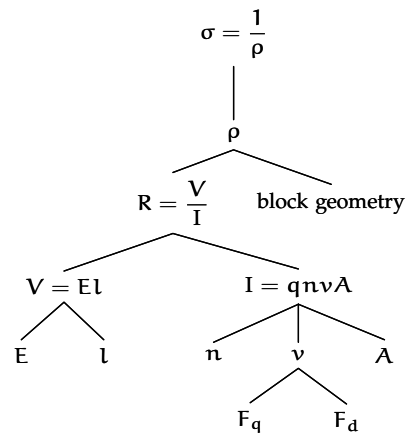### 6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by R = V/I. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qnvA,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so qnvA has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = E\ell \qquad\qquad I = qnvA$$

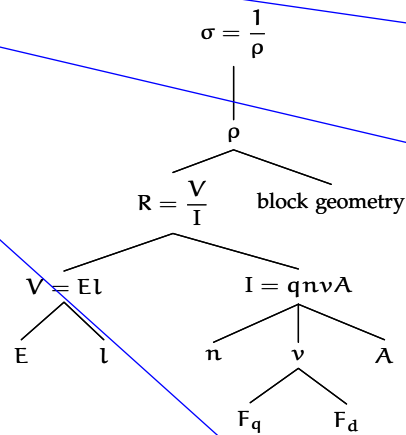$$E \qquad \ell \qquad\qquad n \qquad v \qquad A$$

$$F_q \qquad F_d$$

I thought this method of approximating the conductivity of seawater was difficult. 8.01 and 8.02 were sufficient to allow me to follow the math, but I'm not sure the level of detail used reflects what I would be able to think up on individual problems. I wish there were more frequent ties to the intuitions behind each assumption, as well as connections back to the original goal (how it fits in to the larger context as an example of an easy case).

As someone already said, it can be easy to get caught up in the detailed math and forget purpose/lose interest.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = q n v A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

ρ

$R = \dfrac{V}{I}$ block geometry

$V = E l$ $I = q n v A$

E l n v A

F_q F_d

this saltwater problem seems more like divide and conquer than easy cases.

Read these two subsections for Tuesday's memo.

I thought this was a good example for us to walk through since it's so involved, but I'm very confused: how is this an easy case? In my mind, there are way too many calculations involved here for it to be an easy case, and we don't do too much simplification of the problem.

I agree. I'm not sure where we're headed with this example or what we're trying to show.

It felt that way to me in the previous section also. I don't think I'm fully grasping the concept of "easy case".

I think this is definitely a confusing example, but by easy case we're just looking at an extremely low reynolds number, one close to zero. Because these cases are hard to find in nature it involves a lot of calculations but in the big picture it is an easy case.

I don't think easy case means easy to see, but that with the right simplifications, it makes the calculations and evaluations easy.

I agree, this example is another great way to showcase the relevance of Reynolds numbers and real world cases where they can be very low.

Is this saying that situations with low reynolds numbers are rare or that conductivity in seawater is rare? It seems to me that conductivity in seawater is constitutive and so I would guess that the meaning is the former but could this be clarified in the reading?

I read it as referring to the rarity of low Reynolds numbers.

Particularly after the discussion in class regarding the difficulties in achieving low Reynolds numbers experimentally.

I also think this means that examples with low Reynolds numbers are rare.

Yeah, I agree with this - it makes more sense than the other interpretations of the sentence.

I like that things we've learned earlier keep coming back at a reasonable frequency. When I am about to forget, it gets integrated back to the new material.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.
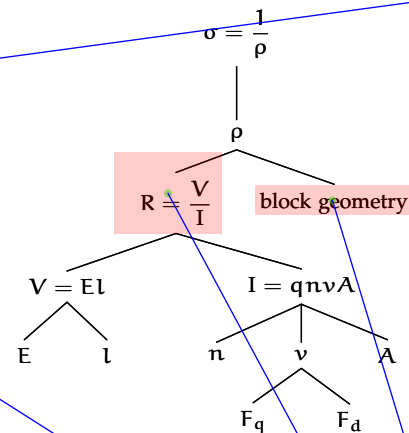
The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qnvA,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = El \qquad I = qnvA$$

$$E \qquad l \qquad n \qquad v \qquad A$$

$$F_q \qquad F_d$$

I'm a little confused, what exactly are we after in this example? The drag force imparted on a charge?

I didnt think about that until you mentioned it, but its a good point...charge flow? or something else? I guess we'll see!

to me, I was just thinking a more traditional approach to conduction (remembering that putting the hairdryer cord in the shower is bad...)

I'm a little confused also, with just a casual glance down the pages(because I thought reynolds number had to do with drag, I didn't see anything about drag or reynolds number). Maybe it would be helpful to very briefly state what the reynolds number will be used for?

The sense I got in lecture is that, given that current in seawater is primarily carried by the flow of Na+ and Cl- ions, rather than by lone electrons, finding the drag force on a single ion moving through the water will allow us to estimate the speed at which those ions flow given a particular electric field, which will tell us the conductivity.

I'm happy to see the return of the tree. We haven't used them in a while.

yeah this is a good place to refresh our memories about trees and the divide & conquer method.

This is a good refresher back to divide and conquer, and also shows us that many problems involve combination of methods we've learned so far.

we are back to unit 1

I always appreciate a good refresher...keep our D&C skills sharp!

Could it be that we're learning to solve these problems through an application of many different methods we've learned?!

is this the resistance?

Is this referring to something like the length and area of a block of seawater?

I think so

Maybe it's a control volume for later calculations.

yes it's mentioned later

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.
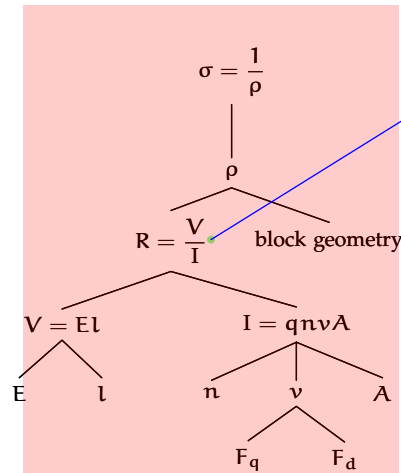
The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qn\nu A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and ν is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and $n\nu A$ has dimensions of T^{-1}, so $qn\nu A$ has dimensions of charge per time or current.

can you define these variables for us?

The variables are defined as you go keep on reading. Perhaps this diagram could be placed later in the section so that it actually makes sense?

I agree, I figured out what "solution tree" was being referenced, but glancing at it right now just made me more confused

This diagram seems a little too early. I didn't understand what it was for until the next page.

Also, it would be helpful if we numbered the different parts of the tree and had them correspond to the text where they are mentioned. That way we could more easily understand the problem.

I'm not sure thats a great idea...I think it would make things overly complicated. Placing the image lower in the text to allow the reader to catch up is definitely helpful though.

It would be helpful to have this tree after all the variables are defined. It would be easier to follow.

I actually think its fine where it is. you may not understand it at first but it is very clear as you go through the reading so in the end you understand it.

Just a thought, but perhaps you could put the questions in bold? Often times I'll start reading and get lost in the details so by then end I'll forget what we were trying to find in the first place. Like a diagram, it would help me stay focused on the main idea.

Interesting- I didn't realize it was the ACTUAL reciprocal

I think that it might be more helpful to define conductivity on its own terms, as opposed to define it relative to another term.

Well, it is logical since resistivity is a previously discussed concept.

You start to explain conductivity but more end up describing resistivity...maybe just put in a sentence to clearly define conductivity.

Conductivity is the inverse of resistivity, so he needs to describe resistivity to describe conductivity.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.
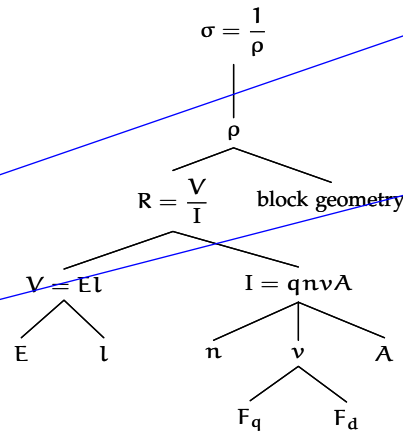
The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = q n v A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \quad \text{block geometry}$$

$$V = El \qquad I = qnvA$$

$$E \quad l \qquad n \quad v \quad A$$

$$F_q \quad F_d$$

I'm not familiar with resistivity, could the term be quickly defined here or even below when resistance is defined?

rho = resistance * (Cross sectional area / length)

Instead of writing an apology here, can you just call resistivity another variable to not confuse it with density? rho sub r? phi? I think saying, we're going to call resistivity phi, as to not confuse it with other variables, and it has units of ohm*m would be good.

Having a subscript would be okay, but I don't think changing variables is a good idea. There are conventions, and then if you see something similar later, it is easier to relate if you stick to the conventions.

I agree with this. I'm already very confused with all the variables and equations in this course.

I feel like resistivity is normally rho and you don't have to apologize or change the variable. It's just convention.

I like how this question is addressed right after conductivity is mentioned, because it's something I asked myself as soon as I read it.

Perhaps this is a vote for early bird gets the worm, but it's really overwhelming to read these paragraphs with myriad grey lines overlapping.

It would be helpful to know how it is related.

I like that the things we're learning are tying together. The initial tree diagram on the first day of classes explaining what we were going to learn made them seem quite independent.

Isn't rho per unit length while R is a total amount of resistance?

I was under this same impression from 8.02 and the like.

I think it will be helpful to show a equation of their relation. It really isnt clear as written here.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.
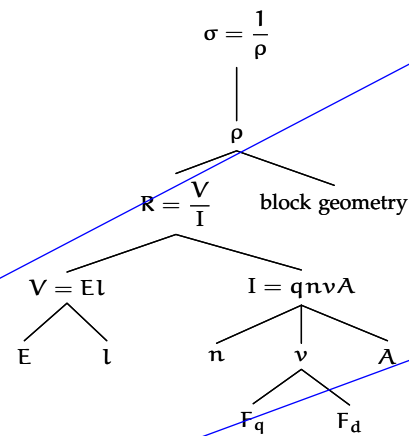
The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qn\nu A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and ν is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and $n\nu A$ has dimensions of T^{-1}, so $qn\nu A$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = E\ell \qquad I = qn\nu A$$

$$E \qquad \ell \qquad n \qquad \nu \qquad A$$

$$F_q \qquad F_d$$

maybe reword as: "then what is the difference between rho and R?"

well i think the question is more about the advantages of having two different related conventions. so it's not so much how they're different. also, i'm glad he answered this question as i was curious

for that matter...why have rho and sigma? if one is just the reciprocal of the other, don't they tell us essentially the same thing...?

So, already knowing the difference between resistance and resistivity, this explanation actually left me more confused. It could be more straightforward... (i.e., maybe getting rid of "Resistivity is related... Then why have..." and just explaining one is based on the specific geometry in addition to the material properties.)

I think it could be changed slightly to say "... for a given wire material, the resistance depends on...". ("material" being the key addition).

What is an intensive quantity? and vs an extensive quantity?

An intensive property is one that doesn't depend on the physical size of the system or the amount of material in the system; an extensive property does depend on those parameters.

