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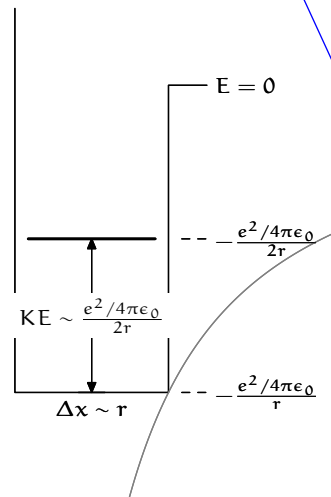
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$\Delta x \sim r$

I JUST WANTED TO SAY:

I USED A DIMENSIONAL ANALYSIS on a complicated physics problem on waves today ON MY EXAM! I forgot the exact equation and was running low on time, didn't want to derive it, so I quickly pulled out dimensional analysis and did the problem in under 30 seconds. Thanks Sanjoy!

Just want to reiterate again how super sweet that was today on the exam!

-OP

if nb had a "like" button, i'd use it here :)

Read Section 7.3 for Tuesday's memo. It revists the hydrogen calculation from Reading 19 (r19-dimensions-hydrogen.pdf on NB), using lumping and easy cases to give a physical explanation of the earlier dimensional-analysis result.

Maybe you could reference the section in which we calculated this for the first time? It would be nice for the readers to be able to easily flip back to see the alternate method.

Agreed, but I also really like the fact that you're repeating earlier examples, just to show that you get similar answers, and as a check on your earlier calculations

Perhaps a brief statement would be nice, but it's best to keep it short. Again, this is a textbook so people can just look back. Maybe just a statement of which section it was in and also the approach used there, in contrast to lumping.

A lot of times he just uses an in-text citation like (1.2.3), I think that would be extremely helpful here.

I agree. I think a brief reminder would be helpful. I couldn't remember what t his was referring to immediately.

I remember the section vaguely, but mostly remember that I didn't understand it fully.

I tried to do that in the introductory note on NB (which says that reading 19 is the last example of it).

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is this specific to different elements? or is it the same for all. and if it is specific does it just measure to the first electron shell or the outermost in the lowest energy state

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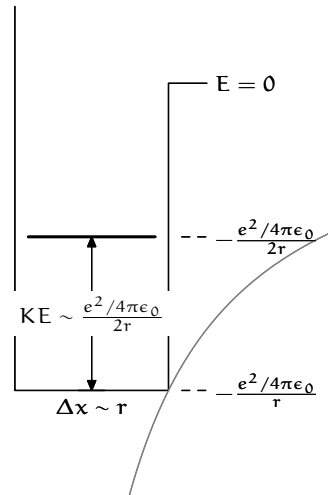
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I feel like this part covers a lot of the things we've learned in this class.

how does d&c relate energy to the bohr radius distance?

D&C is used to analyze the two types of energy. The energy is already related to the Bohr radius as it is the radius of the orbit with lowest energy.

I'm assuming he will connect the total energy to the energy of the ground state. I also think that he was talking about using divide and conquer to figure out what the total energy (potential plus kinetics) was.

I remember an approach similar to this that was explored in my freshman seminar. It seemed to work out well so this seems like a plausible explanation

Is this a divide and conquer approach to the problem or are we simply dividing and conquering by calculating the potential and kinetic energy separating? I recall in divid and conquer we didn't always calculate all the variables involved but sometimes we just approximated with the ones that were greater in value.

This question confuses me – is it asking how one can understand it or where the physical entity that is kinetic energy originates in the universe?

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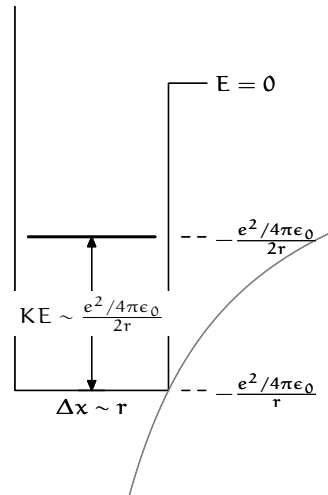
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I don't know much about this stuff – is it in fact accelerating? Why?

is the reading implying that electrons do not accelerate? if so, why is this true?

That's the whole reason that quantum mechanics is necessary. Note the following series of classically true statements:

A body orbiting another body is constantly accelerating (since its velocity is always changing direction). Any accelerating charge emits electromagnetic radiation. Electromagnetic radiation is a form of energy. Energy is conserved. Thus as an electron orbits an atom, it emits (and thus loses) energy. So the electron will eventually run out of energy, spiraling into the nucleus and destroying the atom. But it doesn't (as evidenced by the fact that we're all still here to talk about it). QM provides a way to explain how an electron can be localized in an atom and yet not be constantly emitting energy.

that was a very good explanation. Not overtly tedious but detailed enough to get a general view of things. Thanks

great explanation

Also, can you explain why this would happen? Not sure I remember enough to rationalize it myself...

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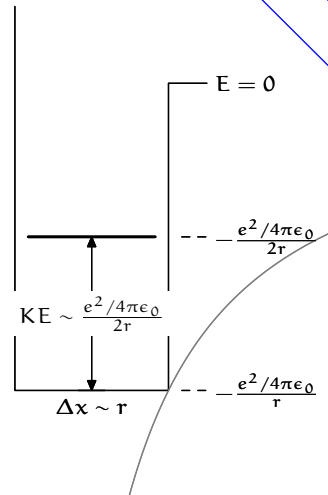
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The fact that it didn't bothered me when I first encountered the idea of an atom in middle school.

I've never heard this explanation before. But after hearing it this whole concept makes much more sense.

In middle school? Wow.

Yeah, that's rather impressive. Especially considering this fact has never bothered me until now (well, technically a few readings ago when this issue was first brought up).

wow you must have been an incredibly smart and knowledged middle schooler to be on the verge of discovering quantum mechanics!

Not really. It was featured on Bill Nye the Science Guy, and my 8th grade science teacher gave an age appropriate introduction to how electrons move around the nucleus of atoms. No rediscovery of quantum mechanics needed.

oh. nevermind then.

Bill Nye is the man... just saying

Yeah, my sister liked Beakman's world, but I always enjoyed Bill Nye better; Beakman was a bit too hipsterish and something was just not right about him. Bill's always been a real bro, the man's man, something I can relate to.

Older sister? I, personally, learned a lot more from beakman...better science.

sarcasm much? but this is an interesting thing to point out; you would think they would mention it when going over the "electron cloud"

This seems depressing. You ask a question, realize that you don't know the answer, or that anyone else does either.

Pretty cool. I'd never heard it put that way.

