

## 6.3 Drag

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When the Reynolds number is high – for example, at very high speeds – the flow becomes turbulent. The high-Reynolds-number limit can be reached many ways. One way is to shrink the viscosity  $\nu$  to 0, because  $\nu$  lives in the denominator of the Reynolds number. Therefore, in the limit of high Reynolds number, viscosity disappears from the problem and the drag force should not depend on viscosity. This reasoning contains several subtle untruths, yet its conclusion is mostly correct. (Clarifying the

This section is too short. I'd like to see 6.3.1 and 6.3.2 both expanded with some more explanation. I'm not lost, but just unfulfilled.

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For Sunday's memo, read this introduction to easy cases and drag (for high and low Reynolds number).

is there a reason we talk about drag so much? I feel like it comes up in every unit

what easy case that you refer to?

I believe he is referring to easy cases in general.

I also don't think there are separate categories within easy cases, so as was mentioned before, it's just easy cases in general for solving complex problems. Whenever we are using an easy, correct solution to suffice for all problems of considerable complexity, there will usually be a loss of some accuracy. That's what easy cases refers to.

Easy cases is one approach to solving difficult problems wherein you consider a simpler scenario and use the technique there to extrapolate how to approach the complex one. A basic example is considering something in 2D, and then adding another dimension to deal with a similar problem in 3D. Unfortunately, using a basic solution in a simpler case, you will sacrifice accuracy in your approach.

maybe say, "meaning, it throws..."

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**Aren't all the estimation methods we have learned somewhat lossy? How is the easy cases method so different?**

nope! some cases are "lossless." for example, symmetry, like how Gauss solved the addition from 1 to 100 problem. the answer he got was EXACTLY correct and didn't lose any info at all :)

I think he means we intentionally discard something with this method. We know that we are throwing something away when we choose some sort of an extreme case. When we use methods we don't explicitly lose information, there is just some "guessing" involved.

I agree with 7:36...was there an explanation of this concept at the start of the book? either way, it might be useful to restate with you mean by "lossy method" at the start of this chapter.

Is there a better word than lossy here? I mean, I do understand it, but I winced a bit reading it. Is a method with loss, is a dropping method; I'm sure, but lossy feels weird.

**lossy is a good way to describe this, although it might be good to explain it at some point to those who do not know what it means**

**I don't know what lossy means...**

I think it means we disregard a lot of information to simplify the problem and we overlook some of the actual parts of it, just piecing it together with our own estimation

I interpreted it as unreliable. like the above said we are making many estimates along the way so things are really not that accurate.

**what was the information it threw away?**

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**It might have been nice to mention this in the above examples - some people were confused before as to where the information was "lost".**

I agree with this. Even though we mentioned we were moving onto "lossy" methods in class I don't think it was mentioned in the previous sections

Yup. This came up in class as well, and I also agree that it's a good point. The previous examples were lossless!

I think we were suppose to infer from the introduction for the previous section that we were moving into lossy territory.

Right, but it still seems strange to label something as such when it isn't. It's also confusing because there are so many labels of different techniques when methodologies can be similar.

**I get what you are saying here, but the wording of this paragraph is kind of awkward.**

**I am not quite sure of the relationship between easy cases to approximation? Do we eventually build off the easy cases to solve more complex cases? or am I confused by the language?**

"easy cases" is the title of this particular approximation method. Although, I feel like there has to be a better title for it...it seems to confuse a lot of people.

**So much drag... why is this class considered course 6 at all?**

Course 2 kids said the same thing when we were doing UNIX.

it's about the methods and I personally like using the same example as it helps me think about the problem when I know the answer as it helps me learn the method.

I agree, it's helpful to use the same problem as an example for 2 main reasons, 1: It shows that one problem can be solved in many different ways, and just because you don't remember the exact way to solve it, you can still apply a variety of methods to arrive at the same answer, and 2) we can check our answer from the new method by checking it with the answer from an old method

i agree it's useful but i'm also tired of reading about drag and i did take 2.006

At least we end up learning about a tough topic in a much easier manner

Yeah I like the fact that we're tackling the same problems using different methods.

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I know this has been brought up before, but a short explanation of what the Reynolds number is (instead of your reader having to go to wikipedia) would go a long way.

We also went over this in class, which was very helpful. Again it might be nice to see this in writing

To prevent another student from having to go to wikipedia: its the ratio of inertial to viscous forces in a flowing fluid. It is used to distinguish between laminar (low Re, smoother flow) and turbulent flow (higher Re, characterized by more randomness, eddies, vortices, and other instabilities)

Thank you!

I think it would be helpful if you provided the equation of the Reynolds number here. I had forgotten the exact form of the equation we derived earlier by the time I read this memo.

since it's already been taught, i don't think it needs to be restated here. we could easily flip back and read about it again. save trees!