In that case, why does the text say that resistance depends on the wire's length and cross-sectional area, yet is not an intensive quantity? Isn't that contradicting?

..no, that's a direct violation of the definition of intensive. Resistance depends on length, area, and resistivity. Since length and area are shape-dependent, so is Resistance. So it is NOT intensive.

Resistivity, on the other hand, is a material property.

so I guess for a fixed size and shape the resistance is know and so it is "intensive" but if the total resistance is yet to be determined (depends on length and width) it is "not intensive"? Seems like with enough information you could turn something that has resistance that is "not intensive" into an "intensive" quantity.
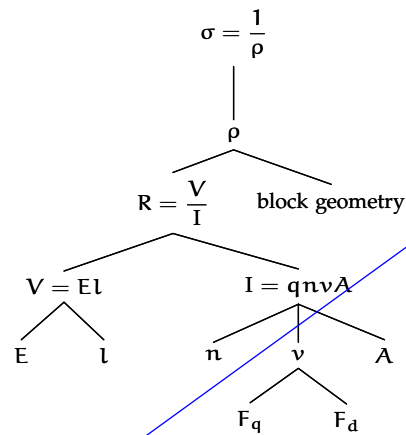
Thanks for the definition!

Thanks for this - it really cleared up this paragraph. I think it would be useful to include this, maybe as a sidebar?

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = q n v A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

ρ

$R = \dfrac{V}{I}$ — block geometry

$V = E l$ — $I = q n v A$

E — l — n — v — A

F_q — F_d

I don't see how these few sentences are relevant since we don't even know what intensive or extensive means yet. You should just sketch the solution tree and not start that little tangent maybe.

that teaser isn't fair! you have us curious, but aren't going to answer why it's not an extensive quantity.

What does it mean for a quantity to be intensive/extensive?

intensive variables don't depend upon size or weight, and therefore can be scaled. extensive variables depend on these properties and are case-specific.

I don't really like this sentence, it seems out of character for him to put something in technical jargon without explaining it.

Agreed, it would probably be better to put a simple definition intensive quantity in parens instead.

These are good terms to bring up though–very good to keep in mind when analyzing physical problems. They relate closely to problems of scale and dimensions that we've been working on.
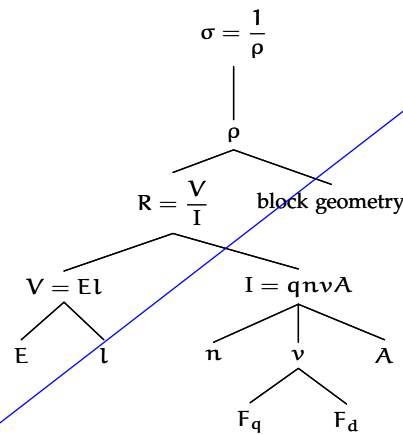
Is it not extensive since even though it depends on size via cross-sectional area and length, it's not a direct relation with volume?

This is a really interesting comment. I think an appendix should be put in at the end of the chapter (a lot of books have answers to weird questions at the end of the chapter) where it shows why this is true. But I didn't know that; very interesting point. Makes sense.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by R = V/I. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = q n v A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = El \qquad I = qnvA$$

$$E \qquad l \qquad n \qquad v \qquad A$$

$$F_q \qquad F_d$$

How can something be neither extensive nor intensive? Please explain.

I think its because it changes but isn't always reliant on the shape or size? I don't think that's true though, so I'm not sure

I agree. This is kind of a perplexing statement and I'd like to see some further explanation on how something can be neither intensive nor extensive. Maybe it just means it's sort of something in between?

what does intensive or extensive mean anyways?

I think it is essentially like the difference between density and length: one is constant for across any quantity of a given material and one is dependent upon physical alteration. This is just a guess, though.

This could definitely be made more clear. Perhaps even including the equation R = rho*d/A and explaining that rho is a property of a given material while R is a property of a given material and geometry would help clarify the difference

This little 2-sentence block of extensive vs. intensive would definitely benefit from some clarification...

does every material have a resistance and a resistivity? or are they mutually exclusive is resistance more of a material property and resistivity more dependent on the size and shape?

What about resistivity? Is it intensive, extensive, or other?

Maybe you should say "R times geometry" since nothing is actually added.

or just "R and geometry"
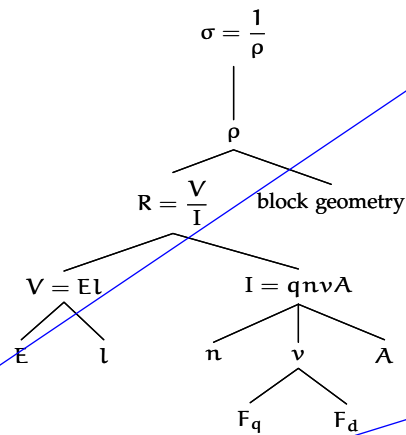
"and" is the clearest thing I can think of too

yeah i agree, the "plus" is confusing but an "and" would make the sentence much clearer.

Thanks for clearing this up. It was confusing.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qnvA,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so qnvA has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = E\ell \qquad\qquad I = qnvA$$

$$E \qquad \ell \qquad\qquad n \qquad v \qquad A$$

$$F_q \qquad F_d$$

solution for what? i'm still confused as to where all this is going.

Agreed, I'm not sure what the point of all this is. I know you mentioned finding the conductivity of seawater, but why do we want to know this? I think a little explanation on why this calculation or result is useful would be helpful.

I agree too, little confused because it seems like conductivity is the purpose but resistivity formulas dominate the intro.

I've seen it used to find the salinity of the water (since that will change from location to location and conductivity is easier to measure). However, we use salinity to find conductivity in this case so I'm not sure why its important

This might be a better place to have the diagram or even just the top section of the diagram.

I agree, the diagram works well but not in the place it's at.

What exactly is a solution tree? I feel like this is a little shady, since afaik, this isn't a standard diagram or method, and it wasn't even mentioned where this solution tree was started...

It's weird to think of rho depending on R, instead of R depending on rho... Maybe a comment on why we are attacking the problem this way?

I think the reason we are doing this is not because it physically relies on R but instead because in our case, finding rho depends on already having R.

I think the coolest part of this class is that it integrates various disciplines... Physics, math, chemistry, and etc.
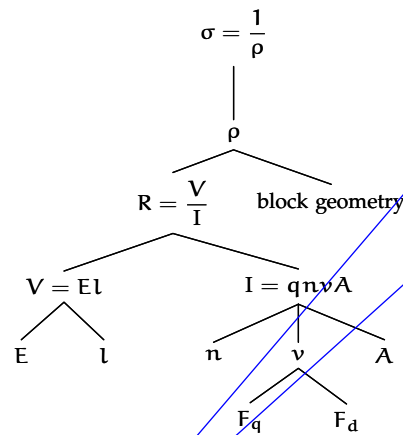
Are we just supposed to know this?

Well, knowing that resistivity is dependent on resistance is a matter of knowing the formula, and the extrapolation to voltage and current is pretty standard when it comes to the study of electricity (witness the entirety of 8.02). The next formula is pretty standard as well.

I like how this model is simple enough that anyone could come up with it (basic electricity knowledge from 8.02)

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qn\nu A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and ν is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and $n\nu A$ has dimensions of T^{-1}, so $qn\nu A$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = E\ell \qquad I = qn\nu A$$

$$E \qquad \ell \qquad n \qquad \nu \qquad A$$

$$F_q \qquad F_d$$

How do you isolate a block of seawater? Isn't there some other unit of seawater that is more appropriate? (that is applicable to liquids?)

I have a feeling we'll figure this out for the "block" and then extend it to a larger value?

So this explains why water and seawater have high conductivities?

Yes. In fact, pure water does not conduct electricity at all as there are no free ions to transport the charge.

Which, from chemistry, implies that the water we drink is not ion-free.

Kind of a random question..but does that mean if you were to use an electronic device in pure water you wouldn't get electrocuted?

I think so!

This read a bit awkwardly until I realized "transport" was the verb. Maybe say "it is the ions" or "...are what transport the charge"

I agree that this is confusingly worded. Perhaps the simplest helpful change would be to change "dissolving" to "dissolved"

Did you get this by dimensional analysis or is this a physics equation I've long forgotten?

As others ask, it would be good to say what the block is.

but if we are talking about the entire ocean how is there a cross sectional block?

but if we are talking about the entire ocean how is there a cross sectional block?
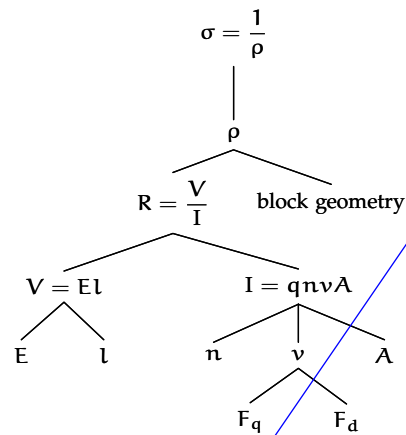
but if we are talking about the entire ocean how is there a cross sectional block? are we doing the calculation per unit volume?

So we are just assuming that all of the ions here are either Na or Cl? so Na+ and Cl- each have a charge of +1 or -1. and theoretically they would be present in equal amounts. wouldn't the + and - charges cancel each other out? or does conductivity only depend on the absolute value of the charge (the magnitude of the charge)

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qnvA,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = El \qquad I = qnvA$$

$$E \qquad l \qquad n \qquad v \qquad A$$

$$F_q \qquad F_d$$

but if we are talking about the entire ocean how is there a cross sectional block? are we doing the calculation per unit volume?

Yes, otherwise the calculations wouldn't make any sense.

if you think about it, we are worried with the resistivity, which will take into account a certain volume and a certain number of things in that volume, so it more like density in that it doesn't matter that we are only using a block of ocean. Assuming that the ocean is uniform in salt concentration, then the whole ocean will have the same resistivity as any size block (its an intensive property).

It might be good to describe the block you're referring to. I guess it's a unit cube?

I think you can think about it this way: Resistivity is an intrinsic property of seawater, so we should always get the same value no matter what configuration of seawater we measure. The easiest way to get an accurate measurement here might be to place two plates of area A on either end of a uniform chamber filled with seawater. That way we know exactly what the relevant cross-section is and can measure the resistivity.

what do you take as the cross sectional block? are we doing the calculation per unit volume?

How do we determine this in a large body of water? Any current would take the path of less resistance right? and not use the full area?

Using the full area does lead to the least resistance. Charge in a wire doesn't just flow down an arbitrary middle core of metal ignoring the outer part of the conductive wire.
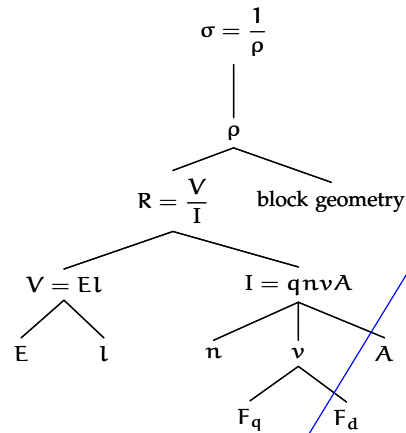
remember that is why we are considering resistivity and not resistance, I'm pretty sure we are assuming things to be uniform also

I am a bit confused by the description of these variables.