'r' is only the radius, right? not a 3d space?

It's interesting how this is so considering protons are not stationary.

i guess this answers my question. hard to believe though!

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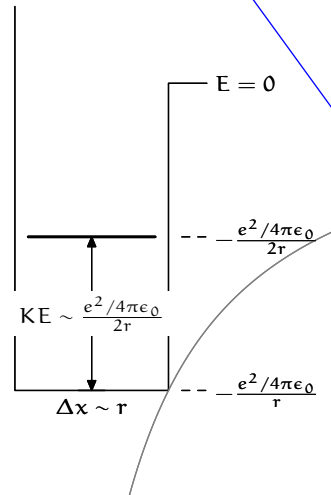
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This I think is actually directly contrary to what I learned in high school. Interesting!

I agree it's amazing how much we've been learning in this class that seems to contradict things we may have learned earlier on. One of the reasons I enjoy it so much

and i've seen at least 5 topics pop up in my other subjects!

these paragraphs are so hard to read when there's a hundred grey lines overlapping

stationary in respect to the center of the nucleus? because the electron is moving around right?

wait–what? do we really not understand this?

Possibly we either can't predict or explain it? At least not in the same way we can explain traditional mechanics

Part of the problem is that we don't have an intuitive feel for quantum mechanics. In fact, that's how quantum came about, sort of: physicists kept observing tings that were really weird and had to come up with new theories to explain them.

I think he says this because you can only explain qm mathematically and it's a little fuzzy.

I believe a lot of physicists in quantum mechanics have their own explanations/intuition on the matter, but there is no consistent 'correct' intuition on the matter, as there is with classical mechanics.

The reason he says that is because QM is really weird. "No one understands it" just means it violates every bit of our classical intuition possible.

I think I can safely say that nobody understands quantum mechanics. – Richard Feynman

And yet, we can still use the tools it provides, even if we don't exactly understand why.

I'm not sure why this is necessary... it seems like the approximation method should just be introduced rather than one nobody can use.

I think you could expand a little more from here?

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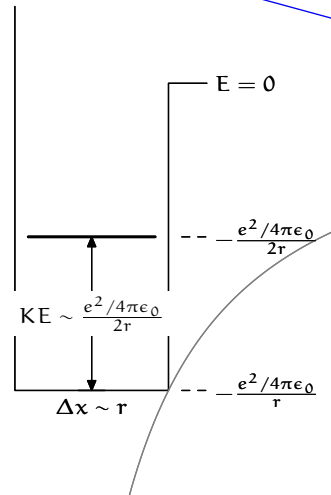
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Having a mathematical model of some phenomenon confers at least SOME understanding, no?

that's an interesting statement. In my mind, a model is a representation of some physical process, but especially in the case of an a model of an unknown phenomenon, even if the model produces similar results - you could still be modeling the wrong thing. But I do agree, having some model tells you more than not having a model at all.

I think the disparity exists between understanding and explanation. A model can explain the behavior, but does not necessarily lead to a full understanding of why, and what individual aspects mean. Rather, it just tells you the end result for a given set of inputs.

When you say only understanding mathematically? Wouldn't they use some sort of approximation for this reasoning? Do you mean that the mathematical reasoning is not 100% accurate or trusted?

this makes me think about the steady state assumptions that we always make? does steady state really exist or is it just to make the problems simpler mathematiclly

At what scale? At the atomic scale, there really isn't steady state, but it isn't practical to look at lots of phenomena at that length scale. Almost any problem is going to involve some lumping.

Stationary state and steady state aren't the same thing, if that's the confusion.

is this paragraph necessary? I feel like its just a lot of background info into quantum.

I really liked this paragraph. It sets up the question well and allows you to visualize the problem. I would like if you did this on all of the problems we face.

Yes, background always helps. This section sure did spur a lot of debate, though. And clarify to me just how much I don't understand about the atomic world....

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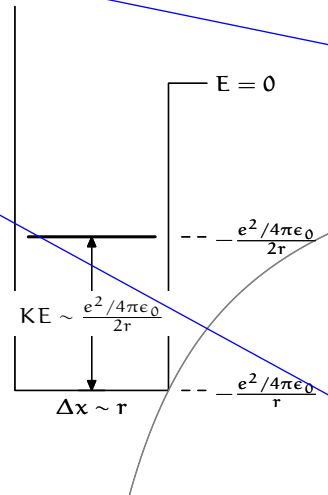
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I remember in chemistry once my teacher talking about how we can model electrons as being stationary but wasn't explained like this and didn't make sense at the time. I have a much better understanding of the rationale now.

I don't really, but is this a form of lumping, to assume that it is stationary?

Correct, they are actually not only not stationary, electrons don't even follow circular orbits. In fact, the location of an electron cannot be predicted without losing some level of precision and information. Thus, we can lump this information and ignore such details.

Stationary in this sense literally means that the expectation value of the position of the electron does not depend on time. This falls out of quantum mechanics, it doesn't require lumping.

I am really having trouble understanding the electron being stationary. Is this a literal or figurative definition?

My understanding is that it's the latter, and I think a lot of people are getting tripped up on this issue in the other comments.

So is this the lumping right here? taking an electron to be "stationary" when we know that it actually can be anywhere in the universe at any given time?

Seems like it - the lumping is only paying attention to the probable location and then reducing that to a stationary location.

this was a funny word for me, I typically do consider stationary things to be able to radiate. When I was reading stationary I was just thinking, "no kinetic energy." (obviously that is not what he meant though)

I wonder exactly then how it does contribute KE?

What do you mean here by radiate? It doesn't radiate what?

Energy. He's saying here, that we assume it doesn't lose energy because it's stationary, so it doesn't radiate away energy and fall into the nucleus.

basically it's not using its KE.

I am still confused about this explanation of an electron

This is a lot of information to digest in one go.

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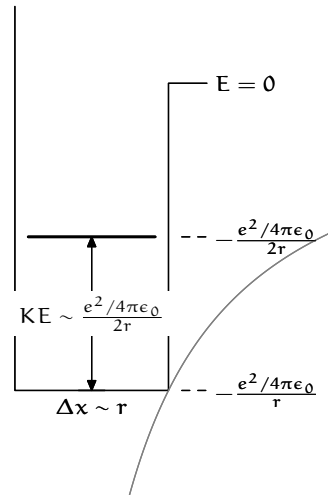
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this seems contradictory to what you just said...if the electron is stationary, shouldn't it exist as a single point? I think you should clarify your definition of radius here...

