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Easy cases

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The first tool for discarding actual complexity is based on the principle that a correct solution works in all cases – including the easy ones. This principle helps us check and, more surprisingly, helps us guess solutions.

6.1 Pyramid volume

As the first example, let's explain the factor of one-third in the volume of a pyramid with a square base:

$$V = \frac{1}{3}hb^2,$$

where h is the altitude and b is the length of a side of the base.

In this reading, I thought the pyramid example was an amazing example that illustrated the concept while teaching us something really worthwhile. However, the Atwood problem I thought was a little tougher to read and was just not as awesome as the first one.

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A new tool! Read these two sections for the memo due Thursday at 10pm.

Grr, I'm sorry this is late. I pulled an all-nighter last night and then collapsed right after classes today, waking up just now.

Sounds cool! I don't know what it is.

I liked this example.

Will this section be part of a later reading?

Similarly, I think what dimensional analysis let us do was pick out the important parts.

Almost all your examples of "scary" equations come from fluids!

That and physics.

it makes me even more scared of 2.005...

well it's true and 2.005 isn't the one you should be afraid of - you don't learn much about Re or cd. You do learn it extensively in 2.006. so don't worry...yet

Does actual complexity refer to problems like those in fluid mechanics where will will have to discount more than just dimensions? I guess I just don't understand what "actual" means in this case..

I think Sanjoy might have meant something more along the lines of 'actually discarding complexity' with this sentence, however I don't really think the word "actual" is relevant to understanding the point, which is that now we're going to look at methods for discarding complexity.

agreed

What were we discarding in dimensional analysis? Virtual complexity?

Nothing was really discarded using dimensional analysis... we are moving onto a new topic (from lossless to lossy discarding of information). Although I'm not sure what information was discarded in the two examples given in the rest of the reading...

It sounds like a necessary skill set to have for complex problem solving.

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But how close would the solution be to the actual solution then? Or is this basically saying, "approximate"? I think I'm a bit confused on this.

also can you quantify the level of reduction

Could you qualify what you mean by reduction exactly? To me, estimating an answer automatically means you accept losing accuracy.

I think he's making a general statement about complex problems. He's saying that, in general, problems are often really difficult, so throwing out some of the hairier stuff will give you a solution but you will lose accuracy.

I think this is saying that what will be presented below is "lossee" estimation instead of what we've been doing so far, which is roughly "lossless estimation".

By reduction, he means he is going to "ignore" some difficult things. Say ignoring friction for something really smooth.

As we discard actual complexity, we lose accuracy by modeling the system as something that isn't identical to what is really happening

yeah, what we have been doing up to this point is approximating difficult problems but trying to model them as closely or accurately as possible. This was lossless (no critical information was lost). What he is now proposing is actively ignoring some things so that we can actually get an answer. This means that we know that we will get a wrong answer (because our model was not accurate but hopefully it will be close to the correct answer.

This principle has definitely helped me in math classes!

Yeah actually in E&M, at least the course 6 versions, we only ever use the 'easy case' of solutions. Like for solutions to the wave equation, we've only seen the easy case solution.

In other words, go with the answer that seems too obvious, right?

True, but its moreso that solving an easy case can give insight into a harder case...which really might not be solveable. It almost sounds like using an easy case (that can be solved) to roughly estimate a harder case

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I've used this idea many times without ever articulating/understanding it as a principle.

I agree. I feel like this has been discussed before in other lessons to check answers but has not been explained like this.

Same here. It's nice to have it be explained in such a way to justify what I've previously considered to be a rather hand-wave-y way to make approximations.

I'm a little confused as to what this idea is trying to convey.

I'm not sure if I understand this...do you mean the correct solution in one problem helps with others? all is it more like working backwards?

Yes I believe you work backwards by finding a solution to an easy problem and then see if it fits the general case as well.

He means that if you can't solve hard cases (like 3D) solve an easier one (like 2D) and extrapolate to more dimensions. This is similar to how we did the solitaire 'box' problem.

It's a method that you can use to solve really complex problems. First find a general solution, and then use that logic and apply it to any additional constraints, since the relationship will hold true.

I think he basically means that if you find an answer, for a problem it has to work for all cases of the problem, including the easy ones. like if you found some equation, to check it, you could plug in zero or 1 for some of the variables (simple case), and if your equation was right it would still work.

This part sounds right but I am not so sure what you really mean in context.

Yea, I had to read over this paragraph several times, but I'm pretty sure he's saying that the solution to a problem will work for simple cases as well so you can just focus on those instead of trying to check with hard or complicated examples

I think if you add a paragraph or two here with some more explanations, it might help people understand where easy cases come from. I feel that if you understand that easy cases can also be boundary conditions, or conditions where symmetry occurs, it will be easier to understand. I immediately think, ok, what are the boundaries, and then what happens at zero and infinity. Explaining this would probably be helpful.

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Good call on starting with this example. The Atwood's machine example is a big step up from this one.

I agree. This example is also very clear and easy to follow. It makes it easier to understand the technique you're trying to describe instead of just jumping into something really complex and confusing us with terminology or semantics.

A quick question about the topic of easy cases.... so the previous 2 opening paragraphs describe how we'll now begin looking at "lossy" methods of problem solving. But I'm a little confused how this pyramid volume problem illustrates lossy complexity?

I think this example as well as the Atwood example are mainly used to show how to develop and work with the "easy cases" but don't really show a lossy result. I think they're setting up for later sections with more complex problems.

I like this recognizable formula. :D

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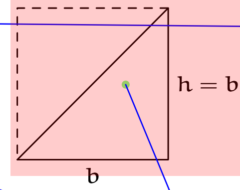

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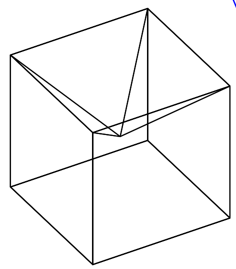

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Hmm...I've never even thought about this problem!

I think we did this back in high school, and it was really interesting!