Oh I see, thank you for the post.

**Why can't dim. analysis solve this?**

Because it can never find those factors like "1/3" that we saw in the last section. it only deals with the groups of variables.

I think easy cases get us more accurate answers than dimensional analysis as it brings in the factors we had originally ignored (like the factor of 1/3 mentioned above).

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I believe that, in class, we were actually able to use dimensional analysis to establish (at least the basic form of) the function,  $f$ , as the relationship between the volume of a pyramid and its height/base.

Is the, perhaps obvious, reason dimensional analysis doesn't work here because the drag coefficient and Reynolds number are, by definition, dimensionless?

If that is the case, then should we consider the methods to come a sort of "Plan B", in general, for when dimensional analysis fails?

Dimensional analysis never finds  $f$  for you. even for the pyramid, dimensional analysis only gave us  $V/hb^2=f(b/h)$ , and then we had to use separate geometric reasoning to find  $f=1/3$ . Dimensional analysis always deals with dimensionless quantities.

Here, Drag coefficient and Reynolds numbers are particular combinations of other variables that yield a net dimensionless products. ( $Re = \text{density} * \text{velocity} * \text{diameter} / \text{viscosity}$ , for example).

Dimensionless analysis has not failed, it's just harder in this case to determine the function  $f$ . We used the physical experiment of dropping the cones to find two data points and we interpolated in a previous reading, but we can't extrapolate just from these points because of the (not particularly nice) behavior of  $f$ .

That was a really good explanation.

I think people are right about having a brief reminder of what Reynolds number is because I am slightly confused reading this paragraph

This was the only method used when I learned dimensional analysis before, but it's obviously somewhat limited in that you need to be able to measure a couple examples before you can find the trend.

Is there a particular reason why the data was limited when using cones? What would be a ideal situation for testing this?

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**I don't remember this result from the cone dropping...I only remember getting a good estimate of their velocity ratio**

Oh, I get what you mean. Perhaps explain more clearly that air is a Reynolds number generally between 2k and 4k?

Maybe this can be found from the cone example, but we just didn't explicitly go through this part in class?

**It'd be nice to see an annotation of the previous section that this was solved in for easy reference (i.e.: (1.2))**

**obviously cannot be proven using experiments, so we'll look at end cases for very small and very large cones?**

Isn't this the idea of easy cases, we are looking at the two ends to see how it will respond in extreme situations.

**I would expect that the surface area of the cones becomes more important for high Reynolds number.**

I think it might be useful to just reiterate and explain what a low or high  $Re$  actually means

and also maybe an example of the extreme cases of the Reynolds number

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### Because they describe the limits?

I think so. Measuring when  $Re$  is in the range 2000-4000 doesn't seem to provide any intuition about the behavior of the function  $f$  in either extreme.

Well, if you do the experiments, you're adding data points and more data points will always give you a more accurate map of  $f$ . However, easy cases look at the extremes which should give a pretty decent map of  $f$ , but may be prone to error since it gives trends.

I think that he means that a map of  $f$  could be created by using a large range of Reynolds numbers, 'such experiments' referring not to only the extreme cases, but to experiments like the cone examples, but over a range of different  $Re$ 's.

Also, how do you know what range is a limited range? How do you know that 2000 is not the minimum and 4000 is not the maximum limit. How do we know that the range goes from 0.1 to  $10^6$  and not from .00001 to  $10^{12}$ ?

You don't. You keep on doing experiments until you see little change, indicating that you're still in the same regime.

### I would think that a high Reynolds number experiment on a paper cone would be nearly impossible

True, although you might be able to do it by dropping in an incredibly inviscid fluid instead of air.



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Not especially, but I get what you're trying to say.

Depends on to which experiments are actually being referred. Conducting experiments where the Reynolds number is  $10^6$  would be rather difficult for the common student/reader.

Yeah, with a very high Reynolds number, the paper cone experiment would be tough.

I don't understand why.

Because Reynolds number is  $\rho v L / \mu$ . So to achieve large Reynolds numbers, you will need extremely large density, velocity, or length (or even some large combination of the three!) parameters. The magnitudes of which are so large, most people would be unable to actually develop an experiment that would allow them to test it.

Probably at least more difficult than estimation...

Maybe just saying they are often not feasible given the average person's available materials - whereas anyone can take out a pen and paper and do the math.

That's a good thing to point out: the extra data points may give you more accuracy, but not necessary more accurate reasoning.

I thought we already knew the Reynolds number and its implications. It seems to me that it would explain quite a lot.

It will be very helpful to have a diagram of a cone in this portion. It would definitely allow me to conceptualize everything better.

Wow, this reminds me of those recursive problems earlier in the book. (in particular the one with the "game")

Could you elaborate on why this reminds you of the recursive problem? I don't see the connection. Also, what section is the "game" problem you mentioned?

