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why linearly?

I'm having trouble seeing what this has to do with lumping...

A thought on the reading: I think this reading would be really helped with a few more pictures/diagrams. Boundary layers are something that most readers do not have intuition on.

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Comments on page 1

Read this section for Wednesay (memo due Wed at 9am) – the endpoint is golf-ball dimples.

Could you mention what boundary layers are, for the uninformed?

The boundary layer is the layer of fluid right next to the surface of whatever object you're considering. If I remember correctly, due to friction, this boundary layer can't be treated the same as the rest of the fluid, since you sort of treat that layer as though it travels with the object. (I think there is a no slip condition at the boundary)

I'm sure there is a no-slip condition.

Basically, drag to the utmost!

I'm one of those uninformed and think it'd be a good idea to clarify this too

Have patience. The section gets to it. It's easier to explain once the problem has been discussed and a diagram drawn with some context.

what paradox?

Yeah....? I thought we had successfully solved the drag problem several times so far this semester..

I agree. The paradox is explained in the next few sentences, however I was unaware that this paradox existed when I began reading the section, and it threw me a little.

Maybe combine this sentence and the one after, to read "...the drag paradox arising from analyzing drag at high Reynolds numbers..."

do you mean that it is another way to look at drag?

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I feel like we've spent the great majority of this class just talking about drag. I thought this class was supposed to discuss several engineering disciplines... we talked about UNIX for a few classes, and then we've spent almost a month on just the concept of drag. I would appreciate it if we could move on to some other topics.

I do agree that we've spent a lot of time on drag, but at the end of the day the drag is always just the example for what we're actually being taught (dimensional analysis, lumping, etc.) I think it's nice to see the same example done using many different approximation methods so we can see exactly what's different about each of them. Maybe it just turns out that drag is one of few examples which all the different approximating methods really work for.

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Yup, but as mentioned, drag is just one example, which we can easily use to apply a wide variety of skills and techniques in thisi class.

Btw, nice triple post there.

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True, but some people don't care too much to be staring at discussions on drag for two months. We could hit a bunch of other cool topics and explore each with a new idea. I suppose the one positive to using drag for all of them is you see all of the approaches to one central answer, but I'm personally a fan of breadth for this class (as it seems that the idea of what we are doing is the ability to solve a large range of problems easily)

I know Reynolds number has been defined several times before, so you shouldn't define it again. But, as a non-course 2 major, I don't know what this number means or what it represents, and every time it's brought up, I have to go back through the old readings to see what it is.

It's defined as the inertial forces / the viscous forces in the fluid. It helps you understand what is physically happening in the system. High Re means that the flow is turbulent, and often little vortices are formed as a result of drag around an object.

basically think: at high reynolds number you have motion similar to a bullet flying in outer space, low reynolds number is like a bullet going through super thick molasses

rho*v*D/mu, or v*D/nu. It's a ratio of inertial to viscous forces.

It might be good here to include a table of common Reynolds numbers, so we can see what makes a high number and what it means to be a high number.

told

You've addressed this before, but I really hate the typesetting in this equation.

I think bolding the viscosity is a quick solution that would get rid of this problem

Its better to bold the velocity than viscosity since velocity is a vector (Frequently vectors will be typeset in bold)

I agree, this is still confusing without looking back.

bold one of them to distinguish the variables

In another section I read, a capital V was used for viscosity. Perhaps do that and bold it...

can you change it to rho*v*D/mu instead of vD/nu? it's often written that way instead.

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I think this would be helpful to us if you wrote out the relation between drag coefficient and reynolds number in the equation that you have done above.

I don't really understand this comment, the equation above gives that relationship exactly.

but it doesn't explicitly relate drag coefficient and Reynolds number. It just tells you what they are in terns of variables in the equation above.

Could you explain the equation a little more? If the left side is the drag coefficient, and rv/v is the Reynolds number... Are you saying that the drag coefficient is the function of the Reynolds number? I guess I don't understand how it can be constant and a function?

So he is saying that the drag coefficient is only a function of the reynolds number; but at high reynolds number (easy to do with the viscosity goes to 0), and so if the viscosity is =0, then the Re=0, and so, the drag coefficient is now not a function of anything.

I still don't understand how we are able to take any variable from a complicated multi-variable equation, and just say that it is zero, when in reality it never is, and expect to get reasonable results from this simplified equation.

This doesn't seem to be adequately justified – I think it made more sense when we covered the topic initially.

what do you mean by as does the Reynolds number? does it vanish too? I am confused.

I think he meant that as viscosity goes to zero, some of the terms in the N-S equation vanish?

I agree that this is a bit confusing as worded. How exactly does that mean that it vanishes? Some terms vanishing means it vanishes? I suspect this actually means something about its influence, that it no longer is variable or something, but I'm not quite sure.

How does dividing by zero make something disappear? it seems to me it invalidates the relation more than anything else.