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qn\nu A,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and ν is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and $n\nu A$ has dimensions of T^{-1}, so $qn\nu A$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

ρ

$$R = \frac{V}{I} \qquad \text{block geometry}$$

$$V = E\ell \qquad I = qn\nu A$$

$E \qquad \ell \qquad n \qquad \nu \qquad A$

$F_q \qquad F_d$

Not sure what it is, but I find this wording very strange. number of ions/m^3 or something along those lines would make it more clear.

I think he prefers to use dimensions over units, though. So perhaps ions/L^3, although I didn't find this confusing.

How do we calculate this?

You can look up the composition of seawater to find out the concentration of each ion.

This seems analogous to mass fluid flow, if ion concentration were density (dimensions of mass/volume) and the left side had dimensions of mass/time.

The "its" confused me because I didn't know what to which quantity "its" was referring.

Yeah I was about to say the same thing.

I think it's ions? Though, I'm not really sure either...

Yes, it is the ions, since moving charges create currents. However, the "its" is quite ambiguous.

I don't remember having to deal with terminal speed when calculating current, maybe a little elaboration?

maybe I'm misinterpreting this, but it seems like this should be the "concentration of the water (ions per volume)" because as it reads now, it doesn't make sense to me to think about concentration of the ion as "ions per volume"

I think it means that we are simply interested in the amount of ions in some arbitrary volume.

We aren't really interested in the concentration of the water, as water isn't the conductor.

The text is probably considering the quantity of ionic units in any volume so it's not really relevant what the concentration of water itself is.
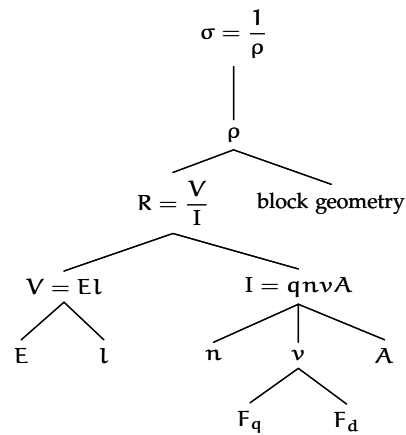
can electricty travel slower than the speed of light?

Its good that we are using all these previous methods, but when are we going to see the easy case thing? I feel like we are just summarizing the semester with this problem so far.

Comments on page 1

6.3.3 Conductivity of seawater

To illustrate a rare example of a situation with low Reynolds numbers, we estimate the electrical conductivity of seawater. Doing this estimate requires dividing and conquering.

The first question is: What is conductivity? Conductivity σ is the reciprocal of resistivity ρ. (Apologies for the symbolic convention that overloads the density symbol with yet another meaning.) Resistivity is related to resistance R. Then why have both ρ and R? Resistance is a useful measure for a particular wire or resistor with a fixed size and shape. However, for a general wire, the resistance depends on the wire's length and cross-sectional area. In other words, resistance is not an intensive quantity. (It's also not an extensive quantity, but that's a separate problem.) Before determining the relationship between resistivity and resistance, let's finish sketching the solution tree, for now leaving ρ as depending on R plus geometry.

The second question is: What is a physical model for the resistance (and how to measure it)? We can find R by placing a voltage V – and therefore an electric field – across a block of seawater and measuring the current I. The resistance is given by $R = V/I$. But how does seawater conduct electricity? Conduction requires the transport of charge. Seawater is mostly water and table salt (NaCl). The ions that arise from dissolving salt transport charge. The resulting current is

$$I = qnvA,$$

where A is the cross-sectional area of the block, q is the ion charge, n is concentration of the ion (ions per volume), and v is its terminal speed.

To understand and therefore rederive this formula, first check its dimensions. The left side, current, is charge per time. Is the right side also charge per time? Do it piece by piece: q is charge, and nvA has dimensions of T^{-1}, so $qnvA$ has dimensions of charge per time or current.

$$\sigma = \frac{1}{\rho}$$

$$\rho$$

$$R = \frac{V}{I} \quad \text{block geometry}$$

$$V = El \qquad I = qnvA$$

$$E \quad l \qquad n \quad v \quad A$$

$$F_q \quad F_d$$

Good step to remember!

This is always a great check to make sure you haven't gone astray.

This seems like it was stated a bit hastily. I find it easier to understand the points about dimensions if they are in a table, or at least some consistent format throughout the entire book.

Does this have dimensions of T^-1 because the units cancel out? Can someone tell me what it is before the units cancel?

feel like it's kinda straightforward without the step by step conclusion. I'd be comfortable with something like: "To understand and rederive this formula, let's check the dimensions. The left side, I, is charge per time and on the right hand side, q is charge and nvA has dimensions of T^-1."

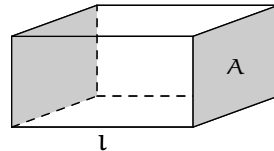it doesn't hurt. it's not that long and may be useful to a reader

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $vA\Delta t$. The ion concentration is n, so the block contains $nvA\Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = qnvA\Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again, $I = Q/\Delta t = qnvA$.

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.

The drag is Stokes drag:

$$F_d = 6\pi\rho_{fl}vvr. \tag{6.6}$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi\eta lr},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

Maybe more time should have been spent on explaining the first method, that is a lot less familiar to me than the check

Is this reading supposed to help us with the pset due tomorrow? I feel like many of the principles overlap.

can you give a real life example of how we will do this? Similar to the example about finding n in a kitchen sink.

I don't get the delta t in the volume formula.

v*delta t gives the width, the distance that the charges have traveled in delta t

This short explanation was helpful to me, maybe this short sentence could be in the text to clarify why there is a delta t in the volume.

basically, we know that the charges are moving at speed v, so over delta(t) they travel the width of the block. also, in terms of dimensional analysis, we want to find a volume here. we need L*L*L. cross sectional area takes care of 2 of those L's. another way to get the 3rd L is v*t.

the delta t is in there because v*deltaT is the width of the rectangular block of volume that we're looking at. As explained earlier, v*deltaT is how far a charge goes in time deltaT.
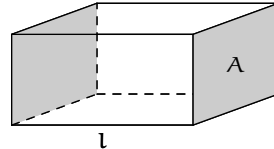
It does seem strange, just keep in mind that v here is velocity and we need to get rid of the time.

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $vA\Delta t$. The ion concentration is n, so the block contains $nvA\Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = qnvA\Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again,

$$I = Q/\Delta t = qnvA.$$

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.

The drag is Stokes drag:

$$F_d = 6\pi\rho_{fl}vvr. \tag{6.6}$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi\eta lr},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

I know what this looks like since it's been seen in 8.02, but for people who don't remember or others who haven't seen this argument before, a pictoral representation would be extremely helpful here.

Yes, I would also like to see a picture here. But I like how you included this section. I feel like it explains where you pulled the equation for current out of.

i really liked this alternative way of analysis. don't need a picture personally.

This doesn't really say anything new to me that the previous verification didn't. Perhaps you could combine the two.

I disagree, I like the having a separate section. It lets there be a clearer delineation of the sections.

Yeah, this made things very clear to me, I liked this explanation a lot.

I don't think a picture is necessarily mandatory here, just because the algebra is explained pretty thoroughly. However, I guess having the picture doesn't hurt, although it might weaken your argument for "easy cases."

Yeah, the picture already presented was what I just used to think of charge flowing through a block

I'd be helped by a picture. I don't really know much physics.

would it just be a picture of an ampere's loop? don't think thats very necessary.

I think they mean a picture of the block of charges passing through a cross-sectional area A in time delta_t.

I personally think the algebra is sufficient; it's very clear.

I really this this explanation is perfect. Please don't change it.

I remember this now

I really liked this second check. I feel that I'm becoming more familiar with manipulating units.

is this the same for any material? I am assuming yes it is just strange to think about sea water at the conductive material

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $vA\Delta t$. The ion concentration is n, so the block contains $nvA\Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = qnvA\Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again, $I = Q/\Delta t = qnvA$.

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.
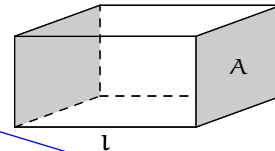
The drag is Stokes drag:

$$F_d = 6\pi\rho_{fl}vvr. \tag{6.6}$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi\eta lr},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

Specify electrostatic force? + bring up that this is like balancing gravity and drag forces for masses in fluid?

This Fq still confuses me.

is this the speed of electrons? i thought they always went the spped of light?

What exactly is the drag force, and what is causing it?

Just checking, but is the drift speed the same as the charge speed from the preceding paragraph?

I think the drift speed is the average speed, it should be the same as the one in the preceding paragraph. Electrons are the charge carriers in metals and they follow an erratic path, bouncing from atom to atom, but generally drifting in the direction of the electric field. (wiki)

I was wondering about this too. i think they are the same. Drift, in EE, usually refers to something caused by and electric field which is what was considered in the previous paragraph.

This example has gotten kind of 8.02 intensive - I have lost where the simplification is.

Yeah, since some of the paragraphs required some re-reading for me, the problem turned out to be more complex than it actually is.

I agree, I don't know if it's just me but I don't know if I can think of a different method of doing this problem.

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $vA\Delta t$. The ion concentration is n, so the block contains $nvA\Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = qnvA\Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again, $I = Q/\Delta t = qnvA$.

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.
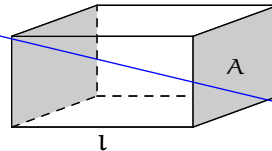
The drag is Stokes drag:

$$F_d = 6\pi\rho_{fl}vvr. \qquad (6.6)$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi\eta l r},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

the divide and conquer tree?

I think so. I don't think there was a point to put the tree at the beginning of the section when we have no clue what it is for.

Perhaps you should show partial trees all the way down, so we can see the tree as you build it up, like you did in the section about trees?

i actually really like the tree at the starting. we've already learned divide and conquer so we can quickly learn where we're going so that the explanations follow suit. I think "preceeding map" should definitely be more specific like "the divide and conquer tree"

Yeah, I don't know if these have been called maps before now.

i agree with how it is now. I took some time and got familiar with where the section was heading and I haven't had to stop and go back or re-read anything

I'm not sure exactly why, but the word "nodes" confused me for a few seconds. Maybe because it is referring to the tree on the previous page? Maybe because I was thinking about circuits and current nodes? It made sense after reading the next sentence and the 3 remaining quantities.

I kind of agree. I actually like the word nodes to describe points on the tree, but either my memory is hazy or you haven't really used node in reference to the tree diagrams in the readings very much. But maybe you should go back and add it in, because it fits with diagrams from comp. sci. and other things.

Ok this answers my earlier question, we still need to calculate n

It's confusing to group these since they are at different parts of the tree. Why not finish out to the leaves and then calculate back up to rho last? After "the preceding map" it might make more sense to jump to "To find v...".

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $vA\Delta t$. The ion concentration is n, so the block contains $nvA\Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = qnvA\Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again, $I = Q/\Delta t = qnvA$.