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I know this was said earlier in another comment, but a general explanation of the Reynold number would be nice. Not only do I not know anything about the Reynolds number, but the following discussion about the meanings of different Reynolds numbers is lost on me.

A lot of what is needed to know about the Reynold's number was explained in earlier sections. It would be redundant to mention it again.

I think the issue is that the readings are spread out over the course of a semester, whereas if it was an actual paper book it would be easy to flip back to the section where it was defined.

### Square or cube??

Good point, the readings about the pyramid first talks about the square base of the pyramid and then it goes into combining the 6 into a cube, whatever case is being referred to should be specified.

Does it really matter? We get the point—it's symmetric.

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For the pyramids, it was quite intuitive as to what would constitute the "easy case". Here though, I can't say that it would occur to me to claim that  $Re \gg 1$  is an easy case. What is the reasoning process here?

He does say that "The physical reasoning in this regime is the subject of Section 6.3.1," so maybe you'll find your answer there.

I think "easy" refers to things like making things extreme or equal to some known quantity or solution (like in the Atwood machine where we made one mass huge and then when we made the two masses equal). In this case, the "easy" part comes from taking "extreme" cases

I think people are reading a bit too far into the use of ' $1$ '.  $Re \gg 1$  and  $Re \ll 1$  simply mean that  $Re$  is relatively huge and relatively small, respectively.

What's important is that for our  $Re \gg 1$  case, viscosity is negligible, and for  $Re \ll 1$ , viscosity dominates.

In general, "huge" for Reynolds numbers can be much larger than thousands or tens of thousands or more, depending on the geometry.

"Small" can mean anything from much less than 1, to "few", to less than a thousand, etc.

I guess someone who's taken thermo would find this fairly obvious, because the cases you're generally looking at are  $Re \gg 1$  and  $Re \ll 1$  (and for  $Re \approx 1$  things get tricky).

is this doing the same thing as the last reading with the pulley? examine two cases and extrapolate from there?

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**More explanation? I don't understand why there are two cases**

just a quick thought..Re could be confused as being the Real part of the Reynolds number...even though it's not possible for it to be imaginary

The reynolds number is a ratio of inertia forces to viscous forces. Low reynolds numbers, especially less than 1 are dominated by viscous forces, meaning that the inertia forces (or density \* velocity \* characteristic length) are not important. I don't know much about these low Reynolds number scenarios, but I think these situations are for micro-organisms and probably many other interesting motions. And so having two cases is important since the motion is dominated by very different forces.

what would be the information lost in this case?

Eek, this is quite difficult to understand without first researching the Reynolds numbers. Granted, this was explained in lecture in class, but as a textbook, for students / instructors that may not have had the corresponding background lecture, this chapter would be confusing. It would be a good idea to have an explanation, especially since this will be compiled into a book.

**I'm not sure where else this has come up, but I like that you actually reference the future sections in the intro. It gives the reader a heads-up**

**Is this saying there is an almost infinite range of the Re?**

**so under what condition do you assume  $Re \ll 1$ ?**

**Maybe I just don't remember seeing this before, but it seems strange to reference these sections when they're about to be introduced to the reader for the first time.**

I think that is a pretty standard text book technique "we will talk more about this in section ... "

**In my experience, turbulent flow means a Reynolds number  $\gg$  about 2300. Why did you choose to simplify it to  $\gg \gg 1$  and  $\ll \ll 1$ ?**

I think 1 is more for the low Re limit, in which  $Re \ll 1$ . In the high Re limit, we're not thinking about laminar/turbulent transition yet, just about very very high Re and the resulting relation between drag and viscosity in that situation.

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People's comments about a definition of Reynolds number hold very true here, seeing at least the equation would make this statement more clear to the reader.

**At what point in terms of Reynolds number does the flow officially become classified as turbulent?**

And also, how do we know that it does?

Perhaps some intuitive explanation would help.

There's a period of transition Reynolds numbers between 2300 and 4000, but it's safe to say anything higher is classified as turbulent flow. In any case, it can just be assumed to be a high number over  $10^{3.5}$ .

2300 is the accepted transition between laminar and turbulent flow. Very few flows are actually laminar, as it turns out.

Is there a background or reason for this cutoff?

**DO we know this from experimental observation or from physical/theoretical law?**

**Is there a method to approximate this value as well or is there an actual value? In thermo, I learned that the transition period is between 2300 and 4000 which seems like a huge range?**

**I really like the description and pictures we had of this in class, it was nice to really see how drag worked visually**

I agree. I also think it would be beneficial to describe what turbulent flow actually looks like and contrast that to uniform flow.

sad I missed that lecture :(

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**make the air thin like at high altitude**

Can air be viscous? I thought this was more talking about fluids, so if  $\nu=0$  then the object is in air

Air is a fluid. It has a viscosity 2 orders of magnitude less than water.

just to further clarify, fluids are things that deform under shear forces, so both liquids and gasses are fluids.