This is a really interesting problem, I've also never thought about it.. I always took equations like this at face value and never looked into them

I agree. This problem presented a really interesting way of looking at something I'd never really considered before.

I think we don't really need this case, it is pretty obvious and it isn't too clear how it shows the 3d case

I like the use of the 2d case. I actually never thought of a problem like this until i read through this quick example. It might be a little clearer if it was fleshed out a bit more as to what exactly we are doing, but it definitely captures the idea of finding a solution that must work simply by looking at a simple case.

yo dude speak for yourself – i like starting with baby steps.

I like this example too, I probably would be lost if you didn't build it as nicely as it is with the 2d then 3d case.

Although I appreciate why we start with easy cases, I don't immediately see the extension from 2d to 3d here.

Need we choose h = b, or could we just use a rectangle where h does not equal b, still draw a diagonal line, and still successfully show that A bh, and the constant in that is still 1/2?

I agree that any triangle would work, but if we are focusing on simple cases, setting h=b is the simplest.

Isn't this method similar to divide and conquer? I say it could be classified in that way

I think divide and conquer is taking a complex problem and breaking it down into smaller pieces. This fits more with the abstraction problems (I think)

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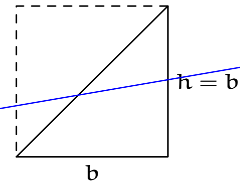

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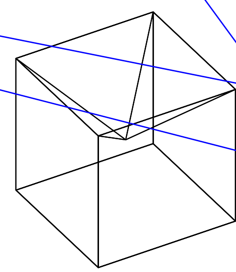

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Once again, this class finds me thinking about answers to questions I never would have even considered asking.

This is such an awesome example. I really like when he grabs formulas that someone made you memorize in 5th grade and you've always taken for granted and shows you where they come from.

I definitely agree with this it is awesome to reevaluate things we've taken for granted for so long and see why they really work the way they do

Also, this makes use of finding proportionality. the area of a triangle is always proportional to bh, with a constant of 1/2

Can this method be used to find the volume of different types of objects?

I like the progression from an easy example to explain a harder one.

Can't this be generalized to the case of a rectangle with two triangles that are reflections of each other?

I like informality here. Most classes I've taken focus formal proof. This one focuses on practical application.

I mean, how much formality do you need for the area of a square?

What I have liked is how visual this class has gotten... its much easier for me to understand something if I see it then if I am handed a bunch of equations.

I think part of that is also the fact that we've memorized the equations in this section like the back of our hands way back when.

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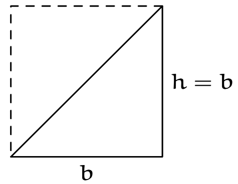

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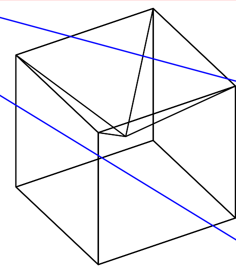

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This may seem like a silly question, especially since everyone seems to understand it already, but it is not entirely clear to me why we can since it works for the easy case, it will work for all cases (I see how if it works for the hard cases, it works for the easy ones, but how does it work going from easy to hard?). I mean, I know what the area of a triangle is, but, I just want to make sure I know how to make the correct assumptions with harder problems.

I think it works because the simplifying assumptions - 45 degrees and b=h - make it easier to reason visually without manifesting themselves in the equations. The two assumptions are just made so it's easier to see and not so that the math is easier. For this reason it doesn't matter if we change b and h, assuming the shape (rectangle) stays the same.

why is this a major emphasis?

I'm pretty lost by this explanation...

Nevermind, unconfused. For some reason though it was the following stuff with the cube that help clear this up. I think the explanation is a bit overcomplicated which makes it a bit confusing.

This is just a minor suggestion, but you might want to include "because if it works in the easy case, it works in all cases" somewhere in here. I know you say it a few lines prior, but it might help solidify things.

I agree- this would bring it back to the main point of the topic.

I disagree, it's only been a paragraph since that statement and the italics at the end kind of reiterate that but with specifics to this problem.

Agreed, although this is common knowledge, maybe it would have been better to use a more complicated triangle to reiterate the point.

How do we know that this is not just a special case and actually is universal? We did say that, only when h=b then A=b^2.

I think what is trying to be said is that the point of these easy cases is that the principle of this estimation is to say that a solutions is valid for all.

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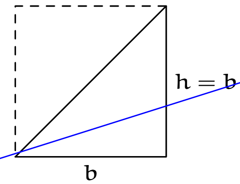

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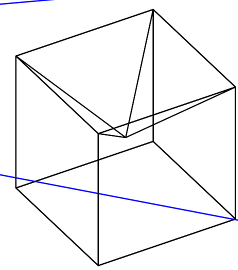

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do you mean A/bh = 1/2 is the constant?

He wrote it correctly. He uses a twiddle and says that the constant in A bh is 1/2, so that precisely A = 1/2*(bh).

heh, it's a bit silly how much time we've spent on this whole and prop to symbols.

Could you make the mathematical connection/transition between the two cases clearer?

how can one be sure that the same principle applies in 3-dimensions? in other words, once we find the constant of 1/3 for a simple case, how can we be certain that it will apply for all hb^2 pyramids?

Extrapolation? Common sense? I dunno, I guess you would just test one or two. Or realize that the geometric properties are true regardless of the lengths of the sides

I agree with the first post here - I definitely don't intuitively grasp how the 1/2 in 2d implies a 1/3 in 3d

is this just finding the easy case? by saying the simplest case is when they combine to make a square?

