct it for the reader so there is less wandering to understand.

Is there also an explanation for why the graph is so level at this point?

Yeah, why is it so level and then why is there a sudden drop around 10ˆ5.5?

The point of this graph, and the text, is not to explain how the Reynold's number changes, but instead to utilize the values given. Thinking too much about it's origins takes you away from the main point of the chapter.

I also wonder why it increases slightly in the middle of this constant range.

same form as listed earlier!

What does a Reynolds number within this range correspond to physically?

The point of using a dimensionless group is that you get this relationship for significantly different physical systems, you could either have a very inviscid fluid and a high velocity, or a viscous fluid and a low velocity, but provided their Reynolds number turns out the same, their drag coefficient will be the same as well.

I really like that we're doing fluids things. It brings my 2.005/2.006 studies home in an intuitive way.

"drag coefficient increases" is ambiguous. It depends if you mean as a function of moving to lower Reynolds numbers or as a standard function of Reynold number, in which case, it is decreasing.

$$\mathrm{Re} \sim \frac{1\,\mathrm{m\,s^{-1}} \times 0.04\,\mathrm{m}}{1.5 \cdot 10^{-5}\,\mathrm{m^2\,s^{-1}}}, \tag{5.7}$$

where $1\,\mathrm{m\,s^{-1}}$ is the fall speed and $1.5 \cdot 10^{-5}\,\mathrm{m^2\,s^{-1}}$ is the kinematic viscosity of air. Numerically, $\mathrm{Re_{small}} \sim 2000$. For the large cone, the fall speed and viscosity are the same as for the small cone, but the diameter is twice as large, so $\mathrm{Re_{large}} \sim 4000$. The result of the cone experiment is, in dimensionless form, that the drag coefficient is independent of Reynolds number – at least, for Reynolds numbers between 2000 and 4000.

This conclusion is valid for diverse shapes. The most extensive data on drag coefficient versus Reynolds number is for a sphere. That data is plotted logarithmically below (from *Fluid-dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance* by Sighard F. Hoerner):

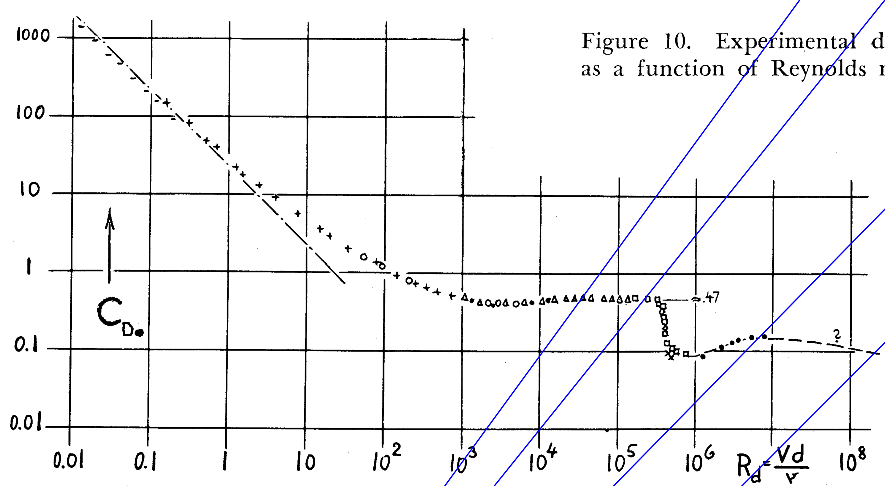

Figure 10. Experimental d
as a function of Reynolds ı

Just like the cones, the sphere's drag coefficient is almost constant in the Reynolds number range 2000 to 4000. This full graph has interesting features. First, toward the low-Reynolds-number end, the drag coefficient increases. Second, for high Reynolds numbers, the drag coefficient stays roughly constant until $\mathrm{Re} \sim 10^6$, where it rapidly drops by almost a factor of 5. The behavior at low Reynolds number will be explained in the chapter on easy (extreme) cases (Chapter 6). The drop in the drag coefficient, which relates to why golf balls have dimples, will be explained in the chapter on lumping (Chapter 8).

I thought this reading was kind of hard. I don't even know what to comment on.

It means that the laminar boundary layer becomes turbulent, thereby decreasing the amount of drag.

This is pretty interesting...thanks for dropping the term for further research.

looking forward to this discussion

Me too!