Or we could just all breathe in deeply at the same time...

**this font similarity for  $\nu$ 's is getting a little too intense**

**not sure if it was mentioned previously but  $\mu$  is also used to represent viscosity, that might be a nice alternative.**

**since the Reynolds number is a function of many variables are you just choosing viscosity to be the independent variable**

**An equation of the Reynolds-number formula would be nice here.**

Yeah, I understand that viscosity is inversely proportional to the Reynolds number, but I have no idea where else the Reynolds number comes from

$Re = \text{density} * \text{velocity} * \text{characteristic length} / \text{viscosity}$ . It's a ratio of inertial forces to viscous forces.

**It would be nice to see the equation so one can clearly see that viscosity disappears in the limit of high Reynolds number**

Agreed. I have no idea where the Reynolds number comes from, and actually being able to see the equation and refer back to it in sections like this would be nice.

I would agree as well. For someone who hasn't internalized/memorized the Reynolds number equation, it would be nice to see the equation each time you bring it up.

I agree as well, it's a little more tiresome flipping back through the previous chapters.

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isn't viscosity the only term in the denominator of the Reynolds number? So if it went to 0, the Reynolds number would go to infinity. I'm not sure how it just "disappears".

It's not the Reynolds number that disappears, it's the viscosity that goes to 0 and thus disappears, or in other words, if there is drag, it doesn't depend on viscosity at that limit.

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is there drag force without viscosity?

only in a world where everything is ideal. viscosity is basically another word for friction.

There is absolutely drag force without viscosity, since the object still has to deflect the fluid in its way. The model  $F_d \rho v^2 A$  still applies.

We can apply this rule to objects in water too right? I believe they have viscosity as well.

But wouldn't it still approach zero as the the viscosity went to zero? I guess there is still some force... like to change the momentum of the fluid since it still has mass..?

I like the earlier comparison to friction...there is still drag on a frictionless surface, if I remember correctly. We tend to assume there is no friction due to air, but there is drag.

Remember that viscosity is a property of the fluid, not a property of the object.

Friction's 'purpose' is to create the laminar boundary layer around the object, 'enabling' the fluid to exert a viscous drag force. (When actually using Navier Stokes, etc., to model the flow you assume that the fluid in contact with the object is not moving relative to the object. Someone correct me if I'm wrong on this.)

But, regardless, there are inertial drag forces, as well, and that has to do with the object having to displace an amount of fluid per unit time proportional to  $v \cdot A$ .

It is the relative importance of these two sources of drag force (inertial and viscous) that the Reynolds number measures.  $Re$  (inertial force)/(viscous force).

(To be a bit more technical:) Inertial force is  $F \rho v^2 D^2$ . Viscous force is  $F \mu v D$ , where  $\mu = \rho \nu$ . Their ratio is  $(\rho v^2 D^2) / (\mu v D) = \rho v D / \mu$ , which is the commonly presented expression for the Reynolds number.



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what do you mean by "mostly" correct?

I think it's on the same order of magnitude but i am not sure

I would love to see an example of how easy cases are actually lossy, since this is the first type of approx. that is technically lossy.

I think that restating how the viscosity affects the Reynolds-number would be very helpful here.

This is a strange to put this sentence.

what are these untruths?

This is a sentence that I hope is explained later on or in class, because it's quite a tease.

what did you throw out at unimportant? and what do you mean by "mostly correct"?

subtleties required two centuries of progress in mathematics, culminating in singular perturbations and the theory of boundary layers [6, 30].)

In other words,  $f$  is constant! The consequence is

$$F_d \sim \rho v^2 A, \quad (6.2)$$

where  $A$  is the cross-sectional area of the object.

Therefore, the drag coefficient

$$c_d \equiv \frac{F_d}{\rho v^2 A} \quad (6.3)$$

is a dimensionless constant. The value depends on the shape of the object – on how streamlined it is. The table lists  $c_d$  for various shapes (at high Reynolds number).

Object	$c_d$
Car	0.4
Sphere	0.5
Cylinder	1.0
Flat plate	2.0

### 6.3.2 Viscous limit

Low Reynolds-number flows, although not as frequent in everyday experience as high-Reynolds number flows, include a fog droplet falling in air, a bacterium swimming in water [20], or ions conducting electricity in seawater (Section 6.3.3). Our goal is to find the drag coefficient in such cases when  $Re$  is small ( $Re \ll 1$ ):

$$c_d = f(Re) \quad (\text{for } Re \ll 1). \quad (6.4)$$

The Reynolds number (based on radius) is  $vr/\nu$ , where  $v$  is the speed,  $r$  is the object's radius, and  $\nu$  is the viscosity of the fluid. Therefore, to shrink  $Re$ , make the object small, the object's speed low, or use a fluid with high viscosity. The means does not matter, as long as  $Re$  is small, for the drag coefficient is determined not by any of the individual parameters  $r$ ,  $v$ , or  $\nu$  but rather only by their combination  $Re$ . So, we'll choose the means that leads to easy physical reasoning, namely making the viscosity huge. Imagine, for example, a tiny bead oozing through a jar of cold honey.