I feel like this method is very case specific. Its the most intuitive process to make a shape out of the pyramid

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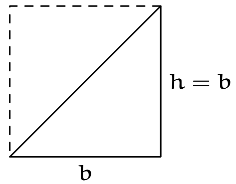

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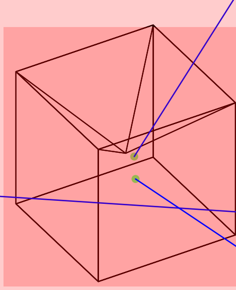

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This is a really helpful picture for visualizing pyramids that have h =1/2 b. However, pyramids that are more extreme, like really large h but really small b, are still difficult for me to visualize forming into a cube.

genius! Coming up with the respresentation of a pyramid in a cube in this way is something that simplifies the calculation of its volume. However, how can we assume that it works for all pyramids? Is it enough to just use the triangle analogy.

this is the image i thought of originally. once you have this its easy to see how it works.

yeah this picture is super helpful, otherwise I think I would not have been able to visualize it in my head very easily

The picture is VERY helpful and VERY well done. The only suggestion that you maybe could play with to show the height better would be to maybe put a pyramid on top and on bottom to show the height of the cube is 2x the height of the pyramid.

unclear.

last time we found an easy case of h and b and made sure the area was correct, now we are doing the analogous thing in 3d.

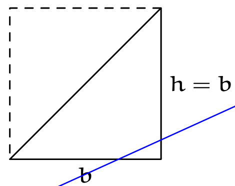
so It's 1/6 the volume of a cube?

yes, for a cube of side length b and pyramids where b=2h, exactly 6 pyramids have the same volume as the cube. he does the calculation below. this is really clever.

That makes sense now.

good image

this graph helps a lot .. i wasn't picturing like this



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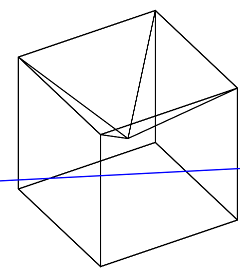

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so where is the loss of information? i'm confused as to how this fits in this section.

I don't think there was necessarily a loss of information in this example, but it definitely showed how we took a seemingly-complex problem and solved it by simplifying it into an easy case (in this particular example, assuming that the base is twice the height and also assuming that the pyramid is square). I guess the "loss of information" would be that we assumed that the base was square and also that 2h = b. This example fits this section quite nicely and does a good job of setting the tone for the chapter.

I would look at it more as losing complexity (sometimes at the cost of losing information). in this example, we're discarding complexity by considering this "simple case" with pyramids with b=2, h=1 (the loss of information would be the limitation of this solution to this case). then, if we find a correct answer, it has to work for other cases too, so we go back to the general case with base b and height h.

How did you determine that only the ratio matters? Or was that something you just made to be so?

well i suppose h/b is a dimensionless group...

We're just trying to find a set of pyramids that will fit inside a cube. Obviously if we scale everything by some constant factor, they'll still fit inside a (scaled) cube, so the absolute size doesn't matter. Thus, only the ratio is important.

I dont really understand how we determine which aspect ratio is "right."

I guess the "right" aspect ration is the one that makes the problem the easiest. The determination of what is easiest, I suspect, is the hard part.

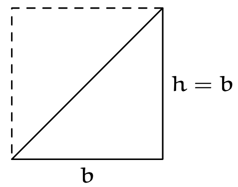
Aha, but you had to know h would be 1/2b to pick b=2. The process can be iterative to make the numbers easier, I know.

It really doesn't matter what you choose, though. Personally I would choose b=1, because that makes the volume trivial to compute.

Shouldn't it just be b/2? so they can meet at the point.

confusing visual for me

I agree. I had to draw a 2D picture from different viewpoints to understand.



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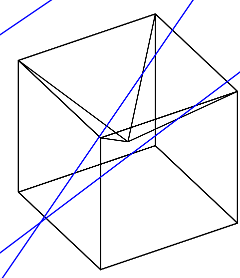

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Very clever.

an somewhat unrelated note, i think kids in elementary schools would find this method very amusing, fun and easy to understand!!!

this is really cool but i also think that it could be explained more concisely – it's pretty intuitive with the right picture

Cool! The picture on the side helps get it across as well.

I agree, it's a nice clarification.

I agree. I got a bit lost in the description, but the picture really cleared things up.

I thought the description made more sense than the picture, so there's something for everyone to understand.

Both helped me, I think

I never realized this! Seems to simple now

Yeah, this was a great example. I always did the 2D version in my head to get the 1/2, but never extended it to get the 3D!

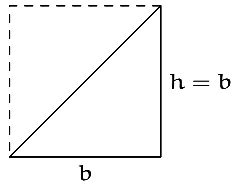
This explanation was a little confusing to me. I would change V to V_pyramid and I would reward the phrase "and must make volume 4/3" to "and must make V_pryamid equal to 4/3."

How did we jump to this?

For a 3-D solid, the volume will be proportional to the product of the three length dimensions, in this case of a pyramid with a square base, h*b*b = hbˆ2. We just don't know what factor (a fraction of 1, since it's smaller than a cube) the formula for this shape has.

Right, the added length from the 2D model is another base, since we are assuming the base is square.

I really like this explanation. It was very easy to follow.



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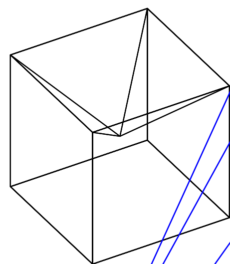

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This is an interesting way to spatially break down the problem. I always found it easier to use 3 triangles with a 90deg corner in the space of a cube, such that all of the square bases lie on the xy, xz, and yz planes.

I'm confused about how the fact that these particular values for h and b are relevant for any pyramid.

He stated earlier that he chose b=2 for this arithmetic but in order to have six pyramids that fit into that cube, the constant must be 1/3.

The way this was arrived at was pretty easy to follow, but I think one more statement about how this easier case allowed us to find the constant for ALL cases would be useful

I'm confused as to how you can claim this is hb^2. How do we eliminate b^2/h as a possibility? We may know what the area of a pyramid should be, but in a problem where we don't know the proper solution, how do we eliminate alternatives?

That's a good point I didn't think about that alternative I'm curious as to how we can discard wrong solutions as well

I think we can intuit that by looking at the dimensions. First clue is that volume needs to have dimensions of L^3. Since b and h are both L dimensions you then have to figure out whats squared and what not. Since height square doesn't really make sense for how we define height, squaring the base dimension works.