I think this is because infinity doesn't depend on v.

meaning that the coefficient of drag is constant at high Re numbers?

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Doesn't that make it a function of infinity? f(infinity)?

I think he's saying that as the denominator goes to 0 you get f(r*nu/0)=infinity.

I agree, but that should be explained more specifically. Those of us who have no prior knowledge of these concepts have little intuition for these extreme cases.

Does this happen because the viscosity is so low that there is, in a manner of speaking, nothing to really impose drag? I'm having trouble grasping the physical implications of this particular "paradox"

yeah, someone please explain. Doesn't removing drag make this calculations useless?

I'm quite confused by this as well. here we're taking the limit as viscosity approaches zero. this makes the reynolds number very high, but how does that remove viscosity and the reynolds number from the picture?

does it imply that at low Reynolds number, drug coefficient is not constant? is it a function of viscosity then/.

Again, it'd be good here to help us get a sense of what a high Reynolds number means.

I'm pretty sure he already mentioned some examples of high and low Reynolds number fluids in a previous section. However I agree, that a quick example of something with a high Reynolds right here would be useful, especially for people like me who don't really know anything about fluid mechanics.

this should either be " at A high reynolds number" or "at high reynolds numberS"

This is what I would expect given how the Reynolds number affects the mechanics of fluids.

I agree but I'm not all that comfortable with how we got there.

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I see you've tried to differentiate between velocity and viscosity by bolding velocity, and that seems to work fine. Do you think you could make this change to the variable sin the written paragraphs too?

I assumed the bolding was to indicate it's a vector, not to differentiate it from nu? If that's not true, than it's also a bit confusing to have it bolded.

I think seeing the notation in this equation made me more confused between nu and v. But I understand the concept.

If person 1 is correct and the bold is to differentiate, then I think it works nicely.

I can't remember earlier notation in the previous chapters but if bolding is used to identify vectors that is another issue I hadn't considered with using bold to differentiate the two symbols. Otherwise I think bolding does the job very nicely

I believe nu is written as a curvy v or a v in italics and velocity is a regular v. I think the bold is to show that it's a vector. If you look at the second term on the right of the equal sign it's an italics v multiplied by the gradient squared multiplied by a bold v.

Maybe a quick note to let the reader know you are bolding velocity to distinguish it would go a long way.

So velocity can be considered a vector (or each cartesian coordinate can be considered independently.. but the grads would change to partial derivatives), so that interpretation wouldn't be incorrect. I think it also serves to differentiate from nu.

You can also try to use the little vector arrow above the v.

using viscosity has always bugged me a little bit I feel like we would want to use velocity as the variable rather than viscosity. which is what we did in 2.005/2.006 and it seems to make for physical sense

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The bolded v here is velocity correct? If velocity is bolded here than it may help to bold velocity above in the Reynolds number equation to help solidify which variables are what

The bolded v here does stand for velocity. In the Reynolds equation, the v, which also stands for velocity is not bolded, and I agree that it might be slightly confusing, but I think the bolded variables are vectors. Perhaps there should be a note about this in the text since so many people are confused about this.

this sentence syntax makes no sense to me.

Yeah – it uses two different appositive constructions (commas and dashes), within a parenthetical statement. Probably too much to be going on in one sentence.

yea the phrase makes sense to me without the – the pressure – and once it's added, i'm not sure what it's referring to

Will this book have an appendix for the requisite mathematical terms and concepts, or will it not be self-contained?

I think terms like gradient and scalar are pretty basic, and information about them is readily available in most math text books of the internet. There are definitely more complicated terms and topics discussed that I would love to see external links to where I can find more information as I often mind myself looking into the things discussed in these chapters more on my own time

Why can't it dissipate energy?

It can't dissipate energy because it is a conservative force, which means essentially that it is reversible without loss of energy. Think of it like gravity: if you take a ball at a height h1 and bring it to a height h2, it loses some amount of energy, but if you repeat the experiment it will always lose the same amount of energy - nothing is dissipated to the surroundings.

Based on my intuition I would expect the drag coefficient to approach a really large value so I agree the drag coefficient cannot approach a constant

oh..this make more sense!

so much better now that this was explained - had no idea where there was a paradox before.

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wait...are you saying it should or should not be 0?

I think he's saying that we know it's not zero, but if you make that assumption this equation implies that it should be zero.

He's saying that the equation says that it should be, but we know from real life that it isn't.

I thought we just said it was infinity?

No, we were taking the Reynold's number to infinity to have a roughly constant drag coefficient.

where does the discrepancy lie? Is there another area where drag coefficient can be measured as a constant?

I think the discrepancy is that according to the equation, drag coefficient (and thus drag) should approach zero as viscosity approaches zero, but in real life drag is never 0

Interesting...I wonder how we resolve this.

I would never have figured out any of this on my own, I would have just ignored N-S!

He also pointed this out on the graph though. The line goes relatively flat, but it's not flat at zero.