In this extremely viscous flow, the drag force comes directly from – surprise! – viscous forces. The viscous force themselves are proportional to the viscosity  $\nu$ . In fact, the viscous force on an object is given by

$$F_{\text{viscous}} \sim \text{viscosity} \times \text{velocity gradient} \times \text{area}, \quad (6.5)$$

nice to see this mentioned. was worried that the generalizations made earlier in the paragraph would have been left without mentioning that there is much more to it than just high speeds leads to turbulent flow.

How did we arrive to  $f$  being constant?

yeah i'm a little confused here

What  $f$  and why is it constant? I agree that viscous forces are constant at zero, but I'm not sure how you can say that drag force is constant, or at least I don't see in what way it's constant.

Agreed, I don't understand how this conclusion was reached. I understand what was said in the previous paragraph, but I don't understand how you can make the jump to this.

I'm a bit confused too... You're saying it's constant with respect to the Reynold's number, but that is a function of velocity as well, isn't it? Or are we assuming velocity is constant also?

Well the previous paragraph talked about the drag force, so perhaps  $f$  is  $F_d$ ?

I don't think it's directly saying that drag force is constant but rather that because viscosity drops out,  $f(Re)$  constant. Therefore using equation 6.1, the drag coefficient =  $f(Re) = \text{constant}$ .

I think that if you say that the function " $f$ " is constant it will be a lot less confusing than just saying that  $f$  is constant. Saying  $f$  is constant makes it sound like some variable in an equation while, here, we're talking about the equation/function itself.

On second thought, I'm not really sure I know what is meant by  $f$  is constant. What does it mean for a function to be constant? I think understanding this could help me understand how we got to the conclusion that  $f$  is constant.

We got to the idea that the viscosity is 0 because we wanted a way to get an infinitely high Reynolds number. We then used this conclusion to eliminate viscosity from the problem. This only works if you continue to assume that the Reynolds number is infinite. So  $f(Re)$  essentially becomes  $f(\text{infinity})$ , which I suppose only makes sense if it's a constant function, not really dependent on the Reynolds number. But the whole thing is extremely poorly worded.

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**This seems like a pretty abrupt conclusion...**

yeah I agree...is it possible to give some additional background on how this conclusion was reached?

i think it sounds abrupt because it's right after a very formal parenthetical. i think the transition could be smoothed

I agree. I have no idea where this came from.

**This kind of seems like it came out of nowhere. Maybe you should mention the equation explicitly somewhere in one of these paragraphs so we can remember it from the last reading?**

**It's still really unclear to me how the drag force remains when the viscosity goes to zero...**

what is the physical basis for the drag constant when there is no viscosity?

I think viscosity is basically how "thick" a fluid is. like how water is more viscous than syrup (it's more free-flowing). but even if something isn't thick, when an object travels through the fluid, it still comes into contact with the fluid, so there is still drag.

Why doesn't 0 describe a vacuum state? It seems to me that the lowest you could ever possibly get is no contact between an object and its medium of transport (i.e. a vacuum). Instead, it seems that 0 describes a superfluid, which seems to be something altogether different.

**in 2.006, we denote crosssectional area as  $A_c$  (A sub c) to remind us, might be helpful**

**How is cross section defined. I think the shape of a rocket has smaller Reynolds number than the shape of a car at the same velocity, even if they have the section area. Am I right? If this is true, why?**

Well right below this it says the value depends on the shape of the object and how streamlined it is.

I think the cross sectional area in this example is the area perpendicular to the direction of motion.

**this reasoning is super confusing**

**doesn't this just always = one? or am I mixing up v's?**

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**I'm a little lost. I know we had previous readings on this drag coefficient but how is the  $c_d$  so small when you make the viscosity 0?**

The  $c_d$  does not get smaller when viscosity gets smaller, it is solely dependent on the shape of the object. It quantifies how "aerodynamic" a shape is.

what makes you say it's "so small"?

**How do we know what the force is without knowing the coefficient?**

We are simply defining the drag coefficient. In order to find the value of the drag coefficient, we will have to measure the force (or vice versa, to find the force, we will have to know the drag coefficient).

**I like this chart, but it may be helpful to mention at about what Reynolds number these are taken at. ( What value constitutes a high Reynolds number)?**

Yeah, I'd like some base values as well to judge these numbers from.

I like this table too. I'd like some more object examples to go with it, I find this very interesting.