In this case, I will agree. It's always nice to summarize after a long problem.

I agree, this conclusion seems hastily drawn...a slower and longer explanation would make things perfectly clear.

I also agree that a summary might help...the example was great, however. As for the uncertainty about why we twiddle hb^2, look at the note surrounding the equation. It comes from the 3 dimensions the pyramid is comprised of. The same base and height as a triangle, but an added 3d dimension outward of b, since we are assuming the base is square.

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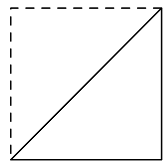

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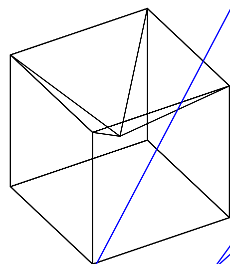

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But this would only work for pyramids where the base is twice the height

ie: An easy case.

Would it then be beneficial / interesting to show how the easy case can be extrapolated so that the base does not have to be twice the height? At least I'd like to see it.

I think the point of this section is to just prove easy cases and say "well, it's a good enough approximation for all other cases" without worrying about proving said other cases.

is this idea perhaps to be used in junction with dimensional analysis at all? or is it a separate thing?

I like this, but for a lot of our uses, won't we end up still leaving the constant out? So it would be V hb^2

Whether we are leaving the constant out in future problems (taking advantage of the dimensional analysis method), we wanted to actually solve for the volume of a square pyramid, so while we did find out that V hb^2, we were really interested in simplifying this case in order to determine that the factor is in fact 1/3. That was the goal of this example problem.

True, we might do so as a way to quickly achieve other goals, but here, the exact formula is our goal.

The point here is not to estimate a volume of some pyramid, the goal is to find an estimate for the formula.

I think in stead of just proving the case, using simple numbers to show how it works helps a lot.

I agree, though I must confess that I do constantly fight the urge to ask "But how does that show that work in EVERY case?" Easy cases seems to go against the urge to make everything like proof.

Like everyone else has said, I think this is clear and useful. It might be tempting to use this all of the time, even when it's not the best way to do a problem.

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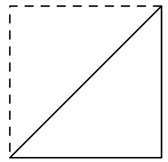

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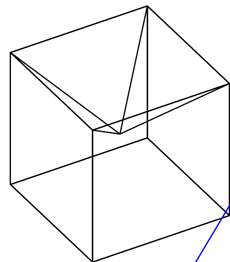

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I agree with what was earlier that it would be nice to see the link on how we can use easy case to understand the concpet and use it to solve more difficult cases. Unless we ware just saying do easy cases and assume them to be close enough. If that's the case then I think maybe stating something like that a little more explicitly would be helpful, that way people are not trying to make the connection.

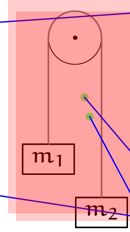
I really like examples like this where you take basic examples we know and 1.) bring up ideas/questions we never thought about and 2.) make us realize how complex the problem can be and 3.) as always, simplify.

Yeah, I like being shown something I already know so I'm 100% sure the method works.

I agree as well. This might have been one of the cleanest sections I've read in a long while

yep it was a great example!

6.2 Atwood machine



The next problem illustrates dimensional analysis and easy cases in a physical problem. The problem is the Atwood machine, a staple of the first-year physics curriculum. Two masses, m_1 and m_2, are connected and, thanks to a pulley, are free to move up and down. What is the acceleration of the masses and the tension in the string? You can solve this problem with standard methods from first-year physics, which means that you can can check the solution that we derive using dimensional analysis, easy cases, and a feel for functions.

The first problem is to find the acceleration of, say, m_1. Since m_1 and m_2 are connected by a rope, the acceleration of m_2 is, depending on your sign convention, either equal to m_1 or equal to $-m_1$. Let's call the acceleration a and use dimensional analysis to guess its form. The first step is to decide what variables are relevant. The acceleration depends on gravity, so g should be on the list. The masses affect the acceleration, so m_1 and m_2 are on the list. And that's it. You might wonder what happened to the tension: Doesn't it affect the acceleration? It does, but it is itself a consequence of m_1, m_2, and g. So adding tension to the list does not add information; it would instead make the dimensional analysis difficult.

Var	Dim	What
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These variables fall into two pairs where the variables in each pair have the same dimensions. So there are two dimensionless groups here ripe for picking: $G_1 = m_1/m_2$ and $G_2 = a/g$. You can make any dimensionless group using these two obvious groups, as experimentation will convince you. Then, following the usual pattern,

$$\frac{a}{g} = f\left(\frac{m_1}{m_2}\right),$$

where f is a dimensionless function.

Pause a moment. The more thinking that you do to choose a clean representation, the less algebra you do later. So rather than find f using m_1/m_2 as the dimensionless group, first choose a better group. The ratio m_1/m_2 does not respect the symmetry of the problem in that only the sign of the acceleration changes when you interchange the labels m_1 and m_2. Whereas m_1/m_2 turns into its reciprocal. So the function f will have

I'm a little confused as to what exactly is the "easy case" in this problem (vs the complex case). In the pyramid example we expanded the triangle case into the harder pyramid case, but I can't seem to distinguish this step in the Atwood example.

The easy cases are taking m1 to be huge and m1=m2. There are lots of other mass ratios that we could use, but those would be harder cases.

The easy cases come at the end when he tries to check to see if the answer is correct.

Don't remember if they ever actually called it this..

My physics teacher did.

I'm glad a figure is here

I am not very comfortable with physics but the way this is explained is very helpful and clear to me.

Ew, 8.01.

Physics is a very valid place to use happy approximation techniques, otherwise it gets VERY nasty very fast.

Agreed. It's definitely easier to work with physics when combined with approximation.

If we model the cow as a sphere...

How did we come to this conclusion? Is this the second use of easy cases that we will describe in this section?