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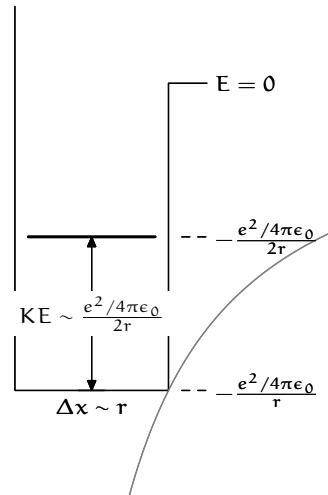
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In that the probability of it being anywhere in the whole universe is non zero?

Yeah I think that's a much better way of putting it - I imagine something literally smeared all over the universe with Sanjoy's wording, like paint over a window or a bug on a windshield.

Which theory does this refer to? This is an amusing way to put it though.

haha I agree. It's a different sort of imagery.

Yeah, so I understand that in order to truly understand atoms, you need to include quantum mechanics. But I'm assuming this textbook is going to be used in other schools, where I'm guessing not all students are familiar with quantum mechanics. I just feel like this example is a bit complicated, or at least, some of the explanations are a bit complicated and unnecessary.

The same was true (and maybe moreso) for the earlier Bohr radius explanation. If this is intended for MIT students I think it's ok, but for younger students or ones without calculus, physics, computer science, or chemistry, there might be a problem. That's a running theme, though, which I think Sanjoy has thought about!

Personally I love these explanations no matter how complicated they get. I love how this class gives us the true explanations of things and not an oversimplified model which may hold many untruths. If anything these explanations make me look into these topics even more, which I would definitely not be doing otherwise.

I actually think this stuff was covered in my AP Chem class in high school. I think what he's talking about refers to the probability density that an electron's position around the nucleus. As I recall from my chem book, this appeared as a cloud around the nucleus when high probability at a certain band. By him saying that its actually smeared over the whole universe, I think he's saying that theoretically the electron could be anywhere just with smaller probability.

Yes, I really also love that these explanations, no matter how approximate they are, are based on intuitions of truths of physics. This brings to mind the paper that Sanjoy wrote about teaching 1st year physics in college I read.

7.3 Quantum mechanics: Hydrogen revisited

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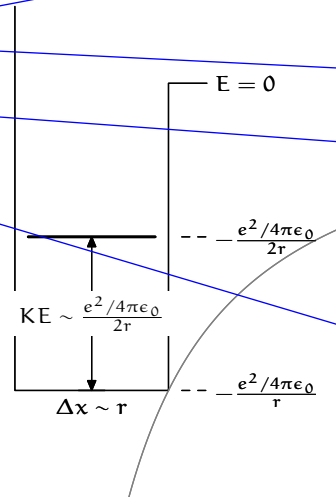
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$$E_{kinetic} \sim \frac{(\Delta p)^2}{m_e} \sim \frac{\hbar^2}{m_e r^2}.$$

This energy is the confinement energy or the uncertainty energy.

$E = 0$

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$KE \sim \dfrac{e^2/4\pi\epsilon_0}{2r}$

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$\Delta x \sim r$

I want Sanjoy to sit in on some of my other classes and give those teachers some pointers!

Agreed.

This phrasing is a little confusing... I'm assuming this means the probability of it is more likely to lie in a typical radius, but the phrasing makes it sounds like this giant blob, a large portion of which lives in a confined area.

haha, I hadn't read it like that but I see what you mean. Would it make it better to say that for a significant amount of the time it lives within a typical radius?

I don't think that significant amount of time is appropriate, just because then it literally implies time. Even though it may live in the radius most of the time, I think it's important that people realize it's with a high probability...

I always wanted to learn this stuff, but never had the time to take an MIT physics class.

I feel like a few classes in course 6 (on the EE side) have teased about this but never really got into it - I'm glad we're looking at it in this one!

Can we get a drawing of this to show what a0 is? Or are r and a0 the same thing?

r is just the variable that we'll be using to eventually approximate a0.

We are trying to show that r=a0.

I like this idea! I'm excited for the section to follow..

How does confinement give energy? Is there no energy if the electron is not confined to a region?

Maybe energy kept because it IS confined? I dunno

It has to do with the allowable energy states for an electron and how the angular momentum has to be an integer multiple of Planck's constant.

What is confinement energy?

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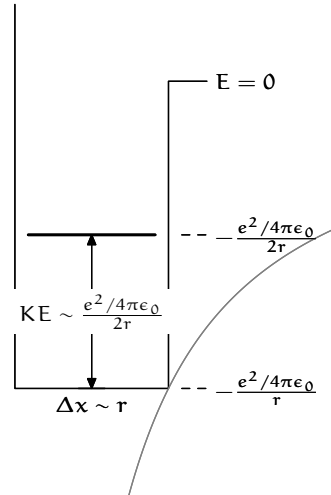
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Insert "Heisenberg" here?

Agreed.

Are there other uncertainty principles?

credit where credit's due!

is this something that must be known before attempting this problem?

I understand this because I already took 8.04, but I think a sentence or 2 explaining what this equation means would be good so people could better understand what you are explaining

Yeah, I could use more explaining on this equation. how do we estimate uncertainty? are we simply relying on the picture and equations to the right?

Are the units of delta_x and delta_p in terms of distance and momentum? If feel like a larger momentum would result in a larger uncertainty in x, not lower. I might be thinking about this wrong.

Yea it would be helpful if there was a chart to follow on the side to help organize what is going on in this equation and the diagram.

can we go over this in class? i read it over a couple times and still don't get what "h" is

yeah I've never really learned this stuff, so a little more explanation of this before having the equation would be useful.

I think even before coming to MIT with very little physics background, I've heard of the uncertainty principle...

The reason he probably didn't put charts for the uncertainty principle, it's because it takes insane amounts of calculus and calculations in order to explain its origin. It is best for him to just mention it

Comments on page 1

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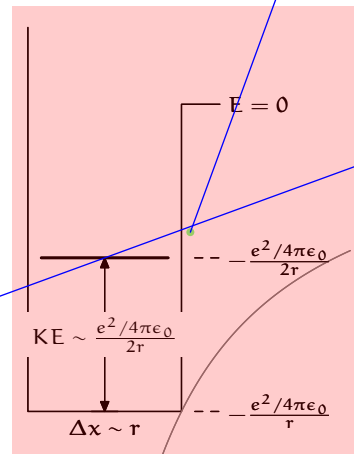
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This graph (to me) is a little more messy than it could be. Labels and maybe making it slightly bigger might help.

and a little confusing- what exactly is it showing?

I agree. It took me a while to understand the picture.

To be honest, I'm still not sure what's going on here.

I understand (I think) the difference in energy due to the shells, but what is the (random?) arc on the right?

I agree, I had problems understanding the diagram since it was neither completely explained nor are there labels for everything.