And by more examples, I mean lower numbers to go with the table below.

yeah this table was super helpful to help me visualize how different shapes/objects are related to their drag coefficient and how the coefficients compare to each other.

**Very helpful to see these in a chart, for quick access.**

**Wow, we do know this is true, but cool that we could reason it out so easily.**

yeah, it was really neat to see a previous concept so easily come across here

**Is there any  $c_d$  that is greater than that of a flat plate? And if so how high is the highest  $c_d$ ?**

**I know we've showed this before but it still surprised me to see how it only depends on shape. The table really helped to drive this home**

Well, after what we've gone through in class, what else COULD it depend on? Mass and weight don't make very much sense, neither does density.

subtleties required two centuries of progress in mathematics, culminating in singular perturbations and the theory of boundary layers [6, 30].)

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Therefore, the drag coefficient

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Can we get a little more here? I'd like to see it done out, maybe as an exercise, but I was a bit lost in this section (at least on first read).

It sounds like these examples are more common everyday (ions in seawater, bacteria swimming- these things are basically happening all the time) but the high reynolds- number flows are definitely more common in experience... interesting

Yeah, it's bit curious if you think about relatively which is more common. Though it's more a matter of scale: both of these cases happen a LOT.

No, when you actually talk about fluid flows, turbulent flow is far more common. I'm not sure bacteria can be defined as a "fluid flow."

Would water droplets forming clouds fall into this category as well?

What are some other characteristics of Reynolds number?

What do you mean by this? We already know what  $Re$  describes.

Now I see the reason for the two cases is because different forces have more impact based on size, so it really is two different problems.

fog droplet falling? this sounds odd...

yeah, what's a droplet of "fog"?

never thought of that as related. cool

How is it related?

Could that be discussed briefly in class tomorrow?

Nevermind. I read the last paragraph.

Maybe add something here about how this will come up later?

are low Reynold's numbers also characterized by shape? fog droplets and bacterium kind of have the same shape...

I'm excited to read this section coming up!!

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Does the  $\ll$  just mean much smaller?

Yes, usually by several orders of magnitude.

Yes.

I've lost sight of the lesson we are learning in this section. What is the easy-cases tool we are sharpening?

I'm also a little confused on this. I think the easy-cases might be the extreme cases (where  $Re \gg 1$  or  $Re \ll 1$ ) which we are trying to generalize to find  $f$ , but I'm not sure.

The easy cases we're looking at are the two limits. when  $Re$  is very large and when  $Re$  is very small. Thus, we are "throwing away information" –everything in between. But we know that in order for us to find a right answer, it must work for all cases. so we can mess around with these two cases to find a solution that can be generalized

I would have liked to see this a little farther up, just as a reminder.

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but not all the things in the table are round (although all the examples given are). How does this work for non-circular objects?

Usually, if something isn't round (at least for pipe flow), you use an approximation for the diameter, called the "hydraulic diameter" (this is for standard Reynolds numbers based on diameter, radius is half that, of course).

The hydraulic diameter is equal to  $4 \times (\text{cross-sectional area}) / (\text{wetted perimeter})$ . The wetted perimeter is the perimeter of the cross-section that is in contact with the fluid. You can easily show that for a circle, the hydraulic diameter is equal to the actual diameter.

I'm not entirely sure if this applies outside of the pipe-flow scenario, but this is the general idea.

To clarify, the hydraulic diameter still falls under the realm of Reynolds number based on diameter. Because the Reynolds number is a dimensionless number which only classifies a flow, you can substitute any length for diameter so long as you then define that its the "Reynolds number based on [that length]." For example, Reynolds numbers based on length (such as over a plate) go turbulent around  $5 \times 10^5$ , as opposed to Reynolds numbers based on diameter go turbulent around 2300.

The short story is that it's all just a matter of defining your parameter so it's clear what the Reynolds number is saying.

Same issue as last time with it being difficult to distinguish  $v$  from  $\nu$ . Heads up, everyone.

Could one of these variables be made bold or something to make them more distinguishable?

I agree. i had to double take

If the object is not spherical, how do we substitute this equation for this condition?

stupid grammar comment: for parallel structure, might just want to add a verb aka "reduce the object's speed"

Alternately, I see how "make" is also applied to "the object's speed low," so just the last part could be changed to "or the viscosity of the fluid high." for a parallel construction.

How high? Would ketchup work?

I like this clarification- it really makes sense

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really effective example

Interesting visual.

I agree. Its a good example of an easy/extreme case. I think the format of this section (comparing extremely low and high Reynolds number) illustrates the point really well.

yep great example to help the reader visualize something with a huge viscosity!

It's a very vivid visual and something we've all had experience with.

an explanation would be helpful for me

Look at the example given in the paragraph before about a tiny bead oozing through a jar of cold honey, that should clarify what viscous forces are.

cute

interesting

Is there any way to quickly justify this, or do we just need to accept it for now?