It took a while to explain what you meant in the intro.

Yeah, but I am glad you did. When I read it in the introduction of the section, I had no idea what it was and thought I was about to not understand this example at all. So I am glad it is included.

I would love to know why that is a paradox

basically because the navier stokes equation (which we assume to be correct) says that if viscosity goes to zero then the drag coefficient should approach the constant zero. However that doesn't happen in real life. So it is a paradox because both of these can't be true.....

7.4 Boundary layers

Boundary layers, which are the final example of lumping, will help us explain the drag paradox. That paradox arises in analyzing drag at high Reynolds numbers (the usual case in everyday fluid motions). Dimensional analysis tells us that the drag coefficient c_d is a function only of the Reynolds number:

$$\frac{F_d}{\frac{1}{2}\rho v^2 A} = f\left(\frac{rv}{\nu}\right), \tag{7.8}$$

where $F_d/\frac{1}{2}\rho v^2 A$ is the drag coefficient, and rv/ν is the Reynolds number. To make a high Reynolds number, take the limit in which the viscosity ν approaches 0. Then viscosity vanishes from the analysis, as does the Reynolds number. The result is that the drag coefficient is a function of nothing – in other words, it is a constant. So far, so good: Empirically, at high Reynolds number, the drag coefficient is roughly constant and independent of the Reynolds number.

The paradox arises upon looking at the Navier–Stokes equations:

$$(\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v}. \tag{7.9}$$

When the viscosity ν goes to zero, the $\nu\nabla^2\mathbf{v}$ viscous-stress term also vanishes. However, the viscous-stress term is the only term that dissipates energy. (The pressure term ∇p is, like gravity, a conservative force field because it is the gradient of a scalar – the pressure – so it cannot dissipate energy.) Without the viscous-stress term, there can be no drag! So, at high Reynolds number, the drag coefficient should approach a constant *but that constant should be zero!*. In real life, however, the drag is not zero; this discrepancy is the drag paradox.

Mathematically, the paradox is a failure of two operations to commute. The two operations are (1) solving the Navier–Stokes equations and (2) taking the viscosity to zero. If we first solve the Navier–Stokes equations, then we find that the drag coefficient is nonzero and roughly constant (i.e. independent of Reynolds number). If we then take the viscosity to zero (by taking the Reynolds number to infinity), no harm is done. Because the drag coefficient is roughly independent of Reynolds number, the drag coefficient remains nonzero and roughly constant.

are there other examples of this? how do we know if it's this or just that we messed up in our analysis?

Agreed, I don't know if I would be able to recognize this paradox myself.

Also agreed. I would be interested to know if there are other such paradoxes in physics/mechanics.

I've run into this problem before for other equations, and realized that it's usually better to solve the equation before plugging in an edge case.

What other examples do you mean?

I definitely agree, but there are cases where it's fine to assume limits before solving. I wonder if there's any more systematic way of knowing when you can and when you can't?

I think this sentence is confusing?

I feel like here is where I finally start to see where boundary layers are talked about, but a definition/earlier introduction might be more helpful for directing this section

wait these seem like they work in harmony instead of dissonance – what am i missing?

Continue reading into next paragraph. The paradox arises then.

The paradox is described in the next paragraph.

wait really? can we see this out in class today? i don't quite get how now hard is done this way

How is the drag coefficient roughly independent but can be a function of Reynolds number?

After a point, it no longer varies significantly with the increasing Reynolds number. It is a function to a point, but becomes roughly independent.

Now imagine applying the two operations in the reverse order. Taking the viscosity to zero removes the viscous-stress term from the Navier–Stokes equations. Because that term is the only loss term, solving these simplified equations – called the Euler equations – produces a solution with zero drag. The mathematical formulation of the drag paradox is that the solution depends on the order in which one applies the operations of solving and taking a limit.

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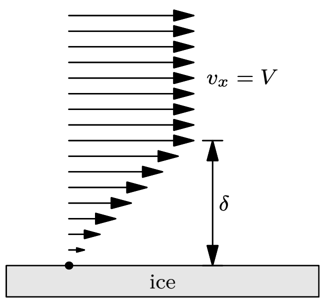

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Comments on page 2

It's so interesting to me that you can use the exactly right equations and still come out with the wrong answer simply due to the order in which you solve them.. Do you have any hints on how to know which equation to approach first (in a general case)?

I'd say it's a decision about when to throw out information. Here we were foiled because we made our lossy assumption first (before using N-S which requires a viscosity).

In other cases, making lossy assumptions first will make problems much more convenient to solve, so it's probably a judgment call. My approach would be try making the assumption first, and sense-check the result to see if your assumption was 'legal'

I think this is a lot of text to explain math. I was having trouble following, especially after the page break.

I agree...it was a bit difficult to follow math via text. I think a small diagram or display of the two methods might help visualize the idea.