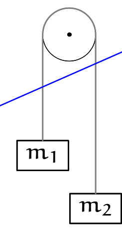
typo

unless you dance when you check the solution.

hehe... you made me laugh :)

NB should have a "Like" option in addition to the "Agree"

6.2 Atwood machine



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what is the point of going through a dimensional analysis and a more standard 8.01 approach? Seems like we aren't saving anything here.

I think the point is to show how you can use it with dimensional analysis and easy cases, but then check the answer with standard 8.01 physics. You may not always be able to check your answer so easily...

And, it's still pretty easy to make a mistake when doing things by normal 8.01 procedures. So having two separate methods allows us to be fairly certain that our answer is correct.

Perhaps as a proof that the dimensional analysis method works?

Shouldn't it still be g?

I believe it should be dependent on m2 because of the rope and pulley. It is not in free fall.

If you drew a free body diagram, there would be a downward force of m1g, but there is also an upward force from the rope. You can imagine if the masses were equal, a would = 0, not g.

This notation is confusing.

I agree. You should say "equal to the acceleration of m1 or -m1" not the equal to the mass itself.

It's just saying that the sign of m_1 is the opposite of the sign of m_2.

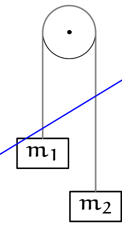
i don't think that's what they're objecting too. more like it should be written a1 or -a1.

Agreed with the person above.

I understand that it's important to continually emphasize previously learned points, but we've just finished the unit on dimensional analysis, and personally I find it hard to wrap my head around this new topic when we're still trying to use the old dimensional analysis problem solving techniques. I'm not sure I could say what "easy cases" is if I can't separate it from dimensional analysis.

For me it helps me to get a better grasp of dimensional analysis. I think it's valuable to see it in following sections because it solidifies my understanding of it and I don't just stop using it as soon as I finish the unit.

6.2 Atwood machine



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Pause a moment. The more thinking that you do to choose a clean representation, the less algebra you do later. So rather than find f using m_1/m_2 as the dimensionless group, first choose a better group. The ratio m_1/m_2 does not respect the symmetry of the problem in that only the sign of the acceleration changes when you interchange the labels m_1 and m_2. Whereas m_1/m_2 turns into its reciprocal. So the function f will have

so we cant always discard mass in gravity problems. you said that in class the other day and it had been bugging me.

It doesn't matter in the general case of accelerating towards earth, as Galileo showed in his experiment dropping two objects. It does matter in this case, however, since the two objects are applying a force on each other, not an acceleration. Think about it...what if the two objects were the same mass? What if 1 object was weightless?

I should be more precise. You can discard the mass – if there's only one mass. With only one mass, it cannot be part of a dimensionless group.

But with two masses, their ratio is dimensionless, so the masses can affect the behavior (but only through their ratio, not their actual values – i.e. if you double each mass, nothing changes).

That's were it gets tricky though – knowing that's it. I feel like if I were to do this I'd think I'd forgotten something...

(to clarify, I wasn't thinking of tension so much as friction of the pulley, etc. – are we just neglecting those?)

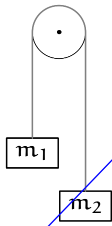
I think we are neglecting friction, but I agree that it's hard to know exactly where to stop. This almost certainly qualifies as a 'use the force' moment, which seems to just be something you have to get a sense for.

I think this 'that's it' decision comes from a feel of what we want to get out of the problem. We only put in what we want to get out. We don't care so much about how friction might affect the solution.

I never would have thought of tension- it's pretty obvious that it's not an external force and would therefore be dependent on m and g

right. it's sort of like if you drew a control volume around the system, it's not an external force acting on the system.

6.2 Atwood machine



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a	LT^{-2}	accel. of m_1
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I think throwing in the buzz word that tension is "dependent" on the other variables. Describing independence helped clarify in the last reading.

Does this even work when we take friction into account?

This assumes a frictionless cord/pulley.

I found it easy to see how tension doesn't matter when I realized that the tension on the m1 side of the pulley canceled out the tension on the m2 side.

I don't know if it's my screen but this line appears to be in a lighter shade of gray than the black

I think it's just your screen

mine too.

It was once in red (before I made the book all black and white), to indicate that "a" is the quantity we are looking for. I think the red got grayscaled, meaning that the color turned into a shade of gray. I'll find another way to indicate that "a" is the goal variable.

I think boldface might do the trick. Or worst case just leave it plain... I don't think it necessarily needs to be called out. (Looking back, it appears the same problem occurred in the tables of other readings, too.)

I really like that we have this box here. It makes thinking about the problem much easier.

I agree, it would be nice to see boxes like this in other parts of the readings.

I agree. These boxes make it really really easy to see the relationships between variables.

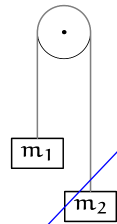
How do we know it's a/g and not g/a?

We don't. g/a would be the same group, to the -1 power. As he mentions in the next sentence, we can make any dimensionless groups from these two groups.

I didn't realize this meant "group 1" right away, I was looking for a variable G or universal gravitational constant.

Haha. That's what i did too.

6.2 Atwood machine



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is this equivalent to writing a/g   (m_1/m_2)?

Not necessarily, since we don't know that it's a linear function. It could be a/g = C*(m1/m2)^2

It's the same as discussed in the last reading memo, where if you have 2 dimensionless groups, 1 is a function of the other, it doesn't specify how it is a function

No. The two might not be linearly related (the twiddle just hides a constant). The function could be linear, or quadratic, or inverse, or some other power of the second group

I'm still unsure how this differs from dimensional analysis.

you could also maybe write or G2 = f(G1) just to remind us of this replacement notation that is later used

This is a much clearer example of a dimensionless function than the previous readings.

I agree. The choice of dimensionless groups in the above paragraph is made very clear as well

What would a function with dimensions be?

I like this idea.

Yeah, this is a very helpful principle in general.

It's analogous to, "The more triage that you do before you pack for a trip, the easier your whole trip will be."