Yeah this was one of the questions off the diagnostic that I want to come out of the class knowing how to solve.

same! i was wondering when we'd get back to it

I'm glad you pointed out when we would learn these things

A comment on the whole section of dimensional analysis.... Generally, I'm pretty confused. At the end of previous sections (D&C, Abstraction, etc.), I felt like I understood the concepts and could effectively use them to solve problems. However, I'm sort of all over the place with dimensional analysis. I understood all the readings, but I just don't understand how to take these concepts and apply them to solve problems.

When we had to do pset/test problems for dimensional analysis using 2.006, the problem would go something like this: You're given a picture and description of some experiment to which you are given results, but you truly do not understand the physics. Using dimensional analysis, you form Pi groups which you know need to be functions of each other. Then you used the experimental results to determine whether the function was linear, quadratic, had some constant, etc.

Hope this helps.

Problem 5.4 Only two groups
Show that F, v, r, ρ, and ν produce only two independent dimensionless groups.

Problem 5.5 Counting dimensionless groups
How many independent dimensionless groups are there in the following sets of variables:

a. size of hydrogen including relativistic effects:

$e^2/4\pi\epsilon_0$, $\hbar$, c, a_0 (Bohr radius), m_e (electron mass).

b. period of a spring–mass system in a gravitational field:

T (period), k (spring constant), m, x_0 (amplitude), g.

c. speed at which a free-falling object hits the ground:

v, g, h (initial drop height).

d. [tricky!] weight W of an object:

W, g, m.

Problem 5.6 Integrals by dimensions
You can use dimensions to do integrals. As an example, try this integral:

$$I(\beta) = \int_{-\infty}^{\infty} e^{-\beta x^2}\, dx.$$

Which choice has correct dimensions: (a.) $\sqrt{\pi}\beta^{-1}$ (b.) $\sqrt{\pi}\beta^{-1/2}$ (c.) $\sqrt{\pi}\beta^{1/2}$ (d.) $\sqrt{\pi}\beta^{1}$

Hints:

1. The dimensions of dx are the same as the dimensions of x.
2. Pick interesting dimensions for x, such as length. (If x is dimensionless then you cannot use dimensional analysis on the integral.)

Problem 5.7 How to avoid remembering lots of constants
Many atomic problems, such as the size or binding energy of hydrogen, end up in expressions with $\hbar$, the electron mass m_e, and $e^2/4\pi\epsilon_0$, which is a nicer way to express the squared electron charge. You can avoid having to remember those constants if instead you remember these values instead:

$$\hbar c \approx 200\,\text{eV nm} = 2000\,\text{eV Å}$$
$$m_e c^2 \sim 0.5 \cdot 10^6\,\text{eV}$$
$$\frac{e^2/4\pi\epsilon_0}{\hbar c} \equiv \alpha \approx \frac{1}{137} \quad \text{(fine-structure constant).}$$

Comments on page 6

Where did these problems come from?

In past sections, he always posted sample questions that relate to the reading.

Maybe from the pset we didn't have?

It seems a bit odd to have so many pages of problems. Perhaps intersperse them more throughout the chapter? The problems are as long as the material itself!

I am sure these problems will come in handy when the next pset comes out.

I thought the reading was extremely long but there are 4 full pages worth of problems - these problems are great, they're a "teaser" for what we'll see on the problem set, and get us thinking a little bit ahead of time.

it's 1, right?' mg=MG/R^2

quanties=4 number of dimensions=3

1 is the correct answer for the same reason explained above when he talked about F = ma

What exactly is this saying here? Is it some function I that is a function of B?

If [x]=L, then [B]=1/L^2, and the answer has to have dimensions of L, and the only choice with dimensions of L is b.

Could you explain why the dimensions of B are 1/L^2 if [x] = L.

B and x^2 are both in the exponent. The entire exponent term cannot have units, so everything in the power of the exponent will be a dimensionless group.

I don't see how this one is a problem; It just looks like an explanation to me. Maybe it should be in the reading?

these problems really help solidify my understanding of this technique

power radiated cannot depend on the origin. The velocity cannot matter because of relativity: You can transform to a reference frame where $v = 0$, but that change will not affect the radiation (otherwise you could distinguish a moving frame from a non-moving frame, in violation of the principle of relativity). So the acceleration a is all that's left to determine the radiated power. [This line of argument is slightly dodgy, but it works for low speeds.]

a. Using P, $q^2/4\pi\epsilon_0$, and a, how many dimensionless quantities can you form?
b. Fix the problem in the previous part by adding one quantity to the list of variables, and give a physical reason for including the quantity.
c. With the new list, use dimensionless groups to find the power radiated by an accelerating point charge. In case you are curious, the exact result contains a dimensionless factor of 2/3; dimensional analysis triumphs again!