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.
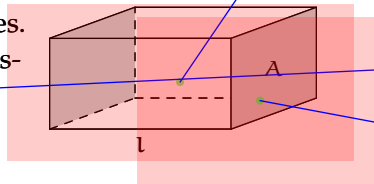
The drag is Stokes drag:

$$F_d = 6\pi\rho_{fl}vvr. \tag{6.6}$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi\eta lr},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

---

This figure doesn't tell me much about rho and R.

I would say this is more confusing than helpful, the way it is currently labeled

It just shows that rho is R times the factor of A/l, which is represented by the box in the figure, but I'm not sure if it really adds any insight that the equation doesn't already do. I guess it just helps to visualize it as well.

how does it show that, exactly?

This figure does little to illustrate anything; I can assume, a bit, what is trying to say to me (in terms of the relationship), but I think just taking it out and taking out the sentence about the figure, or labeling the figure much better, would be better.

I don't see p or R in the figure. How does the figure illustrate their relationship?

Maybe dumb question, but how would one physically measure a block of water and the charge going through it like this?

I don't get were this comes from?

Yeah, I'm surprised there wasn't an explanation given the earlier intro about the different between resistivity and resistance.

This isn't a particularly analytical explanation, but I like to think about it like this: Rewrite it in terms of R= l*rho/A. If there is a large area, it is easier for ions/fluid/whatever to flow through, so resistance based on that geometry would be lower. If the length is long (say, you have a long pipe) it will make flow more difficult than over a very short distance, so resistance should increase.

This is a really helpful way of thinking about it and I feel like it should probably be included in the text. Thanks!

I'm sure we all learned this in 8.02, but short explanations can't hurt.

Why are we multiplying by A? do the other constants above turn into R and we're still left with A?

I agree this relation should have been explained earlier when you explained the tree diagram.

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $v A \Delta t$. The ion concentration is n, so the block contains $n v A \Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = q n v A \Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again, $I = Q/\Delta t = q n v A$.

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.
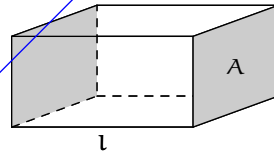
The drag is Stokes drag:

$$F_d = 6\pi \rho_{fl} v v r. \tag{6.6}$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi \eta l r},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

Why are we using stokes drag here?

We're near the low Reynolds number limit.

this is because ions are small right?

Yes, Re is small because the ions small, the resulting velocity is small, and the viscosity is moderately large.

Good practice is to check this type of assumption after finding a solution.

Thanks for the explanation. I've struggled all year with the "drag" stuff. Bring back the UNIX!

Couldn't disagree with the previous statement more..

When I read this, I wondered this as well.

How many types of drag are there?

Numerically Stokes drag is different than regular air drag, but why is this the case?

I did not find the explanation up to this point clear enough and have to read it again now :(

So we model the charges as solid spheres moving through the water?

It seems that way. It's probably not a bad model. It'd be nice to have a little justification...

I think we're comparing it to the marble in class, which is also a sphere, so modeling the ions as spheres makes it easier.

At least at some minimum length scale, ions appear like points or small spheres, and the only parameter that matters is radial distance from the center.

That's sort of rule 1 for physics: when in doubt, assume your system has the maximum number of symmetries possible, which in this case means assume ions are spheres.

how were we supposed to know this existed?

As a second check, watch a cross-section of the block for a time Δt and calculate how much charge flows during that time. The charges move at speed v, so all charges in a rectangular block of width $v\Delta t$ and area A cross the cross-section. This rectangular block has volume $vA\Delta t$. The ion concentration is n, so the block contains $nvA\Delta t$ charges. If each ion has charge q, then the total charge on the ions is $Q = qnvA\Delta t$. It took a time Δt for this charge to make its journey, so the current is, once again, $I = Q/\Delta t = qnvA$.

The drift speed v depends on the applied force F_q and on the drag force F_d. The ion adjusts its speed until the drag force matches the applied force. The result of this subdividing is the preceding map.

Now let's find expressions for the unknown nodes. Only three remain: ρ, v, and n. The figure illustrates the relation between ρ and R:

$$\rho = \frac{RA}{l}.$$

To find v, we balance the drag and electrical forces. The applied force is $F_q = qE$, where q is the ion charge and E is the electric field. The electric field produced by the voltage V is $E = V/l$, where l is the length of the block, so

$$F_q = \frac{qV}{l}.$$

This expression contains no unknown quantities, so it does not need further subdivision.
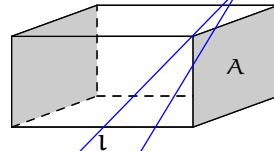
The drag is Stokes drag:

$$F_d = 6\pi\rho_{fl}vvr. \tag{6.6}$$

Equating this force to the applied force gives the terminal velocity v in terms of known quantities:

$$v \sim \frac{qV}{6\pi\eta lr},$$

where r is the radius of the ion.

The number density n is the third and final unknown. However, let's estimate it after getting a symbolic result for σ. (This symbolic result will

I really like this method of finding the speed but I think I could have been more simply detailed prior to the method. You mention that we balance the forces, but why?

so what is actually going on?

i'm confused as to where all these (undefined) variables are coming from.

V and l are defined a few paragraphs above, r is defined here, and q is charge. Eta is the dynamic viscosity, and I agree that that switch is unexplained, though it is useful to wrap up the density * nu into one parameter (eta).

Some of the variables tend to be standard when talking about certain types of problems: I will always be current, V voltage, q charge, and so on.

Was wondering about the eta. My question then becomes why is it proportional if they have to equal to balance each other? Are there other forces involved?

The missing step is that Fd = F from the electric field. Then you rearrange terms to get the v from the Fd by itself on one side of the equation. Try writing it out in a few lines, you'll see.

I think that was a rho*nu disappearing into an eta? I guess this is now the dynamic viscosity.

yeah. eta = mu = dynamic viscosity (engineers use mu, scientists use eta... generally).

Symbolic result?

I think he means in terms of variables, instead of just trying to estimate it from the cuff using numbers.

contain n.) To find σ climb the solution tree. First, find the current in terms of the terminal velocity:

$$I = q n v A \sim \frac{q^2 n A V}{6\pi\eta l r}.$$

Use the current to find the resistance:

$$R \approx \frac{V}{I} \sim \frac{6\pi\eta l r}{q^2 n A}.$$

The voltage V has vanished, which is encouraging: In most circuits, the conductivity (and resistance) is independent of voltage. Use the resistance to find the resistivity:

$$\rho = R\frac{A}{l} \sim \frac{6\pi\eta r}{q^2 n}.$$

The expressions simplify as we climb the tree. For example, the geometric parameters l and A have vanished. Their disappearance is encouraging – the purpose of evaluating resistivity rather than resistance is that resistivity is independent of geometry.

Now use resistivity to find conductivity:

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}.$$

Here q is the electron charge e or its negative, depending on whether a sodium or a chloride ion is the charge carrier. Thus,

$$\sigma = \frac{1}{\rho} \sim \frac{e^2 n}{6\pi\eta r}.$$

To find σ still requires the ion concentration n, which we can find from the concentration of salt in seawater. To do so, try a kitchen-sink experiment. Add table salt to a glass of water until it tastes as salty as seawater. I just tried it. In a glass of water, I found that one teaspoon of salt tastes like drinking seawater. A glass of water may have a volume of $0.3\,\ell$ or a mass of $300\,g$. A flat teaspoon of salt has a volume of about $5\,m\ell$. Why $5\,m\ell$? A teaspoon is about $4\,cm$ long by $2\,cm$ wide by $1\,cm$ thick at its deepest point; let's assume $0.5\,cm$ on average. Its volume is therefore

$$\text{teaspoon} \sim 4\,cm \times 2\,cm \times 0.5\,cm \sim 4\,cm^3.$$

It would be nice if the solution tree was reproduced here

That's true, or maybe just the relevant part of it.

I'm not sure I would know to go back through the solution tree and find sigma in order to determine n. Should something be a clue to this or simply the fact that we cannot find it another way?

maybe it would be better to just call it "the tree" instead of "the solution tree"?

Well, that would take up a lot of space, but I definitely agree that some of it should be reproduced for ease ofa ccess.

shouldn't this be = ?

Maybe at some point we decided we were just estimating, and so it might be off by a constant somewhere

But there are no constants in this first part of the equation. R = V / I. Maybe its irrelevant to place an = (since we are only concerned with twiddles), but why would the following equations also use an = and not this one?

Yes.

In what circuits are they dependent on voltage?

I believe the resistance of diodes depend on voltage.

MOSFETS

there are many voltage controlled resistors

Maybe show the step in which it vanishes?

I like the way this is stepped out; it's a good progression and makes it easy to follow. I think the derivations in this section are clear and are important. I'm not lost at all, so I would keep the derivation part as is.

I agree. Sometimes long derivations are easy to get lost in and wonder if you are following correctly. This makes it easy to know that you are on the right path without having to actually work out the derivation on paper

contain n.) To find σ climb the solution tree. First, find the current in terms of the terminal velocity:

$$I = qnvA \sim \frac{q^2 nAV}{6\pi\eta lr}.$$

Use the current to find the resistance:

$$R \sim \frac{V}{I} \sim \frac{6\pi\eta lr}{q^2 nA}.$$

The voltage V has vanished, which is encouraging: In most circuits, the conductivity (and resistance) is independent of voltage. Use the resistance to find the resistivity:

$$\rho = R\frac{A}{l} \sim \frac{6\pi\eta r}{q^2 n}.$$

The expressions simplify as we climb the tree. For example, the geometric parameters l and A have vanished. Their disappearance is encouraging – the purpose of evaluating resistivity rather than resistance is that resistivity is independent of geometry.

Now use resistivity to find conductivity:

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}.$$

Here q is the electron charge e or its negative, depending on whether a sodium or a chloride ion is the charge carrier. Thus,

$$\sigma = \frac{1}{\rho} \sim \frac{e^2 n}{6\pi\eta r}.$$

To find σ still requires the ion concentration n, which we can find from the concentration of salt in seawater. To do so, try a kitchen-sink experiment. Add table salt to a glass of water until it tastes as salty as seawater. I just tried it. In a glass of water, I found that one teaspoon of salt tastes like drinking seawater. A glass of water may have a volume of $0.3\,\ell$ or a mass of $300\,g$. A flat teaspoon of salt has a volume of about $5\,m\ell$. Why $5\,m\ell$? A teaspoon is about $4\,cm$ long by $2\,cm$ wide by $1\,cm$ thick at its deepest point; let's assume $0.5\,cm$ on average. Its volume is therefore

$$\text{teaspoon} \sim 4\,cm \times 2\,cm \times 0.5\,cm \sim 4\,cm^3.$$

A comment about all this math.... if you could add in some reminders around here about what our original goal is, it would help a lot. All this math gets me a little confused, and I spend so much time figuring out what's going on in the math that I forget why we're doing it.

It just makes me get confused and lose interest when I don't know why we are doing the math.

I think the statements in between the lines do enough to clarify the intent.

Yea I think that if you started adding more "clarifying" between equations it would actually be more confusing because the flow of the equations could be lost.

It could be useful to restate the equations that we are using to find these new equations. Even using reference numbers and saying "from equation 1 and 2" could do the trick.