I too am a little confused here. It would be nice if this graph had the same kind of breakdown as on the next page for E vs r.

It would also help to mention this figure at some point in the text. I am not familiar with quantum mechanics and don't fully understand what's going on in this graph.

I also don't know what the arc is, but could it be the orbit of highest probability?

What is going on in this graph? I tried to make sense of it but ended up having to ignore it instead.

agreed on the confusion of this picture

I assume the intermediate step here starts with E mv^2?

Yeah the equations aren't immediately obvious from the description in the text.

I was a little confused here too and had to stop reading and think about it for a bit. It might be helpful to slip in an equation or two.

And p = mv

Yeah, it would definitely be helpful here to mention that E m*(v^2), and also to mention that p = mv. Thank you to the person above who noted that p = mv, otherwise I still would have been lost. I'd definitely make these additions to the text, it'll flow nicer.

I think a better way to state this is that the rms value (?) for the momentum is approximately delta p. Then KE = p^2/2m.

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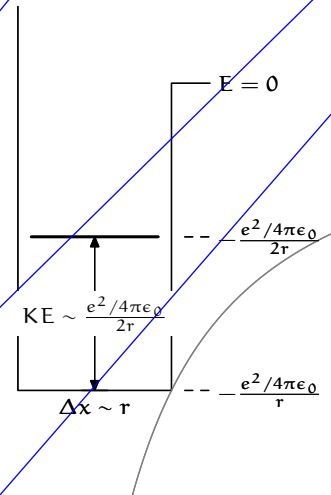
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Should this be a delta E, if we're using delta p? (to signify the range of kinetic energy variations)

Why do we know that the 'confinement' energy is kinetic energy, as opposed to potential energy?

It comes out of the uncertainty relationship given above – "confinement" means restricting delta x, so delta p must be large enough that the product of those two terms is at least h. Delta p just means the distribution of momentum, so increasing it translates to increasing potential energy (which is, of course, due to momentum).

That's a good question...I'd like to know the answer as well.

The energy derived uses momentum and is thus related to motion and kinetic energy. That word "confinement" has to do with the model of an atom. The specified energy confines the electron to the atom.

I feel like confinement just kind of gives the idea of not kinetic... I guess that is not really a helpful explanation

This section had me reading all the peer notes I could, hoping for some clarifications that I couldn't find in the readings...

This estimate uses lumping twice. First, the complicated electrostatic potential, which varies with distance, is replaced by the simple potential well (with infinitely high sides). Second, the electron, which in reality is smeared all over, is assumed to be at only one spot that is at a distance r from the proton.

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This is sort of a strange way to introduce the use of lumping in this section on lumping. It almost treats lumping as a afterthought to the example. I like (but don't entirely understand) this example, but I would like to see more emphasis on lumping.

I like it when the text states how the current problem is being solved by the overall section's new method. It's a good reminder and helps relate the problem to the approximation method.

> I agree, it allows me pay attention to what he is doing as opposed to trying to figure out what method he is using.

maybe reference the lumping section here (i.e. (1.2.3)) just for people who skip around sections and might not know what this refers to

I'm interested to see how using lumping twice affects the accuracy of the results.

> It seems like when we make multiple approximations, they are frequently tailored such that they will, to some extent, cancel each other out. It'll be interesting to see what happens here.

> There's some sort of a trade-off. The less you lump, the fewer places to introduce error. The more you approximate, the better the chances that the errors cancel out.

>> I've never completely accepted this - although I can see how in _some_ estimations this would happen, I hardly think we can take that as an absolute rule.

when did we introduce this well?

The idea of "infinity" in physics and engineering always slightly bothered me because I can't really picture something with "infinite" anything.

> I think a picture would really help here. I learned about this briefly in 6.007 but I still don't really understand it. Pictures help though, especially for such a theoretical subject.

>> Is this sort of represented in the diagram on the previous page? There are high sides to a well in that picture. On the difficulty picturing anything infinite, I think this is like a step function instead of one that's more curved. A step function has infinite slope but you can picture it.

> I think he just means the electron is stuck in this shell. It isn't going anywhere. So it is in constant potential well with no chance of escaping.

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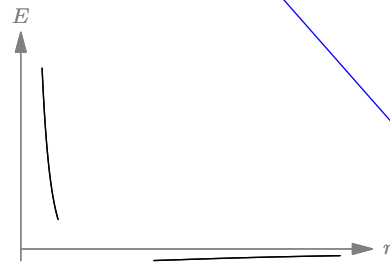
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Based on this which things are still left uncertain. Just velocity?

What do you mean by high sides?

it means, that there is zero probability for it to exist outside the well

This paragraph makes how we used lumping very clear. I was initially confused, but these two sentences helped a lot.

Is there a better way to explain an "electron being smeared all over"? I understand what you're saying, but I'm guessing quite a few people in this class don't understand, and therefore plenty of people elsewhere reading this text would also be confused.

Maybe just "equal probability of being anywhere within the radial distance" or even "an even distribution of possible locations" or something

I think smeared is probably the easiest way to describe the phenomenon as far as readability goes for an average reader.

I'd personally find some comment regarding the fact that there's a non-zero probability of it existing in various places, but significantly greater probability at "r" (although this may not be quite right.)

Perhaps use "spread"?

I think you should mention that this is a probability density thing. It's most likely at radius r, so that's why we assume it, but it really could be anywhere.

I think you should mention that this is a probability density thing. It's most likely at radius r, so that's why we assume it, but it really could be anywhere.

Perfect, answered my earlier question.

Yep, good example of lumping approxmation.

I feel like it would be a little more helpful and clearer if this was mentioned when the approx. was made.

I really dislike this phrasing.

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I thought the first part of our lumping was to replace the electrostatic potential with potential well... but the entire problem seems to use the electrostatic potential and not a potential well?

Yeah, I am confused by this as well.

I don't remember why this is negative.

I believe its because of the direction at which you define your E field.

I think it has something to do with an electron having a negative charge.

The general equation has q1*q2 in the numerator, so in this case we have the 2 charges of the electron (-e) and the proton (e), so we get -e^2. Two oppositely charged particles very far apart then have a potential energy approaching 0 and as they get closer, it gets more negative, sort of the reverse of gravity. If they were attracting each other starting from very far apart, their original potential energy (zero) would be converted into KE as they accelerated toward each other, thus LOWERING PE and making it negative.

also, this is so that likes repel and opposites attract.

Because the potential energy is attractive (i.e. between an electron and a proton). It would be positive if the potential energy was repellent.

Basically, it's negative if the charges have opposite sign and positive if the charges have the same sign.