Problem 5.10 Yield from an atomic bomb
Geoffrey Taylor, a famous Cambridge fluid mechanic, annoyed the US government by doing the following analysis. The question he answered: 'What was the yield (in kilotons of TNT) of the first atomic blast (in the New Mexico desert in 1945)?' Declassified pictures, which even had a scale bar, gave the following data on the radius of the explosion at various times:

t (ms)	R (m)
3.26	59.0
4.61	67.3
15.0	106.5
62.0	185.0

a. Use dimensional analysis to work out the relation between radius R, time t, blast energy E, and air density ρ.
b. Use the data in the table to estimate the blast energy E (in Joules).
c. Convert that energy to kilotons of TNT. One gram of TNT releases 1 kcal or roughly 4 kJ.

The actual value was 20 kilotons, a classified number when Taylor published his result ['The Formation of a Blast Wave by a Very Intense Explosion. II. The Atomic Explosion of 1945.', *Proceedings of the Royal Society of London. Series A, Mathematical and Physical* **201**(1065): 175–186 (22 March 1950)]

Problem 5.11 Atomic blast: A physical interpretation
Use energy densities and sound speeds to make a rough physical explanation of the result in the 'yield from an atomic bomb' problem.

Problem 5.12 Rolling down the plane
Four objects, made of identical steel, roll down an inclined plane without slipping. The objects are:

Just curious, if we have any extra time in class could we solve this?

It's not too bad. Give it a shot. Also, here's a link to the paper: http://tinyurl.com/y9gewzd (opens a PDF)

it's pretty interesting stuff, I just skimmed though it but it seems like a good read if you can find the time...

Oh my.

Possible answer:

E t^2 / (p R^5) = const.a

and then you can rearrange to find:

R^5 = const.b * (E/p) * t^2

where const.b is the reciprocal of const.a

this is of the form:

R = A t^(2/5)

Using the table, we fit a curve with the best guess on A.

However, here is where I run into trouble. There is still that "const." from the beginning to be dealt with.

A^5 = const.b * (E/p)

I can rearrange to calculate E:

E = const.a * p * A^5

but without any information about const.a, E could be arbitrarily large/small....

to clarify my point in a simpler/similar rephrasing:

the table only lets us initialize the RATIO of a combination of R to t, and thus we can substitute that number in to get rid of the R and t from the equation we find in part a).

But, we need some other information to initialize the arbitrary constant which our dimensionless group is equal to.

Turns out (after reading his solution), he arbitrarily set the dimensionless group to equal 1. I'm not sure why.

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I always liked this problem. It causes some distress in thinking for me since it seems both intuitive and counter-intuitive at the same time. But I loved solving it in my AP physics class.

When I think of dimensions I automatically think of numbers and variables? This problem has neither....so where would i start? Would we begin assigning variables and estimated number measurement? Should I draw something or understand physical nature of the problem?

1. a large spherical shell,
2. a large disc,
3. a small solid sphere,
4. a small ring.

The large objects have three times the radius of the small objects. Rank the objects by their acceleration (highest acceleration first).

Check your results with exact calculation or with a home experiment.

Problem 5.13 Blackbody radiation

A hot object – a so-called blackbody – radiates energy, and the flux F depends on the temperature T. In this problem you derive the connection using dimensional analysis. The goal is to find F as a function of T. But you need more quantities.

a. What are the dimensions of flux?

b. What two constants of nature should be included because blackbody radiation depends on the quantum theory of radiation?

c. What constant of nature should be included because you are dealing with temperature?

d. After doing the preceding parts, you have five variables. Explain why these five variables produce one dimensionless group, and use that fact to deduce the relation between flux and temperature.

e. Look up the Stefan–Boltzmann law and compare your result to it.

What's the more specific definition of a blackbody?

If by flux you mean the energy flux density, then the dimensions are Energy/Time/Area, or M/T^3 where T is the TIME not the temperature.

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reduced planck's constant h(bar) and speed of light c

Probably Kb the Boltzman constant

There are 4 dimensions, Mass, Time, Temperature and Length. 5-4=1 dimensionless group.

Flux k^4*T^4/(h^3*c^2)

Looks good! Just missing some pi's.

why so many problems?

Kinda daunting, but if you read through I think it helps. Def. dense tho.

I think that, as far as the book is concerned, yes these problems are helpful...however, to have 4 pages of problems in the reading was a little much.

That is my fault. I meant to say in the introductory NB comment, "ignore the problems,they are just there so you know what kinds of questions will be asked on the pset."