Do you need this word here? I feel you can omit it without any loss of information (pun intended).

It's an important point, though. It implies his next point, that resistivity is independent of geometry. It might be nice to note that, therefore, resistivity is an intensive property (since the terms were brought up before).

you should have written this above as an explanation to the "intensive" property description.

Or just make a note here bringing it back to the earlier point, considering we hadn't derived this yet

I thought this was clearly stated when resistance and resistivity were introduced in the beginning of the section.

I think having a tree in the margin that gets filled in as these values are found would help. The benefit of the tree is lost and confused when the solutions are presented linearly and don't iterate back to the nodes of the tree.

Agreed. I think seeing a tree build up is one of the most useful things - watching them go up in class was usually an epiphany moment to me.

contain n.) To find σ climb the solution tree. First, find the current in terms of the terminal velocity:

$$I = q n v A \sim \frac{q^2 n A V}{6\pi\eta l r}.$$

Use the current to find the resistance:

$$R \sim \frac{V}{I} \sim \frac{6\pi\eta l r}{q^2 n A}.$$

The voltage V has vanished, which is encouraging: In most circuits, the conductivity (and resistance) is independent of voltage. Use the resistance to find the resistivity:

$$\rho = R\frac{A}{l} \sim \frac{6\pi\eta r}{q^2 n}.$$

The expressions simplify as we climb the tree. For example, the geometric parameters l and A have vanished. Their disappearance is encouraging – the purpose of evaluating resistivity rather than resistance is that resistivity is independent of geometry.

Now use resistivity to find conductivity:

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}.$$

Here q is the electron charge e or its negative, depending on whether a sodium or a chloride ion is the charge carrier. Thus,

$$\sigma = \frac{1}{\rho} \sim \frac{e^2 n}{6\pi\eta r}.$$

To find σ still requires the ion concentration n, which we can find from the concentration of salt in seawater. To do so, try a kitchen-sink experiment. Add table salt to a glass of water until it tastes as salty as seawater. I just tried it. In a glass of water, I found that one teaspoon of salt tastes like drinking seawater. A glass of water may have a volume of $0.3\,\ell$ or a mass of $300\,\mathrm{g}$. A flat teaspoon of salt has a volume of about $5\,\mathrm{m}\ell$. Why $5\,\mathrm{m}\ell$? A teaspoon is about $4\,\mathrm{cm}$ long by $2\,\mathrm{cm}$ wide by $1\,\mathrm{cm}$ thick at its deepest point; let's assume $0.5\,\mathrm{cm}$ on average. Its volume is therefore

$$\text{teaspoon} \sim 4\,\mathrm{cm} \times 2\,\mathrm{cm} \times 0.5\,\mathrm{cm} \sim 4\,\mathrm{cm}^3.$$

This makes sense here, maybe explain above that you will use resistivity to find the conductivity

I agree. After reading this part, the intro makes more sense to me when I reread it.

So I looked ahead and saw that finding conductivity is going to take 2.5 pages. It would be great it you could state initially what form we need it in because I got lost in the numbers.

if e is going to be squared, then its sign doesn't matter, does it?

haha, good point. I think that might just have been for explanation of the physical reality

why do we need to do an experiment to calculate concentration?

Do you know the concentration of salt ions in sea water off the top of your head?

is this really necessary?? It's not like this is the type of approximation we can do while taking an exam

I don't think this would be on the exam....and its to demonstrate that it is possible to do simple experiments sometimes to find numbers we want to know.

Yeah, I think this is actually really helpful to see this because too often we jump to the conclusion that we don't know something when there are actually practical ways of guessing.

I don't think the point of this class is to learn how to take an exam.

while I don't think exams are the main focus of this class...I do think this approximation is a bit unnecessary. is it plausible to assume that seawater is saturated with salt?

This sentence is a little weird here. Don't really think it needs to be here.

not necessary but still made me LOL

I think the adds an element of comedy into the text, which is nice. If there are strong objections to having it as a separate sentence, perhaps you could put it in parentheses at the end of the previous sentence and it would flow nicely.

Learning by doing!

contain n.) To find σ climb the solution tree. First, find the current in terms of the terminal velocity:

$$I = qnvA \sim \frac{q^2 nAV}{6\pi\eta lr}.$$

Use the current to find the resistance:

$$R \sim \frac{V}{I} \sim \frac{6\pi\eta lr}{q^2 nA}.$$

The voltage V has vanished, which is encouraging: In most circuits, the conductivity (and resistance) is independent of voltage. Use the resistance to find the resistivity:

$$\rho = R\frac{A}{l} \sim \frac{6\pi\eta r}{q^2 n}.$$

The expressions simplify as we climb the tree. For example, the geometric parameters l and A have vanished. Their disappearance is encouraging – the purpose of evaluating resistivity rather than resistance is that resistivity is independent of geometry.

Now use resistivity to find conductivity:

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}.$$

Here q is the electron charge e or its negative, depending on whether a sodium or a chloride ion is the charge carrier. Thus,

$$\sigma = \frac{1}{\rho} \sim \frac{e^2 n}{6\pi\eta r}.$$

To find σ still requires the ion concentration n, which we can find from the concentration of salt in seawater. To do so, try a kitchen-sink experiment. Add table salt to a glass of water until it tastes as salty as seawater. I just tried it. In a glass of water, I found that one teaspoon of salt tastes like drinking seawater. A glass of water may have a volume of $0.3\,\ell$ or a mass of $300\,g$. A flat teaspoon of salt has a volume of about $5\,m\ell$. Why $5\,m\ell$? A teaspoon is about $4\,cm$ long by $2\,cm$ wide by $1\,cm$ thick at its deepest point; let's assume $0.5\,cm$ on average. Its volume is therefore

$$\text{teaspoon} \sim 4\,cm \times 2\,cm \times 0.5\,cm \sim 4\,cm^3.$$

you may not want to recommend trying this. drinking salt water can cause you to throw up.

A sip of it won't. He's not telling you to chug the whole glass. Besides. You can spit it out after you taste it.

I'm not sure I've tasted seawater enough times to know what it tastes like....

Class field trip to the beach!

Yeah, and I'm not sure that taste alone is enough to actually call it close to seawater.

haha I liked this!

great experimental method!

Why didn't we solve for n at the same time we were solving for other variables, such as rho, earlier in the problem? This seems out of order to me.

why do you need to describe this here? we're doing things far more complicated than volume estimates.

Also, my teaspoons at home say 5mL right on them.

while I need no further proof that I learn at a slower pace than the majority of my classmates, I like it when you break it down into small steps. I'm still baffled when you state facts out of what otherwise seems like omniscience.

it also helps people stay focused on what he is trying to prove instead of getting lost on the fact that a teaspoon is 5mL

I know this is simple, but it is rather nice to have a call-back to what we did at the very beginning!

Sometimes I just sit back and enjoy some of the little, even silly, estimations we do in this class.

The density of salt is maybe twice the density of water, so a flat teaspoon has a mass of roughly $10\,\mathrm{g}$. The mass fraction of salt in seawater is, in this experiment, roughly $1/30$. The true value is remarkably close: 0.035. A mole of salt, which provides two charges per NaCl 'molecule', has a mass of $60\,\mathrm{g}$, so

$$n \sim \frac{1}{30} \times \underbrace{1\,\mathrm{g\,cm^{-3}}}_{\rho_{\text{water}}} \times \frac{2\,\text{charges}}{\text{molecule}} \times \frac{6\cdot 10^{23}\,\text{molecules}\,\text{mole}^{-1}}{60\,\mathrm{g\,mole^{-1}}}$$

$$\sim 7\cdot 10^{20}\,\text{charges}\,\mathrm{cm^{-3}}.$$

With n evaluated, the only remaining mysteries in the conductivity

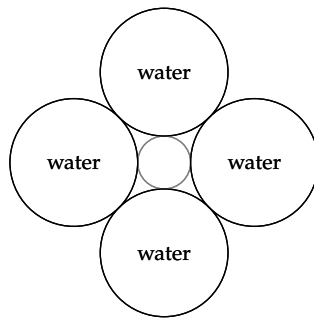$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}$$

are the ion radius r and the dynamic viscosity η.

Do the easy part first. The dynamic viscosity is

$$\eta = \rho_{\text{water}}\nu \sim 10^3\,\mathrm{kg\,m^{-3}} \times 10^{-6}\,\mathrm{m^2\,s^{-1}} = 10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}.$$

Here I switched to SI (mks) units. Although most calculations are easier in cgs units than in SI units, the one exception is electromagnetism, which is represented by the e^2 in the conductivity. Electromagnetism is conceptually easier in cgs units – which needs no factors of μ_0 or $4\pi\epsilon_0$, for example – than in SI units. However, the cgs unit of charge, the electrostatic unit, is unfamiliar. So, for numerical calculations in electromagnetism, use SI units.

The final quantity required is the ion radius. A positive ion (sodium) attracts an oxygen end of a water molecule; a negative ion (chloride) attracts the hydrogen end of a water molecule. Either way, the ion, being charged, is surrounded by one or maybe more layers of water molecules. As it moves, it drags some of this baggage with it. So rather than use the bare ion radius you should use a larger radius to include this shell. But how thick is the shell? As a guess, assume that the shell includes one layer of water molecules, each with a radius of $1.5\,\text{Å}$. So for the ion plus shell, $r \sim 2\,\text{Å}$.

Comments on page 4

Where does this come from? Just an approximation?

I agree without looking that up, i would not have guessed that

most rocks are 2-4 times as dense as water. it's a fair guess

well, from the beginning of the class, water is about 1 and rock is about 3 (g/cm^3), and halite (a rock, or table salt) is probably on the less dense side.

it comes from talking to the gut

talking to the gut?

A reference to a comment in an earlier lecture

It's a reference to the "technique" of getting enough intuition about problems to just pull these sorts of numbers out.

I understand how maybe we should be able to use our gut for this one, but my gut leads me in the wrong direction. Just imagining holding two containers, one with water and one with salt, I want to say the water would be heavier but apparently that's wrong.

thats why you must train your gut by doing several examples and problems and gathering information from around you. the more you do this and talk to your gut, the more accurate it becomes

It's been a while since we've seen something this off-the-cuff and filled with daily experience. I like it.

I find it amazing how these results end up so close to actual values

I think it's also amazing how we can look at a value, like volume of a teaspoon, and have no idea off the top of our heads. then when we do a little bit of applying a known situation and some of our estimation, and we get a feasible answer. also, it's amazing how the estimation of the teaspoon as rectangular works so well.

Why 1/30? It seems like most predictions used in this class are often nicer than this.

This was an extremely clever way to get this value. I would have never thought of making my own seawater to find the mass fraction of salt in it. Cool stuff.