What's wrong with taking the derivative of the energy to find the value of r that minimizes the energy? I don't think it's overly-complex using calculus to find a First Order Condition that yields the r we are looking for. Plus, it eliminates any errors we might have made in figuring out what term dominates at large and small r's.

Because this section is all about anti-calculus! Wouldn't it be more fun to approximate it?

but it's really easy to find the max of A/r^a + B/r^b=C. It's just r^(b-a)=-(b*B)/(a*A). (A handy abstraction for you.)

Here a=1, b=2, so r=2*(h^2/m)/(e^2/4.pi.eps)

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This is a familiar technique for sketching out curves. I never thought of it as it's own approximation technique however.

Yeah, this is the whole idea of easy cases–to take a complicated equation or problem and apply it to some simple cases that may cause several variables to drop out, etc.

I really like how this section is pulling out a lot of old techniques like divide and conquer and easy cases.

I like how this example uses both the "easy cases" method as well as lumping. It helps tie the two different methods together and reinforce the stuff we learned earlier.

I like this simple explanation.

Yeah, it was super easy for me to follow and understand despite my lack of knowledge in this area.

is this the lumping technique? simplifying all cases as one of these two?

This almost feels like a sub-step using easy cases, and then we use this to lump results together. Just my guess.

How do we have any idea what the scale of r is, does this change the calculations at all?

I really like how the readings apply previous concepts to solving new and more complicated problems as we go along. It really helps to demonstrate the power these techniques have when used in conjunction with eachother.

Again, I like the simple breakdown and repeat

I agree, this made what could have been a very complicated section much easier to follow.

Is this an example of how the math explains the physics, i.e. stating how we don't understand quantum physics?

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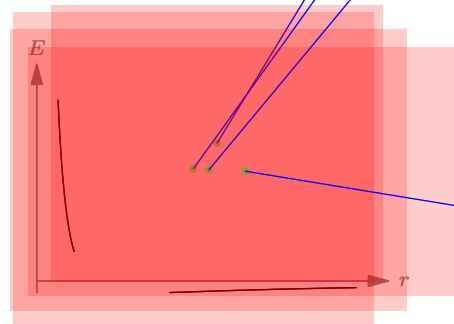$$E_{\text{potential}} \sim -\frac{e^2}{4\pi\epsilon_0 r}.$$

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Now we can sketch E in the two extreme cases. The sketch demonstrates the result in which we are interested: that there must be a minimum combined energy at an intermediate value of r. There is no smooth way to connect the two extreme segments without introducing a minimum. An analytic argument confirms that pictorial reasoning. At small r, the slope dE/dr is negative. At large r, it is positive. At an intermediate r, the slope must cross between positive and negative. In other words, somewhere in them middle the slope must be zero, and the energy must therefore be a minimum.

I figured that we always do this in engineering

Yeah, lumping is used really often in engineering and even physics.

I agree but I think easy cases is the more applicable technique. I have always found that I have been able to solve problems that others can't simply by looking at the easy cases if I don't know the formula necessary for finding the exact answer.

agreed that we do this in engineering all the time though, is 1st(2nd?) order taylor approximation necessary for this since there i some degree of r in the denominator in both terms?

This graph and explanation behind it are really cool. The graph is really similar to those created using wavefunctions in chemistry. I'm assuming this is no coincidence.

I remember talking about this in chemistry and 2.002

At first glance this could almost seem like random marks on the graph, but thinking about it more helps me understand why the extreme cases can help us figure out what goes on in the middle.

I agree. Based on the two lines here, my intuition tells me that there is really only one *likely* solution (granted, it could do all kinds of crazy stuff in the middle... but let's hope not...) - which was confirmed when I looked at the final sketch.

I think this graph is really helpful. since we know that initially the graph is decreasing and for large r, the graph is increasing, there must be a minimum somewhere in between if the graph is continuous. we know it's continuous because the value we're looking for here is Energy (kinetic+potential), and energy is conserved, without any external forces

Yeah this is a good example of putting down what you know and then kind of connecting the dots. It works well.

why wasn't this done on a log-log plot? Theyre quite useful, and it seems like this course is trying to emphasize them somewhat. This seems like a prime opportunity outside of the realm of bode plots.

Log-log plots can't handle negative numbers.

Comments on page 2

This estimate uses lumping twice. First, the complicated electrostatic potential, which varies with distance, is replaced by the simple potential well (with infinitely high sides). Second, the electron, which in reality is smeared all over, is assumed to be at only one spot that is at a distance r from the proton.

This second lumping approximation also helps us estimate the potential energy. It is the classical electrostatic energy of a proton and electron separated by r:

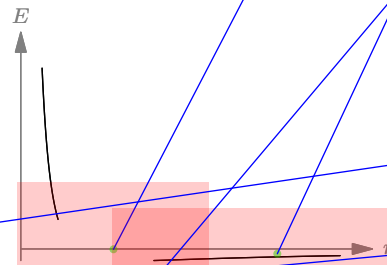$$E_{\text{potential}} \sim -\frac{e^2}{4\pi\epsilon_0 r}.$$

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maybe emphasize the importance of the energy value and corresponding radius?

I don't think there is any need to – this makes sense as is, especially to someone who knows little about these things. It's interesting from an analytic perspective.

What if the line is horizontal? That doesn't count as minimum, does it?

I'm not positive it would apply here, but it would be nice to see a dotted line connecting the two cases on this graph

But then we wouldn't have our own mind approximate what might happen in between the end behavior and I think that's the point here.

Yes, maybe more of a point-out in the text to the missing chunk then.

why is this line almost horizontal, if it had a slightly negative slope, then the function wouldn't have a minimum except for r = infinity, how do we know that this isn't the case from the drawing?

To me it looks like the slope dE/dr is 1/r^2 for small values and -1/r^3 for large values.

Meaning that the slope at small r is positive and negative for large r, opposite of what the text says.

Someone please point out if I'm making a stupid mistake

take a really close look at the graphs - what the text says is correct.

The pictures are E(r) functions. For small values of r, the value of E decrease as r increases, so there would be a negative slope. For large values of r, E would increase as r increases, so there's a positive slope.

You can also see this by taking the derivative of 1/r^2 and -1/r.

Assuming the derivative is monotonic increasing...

Maybe give a brief note about how large r physically makes sense to have increasing energy and smaller r values are associated with decreasing energy... It makes sense on the graph though.