Yes it is a lot of text, but it seems necessary to explain this. as mentioned a comparison of the two methods (and how the viscous-stress term drops out) would help to tidy this up

I was totally confused until this point...I think that this would work better with a picture/graph or actually writing the equations.

I agree, seeing the two methods of solving this problem would help a lot here.

Now imagine applying the two operations in the reverse order. Taking the viscosity to zero removes the viscous-stress term from the Navier–Stokes equations. Because that term is the only loss term, solving these simplified equations – called the Euler equations – produces a solution with zero drag. The mathematical formulation of the drag paradox is that the solution depends on the order in which one applies the operations of solving and taking a limit.

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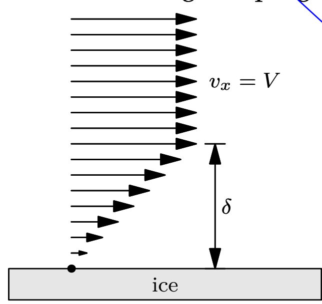

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Even after this is explained, I still find it incredulous that such important math equations work out differently depending on order!

It has to do with the lossy approximation of viscosity going to zero and striking out that term from N-S before solving it.

I think this relates to the problems one can run into if you throw out the zeroes too early in certain problems.

Yeah, it has to do with the order of the problem (wow, English is confusing... by order of the problem I mean removing the highest-order derivative not order of solving equations). THis is explained in the next paragraph.

I'd go with 'shocking' over 'incredulous' ... I understand why, it's just that it's a little hard to believe without really thinking about it.

yeah i think this has to do with our sketchy math at the beginning more than anything else.

are there other equations like this?

yeah I would be really curious to hear about some other example of equations like this in which the solution depends on the order!

Just to clarify, you follow this with an explanation but it's not immediately clear if that's a definition of what a singular limit is here, or just this specific case with drag...

Perhaps change the next sentence to "It is the limit that..." would make it more clear.

I agree with the previous statement; I'm not sure if it's a definition for singular limit, but I have to assume it is because I don't know what a singular limit is (though I think it removes the highest-order-derivative...)

Now imagine applying the two operations in the reverse order. Taking the viscosity to zero removes the viscous-stress term from the Navier–Stokes equations. Because that term is the only loss term, solving these simplified equations – called the Euler equations – produces a solution with zero drag. The mathematical formulation of the drag paradox is that the solution depends on the order in which one applies the operations of solving and taking a limit.

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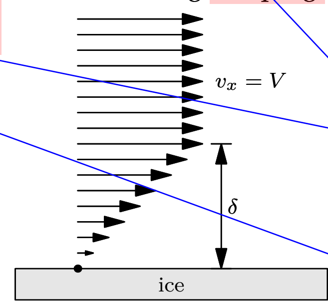

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I thought this idea was really cool - that the operations can't commute because you're reducing the dimensions of the equation by switching the orders.

That does seem really crazy- by doing the math in a certain order you can actually obtain a contradiction.

Does this relate to the problem with taking a square root in an equation? For instance, reducing x^2 = 4 to x=2. I understand its not an issue of limits, but its a mathematical operation that is communicable.

That's really cool. The first thing I thought of was operators in quantum mechanics, but I never really got that some operators can't commute because the commutation changes the order of the derivative until I read this.

This sentence is confusing as it is worded now... maybe say, "this qualitative change in the order of the equation produces a qualitative change in behavior."

I have a feeling its a type. Maybe he meant to say that "this quantitative change produces a qualitative..."

Ahh, so this is the origin of the boundary layer. Very cool!

Is this an example of a mathematician using physical reasoning to solve an equation?! Gasp!

good indicator of next steps.

how true is the no slip condition?

Now imagine applying the two operations in the reverse order. Taking the viscosity to zero removes the viscous-stress term from the Navier–Stokes equations. Because that term is the only loss term, solving these simplified equations – called the Euler equations – produces a solution with zero drag. The mathematical formulation of the drag paradox is that the solution depends on the order in which one applies the operations of solving and taking a limit.

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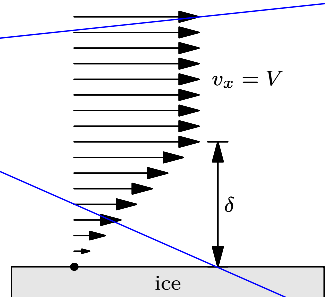

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Why is the type of surface not a significant factor in the size/significance of the boundary layer. I would think that more slippery objects would cause the boundary layer to be much thinner, and therefore change the drag significantly. Is this the case or is it not really that important?

"Slippery object" isn't really an appropriate concept. First of all, we are assuming what's called a "no-slip" condition. This means that the fluid that is in contact with the object is not moving with respect to the object. Slightly more explanation here: http://en.wikipedia.org/wiki slip_condition

Second, given that no-slip condition, any non-geometric properties of the object won't have an effect. Fluid that is moving past other fluid is only affected by the fluid's own viscosity, not by any "slipperiness" of the object. I kind of like this example: http://en.wikipedia.org/wiki/Couette_flow

Maybe you could say "In a fluid there is viscosity, and the fluid is..." instead of the current phrasing which requires an explanation in parenthesis.