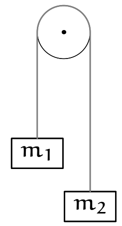
I figured this out the hard way many times.

Same here! I remember erasing profusely on these questions in 8.01

How can we see that this group isn't already good enough?

This is tricky. Looking at the symmetry helps, but it's not something I would have thought of at first.

6.2 Atwood machine



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Easy cases, dimensional analysis, and now symmetry? Looks like we're using many tools to solve this problem!

It's like a meta-problem! I do like that he's back referencing previous units though. It makes the order we learned them in make more sense

Yep, that seems to be the case. Admittedly, I stlil probably would ot approach this correctly though.

I'm just confused as to what symmetry exactly we're violating.

I agree, I feel like we're using too many tools. Sometimes I like references to previous units, but this time it feels like too much. The references to the previous units are overpowering the relevant "easy cases" content.

The symmetry in the way the problem is set up states that if a/g=f(m1/m2), then f(1/x)=-f(x). (i.e. switching the masses reverses the acceleration.) This is a much harder function to construct than one in which, say, f(-x)=-f(x), as in the bending light example.

i'm confused about what this means...

how do you know this? would it still work? are we looking for a new dimensionless group or just going to re arrange the ones we have?

this sentence seems awkward by itself; maybe combine it with the last sentence via a comma?

I don't understand what he's trying to say.

Agreed, it doesn't really seem to connect well with the previous section.

Agreed it could be worded better, but in case you didn't catch it from the rest of the reading: He's saying that the way the function is written doesn't match the way the system acts in real life. In real life if you switched the masses attached to the pulley, it would accelerate in the opposite direction. The way the current function is written, switching the masses creates a reciprocal instead of switching the sign of the acceleration.

i definitely did not get that from this sentence.

to do lots of work to turn the unsymmetric ratio m_1/m_2 into a symmetric acceleration.

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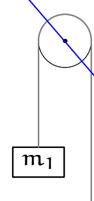

Here is a plot of our knowledge of f:

That is a good point. But is it intuitive like this case most of the times? I feel like I might have trouble figuring it out when it comes to real problems.

Would this even be possible?

G1?

Why aren't you using the capital Pi from class?

referring to group 1 = m_1 / m_2

I know, but I was wondering why he didn't use the capital Pi he used in class. It seems like his variables for 'groups' are shifting. And I don't like G. It does look like the gravitational constant.

isn't this not dimensionless?

I like this process of dimensionless analysis, but I'm still uncomfortable with it sometimes. It feels like it requires a lot of creativity, or it feels like cheating...or maybe using creativity to cheat, lol.

Haha that's exactly how I feel about it...I feel like there would definitely be times where I'd neglect a variable and make some strange and wonky dimensionless groups.

I agree. I often find myself including unneeded or redundant variables.

I feel like using this method will become easier and more intuitive with practice.

It just seems strange that we can use this instead, just because it's symmetrical and it won't affect the outcome

We have to start somewhere and this is just kind of the starting point. In order to get it to be dimensionless, we will have to do more things to it later.

i don't understand what this sentence means.

I don't either; why exactly do you choose m1 - m2, why not m2 - m1, or m1+m2? It is just because m1 is moving in the opposite direction of m2? Then, why do you divide by m1+m2 lower down?

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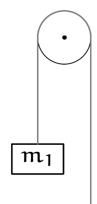

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Here is a plot of our knowledge of f:

I have no idea what you mean by this little figure...

I think its just to show that acceleration symmetry mentioned earlier on, since acceleration depends on m1 and m2

If the values of m1 and m2 are switched, then the acceleration of m1 will be of the same magnitude, but in the opposite direction, so the sign will change. That's what "mass interchange" means, though it's a poor term.

This is a cool way to thing about the physics without really thinking about the physics.

I remembered the solutions in 8.01 always had this sort of form!

so in the examples in the previous readings/in class, we could have done this too, like (var 1+ var2)/var1 or other types of manipulations such that it ends up to be dimensionless, why didn't we do all these before, why did we just use simple multiplication or division?

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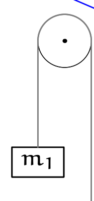

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yeah if this was always allowed, then I feel like we should have done an example like this in which the variables are added or subtracted in the unit about dimensional analysis.

I think this problem is harder then some of the other ones we have done so we need to use a bit more complex method. We cannot just divide or multiply so it becomes dimensionless.

I agree, but in this case there are clearly two separate variables of the same dimensionality, so it makes more sense. Before, you would have had to make strange groups like (2F - ma)/F, which would just give a dimensionless constant. It usually makes more sense to multiply to get dimensionless groups unless you have two separate things like lift and drag or two different masses.

The rough intuition is that the more closely the dimensionless group matches the physics of the problem, the less work the dimensionless function f has to do. In other words, the simpler the function f is, and the easier it is to guess.

So, the whole approach is a kind of divide and conquer. Instead of using the simplest group, e.g. m1/m2, and then having a hard problem to guess f, you do some work to turn m1/m2 into a nicer group, and then some work to guess f.

This is all fine to understand once it's already done, but I sincerely doubt I would go through the same rationale to get the result that ends up working.

I definitely feel that way the first time I see a problem solved a new way. But I think that once you see it done a couple of times, you figure out how to apply it to solving your own problems.

hopefully he'll give a few other examples in class, and this will become more intuitive

and now we know to keep it in mind as we solve other problems.

True - sometimes on the problem sets I see how a concept could be applied, but I often feel myself struggling because I've missed some important step.

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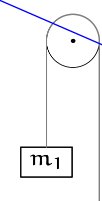

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It almost feels like "easy cases" is taking a back seat to dimensional analysis here. I realize you need to do dimensional analysis to get to the easy cases part, but is there a way to condense the DA to focus on the easy cases?

I agree - this seems like we're just doing work from the previous section rather than working on "easy cases"

I dunno, I kind of like this, as it's illustrating how the previous topic interplays with the current topic. We don't know much about "easy cases", so it makes sense that we'll need to rely on our other techniques at first. Plus, this actually tied together a few more knots for me with regards to dimensional analysis

I am a little confused on what you mean by symmetry here

Why does putting m_1 + m_2 keep symmetry?

does this violate the equation? dividing one side but not the other?