The density of salt is maybe twice the density of water, so a flat teaspoon has a mass of roughly 10 g. The mass fraction of salt in seawater is, in this experiment, roughly 1/30. The true value is remarkably close: 0.035. A mole of salt, which provides two charges per NaCl 'molecule', has a mass of 60 g, so

$$n \sim \frac{1}{30} \times \underbrace{1\,\mathrm{g\,cm^{-3}}}_{\rho_{water}} \times \frac{2\,\text{charges}}{\text{molecule}} \times \frac{6 \cdot 10^{23}\,\text{molecules}\,\mathrm{mole^{-1}}}{60\,\mathrm{g\,mole^{-1}}}$$

$$\sim 7 \cdot 10^{20}\,\text{charges}\,\mathrm{cm^{-3}}.$$

With n evaluated, the only remaining mysteries in the conductivity

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}$$

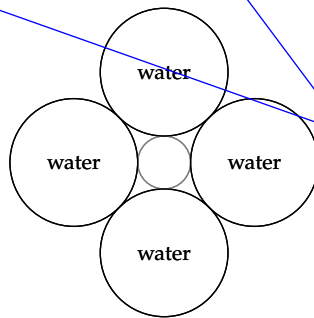are the ion radius r and the dynamic viscosity η.

Do the easy part first. The dynamic viscosity is

$$\eta = \rho_{water}\nu \sim 10^3\,\mathrm{kg\,m^{-3}} \times 10^{-6}\,\mathrm{m^2\,s^{-1}} = 10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}.$$

Here I switched to SI (mks) units. Although most calculations are easier in cgs units than in SI units, the one exception is electromagnetism, which is represented by the e^2 in the conductivity. Electromagnetism is conceptually easier in cgs units – which needs no factors of μ_0 or $4\pi\epsilon_0$, for example – than in SI units. However, the cgs unit of charge, the electrostatic unit, is unfamiliar. So, for numerical calculations in electromagnetism, use SI units.

The final quantity required is the ion radius. A positive ion (sodium) attracts an oxygen end of a water molecule; a negative ion (chloride) attracts the hydrogen end of a water molecule. Either way, the ion, being charged, is surrounded by one or maybe more layers of water molecules. As it moves, it drags some of this baggage with it. So rather than use the bare ion radius you should use a larger radius to include this shell. But how thick is the shell? As a guess, assume that the shell includes one layer of water molecules, each with a radius of 1.5 Å. So for the ion plus shell, $r \sim 2$ Å.

water
water water
water

This part is the hardest for me to figure out (chemistry was never my favorite). Where does this number come from?

A single NaCl pair will split up in water into an Na+ and a Cl- (the Na donates one electron to the Cl), each of which has a charge of +e or -e (the charge of one electron). Thus, we now have two charges which can move around and conduct electricity.

why is molecule in 's?

In solution and in crystals they are separate ions, so even though they come 1:1, they aren't considered a molecule.

I haven't seen chemistry in a while and i can't exactly remember how much a mole is..

a mole is 6*10^23. the reason we're using moles here is because we know molar mass, so we can divide the two

OK so this was very well explained. I cringed at the thought of two pages to explain conductivity, but it doesn't seem too bad.

when did this appear?

this variable should have been defined earlier...

I'm not familiar with this unit, whats mks stand for?

i hadn't even really realized that you'd changed units until you mentioned it. i'm not sure you need to explicitly state and explain why you choose to switch units.

i think it's a very important thing to keep track of. half of this entire course is making sure that the units match up, and this is an example of that.

yeah I agree, its really important to realize what units you're working in so this sentence is really helpful

It'd be helpful if you mentioned what cgs units are because I have no clue. Is it some common knowledge I am missing?

It's a convention for units referring to centimeters, grams, seconds.

interesting point, I would have just tried to continue using cgs, and avoided the confusion of having to switch back and forth.

Comments on page 4

The density of salt is maybe twice the density of water, so a flat teaspoon has a mass of roughly $10\,\text{g}$. The mass fraction of salt in seawater is, in this experiment, roughly $1/30$. The true value is remarkably close: 0.035. A mole of salt, which provides two charges per NaCl 'molecule', has a mass of $60\,\text{g}$, so

$$n \sim \frac{1}{30} \times \underbrace{1\,\text{g}\,\text{cm}^{-3}}_{\rho_{\text{water}}} \times \frac{2\,\text{charges}}{\text{molecule}} \times \frac{6 \cdot 10^{23}\,\text{molecules}\,\text{mole}^{-1}}{60\,\text{g}\,\text{mole}^{-1}}$$

$$\sim 7 \cdot 10^{20}\,\text{charges}\,\text{cm}^{-3}.$$

With n evaluated, the only remaining mysteries in the conductivity

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}$$

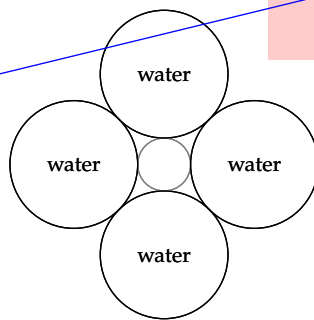are the ion radius r and the dynamic viscosity η.

Do the easy part first. The dynamic viscosity is

$$\eta = \rho_{\text{water}}\nu \sim 10^3\,\text{kg}\,\text{m}^{-3} \times 10^{-6}\,\text{m}^2\,\text{s}^{-1} = 10^{-3}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}.$$

Here I switched to SI (mks) units. Although most calculations are easier in cgs units than in SI units, the one exception is electromagnetism, which is represented by the e^2 in the conductivity. Electromagnetism is conceptually easier in cgs units – which needs no factors of μ_0 or $4\pi\epsilon_0$, for example – than in SI units. However, the cgs unit of charge, the electrostatic unit, is unfamiliar. So, for numerical calculations in electromagnetism, use SI units.

The final quantity required is the ion radius. A positive ion (sodium) attracts an oxygen end of a water molecule; a negative ion (chloride) attracts the hydrogen end of a water molecule. Either way, the ion, being charged, is surrounded by one or maybe more layers of water molecules. As it moves, it drags some of this baggage with it. So rather than use the bare ion radius you should use a larger radius to include this shell. But how thick is the shell? As a guess, assume that the shell includes one layer of water molecules, each with a radius of $1.5\,\text{Å}$. So for the ion plus shell, $r \sim 2\,\text{Å}$.

water

water   water

water

what are cgs units? i'm assuming the English system...

No, it's cm/g/sec vs. m/kg/s for SI. It's the EM units that are more different.

As an 8.022 buff, I disagree. I actually found cgs units better (for the reasons you mentioned). If you need to, you can convert from cgs to mks at the end.

I like cgs also, but conversions are tricky because the units are actually fundamentally different. It's not like converting feet to meters, it's almost like converting feet to kilograms. That's not the best analogy, since there is a conversion, but it's more like values in cgs correspond to values in SI.

Earlier you said the last unknown was n. I think this explanation could use some restructuring for consistency and to better reflect the tree.

nice illustration here

I agree as well, simple but effective!

This definitely makes sense, but perhaps you could help us visualize it with a picture.

This is a pretty hard problem. I am encouraged by the attempts to simplify solvation...

I agree, I kind of assumed this but any type of approximation skills I have are greatly undermined when I have to give a value for the radius of an ion.

Wouldn't there be some repulsion to keep it away? ie. the hydrogen will move away from the positive sodium and keep the molecules not to far. Or will the molecules constantly be moving trying to find the unit they are attracted to?

The density of salt is maybe twice the density of water, so a flat teaspoon has a mass of roughly 10 g. The mass fraction of salt in seawater is, in this experiment, roughly 1/30. The true value is remarkably close: 0.035. A mole of salt, which provides two charges per NaCl 'molecule', has a mass of 60 g, so

$$n \sim \frac{1}{30} \times \underbrace{1\,\mathrm{g\,cm^{-3}}}_{\rho_{\text{water}}} \times \frac{2\,\text{charges}}{\text{molecule}} \times \frac{6 \cdot 10^{23}\,\text{molecules mole}^{-1}}{60\,\mathrm{g\,mole^{-1}}}$$

$$\sim 7 \cdot 10^{20}\,\text{charges cm}^{-3}.$$

With n evaluated, the only remaining mysteries in the conductivity

$$\sigma = \frac{1}{\rho} \sim \frac{q^2 n}{6\pi\eta r}$$
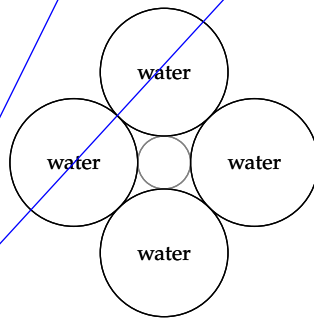
are the ion radius r and the dynamic viscosity η.

Do the easy part first. The dynamic viscosity is

$$\eta = \rho_{\text{water}} \nu \sim 10^3\,\mathrm{kg\,m^{-3}} \times 10^{-6}\,\mathrm{m^2\,s^{-1}} = 10^{-3}\,\mathrm{kg\,m^{-1}\,s^{-1}}.$$

Here I switched to SI (mks) units. Although most calculations are easier in cgs units than in SI units, the one exception is electromagnetism, which is represented by the e^2 in the conductivity. Electromagnetism is conceptually easier in cgs units – which needs no factors of μ_0 or $4\pi\epsilon_0$, for example – than in SI units. However, the cgs unit of charge, the electrostatic unit, is unfamiliar. So, for numerical calculations in electromagnetism, use SI units.

The final quantity required is the ion radius. A positive ion (sodium) attracts an oxygen end of a water molecule; a negative ion (chloride) attracts the hydrogen end of a water molecule. Either way, the ion, being charged, is surrounded by one or maybe more layers of water molecules. As it moves, it drags some of this baggage with it. So rather than use the bare ion radius you should use a larger radius to include this shell. But how thick is the shell? As a guess, assume that the shell includes one layer of water molecules, each with a radius of 1.5 Å. So for the ion plus shell, $r \sim 2$ Å.

This is so involved- I definitely would not have thought of doing this to calculate the ion radius

I agree. Although I think this is cool, this problem transcends basic approximation to become more of a technical science problem

Yeah there is a lot more 8.01,8.02 and 5.111 in this problem than there should be for this to be considered a basic approximation problem.

Yup, I never would have thought of it either. On the other hand, completely ignoring this fact would have probably led me to guess an ion radius of about .5 or 1 Angstrom, which is only off by a factor of 2. So the danger from missing this fact is pretty low.

Wow very true...I guess that's why its our homework to read and learn about it.

Agreed, so makes me wonder, would we actually include this if we were calculating this? i understand why it's included in the text for completeness, but perhaps it should be mentioned, after arriving at the answer, that perhaps intricacies like this could be skipped over next time?

We found before that a hydrogen atom has a diameter of 1A. How could we extrapolate to this number?

This confused me also. I think earlier we did the hydrogen atom (not ion). I think this is one of those numbers you may have to look up.

But what is the deciding factor in looking up a value? Looking up the volume of a teaspoon would have been just as easy...

You could model water as a space-filling model (little Mickey Mouse with hydrogen ears) or ball + stick model, so its radius would be something a little bigger than 1A. http://www.sacredbalance.com/web/flashplayer.html?id=h2o

I agree. You know that the hydrogen atoms sit on the oxygen, not straight across but at an angle. If you really remembered your chemistry you would know that angle, but even without it you can assume if each atom is 1A across, then the total has to be less than 3, so maybe a result in the 1.5-2A range is reasonable.