I don't think that change would make it better...it seems a bit awkward. Why not just say "All fluids have viscosity and is at rest next to a solid object."

OK, I was just trying to make a helpful suggestion. Since you want to get picky about it, your sentence isn't even grammatically correct. The sentence: "All fluids have viscosity and ARE at rest..." has the appropriate plurality.

And I am just giving my views on it. There's no needs to start an argument.

hahaha. fight on nb. but seriously...you go back and read your replies?? weird.

didnt we just make a whole argument assuming that the viscosity didn't exist??? i dont really understand why we didnt start with this statement.

I think the point is to show how the previous model does not explain the paradox - that we need to use another method to figure this out.

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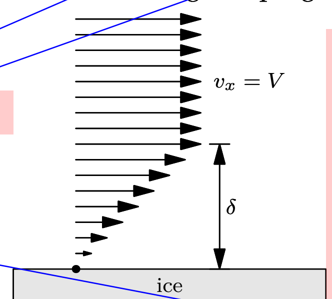

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Is the no-slip condition true of all fluids in the physical world, or is at an analytical shortcut - like "imagine an object falling in a vacuum"?

I think this has to be a true condition, or at least a good approximation to reality. It seems to me that molecular interactions between the "object" and the "fluid" would cause the object to drag a very thin layer of the fluid along with it, and this layer would then have zero velocity with respect to the object.

This scenario seems like it has a large number of application to everyday events.

This is a great example because it's so easy to picture and everyone is able to relate it

This is a great description! I can definitely visualize the wind and the frozen lake interaction.

Agreed. I feel this way about a lot of explanations in this class - I wish they'd be given earlier

excellent diagram- why wait till now to show it? maybe have a diagram of the paradox. visuals i feel are key in explaining this content

Yeah I really like this picture. It's really helpful in understanding this example. It reminds me of the difference between Newtonian and relativistic mechanics.

Agreed. I think its a great diagram and these comments just show that maybe more diagrams are needed in areas that involve heavy mathematical explanations.

I'd say: "...frozen lake. Despite the wind, the air has zero velocity in the horizontal direction just above the ice, and . (In the vertical..."

I had to re-read this paragraph to remember that you were taking about just above the ice...switching the order would have really helped.

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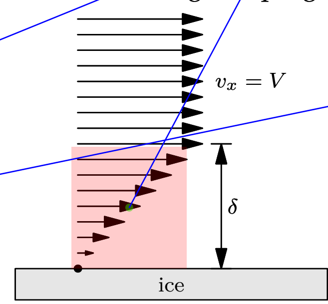

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Is this diagram showing the linear function that we estimated with lumping or is this a more accurate function?

This is the lumping approximation. And the consant part above is also an example of lumping.

This diagram is excellent for understanding our lumping model.

I agree that this is an excellent visual of our lumping model, but it may help to put the equation for the boundary layer region the same way the our upper part is v_x = V

I like the picture here, it helps make these sentences very easy to understand.

Definitely. I was somewhat picturing a similar graph in my mind, but that one here is quite helpful in understanding the phenomenon.

I agree, this is similar to pictures we used in 2.005 and pictures like this probably could have been used in earlier sections as well.

I don't understand this... why does the air have zero horizontal velocity next to the ice?

This was new information for me too. Is this something we could have observed on our own? How big is the boundary layer in this case?

why is the speed slower (or zero) closer to the ice? is it just because the ice is cold that it slows down the wind somehow?..or..what.

I am confused by this as well. conceptually this may need a little more explanation. how can the horizontal velocity be zero by the ice, and what causes the boundary layer?

I think I left out an important intuition that belongs the explanation, namely that viscosity prevents the horizontal flow speed from changing too rapidly. So, the flow speed is zero next to the ice and is at full speed far away, and it varies smoothly in between thanks to viscosity. I'll explain that more in lecture...

This is really cool. Another interesting fact that I never knew.

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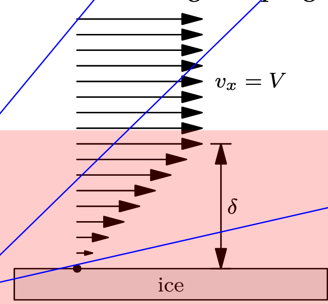

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I love this example- I was starting to get lost, but I understand exactly what you mean now with this application

I agree. This simple example, coupled with the diagram on the right, help clarify some confusing points above.

I also agree. It's easy to get lost in the math, but the example here and those in class make it all click.

I like this explanation in the context of the example, but it may be helpful to have a more formal explanation of what a boundary layer is as well.