Not really – remember, we're still using an unknown function f() to relate one side to the other. This just means that f() will be different.

No, we are still working inside the f(). As long as that remains dimensionless, the equation is satisfied.

Interchanging m1 and m2 should give an acceleration with the opposite sign because of the way the system is physically connected. In order to make our function f() as simple as possible, we are making the argument to f() similarly symmetric.

I forgot to take momentum into account when I first thought about this.

how does it respect he symmetry of the problem, can you explain this a bit more

it respects the symmetry, in that if you interchange m1 with m2, you get the negative of the exact same value (which we want–intuitively, this makes sense).

i really like this idea of "the symmetry of the problem." AKA we know that nature doesn't care about our imposed coordinates systems or signs.

@ previous post: that was a really interested way of looking at it, that nature doesn't care about our imposed coordinate systems or signs, and the symmetry of problems. That puts it in a great perspective!

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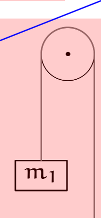

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I am kind of confused with this - it seems like the groups are just chosen at random. How do we chose without having a lot of prior knowledge of the atwood physics.

These formulas are all the same if you were to do it algebraically, I don't see the benefit of using this new method (it's somewhat confusing)

I agree, I remember finding the derivation rather helpful in 8.01

I think the point here is to give an example of how you could use this method to solve a problem we know how to solve so that when we get tougher problems we'll know what to do. Since we already understand how to solve these problems we can trust the answers we get here and understand the reasoning better than if we were given something that we had never seen before.

This is very clearly explained. Even when I thought I had questions or problems, all I had to do was go back and re-read the particular part and my question was answered.

I don't understand why x is introduced here, it just seems to make things harder to understand.

This is very helpful because we can easily plug in m2=0, m1=0, or m1=m2, to make sure the magnitudes of our answers are relatively correct

This is always the "last problem" on any part of any physics test. "What sort of behavior will you expect if m approaches infinity? if m approaches zero? Does your equation give the expected results?"

I agree that this was very helpful. It allows us to get a tangible feeling for the physics governing the problem.

I hate to be a brat, but do you think you could avoid highlighting an entire paragraph in your comments? It makes it so we can't highlight specific portions to comment on. Thank you and happy approximating!

I too thought of limits when I read this paragraph. Isn't this one of the upcoming chapters?

to do lots of work to turn the unsymmetric ratio m_1/m_2 into a symmetric acceleration.

Back to the drawing board for how to fix G_1. Another option is to use $m_1 - m_2$. Wait, the difference is not dimensionless! I fix that problem in a moment. For now observe the virtue of $m_1 - m_2$. It shows a physically reasonable symmetry under mass interchange: $G_1 \to -G_1$. To make it dimensionless, divide it by another mass. One candidate is m_1:

$$G_1 = \frac{m_1 + m_2}{m_1}.$$

That choice, like dividing by m_2, abandons the beloved symmetry. But dividing by $m_1 + m_2$ solves all the problems:

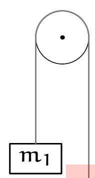
$$G_1 = \frac{m_1 - m_2}{m_1 + m_2}.$$

This group is dimensionless and it respects the symmetry of the problem.

Using this G_1, the solution becomes

$$\frac{a}{g} = f\left(\frac{m_1 - m_2}{m_1 + m_2}\right),$$

where f is another dimensionless function.



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Here is a plot of our knowledge of f:

It might still be nice to see an 'm2' here, especially since this paragraph also talks about a case when m1=m2 and not just the case when m1>>m2

I agree, a more detailed figure then this might be even more helpful. Possibly showing the result of the easy cases.

I think an arrow may be helpful, or some sort of force directional.

I liked how this section was a good synthesis of the things we've learned, but I feel that the easy cases part was thrown in at the end like an afterthought, rather than the main substance of this section. Though, that might be pretty representative of how easy cases are usually used.

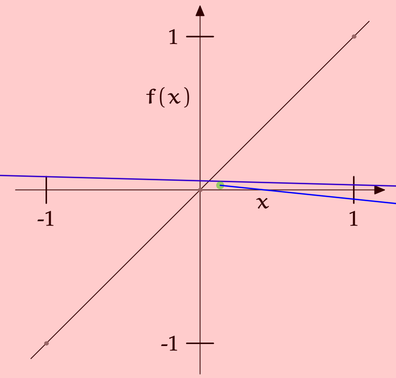
these simple cases are excellent proof-of-concept demonstrators. will we see an example of a fluid mechanics problem?

This is not technically a plot of our knowledge of f. Such a plot would have only the 3 easy points described. Drawing a straight line connecting them is our "educated guess" as you said below. It might be nice to literally draw what you say; or you could rephrase what you said to match the plot... "say what you mean, mean what you say"

I agree...how exactly would this be a plot of our knowledge? would it be in relation to the problem that we are knowlegable about?

Also agree. There should be a step where the leap from three collinear points to the assumption a linear function occurs. Because it could be, for example, f(x)=x^k for any odd k, or f(x)=sin(2x/pi), etc.

This line would look better if it were on the next page, above the plot.



The simplest conjecture – an educated guess – is that $f(x) = x$. Then we have our result:

$$\frac{a}{g} = \frac{m_1 - m_2}{m_1 + m_2}.$$

Look how simple the result is when derived in a symmetric, dimensionless form using easy cases!

OK I see how m_1-m_2 solves this now.

Plotting the data definitely puts the info into a more readable format

Yay for diagrams!

it's be nice to plot the points we just discussed that we used to form this graph

I always found it cool how plots make things so much easier to see.

definitely - especially when they're nice linear relationships like this one

Yes, it especially helps to check the special cases and extremes on the graph and make sure they make sense to the problem.