I like this reasoning

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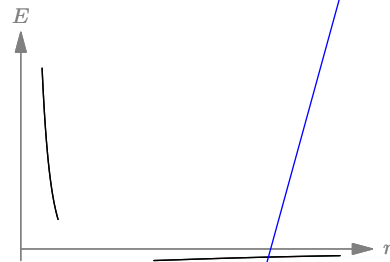$$E_{\text{potential}} \sim -\frac{e^2}{4\pi\epsilon_0 r}.$$

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typo

I really appreciate the detailed walkthrough of the graph and it's meaning. It helped to slow things down and allow me to digest the problem so far.

Yeah, I never thought of thinking about deriving the wave functions in this way.

There are two approximate methods to determine the minimum r. The first method is familiar from the analysis of lift in Section 3.5.2: When two terms compete, the minimum occurs when the terms are roughly equal. In other words, at the minimum energy, the potential energy and kinetic energy (the two competing terms) are roughly equal in magnitude. Using the Bohr radius a_0 as the corresponding separation, this criterion says

$$\underbrace{\frac{e^2}{4\pi\epsilon_0 a_0}}_{\text{PE}} \sim \underbrace{\frac{\hbar^2}{m_e a_0^2}}_{\text{KE}}. \tag{7.6}$$

The result for a_0 is

$$a_0 \sim \frac{\hbar^2}{m_e(e^2/4/4\pi\epsilon_0)}. \tag{7.7}$$

The second method of estimating the minimum-energy separation is to use dimensional analysis, by writing the energy and radius in dimensionless forms. Such a rewriting is not mandatory in this example, but it is helpful in complicated examples and is therefore worth learning via this example. To make r dimensionless, cook up another length l and then define $\bar{r} \equiv r/l$. The only other length that is based on the parameters of hydrogen (and the relevant constants of nature such as ϵ_0) is

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simplifies greatly:

---

should this be "approximation methods"?

Does this mean for two terms that equivalent to something? ie. two terms that are added, multipled, together

I need to remember this!

agreed- i wouldn't have known that otherwise.

This reminds me of the minimization/maximization problems from middle and high school where we had to graph out the functions constraining us to solve the problem visually, and always the answer was at the intersection of the functions.

Reminds me of the drag problem from way back when about the ideal velocity or height for a plane to fly.

I'm pretty sure that's the exact example he's talking about

This is pretty intuitive.

Yea, couldn't we just observe the derivative graph and determine the minimum?

I'd recommend against memorizing this. It's wrong when the powers aren't equal in magnitude and opposite in sign. It's tempting to use this sort of trick more generally, but it can easily put you off by a factor of few.

I wrote this elsewhere, but I find it much more useful: the max of A/r^a+B/r^b=C is at r^(b-a)=-(b*B)/(a*A). It differs by a factor of b/a from assuming the two terms are equal, but it's exactly right and applicable everywhere, so it's a much more useful abstraction.

I'm a bit confused and not quite sure what this means?

are you saying this is true at the minimum r?

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Does it matter that we just dropped the negative here?

I doubt it, since we're just using twiddle and not equals.

He's just setting the magnitude of the energies equal here. I don't think the sign matters in this situation.

with a twiddle, "dropping" the negative would be like making the constant negative.

I love how twiddle is now acceptable in mathematical discussion.

i don't fully understand where the lumping came in

An extraneous /4 here?

Ooh good catch. I totally missed it on my pass through of the equation.

I think so

Yeah, pretty sure.

I was actually just thinking to myself if we were going to do this!

can you do this?

I think he just wants a unitless value...but right now it seems sketchy

What exactly does "L" represent?

I don't remember ever cooking anything up in the dimension section. Why is this necessary, how is it legit, and how do we know when to apply this?

I think he did this because most equations become easier to work with when you take out their dimensions. He cooked up this value because it is a length specified only by the quantities relevant to the problem and therefore defines some sort of length scale the problem has.

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I am confused... in dimensional analysis we looked at the variables we already have and made dimensionless groups using those.. why are we now making up new variables? it seems like it's going to complicate things and is also not an intuitive thought...

We're nondimensionalizing. It's one of the common techniques to make equations and variables simple:

http://en.wikipedia.org/wiki/Nondimensionalization

nondimensionalizing equations is one of the best skills I've attained at MIT so far.

Would this section be better served by being moved to the dimensional analysis section of the book? Then, you can just refer to it here. I understand the importance of doing things multiple ways, but the switching back and forth between methods in a section devoted to lumping makes the entire section a bit disjointed. This might just be a personal thing though.

I think it's fine here...I don't think it's great to limit a certain technique to a certain section and only focus on that technique because all of these techniques are meant to complement the others/double check yourself.

Is l just a0 * 4? So confused as to what l represents...

I don't understand where this came from. I thought we were using a new method to derive this. Or are we just using this result from above?

I think this is the "cooked-up" l? However, I don't really understand how it is chosen...

Perhaps he used dimensional analysis to get something with the dimension of length, and it happened right off to be this? (He'd have to have done the assumed grouping from a previous reading to get the constants in there.)

I believe he is using a length which is on the same magnitude as the radius and more importantly one that depends on hydrogen, which we are looking to solve for.

I think you are right. He probably used dimensional analysis and guessed the other variables that were important and used them to create a dimensionless group with l.

i would have never been able to come up with this – it's slightly frustrating in that sense

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typo

How would I know what is reasonable or not?

Why is this the case, How are these parameters chosen?

Can you maybe explain here why it is a reasonable candidate?

despite the fact that these are familiar terms and equations, i feel like estimation problems are more fun with tangible figures and values.

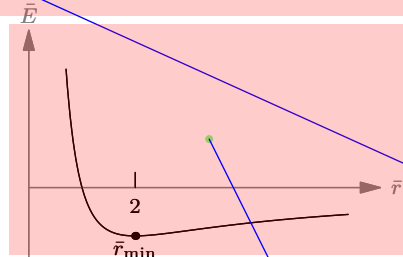$$\bar{E} \sim -\frac{1}{\bar{r}} + \frac{1}{\bar{r}^2}.$$

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Calculus locates this minimum-energy $\bar{r}$ at $\bar{r}_{\min} = 2$. Equating the two terms $\bar{r}^{-1}$ and $\bar{r}^{-2}$ gives $\bar{r}_{\min} \sim 1$. In normal, unscaled terms, it is

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Engineers understand this idea of not over-engineering a system. If a bicycle most often breaks at welds in the frame, there is little point replacing the metal between the welds with expensive, high-strength aerospace materials. The new materials might last 100 years, but such a replacement would be overengineering because something else will break before 100 years are done.