I don't think it should be too formal. If you read on, he kind of supplements his explanation here with more examples and explanation. I think it's sufficient.

This is I think as formal as you can get by example...I like it

I feel like the way he explains it is great because it helps you remember it in a visual way. I personally probably would skim over a formal definition and forget it, but I won't forget this.

I thought this explanation was as good as it could be.

i understand the concept of the boundary layer but i don't understand how this solves the paradox

"Actually, the horizontal velocity never fully reaches the full wind velocity V (called the free-stream velocity)."

Why not?

yeah this seems like a strange digression.

Maybe because of interference from the vertical velocity? Just a wild guess.

This is really where this section start making sense to me, the example is very clear, and I finally understand was is meant by boundary layer

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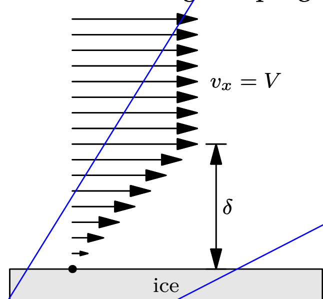


The explanation begins with the no-slip condition: In a fluid with viscosity (which means all fluids), the fluid is at rest next to a solid object. As an example, imagine wind blowing over a frozen lake. Just above the ice, and despite the wind, the air has zero velocity in the horizontal direction. (In the vertical direction, the velocity is also zero because no air enters the ice – that requirement is independent of the no-slip condition.) Far above the ice, the air has the speed of the wind. The boundary layer is the region above the ice over which the horizontal velocity v_x changes from zero to the wind velocity. Actually, the horizontal velocity never fully reaches the full wind velocity V (called the free-stream velocity). But make the following lumping approximation: that v_x increases linearly from 0 to V over a length δ – the thickness of the boundary layer.

The boundary layer is a result of viscosity. The dimensions of viscosity, along with a bit of dimensional analysis, will help us estimate the thickness δ. The dimensions of ν are L^2T^{-1}. To make a thickness, multiply by a time and take the square root. But where does the time come from? It is the time which the fluid has been flowing over the object. For example, for a golf ball moving at speed v, the time is roughly $t \sim r/v$, where r is the radius of the ball. Then

Do you mean just in the boundary layer it doesn't reach V, or is there some bigger point I'm missing?

I think he means in the boundary layer, otherwise V wouldn't exist at all.

It looks like just in the boundary layer in his diagram

I agree there is some slight confusion. But I do think he means just in the boundary layer.

I thought this had something to do with the air resistance.

It's analogous to an object never reaching terminal velocity – it just approaches that speed more and more as it falls. Similarly, as you go farther away from the ice, the speed gets closer to V but it never reaches it fully. The boundary layer is, intuitively, the region over which it mostly gets to V.

I'm not sure this sentence to the left is actually a sentence. I'm not sure if it was meant this way, or it is meant to be one.

I know we're assuming the relationship is linear for the sake of a lumping approximation, but what shape does this relationship take empirically?

It's been about a year since I did boundary layers, but I'm pretty sure it resembles the shape of the f(x) = 1 - (eˆ-x) function.

That sounds right. It's also commonly approximated by a parabola instead of a line. The parabola gives a quite good fit to the actual velocity profile, without adding too much complexity as to make the governing equations unsolvable.

This is a nice refresher on material from 2.005

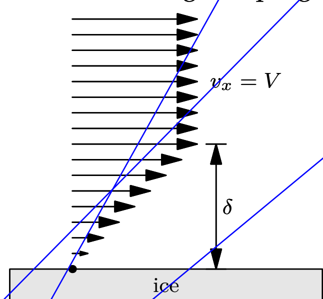
So in general a boundary layer is the space that doesn't reflect the surrounding velocity of air?

I say: "...viscosity. The dimensions of viscosity (L2T-1), along with a bit of dimensional analysis, will help us estimate the thickness. To make..."

where is the dimensional analysis for this? I am a little confused trying to follow what is going on

Now imagine applying the two operations in the reverse order. Taking the viscosity to zero removes the viscous-stress term from the Navier–Stokes equations. Because that term is the only loss term, solving these simplified equations – called the Euler equations – produces a solution with zero drag. The mathematical formulation of the drag paradox is that the solution depends on the order in which one applies the operations of solving and taking a limit.

This paradox, which remained a mystery for over 100 years, was resolved by Prandtl in the early 1900s. The resolution identified the fundamental problem: that taking the viscosity to zero is a *singular limit*. That limit removes the highest-order-derivative from the differential equation, so it changes the equation from a second- to a first-order equation. This qualitative change produces a qualitative change in behavior: from nonzero drag to zero drag. To handle this change properly, Prandtl devised the concept of a boundary layer – which we can understand using lumping.



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Why is the L dimension squared?

I think that's the viscosity v, not velocity.

In case you knew it was viscosity, it makes sense because the area matters when two things rub against each other.

This feels a bit arbitrary.