You can always check the units.... if that works, it's at least mildly justified.



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Isn't this acceleration?

This is pretty intuitive, isn't it?

It is, but it may not be obvious when reading. I think this is very important stated here. Thanks for it!

how is it sufficient? and why didn't we know this 2 pages ago?

It seems a lot of "easy cases" are linked to dimensional analysis in some way.

You probably have defined the variables before, but please define them again. A disadvantage of an e-book is that it is difficult to look things up.

I really like the tables that show variables' meaning, just like in the last sections. Perhaps a table at the beginning (or end) of every chapter with all of the variable definitions used in that chapter would be helpful – that way we would only need to refer to one particular page, rather than looking through the entire text. Things like the formula for  $Re$  could also possibly be included (but not derived there), just so that it's easier to find for later reference.

what was fl? flat plate?

Or maybe a page of formulas that is added to each we learn something new.

Although I agree with this as he discussed in class I'm not sure how this could be easily fixed, maybe make one bold? I don't think it's too big of a problem if you've been following the previous steps though

Once again, the nu and the v look very similar, and it's still quite confusing.

What if we used  $\mu = \nu \cdot \rho$  in place of  $\nu$ , throughout? (dynamic instead of kinematic viscosity?) The math works the same, the Reynolds number is more familiar (at least to course 2-ers, I think), and we avoid the tricky  $\nu$ - $\nu$  similarities.

I think this is a great idea.

Quick review of  $f(\nu r/\text{viscosity})$ . The  $f()$  does not change the units at all right? So then we get  $F_d/\rho r^2 v^2$  through dimensional analysis? Also, are we just supposed to know that  $f(x)=1/x$  when the  $Re \ll 1$ ?

where velocity gradient is the rate of change of velocity with distance (so if the velocity does not vary, then there is no viscous force), and the area is the surface area of the object. Because the drag is due directly to viscous forces, the drag force is also proportional to viscosity:

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I had to re-read this a few times understand what was being said. It was not apparent what was going on

This might have been covered earlier, but why do we want to make  $F_d$  proportional to  $\nu$ ?

Two paragraphs earlier, we found that for low  $Re$  flows, drag force is proportional to viscosity.

Thanks, this helped me too!

This might be extremely nitpicky, but here you have two different ways of expressing  $1/\text{something}$ . Shouldn't you just pick a convention and stick with it?

Why is the function  $f$  given by this for low  $Re$ ? Could someone please elaborate on this, I'm a little confused.

why didn't we look at  $Re \gg 1$ ?

Yeah I was waiting for that case to be explain after  $Re \ll 1$

We did look at  $Re \gg 1$ ; its the section before - the turbulent case.

Er, we did.  $Re \gg 1$  is the turbulent case - the first one discussed.

I'm a little confused on how these constants are in the equation.

In the equation prior to the above paragraph, it gives  $F_d / (\text{this expression}) = f(\nu r / \nu)$ . Solving for  $F_d$  gives this expression in front.

I think you've lost a factor of  $r$  here.

It's also interesting that nothing else about the material matters...it really is the fluid at low  $Re$

This equation does seem dense.

so did this get you any farther than where you started? I feel like were in the same place-not completly knowing the relationship all we did was move some variables around

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**Maybe I missed something but when did this missing constant come into play? Also it would have been great for a last paragraph summarizing our methods of using easy cases.**

We were originally solving for the dimensionless drag coefficient  $c_d$ , but it looks like we found  $F_d$  instead..I guess I'm kind of confused as well.

Same here, I think a summarizing paragraph to wrap up this section is really needed. It says dimensional analysis alone is insufficient which I agree with and understand but wasn't the point of easy cases to help us finish what dimensional analysis couldn't, or are you showing here that there are cases like with drag where you need complicated equations to actually figure out the missing constants.

Yup, that would be useful. This is a relatively short chapter so it's won't be ridiculous to add to the end of it.

yeah, that sentence directly after the above explanations and equations kind of confused me about the point of easy cases.

I totally agree. I was really confused after reading this section, even after rereading for the 3rd time! What I think is going on is that we said (from dimensional analysis) that  $F_d / (\rho_{fl} r^2 V^2) = \text{function of } Re$ . The missing coefficient should represent that "function" of  $Re$ . And even after doing dimensional analysis, we still can't find it. So, my conclusion is that he's concluding that this is a case when an "easy case" isn't so easy.

**I'm confused by this. I thought we got the answer using dimensional analysis, but this is now telling us that we didn't?**

Yeah this is also a little unclear to me.