Regarding the question above if we plot our our 3 simple cases we get three points, but how do we know that the the relationship is linear? Is it just a case of the simplest guess is usually correct?

This was just a set-up, right, for the less elegant problems we're going to encounter shortly?

Yea are all the cases this "easy?"

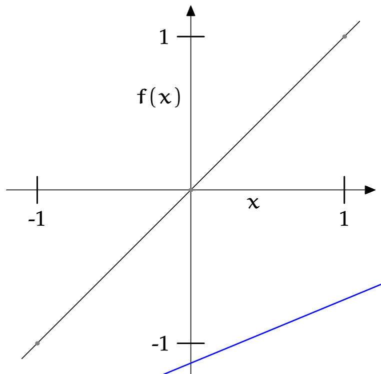
this clears up how we solve for f from the previous readings

I intuitively understand this, however, another step would have been nice to get from the graph to the ultimate equation.

I don't see how this is valuable enough to extend to solving most types of problems, this seems to only work when the solution is very simple anyway

Well this is how you would approach these types of problems, if you recall from 8.01. The answer / relationship here is that same that you would get fro mforce diagrams. That's really what we did, we just didn't draw them out.

I thought that too, but I believe the whole point was to show how we can use common knowledge and previous techniques to simplify problems.



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Look how simple the result is when derived in a symmetric, dimensionless form using easy cases!

I think we need to check m1=m2 to show that the correct dimensionless quantity is a/g and not g/a?

If the dimensionless quantity was g/a, the plot would asymptotically approach -infinity in the left half plane, and +infinity in the right half plane as x tended towards zero, making it a not linear relationship.

how does this differ from the case where your second dimension less group was m/m? I don't really see the benefit of changing the groups here

This section was awesome. It's great to see how all the concepts we've learned so far can be put together and applied to solve hard problems.

I still think the algebraic method would have been easier in this case.

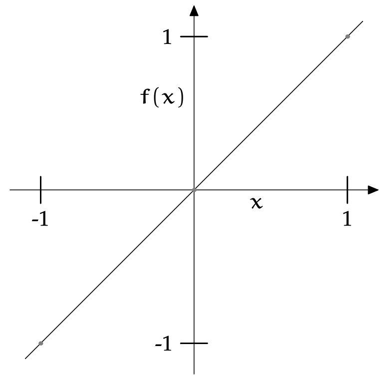
Well, I think the point is to show a simple case so you can then use this for harder cases. (like, rewriting G to make is symmetrical seems like a bit of extra work for a problem this simple, but it makes sense for more complicated problems)

I agree. I think the algebraic method would have been faster and easier for this one.

Actually, I disagree. I think we're comparing an algebraic method which we've been trained on for years to a new method that we've only known for a few weeks at most. Imagine how long it would have taken to go through this rigorously if we hadn't all had years of similar math and physics problems.

It looks like their is a lot of trial and error in these cases. I guess that's why it is easy cases.

so what would be the easy case here? the fact that we were choosing which variables to look at in an easy way?



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Look how simple the result is when derived in a symmetric, dimensionless form using easy cases!

After reading this section, I'm still not sure where the "loss" is. It seems we reasoned through things, using easy cases, and we came out with pretty solid formulas.

Agreed - what was lost? I thought we were pretty thorough and looked like we came to a reasonably correct solution.

In this case I don't think anything was lost but I can imagine cases where we can not predict f(x) exactly, and maybe this is where the loss comes in?

That's true; in this case our guess was 100% correct. But it's not guaranteed to be right. And in the upcoming fluid-mechanics examples, you'll see that you get reasonably good predictions, but not exact ones – because we've thrown away information (by assuming that we are in one of the special cases).

So in simple cases there is no loss, but in regressing to simple cases from more complicated ones we lose information. ok.

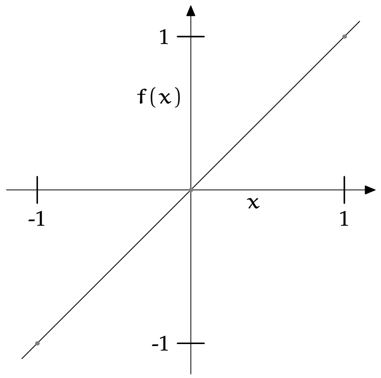
But is something like throwing out friction a loss here? The two easy cases of no mass on the other end (or no rope) and equal masses would remain the same, but friction would change the shape of our plot.

This was no more lossy than the bending of light problem, for example. In all of these cases we have performed dimensional analysis and then guessed a (usually conveniently linear) function. Also, in both cases, we have predicted values for the coefficient of the linear term (although in the bending of light problem, there were several theories).

Perhaps it would serve to calm any slight ambivalence the reader might have by stating explicitly:".. in this case our guess was 100% correct. But it's not guaranteed to be right. And in the upcoming fluid-mechanics examples, you'll see that you get reasonably good predictions, but not exact ones "

I wish I had this text available when I was in physics! This concept seemed so difficult and complex when I learned it in 8.01 but after reading this section, I feel so much more confident with the concepts and feel like I actually understand them. This is one of the most concise and complete sections of any of our assignments.

I think this is interesting, but it seems like easy cases like this may as well just be done out the bruce force method.



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$$\frac{a}{g} = \frac{m_1 - m_2}{m_1 + m_2}.$$

Look how simple the result is when derived in a symmetric, dimensionless form using easy cases!

So what is "easy cases"? After reading this introduction to the topic, I'm still not sure exactly what it is...

yeah i agree..I feel like this was more about dimensionless groups. I'm not sure I understand what easy cases are.

Easy is not trying to think about cases like m1 = 40 and m2 = 60 and what the acceleration would be, but rather thinking about m1 = 40 and m2 = 0 or 40. It could also be called extreme cases here. Then we can find the solution for all the intermediate states from those two situations + symmetry.

I think doing a bit more of a formal intro to Easy Cases in the beginning of this lecture would be helpful. I realize you might reach some conclusion at the end of the section, but I think it'd be nice to have some guiding/overaching principle from the beginning.