Now, given that v=velocity is an important parameter, why not just divide by velocity? [nu/v]=L. Why would we guess to include a diameter as well? Or at least, why would we ignore the dimensionless group v*delta/nu?

Im a little confused about the time dimension here.

It is very unclear to me what this time is. Does this only apply to the fluid at the bottom of the boundary layer, or is it the average time that the average fluid involved in the boundary layer takes to get around the object, or something else entirely. I would appreciate a little more explanation about this.

Also, if we were to take the frozen lake example, do we use a T from the time the ice was formed to the time of measurement? But I guess this might make sense, for if you take antarica, a larger time, will decrease the boundary layer, making surface winds very high, which is true...

what time are looking at? what is the reasoning behind choosing r?

Nevermind, I understand now that you want the time it takes to pass over the object.

$$\delta \sim \sqrt{\nu t} \sim \sqrt{\frac{\nu r}{v}}. \tag{7.10}$$

Relative to the size of the object r, the dimensionless boundary-layer thickness δ/r is

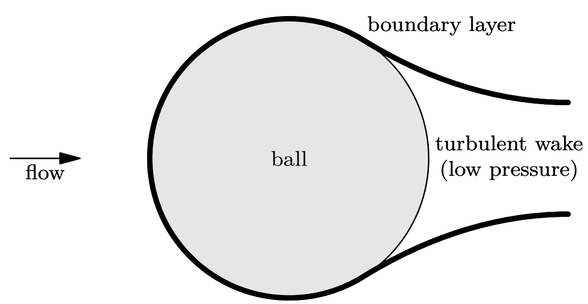
$$\frac{\delta}{r} \sim \sqrt{\frac{\nu}{rv}}. \tag{7.11}$$

The fraction inside the square root looks familiar: It is the reciprocal of the Reynolds number Re! Therefore

$$\frac{\delta}{r} \sim \mathrm{Re}^{-1/2}. \tag{7.12}$$

For most everyday flows, $\mathrm{Re} \gg 1$, so $\mathrm{Re}^{-1/2} \ll 1$. The result is that the boundary layer is a thin layer.

This thin layer will resolve the drag paradox. The intuition is that the boundary layer separates the flow into two regimes: inside and outside the boundary layer. Inside the boundary layer, viscosity has a large effect on the flow. Outside the boundary layer, the flow behaves as if viscosity were zero; there, the flow is described by the Euler equations (by the Navier–Stokes equations without the viscous-stress term). However, the boundary layer does not stick to the object everywhere. Generally, it detaches somewhere on the back of the object. Once the boundary layer detaches, a wake – the region behind the detached layer – is created, and the wake is turbulent. The wake has high-speed and therefore low-pressure flow: Bernoulli's principle says that pressure p and velocity v are related by $p + \rho v^2/2 = \text{constant}$, so high v implies low p. Therefore, the front of the object experiences high pressure and the back experiences low pressure. The result is drag. Intuitively, the drag coefficient is the fraction of the cross-sectional area covered by the turbulent wake.



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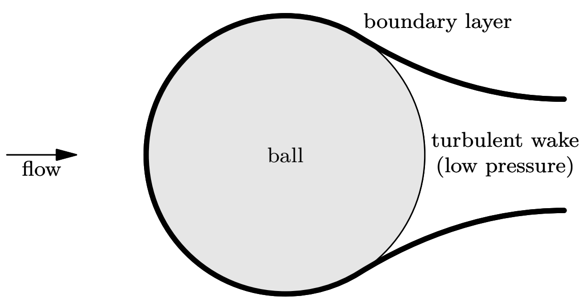

haha, absolutely amazing.

I never would have guessed that the Reynolds number would be so important and useful in all of these applications.

Also, I'm still impressed that we continue to determine all of these things via dimensional analysis.

Totally agree, the dimensional analysis techniques we've learned in this class have helped me across the board in all of my other classes.

I suppose that depends on what major you are. I haven't been able to apply any of this to my other classes. I wish I could...

What major are you that none of this is applicable?

I agree with the earlier comment on not being able to apply any of this material.... All this stuff we've been learning is very interesting, but as a course 6 major, it simply has nothing to do with our curriculum.

I find that hard to believe. Surely, as a course 6 major, you've had to write unit tests before? Divide and conquer, easy cases, and lumping are applicable there.

Perhaps in your future career, as a course 6 major, you will be asked by your client/employer to develop a system capable of handling hundreds or thousands of users. Divide and conquer, along with some lumping, could allow you make a rough estimate as to how scalable your system needs to be.

Perhaps you are right in that you may never have to work with a Reynolds number in your curriculum or career, but it's a way of thinking, not knowledge of this specific example, that this class is trying to develop. :)

As well as all the stuff we did at the beginning with unix. Or the stuff we have been doing with circuitry.