**The easy case turned into no longer easy at this point for me.**

where velocity gradient is the rate of change of velocity with distance (so if the velocity does not vary, then there is no viscous force), and the area is the surface area of the object. Because the drag is due directly to viscous forces, the drag force is also proportional to viscosity:

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**What are the other shapes, out of curiosity?**

I'd guess the ones seen in the table above but I'm not sure

Except for the car..

yup

or the ones with defined/simple boundary conditions (haha basically the ones that we would do in a fluid mechanic class, but in the real world, i guess a lot of conditions are much harder to define so you guesstimate a lot)

**Did he reason this mathematically or experimentally (or both)?**

It seems like we jumped to this, our methods got us most of the way but I'm curious as to how exactly he came up with this number

I would also be interested in knowing how he came up with this value, as a comparison with the approximation methods we've used here.

Given he is a mathematician, I would say that he solved it by messy integrals of the unpleasant kind..

I think we need more here too. I think this section needs to be expanded.

**referring to the Navier-Stokes Eq?**

so "stoked"

is this related to the stokes shift?

That's cool, I like this example.

So do I, it's cool to learn the progression of how different scientific methods and discoveries came about.

Would we approximate the 6 pi?

I think dimensionless constants in general are almost impossible to approximate. For a sphere, you could've seen the pi coming, but that 6 is not as simple.

Agreed - the dimensions are a easier to come by.

**I'm pumped to apply this to Navier-Stokes.**

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**I'm interested to see how this is used for this application.**

Conditions where  $Re \ll 1$  also lead to neat things like reversible flow, which is pretty uncommon in fluid mechanics studies. You could put a drop of dye in some viscous liquid, stir, then reverse your path and the dye would regain the original shape. I suppose this also depends on the diffusion time constant being relatively long, which is captured by another dimensionless parameter.

**Again, I'm confused about what exactly is meant by "easy cases". I don't really know what methods were used to solve the problem here, and I certainly couldn't try to use this method to solve a problem myself.**

The only easy case I caught was given right at the end for  $Re \ll 1$ , but I'm don't really even understand that one.

I agree, I don't think I understand the basic concept of easy cases, or how it is applied in this case. It just seems to be more dimensional analysis

I think a short "wrap up" paragraph at the end could go a long way in describing how the method was used, etc., and just overall clarification.

At the beginning of the section, I think Sanjoy defined an "easy case" to be a problem in which we have to sacrifice accuracy because the problem is so complex. However, in some of the first "easy cases," no accuracy was sacrificed. In this one, his definition holds. I guess I'm confused about this also, just in a different way.

Easy cases tend to be extremes. Like  $Re \ll 1$  and  $Re \gg 1$ . Sometimes numbers like 0 and infinity are easy cases.

**I was expecting you to show a comparison between these 2 functions and the actual behavior observed (the graph displayed in the last section) so we could see just how close our estimations got to the actual thing.**

Yeah, I agree, I'd like to see a graph... I find I tend to lose some conceptual meaning to things when I'm just staring at equations.

That plot comes next (as we saw in class).

where velocity gradient is the rate of change of velocity with distance (so if the velocity does not vary, then there is no viscous force), and the area is the surface area of the object. Because the drag is due directly to viscous forces, the drag force is also proportional to viscosity:

$$F_d \propto \nu.$$

This constraint is sufficient to determine the form of the function  $f$  and therefore to determine the drag force. Start with the result from dimensional analysis:

$$\frac{F_d}{\rho_{fl} r^2 v^2} = f\left(\frac{\nu r}{\nu}\right).$$

The viscosity  $\nu$  appears only in the Reynolds number, where it appears in the denominator. To make  $F_d$  proportional to  $\nu$  requires making the drag coefficient proportional to  $Re^{-1}$ . Equivalently, the function  $f$ , when  $Re \ll 1$ , is given by  $f(x) \sim 1/x$ . For the drag force itself, the consequence is

$$F_d \sim \rho_{fl} r^2 v^2 \frac{\nu}{\nu r} = \rho_{fl} \nu v r.$$

Dimensional analysis alone is insufficient to compute the missing magic dimensionless constant. A fluid mechanic must do a messy and difficult calculation that is possible only for a few special shapes. For a sphere, the British mathematician Stokes showed that the missing constant is  $6\pi$ ; in other words,

$$F_d = 6\pi \rho_{fl} \nu v r.$$

This result is called Stokes drag. In the next section, we will use this result to study electrical properties of seawater.

**From this example, I don't know if this section should be called easy cases. Aren't they just complex cases that can be solved experimentally?**

I think it would be a lot easier to do empirical studies of the mid-range of Reynolds numbers. Was it possible to solve the low or high  $Re$  case experimentally and easily? We've taken the problem of drag, which is complex overall, and limited our study to just the easy cases within it.

**Overall, I thought this section was rather ironic: While it is based on easy cases, I found these examples to be some of the more complicated ones we've seen thus far in the course.**

But these easy cases are still much easier than solving for drag along the entire range of  $Re$  using Navier-Stokes or constructing experimental apparatus.