In estimating the Bohr radius, the kinetic-energy estimate uses a crude form of the uncertainty principle, $\Delta p \Delta x \sim \hbar$, whereas the true statement is that $\Delta p \Delta x \geqslant \hbar/2$. The estimate also uses the approximation $E_{\text{kinetic}} \sim (\Delta p)^2/m$. This approximation contains m instead of $2m$ in the denominator. It also assumes that Δp can be converted into an energy as though it were a true momentum rather than merely a crude estimate for the root-mean-square momentum. The potential- and kinetic-energy estimates use a crude definition of position uncertainty Δx: that $\Delta x \sim r$. After making so many approximations, it is pointless to minimize the result using the elephant gun of differential calculus. The approximate method is as accurate as, or perhaps more accurate than the approximations in the energy.

I dont quite get how you made this jump. Could you maybe show the simplification in class?

I'm actually surprised we waited as long as we did to reduce the equation to this. I feel as though we carried the constants for awhile when we weren't really using them.

Agreed. Didn't we come to this conclusion 2 pages ago in (7.5).

I think it was necessary though to show the entire process.

Yeah I found seeing the process drawn out to be helpful.

Lumping doesn't seem to be a technique that is useable on its own, most of this problem was still done with dimensional analysis- we're just naming the assumptions we've made all along as lumping

Well, many of the other techniques aren't necessarily useful "on their own" either. It depends on the problem and how you approach it. Often here, we've dealt with lumping and dimensional analysis or easy cases but that doesn't mean lumping is exclusively usable in conjunction with other skills.

Lumping is a way to simplify a complex problem to the sort that is more easily solvable, but you still need other methods to reach an answer.

I thought we were trying to avoid calculus? Wasn't lumping the "opposite" of calculus? Seems a little silly if we can't finish the problem without use of calculus ;)

Maybe because it's easy calculus?

I think he's just showing how 'close' our answer gets. Sort of like telling us the answer to book sales after we estimate it.

so is it 2 or 1 that we are gonna use?

Nice to see this graph again with the kinetic and potential energies connected

Great. Now it makes sense graphically.

can't beat graphical learning

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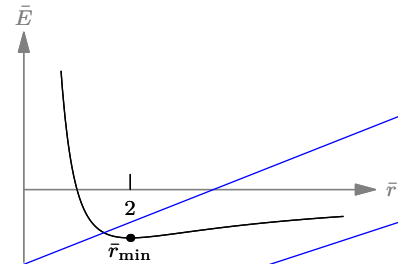
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somehow i like this method more because it seems more accurate because we are actually doing derivatives

Messy ones, but I see the point.

Is there some reason why this happened other than pure luck? Is the lesson that you are trying to tell us that we can be sloppy and it all will probably just work out, or was this a special case?

I don't think I understanding this. Are you saying that since we're only looking for the minimum, the sloppiness sort of doesn't matter?

What does this refer too?

equating the two energies to get an approximate minimum

This should be a lower case t.

There's a missing period here.

due to the fact that in quantum there are discrete levels of energy, (not continuous).

I'm not sure that's what he means. I think he might just mean a proper calculus method is like carrying 10 significant figures, when all of the rest of derivation only has accuracy to 2 significant figures.

But why does that make the answer worse?

It doesn't make the answer worse so much as it adds specificity where there was none to begin with.

it's similar to what we learn with sig figs, "you can't be more precise than your measuring device" – if we're going to estimate for the rest of the derivation, no point in being precise for this one part

Oh, that makes more sense.

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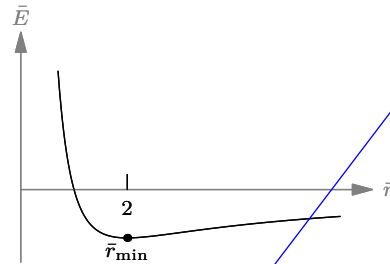
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do you mean over complicating?

Over engineering could include over complicating but it's more broad than that. http://en.wikipedia

I think he's using this to mean the same thing as over-complicating, he uses the same phrase again later in the paragraph so i think he's just coining a new term :)

the example he gives of the bike sounds like overengineering has less to do with over complicating and more to do with needless redundancy and inefficiency.

Sounds like its just not balanced, if you make the whole bike with comparably good materials it should all last 100 years until the weld breaks but if you make the other parts with materials that will last as long as the estimated time to failure of the welds then you can be efficient.

It's basically the idea of if it's not the weakest link, don't fix it yet.

Huh? how is this relevant?

I'm not entirely sure it's exact relevance, but it is a cool and truthful anecdote!

I think this is following up on the "calculus method is too accurate" comment, meaning that we shouldn't "over-engineer" by using calculus when the rest of approximation is inaccurate. Likewise, we shouldn't use aerospace materials to patch a cheap bicycle.

That explanation helps a bit in understanding how not to do too many complicated calcutions. It is just hard to draw any parallels between the two examples.

I think this analogy is very fitting for the situation.

Yeah I like it.

Great example to show why it's bad to over-engineer a system. I guess I've always been a fan of using calculus or "over-engineering" problems in this course thus far, but this analogy sheds some light on not doing that.

Haha, yeah. Imagine the costs.

Agreed. I like how we are relating our choice of math/engineering to practical applications.

$$\bar{E} \sim -\frac{1}{\bar{r}} + \frac{1}{\bar{r}^2}.$$

The ugly constants all live in the definitions of scaled length and energy. This dimensionless energy is easy to think about and to sketch.
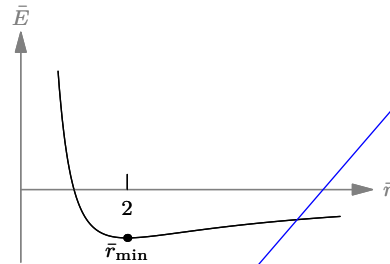
Calculus locates this minimum-energy $\bar{r}$ at $\bar{r}_{min} = 2$. Equating the two terms $\bar{r}^{-1}$ and $\bar{r}^{-2}$ gives $\bar{r}_{min} \sim 1$. In normal, unscaled terms, it is

$$r_{min} = l\bar{r}_{min} = \frac{\hbar^2}{m_e(e^2/4\pi\epsilon_0)},$$

which is the Bohr radius as computed using dimensional analysis (Section 5.3.1) and is also the exact Bohr radius computed properly using quantum mechanics. The sloppiness in estimating the kinetic and potential energies has canceled the error introduced by cheap minimization! Even if the method were not so charmed, there is no point in doing a proper calculus minimization. Given the inaccuracies in the rest of the derivation, The calculus method is too accurate

Engineers understand this idea of not over-engineering a system. If a bicycle most often breaks at welds in the frame, there is little point replacing the metal between the welds with expensive, high-strength aerospace materials. The new materials might last 100 years, but such a replacement would be overengineering because something else will break before 100 years are done.