With these numbers, the conductivity becomes:

$$\sigma \sim \frac{\overbrace{(1.6 \cdot 10^{-19}\,\text{C})^2}^{e^2} \times \overbrace{7 \cdot 10^{26}\,\text{m}^{-3}}^{n}}{\underbrace{6 \times 3 \times 10^{-3}}_{6\pi} \underbrace{\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}}_{\eta} \times \underbrace{2 \cdot 10^{-10}\,\text{m}}_{r}}.$$

You can do the computation mentally. First count the powers of ten and then worrying about the small factors. Then count the top and bottom contributions separately. The top contributes -12 powers of 10: -38 from e^2 and $+26$ from n. The bottom contributes -13 powers of 10: -3 from η and -10 from r. The division produces one power of 10. Now account for the remaining small factors:

$$\frac{1.6^2 \times 7}{6 \times 3 \times 2}.$$

Slightly overestimate the answer by pretending that the 1.6^2 on top cancels the 3 on the bottom. Slightly underestimate the answer – and maybe compensate for the overestimate – by pretending that the 7 on top cancels the 6 on the bottom. After these lies, only $1/2$ remains. Multiplying it by the sole power of ten gives

$$\sigma \sim 5\,\Omega^{-1}\,\text{m}^{-1}.$$

Using a calculator to do the arithmetic gives $4.977\ldots\,\Omega^{-1}\,\text{m}^{-1}$, which is extremely close to the result from mental calculation.

The estimated resistivity is

$$\rho \sim \sigma^{-1} \sim 0.2\,\Omega\,\text{m} = 20\,\Omega\,\text{cm},$$

where we converted to the conventional although not fully SI units of $\Omega\,\text{cm}$. A typical experimental value for seawater at $T = 15\,^\circ\text{C}$ is $23.3\,\Omega\,\text{cm}$ (from [15, p. **14**-15]), absurdly close to the estimate!

Probably the most significant error is the radius of the ion-plus-water combination that is doing the charge transport. Perhaps r should be greater than $2\,\text{Å}$, especially for a sodium ion, which is smaller than chloride; it therefore has a higher electric field at its surface and grabs water molecules more strongly than chloride does. In spite of such uncertainties, the continuum approximation produced accurate results.

This notation is EXTREMELY helpful in understanding the equation- I would love to see this in other equations!

Agreed! This is useful, especially if its 12am and I can't remember exactly what I read a few pages ago

I enjoy these approximations from the beginning.

Yes me too. Can we have more readings like this where we incorporate past concepts? I think it would hone in everything better.

I cannot agree to this enough (even if it's difficult to function do in latex)

yeah i definitely agree too! sometimes after a long derivation, a big equation with numbers is shown and I don't really remember what each number is referring to, but this way of representing the equation makes it very clear!

I think this is a good touch because it relates to thinks we did before. It's always good to see a course building among itself.

This sentence doesn't make sense?

I think it makes sense once you change "worrying" to "worry."

I like that this references back to one of the first lessons!

Agreed. It never hurts to a refreshing lesson every now and then.

I find that I make so many more careless mistakes when I do things in my head instead of writing them down. Does it still count as doing it in your head if you write down one or two numbers to help yourself along as you go?

I think this method goes without saying..

i like that you explain this again here. it reminds us of the simple stuff we learned at the beginning and how it's always useful.

I also find the quick review nice, it's easy to forget with all the different approximation methods we've used in between.

Very true, going back to the first few things we learned keeps us on our feet

With these numbers, the conductivity becomes:

$$\sigma \sim \frac{\overbrace{(1.6\cdot 10^{-19}\,\text{C})^2}^{e^2} \times \overbrace{7\cdot 10^{26}\,\text{m}^{-3}}^{n}}{\underbrace{6\times 3}_{6\pi} \times \underbrace{10^{-3}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}}_{\eta} \times \underbrace{2\cdot 10^{-10}\,\text{m}}_{r}}.$$

You can do the computation mentally. First count the powers of ten and then worrying about the small factors. Then count the top and bottom contributions separately. The top contributes -12 powers of 10: -38 from e^2 and $+26$ from n. The bottom contributes -13 powers of 10: -3 from η and -10 from r. The division produces one power of 10.

Now account for the remaining small factors:

$$\frac{1.6^2 \times 7}{6 \times 3 \times 2}.$$

Slightly overestimate the answer by pretending that the 1.6^2 on top cancels the 3 on the bottom. Slightly underestimate the answer – and maybe compensate for the overestimate – by pretending that the 7 on top cancels the 6 on the bottom. After these lies, only 1/2 remains. Multiplying it by the sole power of ten gives

$$\sigma \sim 5\,\Omega^{-1}\,\text{m}^{-1}.$$

Using a calculator to do the arithmetic gives $4.977\ldots\,\Omega^{-1}\,\text{m}^{-1}$, which is extremely close to the result from mental calculation.

The estimated resistivity is

$$\rho \sim \sigma^{-1} \sim 0.2\,\Omega\,\text{m} = 20\,\Omega\,\text{cm},$$

where we converted to the conventional although not fully SI units of $\Omega\,\text{cm}$. A typical experimental value for seawater at $T = 15\,^{\circ}\text{C}$ is $23.3\,\Omega\,\text{cm}$ (from [15, p. **14**-15]), absurdly close to the estimate!

Probably the most significant error is the radius of the ion-plus-water combination that is doing the charge transport. Perhaps r should be greater than 2 Å, especially for a sodium ion, which is smaller than chloride; it therefore has a higher electric field at its surface and grabs water molecules more strongly than chloride does. In spite of such uncertainties, the continuum approximation produced accurate results.

Where did we define Omega?

I think he means Ohms, which is a measurement of resistance. It may help to state that though.

is there something up with the NB system. i just tried deleting the comment right under this one and it doesn't seem to fully work.

Often times you have to refresh the page to make changes like deleted comments actually register. The NB website doesn't update in realtime, it just loads the most current version of the document when you sign on.

I find it funny after the long argument to use mental math, you resort to a calculator in the end. This is much easier to enter though, so I see the point.

I like how you encouraged mental calculation. I feel like the computing part of our brain is atrophying because of our dependence on machines to do even simple calculations.

Yep - this part is good too since it definitely ties back into the "few" arguments we were making at the beginning of the term.

I completely agree with this I often find myself doing the simplest calculations on a calculator which ends up making more mistakes due to typos and it's good to have a class where we exercise our brains again

what do you mean by this?

ohms and cm are not SI. mhos and meters are.

With these numbers, the conductivity becomes:

$$\sigma \sim \frac{\overbrace{(1.6 \cdot 10^{-19}\,\text{C})^2}^{e^2} \times \overbrace{7 \cdot 10^{26}\,\text{m}^{-3}}^{n}}{\underbrace{6 \times 3}_{6\pi} \times \underbrace{10^{-3}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}}_{\eta} \times \underbrace{2 \cdot 10^{-10}\,\text{m}}_{r}}.$$

You can do the computation mentally. First count the powers of ten and then worrying about the small factors. Then count the top and bottom contributions separately. The top contributes -12 powers of 10: -38 from e^2 and $+26$ from n. The bottom contributes -13 powers of 10: -3 from η and -10 from r. The division produces one power of 10.

Now account for the remaining small factors:

$$\frac{1.6^2 \times 7}{6 \times 3 \times 2}.$$

Slightly overestimate the answer by pretending that the 1.6^2 on top cancels the 3 on the bottom. Slightly underestimate the answer – and maybe compensate for the overestimate – by pretending that the 7 on top cancels the 6 on the bottom. After these lies, only $1/2$ remains. Multiplying it by the sole power of ten gives

$$\sigma \sim 5\,\Omega^{-1}\,\text{m}^{-1}.$$

Using a calculator to do the arithmetic gives $4.977\ldots\,\Omega^{-1}\,\text{m}^{-1}$, which is extremely close to the result from mental calculation.

The estimated resistivity is

$$\rho \sim \sigma^{-1} \sim 0.2\,\Omega\,\text{m} = 20\,\Omega\,\text{cm},$$

where we converted to the conventional although not fully SI units of $\Omega\,\text{cm}$. A typical experimental value for seawater at $T = 15\,^\circ\text{C}$ is $23.3\,\Omega\,\text{cm}$ (from [15, p. **14-15**]), absurdly close to the estimate!

Probably the most significant error is the radius of the ion-plus-water combination that is doing the charge transport. Perhaps r should be greater than $2\,\text{Å}$, especially for a sodium ion, which is smaller than chloride; it therefore has a higher electric field at its surface and grabs water molecules more strongly than chloride does. In spite of such uncertainties, the continuum approximation produced accurate results.

Why did we not figure temperature in?

Maybe resistivity changes? Or ion density?

I'm not sure. From http://sam.ucsd.edu/sio210/lect_2/lecture_2.html

" Conductivity of sea water depends strongly on temperature, somewhat less strongly on salinity, and very weakly on pressure. If the temperature is measured, then conductivity can be used to determine the salinity. Salinity as computed through conductivity appears to be more closely related to the actual dissolved constituents than is chlorinity, and more independent of salt composition. Therefore temperature must be measured at the same time as conductivity, to remove the temperature effect and obtain salinity. Accuracy of salinity determined from conductivity: 0.001 to 0.004. Precision: 0.001. The accuracy depends on the accuracy of the seawater standard used to calibrate the conductivity based measurement. "

this is crazy how close it is

wow that is impressive

I really like this outline of the possible flaws of the estimation.

I agree, I don't think I'd be able to get through of these steps on my own but I can understand the reasoning because of this.

Since the net charge is the same, the field as a function of distance from the center of the ion should be the same (outside the volume of the ion itself, that is). I would have guessed that the size of the water/ion ball, therefore, wouldn't depend on the ion size (unless the ion were very large).

I'm assuming here that the radius of the water/ion ball is determined by when the electric field drops below some value, and can no longer attract water molecules. Perhaps that assumption is flawed.

With these numbers, the conductivity becomes:

$$\sigma \sim \frac{\overbrace{(1.6 \cdot 10^{-19}\,\text{C})^2}^{e^2} \times \overbrace{7 \cdot 10^{26}\,\text{m}^{-3}}^{n}}{\underbrace{6 \times 3}_{6\pi} \times \underbrace{10^{-3}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}}_{\eta} \times \underbrace{2 \cdot 10^{-10}\,\text{m}}_{r}}.$$

You can do the computation mentally. First count the powers of ten and then worrying about the small factors. Then count the top and bottom contributions separately. The top contributes -12 powers of 10: -38 from e^2 and $+26$ from n. The bottom contributes -13 powers of 10: -3 from η and -10 from r. The division produces one power of 10.

Now account for the remaining small factors:

$$\frac{1.6^2 \times 7}{6 \times 3 \times 2}.$$

Slightly overestimate the answer by pretending that the 1.6^2 on top cancels the 3 on the bottom. Slightly underestimate the answer – and maybe compensate for the overestimate – by pretending that the 7 on top cancels the 6 on the bottom. After these lies, only 1/2 remains. Multiplying it by the sole power of ten gives

$$\sigma \sim 5\,\Omega^{-1}\,\text{m}^{-1}.$$

Using a calculator to do the arithmetic gives $4.977\ldots\,\Omega^{-1}\,\text{m}^{-1}$, which is extremely close to the result from mental calculation.