In estimating the Bohr radius, the kinetic-energy estimate uses a crude form of the uncertainty principle, $\Delta p \Delta x \sim \hbar$, whereas the true statement is that $\Delta p \Delta x \geqslant \hbar/2$. The estimate also uses the approximation $E_{kinetic} \sim (\Delta p)^2/m$. This approximation contains m instead of $2m$ in the denominator. It also assumes that Δp can be converted into an energy as though it were a true momentum rather than merely a crude estimate for the root-mean-square momentum. The potential- and kinetic-energy estimates use a crude definition of position uncertainty Δx: that $\Delta x \sim r$. After making so many approximations, it is pointless to minimize the result using the elephant gun of differential calculus. The approximate method is as accurate as, or perhaps more accurate than the approximations in the energy.

Why did you bother to make this 'approximations' when the exact equation was known? I think for many people you already sort of pulled these equations out of a hat, so why not pull out the correct equations to begin with?

he explains below, that that ould be overengineering. We know the uncertainty principle so having some sort of approximation that deviates from its actual value by less than the uncertainty value, doesn't affect out answer at all.

I like the recap of everything you approximated.

I'm not sure what I'm supposed to be learning from these paragraphs – there is so much information and so many things I don't know.

tis is a very useful point to remeber. u should bold it to make sure readers get the point

I'm pretty surprised by this, it just seems like something that we would have already figured out the exact method for solving.

I wish this was gone over in more detail, with explanations as to why it is more accurate

Same here, perhaps a little more explanation on accuracy would help ease my unrest here.

The method of equating competing terms is called balancing. We balanced the kinetic energy against the potential energy by assuming that they are roughly the same size. Nature could have been unkind: The potential and kinetic energies could have differed by a factor of 10 or 100. But Nature is kind: The two energies are roughly equal, except for a constant that is nearly 1 (of order unity). This rough equality occurs in many examples: You often get a reasonable answer simply by pretending that two energies (or two quantities with the same units) are equal. [When the quantities are potential and kinetic energy, as they often are, you get extra safety: The so-called virial theorem protects you against large errors (for more on the virial theorem, see any intermediate textbook on classical dynamics).]

Yep. I find myself doing this all the time.

is this another method to think about? or just an explanation of above?

In general, will we be able to assume that KE and PE are the same size? Or should we assume that it only worked in this case?

Only in this case, I think

Yeah, I like this method but am still a bit unsure about how often I can use it and in what cases. It would be nice if this was a section in itself.

Nature seems to be kind to us more often than not when we do these estimations.

What happens when nature is unkind? ie. we can assume that it will be a factor of 10 or 100 difference?

I have the same question. How do you decide when they are approximately the same or not?

Is there some way of knowing if this is true in an individual case? Or throughout this class (and life beyond) do we just assume it to be true and cross our fingers?

I think the idea is to assume it's true and cross our fingers, because it usually is for the degree of accuracy we need. If it's off it's usually a quick fix of a constant, and at least we've already done most of the work.

I think this goes in the category of its better to make a model and get some sort of an answer then get stuck trying to be more exact and getting no answer at all.

just use 1 here, stick with your defined conventions

also, can you talk about the limitation of using balancing? when should you NOT use it

when the ratio of the magnitudes of the powers is too large of an error factor (since that is precisely how much the answer from balancing differs from the calculus answer).

it would be great if you can just briefly mention some examples, it just makes the arguments more convincing

it would be interesting to hear more about why these energies are equal

The method of equating competing terms is called balancing. We balanced the kinetic energy against the potential energy by assuming that they are roughly the same size. Nature could have been unkind: The potential and kinetic energies could have differed by a factor of 10 or 100. But Nature is kind: The two energies are roughly equal, except for a constant that is nearly 1 (of order unity). This rough equality occurs in many examples: You often get a reasonable answer simply by pretending that two energies (or two quantities with the same units) are equal. [When the quantities are potential and kinetic energy, as they often are, you get extra safety: The so-called virial theorem protects you against large errors (for more on the virial theorem, see any intermediate textbook on classical dynamics).]

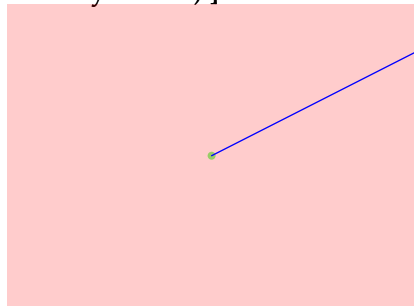**it'd be nice to have a better idea of when it isn't appropriate to do this**

http://en.wikipedia.org/wiki/Virial_theorem

Thanks, this was helpful

Thanks.

I was wondering if that was a typo.

The method of equating competing terms is called balancing. We balanced the kinetic energy against the potential energy by assuming that they are roughly the same size. Nature could have been unkind: The potential and kinetic energies could have differed by a factor of 10 or 100. But Nature is kind: The two energies are roughly equal, except for a constant that is nearly 1 (of order unity). This rough equality occurs in many examples: You often get a reasonable answer simply by pretending that two energies (or two quantities with the same units) are equal. [When the quantities are potential and kinetic energy, as they often are, you get extra safety: The so-called virial theorem protects you against large errors (for more on the virial theorem, see any intermediate textbook on classical dynamics).]

So I'm a little confused at the end here... this reading is supposed to be about lumping, and yet most of the analysis is done with dimensional analysis and easy cases. The only "lumping" I remember was ignoring the implications of quantum mechanics on electrons, which seems like a necessary reduction given the complexity of quantum mechanics. Would this example be better in another section?

I was also confused by our use of "lumping" in these calculations.

I think the idea is that lumping is just one more tool that we can use.

Based on all of the examples seen here and in class, I believe that there are not many (interesting) problems that can be solved by lumping alone. But, when used in conjunction with our other tools, it complements them quite nicely.

It would have been since to have a final summary (digram!) of the different methods we used. And also the discussion of our final answer is ???

Yea I was confused as to where we used lumping in these examples. I think that lumping is used to help make the problem easier to solve (like with easy cases), but you still have to use other methods like dimensional analysis or setting the energies equal to each other in order to find a solution to the problem

I think the main idea is that for complex problems we can use lumping to help find a place to start. in this case, lumping didn't solve the problem for us but it put us in the right direction. we used the model of the potential well and then we assumed that electrons are stationary with distance r from the origin. we used this new info to come up with that graph that was missing its middle section, then we used dimensional analysis to actually come to a conclusion

Lumping as a 'tool' does seem really rather vague to me. It isn't really definable, and it seems like any way of making a problem easier can be considered 'lumping'.

I agree. This section seemed like it used techniques which we are familiar with from the class, but I'm not sure what I got out of it about lumping specifically.