The estimated resistivity is

$$\rho \sim \sigma^{-1} \sim 0.2\,\Omega\,\text{m} = 20\,\Omega\,\text{cm},$$

where we converted to the conventional although not fully SI units of $\Omega\,\text{cm}$. A typical experimental value for seawater at $T = 15\,°\text{C}$ is $23.3\,\Omega\,\text{cm}$ (from [15, p. **14-**15]), absurdly close to the estimate!

Probably the most significant error is the radius of the ion-plus-water combination that is doing the charge transport. Perhaps r should be greater than $2\,\text{Å}$, especially for a sodium ion, which is smaller than chloride; it therefore has a higher electric field at its surface and grabs water molecules more strongly than chloride does. In spite of such uncertainties, the continuum approximation produced accurate results.

cool results! but how can this be considered a "simple case." it just took you 6 pages to cover it, whereas some of the other examples we've seen takes like half a page.

I agree. I'm also wondering, are we still looking at lossy examples here? I'm confused as to what terminology you're using for all of these.

Although this section took me a couple reads, I think it definitely still qualifies as an "easy case." Take a look at the question again, it's pretty complex. But we were still able to find an answer by drawing a tree diagram and using simple cases. Although it took 6 pages, you have to remember that this particular problem required a lot of algebra, which is why it spanned so many pages (plus it was thoroughly explained which was nice).

As I understand it, "simple cases" doesn't mean "doing problems which are simple." Rather, it seems to mean "doing hard problems using the simplest possible setup." In this case, we chose a very simple setup of imagining conduction between two plates of area A, which made all the geometric calculation much easier.

That accuracy is puzzling. At the length scale of a sodium ion, water looks like a collection of spongy boulders more than it looks like a continuum. Yet Stokes drag worked. It works because the important length scale is not the size of water molecules, but rather their mean free path between collisions. Molecules in a liquid are packed to the point of contact, so the mean free path is much shorter than a molecular (or even ionic) radius, especially compared to an ion with its shell of water.

The moral of this example, besides the application of Stokes drag, is to have courage: Approximate first and ask questions later. The approximations might be accurate for reasons that you do not suspect when you start solving a problem. If you agonize over each approximation, you will never start a calculation, and then you will not find out that many approximations would have been fine – if only you had had the courage to make them.

6.3.4 Combining solutions from the two limits

You know know the drag force in two extreme cases, viscous and turbulent drag. The results are repeated here:

$$F_d = \begin{cases} 6\pi\rho_{fl}\nu v r & \text{(viscous),} \\ \frac{1}{2}c_d\rho_{fl}Av^2 & \text{(turbulent).} \end{cases}$$

Let's compare and combine them by making the viscous form look like the turbulent form: Multiply by the Reynolds number rv/ν (basing the Reynolds number on radius rather than diameter). Then

$$F_d = \underbrace{\left(\frac{rv/\nu}{Re}\right)}_{1} \times \underbrace{6\pi\rho_{fl}\nu v r}_{F_d} = \frac{1}{Re}6\pi\rho_{fl}v^2r^2 \qquad \text{(viscous).}$$
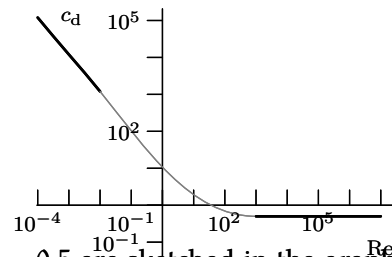
With $A = \pi r^2$,

$$F_d = \frac{6}{Re}\rho_{fl}v^2A \qquad \text{(viscous),}$$

Since $c_d \equiv F_d/(\frac{1}{2}\rho_{fl}v^2A)$,

$$c_d = \frac{12}{Re} \qquad \text{(viscous).}$$

That limit and the the high-speed limit $c_d \sim 0.5$ are sketched in the graph (with a gray interpolation between the limits). Almost all of the experimental data is explained by this graph, except for the drop in c_d near the $Re \sim 10^6$.

I was just about to leave a comment to this effect.

I feel like all the calculations done in these notes turn out eerily accurate- if I were to try and estimate the same values I would probably be way more off.

I wonder what the average percent accuracy is with these estimation (such as, answers differed from actual results by x%)

did we assume stokes drag because of the low reynolds number?

How does short mean free path make it more like a continuum? I have a gut agreement with this. Is it the finer scale, the way moving my hand through sand is more like a continuum than moving through gravel?

Will this idea be repeated for future concepts?

yea, i think that's become very clear over this course!

But what if your approximation wasn't done correctly, but still numerically accurate?

I really like how this tends to work out, we've seen it a couple times now

I like this focus on the ends instead of the means

This seems applicable to life decisions in general haha

I really like this paragraph. This has always been my main problem with approximations in the past. This is something this class has taught me and even if it can get some wrong answers, they eventually lead to correct ones.

That accuracy is puzzling. At the length scale of a sodium ion, water looks like a collection of spongy boulders more than it looks like a continuum. Yet Stokes drag worked. It works because the important length scale is not the size of water molecules, but rather their mean free path between collisions. Molecules in a liquid are packed to the point of contact, so the mean free path is much shorter than a molecular (or even ionic) radius, especially compared to an ion with its shell of water.

The moral of this example, besides the application of Stokes drag, is to have courage: Approximate first and ask questions later. The approximations might be accurate for reasons that you do not suspect when you start solving a problem. If you agonize over each approximation, you will never start a calculation, and then you will not find out that many approximations would have been fine – if only you had had the courage to make them.

6.3.4 Combining solutions from the two limits

You know know the drag force in two extreme cases, viscous and turbulent drag. The results are repeated here:

$$F_d = \begin{cases} 6\pi\rho_{fl}\nu vr & \text{(viscous)}, \\ \frac{1}{2}c_d\rho_{fl}Av^2 & \text{(turbulent)}. \end{cases}$$

Let's compare and combine them by making the viscous form look like the turbulent form: Multiply by the Reynolds number rv/ν (basing the Reynolds number on radius rather than diameter). Then

$$F_d = \underbrace{\left(\frac{rv/\nu}{Re}\right)}_{1} \times \underbrace{6\pi\rho_{fl}\nu vr}_{F_d} = \frac{1}{Re}6\pi\rho_{fl}v^2r^2 \qquad \text{(viscous)}.$$
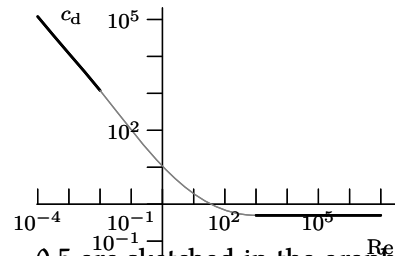
With $A = \pi r^2$,

$$F_d = \frac{6}{Re}\rho_{fl}v^2A \qquad \text{(viscous)},$$

Since $c_d \equiv F_d/(\frac{1}{2}\rho_{fl}v^2A)$,

$$c_d = \frac{12}{Re} \qquad \text{(viscous)}.$$

That limit and the the high-speed limit $c_d \sim 0.5$ are sketched in the graph (with a gray interpolation between the limits). Almost all of the experimental data is explained by this graph, except for the drop in c_d near the $Re \sim 10^6$.

So.... yet again, how is this "easy cases"? Nothing about this really seemed "easy". I was pretty confused throughout. Also, I'm still not sure how this unit is about lossy complexity.

> i agree...

>> I was thinking this the whole time I was reading this. I guess the easy case was stated in the beginning that we are looking at low Reynolds number. But no where in the reading did he expound on the fact that this conclusion came about because of this easy case.

>> I'm going to be so lost when we get a hard case.

In real life how would you normally go about finding out if you were close to the true answer? It seems like in a lot of situations you might not ever find out...

deep, I can totally relate to times when i didnt even know how to start approximating

This is how I feel about the homework assignments! I feel like each time I surprise myself by being close, I'm more likely to trust my gut instinct.

who would have known that my engineering class was meant to teach me courage?

It is better to have approximated and lost (accuracy) than to have never approximated at all...

> Haha it's funny, his word choice reminded me a lot of that love quote too.

Maybe here you could have a quick recap of how our results relate to the Reynolds number, especially before we try to integrate these results in the next section.

> Agreed. At least we should check the laminar-regime assumption.

could you use this same reasoning to approximate the resistivity of any material? and I wish you explained resistivity a little better

That accuracy is puzzling. At the length scale of a sodium ion, water looks like a collection of spongy boulders more than it looks like a continuum. Yet Stokes drag worked. It works because the important length scale is not the size of water molecules, but rather their mean free path between collisions. Molecules in a liquid are packed to the point of contact, so the mean free path is much shorter than a molecular (or even ionic) radius, especially compared to an ion with its shell of water.

The moral of this example, besides the application of Stokes drag, is to have courage: Approximate first and ask questions later. The approximations might be accurate for reasons that you do not suspect when you start solving a problem. If you agonize over each approximation, you will never start a calculation, and then you will not find out that many approximations would have been fine – if only you had had the courage to make them.

6.3.4 Combining solutions from the two limits

You know know the drag force in two extreme cases, viscous and turbulent drag. The results are repeated here:

$$F_d = \begin{cases} 6\pi\rho_{fl}\nu\nu r & \text{(viscous)}, \\ \frac{1}{2}c_d\rho_{fl}Av^2 & \text{(turbulent)}. \end{cases}$$

Let's compare and combine them by making the viscous form look like the turbulent form: Multiply by the Reynolds number rv/ν (basing the Reynolds number on radius rather than diameter). Then

$$F_d = \underbrace{\left(\frac{rv/\nu}{Re}\right)}_{1} \times \underbrace{6\pi\rho_{fl}\nu\nu r}_{F_d} = \frac{1}{Re}6\pi\rho_{fl}v^2r^2 \qquad \text{(viscous)}.$$
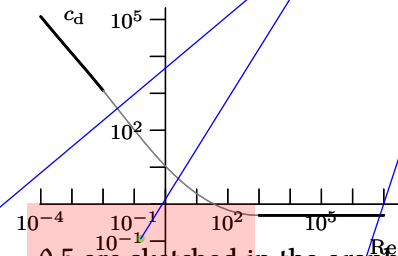
With $A = \pi r^2$,

$$F_d = \frac{6}{Re}\rho_{fl}v^2A \qquad \text{(viscous)},$$

Since $c_d \equiv F_d/(\frac{1}{2}\rho_{fl}v^2A)$,

$$c_d = \frac{12}{Re} \qquad \text{(viscous)}.$$

That limit and the the high-speed limit $c_d \sim 0.5$ are sketched in the graph (with a gray interpolation between the limits). Almost all of the experimental data is explained by this graph, except for the drop in c_d near the $Re \sim 10^6$.

I like this section- to the point and very understandable.

I agree. It is also interesting to see how we are able to come from those two equations to make a coherent plot.

Yup, although very brief, it was really nice at summing everything up and tying it all together.

I also strongly agree. I actually got so caught up in details above that I had lost sight of the point of what we were doing. This section nicely brought everything back together.

Is this supposed to be "now"?

why are we doing this?

just a note, the graph is too close to the writing.

only near 10^6, how about greater than that? why is this an exception

I believe there is a plan afoot to show us this soon.

Did I miss something, or was there no actual calculation of Re?