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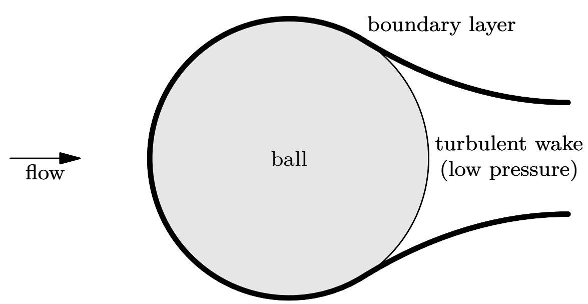

Yeah, I understand this class is trying to develop a "way of thinking"... my point is that, as a course 6 person, this class hasn't taught me anything I didn't already know. If you can't understand a basic concept like D&C in course 6, you're screwed. But that's why I learned it and UNIX and circuitry 3 years ago. Most of the "handy tricks" that he's shown us that aren't taught in other classes are for course 2 majors. So yeah, nothing I've LEARNED in this class has helped me. Because discussing the same drag problem for a month doesn't help course 6 people.

This is awesome. Great movement from the earlier sections to here; and then right into the golf balls. Good section.

What kinds of topics and problems are you doing in the course 6 courses? (And, are you 6-1, 6-2, or 6-3?)

I might like to see this particular problem with the golf ball solved so we can see what a reasonable number is... I know we already know delta and r and could do this ourselves but it would be fast and cool to see the calculation done here to show that this statement is valid.

I intuitively would hope this to be the case.

I would suspect that this would be true with air, but I would be curious to see how large the boundary layer is with objects flowing in water, a very everyday flow. I would guess that it might be quite large.

I would love to see the thickness of this layer calculated, and how it is affected by different changes in viscosity or speed

Oh, that's a good point. Those would be interesting calculations to see for sure.

i'd say "the boundary layer is thin"

..."the boundary layer is a thin layer" sounds funny

how does this resolve the paradox?

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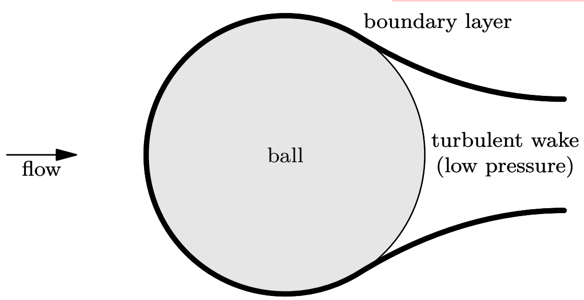

Does the surface friction of the object also play a role here, or is the relationship always no-slip?

It's always a no-slip requirement regardless of material or finish.

I don't really understand this. can you clarify what you mean by 'sticking', and why this happens?

I sort of understand the idea for the ball, but what about for the ice example? Where is the wake?

I agree it's pretty easy to see for the ball, but what about in other situations?

really helpful to have the picture of the flow around the ball, otherwise I might have been more confused.

This is sort of a deceptive use of the term "turbulent." Technically any flow with Re > 2300 is turbulent (not laminar). We're assuming that the flow outside the boundary layer is turbulent, but I think you mean to distinguish the flow in the wake as having vortices.

This is very interesting, and another great intuitive explanation for drag. I like this idea of taking a concept and studying it from a variety of different perspectives throughout the term.

The fraction explanation of the drag coefficient is also very easy to visualize.

I agree.

I enjoy seeing the same concept explained in a variety of similar, yet different, ways. It really helps hammer out the uncertainties and increases understanding.

We just derived Bernoulli's equation in 2.005, and it's great to see it broken down and applied in a different way. Really helps to get a better understanding of the concept

That certainly wasn't my intuition...because I still don't have any about drag. Can you explain this more?

I's really like an extra sentence or 2 to explain this. I think it makes sense in that something with a really turbulent wake also have a really high drag force.

I agree, just thinking about the situation and the idea of a turbulent wake, this makes it all easier to understand.

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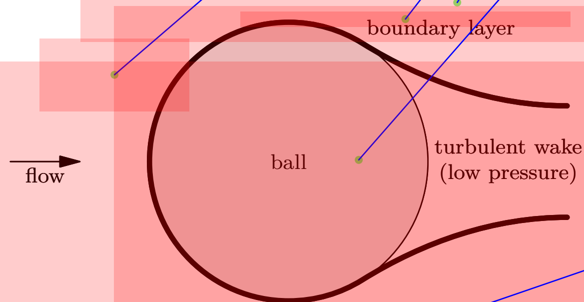

and turbulent is related to a higher reynolds number?...how does that relate to this sentence?

I'm not 100% sure what covering a cross-sectional area with turbulent wake means. It would make more sense to me for it to be the fraction of the _perimeter_ covered by turbulent wake.

how would dimples on a golf ball affect the size of the boundary layer on a golf ball?

to make the diagram perfectly clear, it'd be helpful to have the boundary layer specified as the thick line (arrow + circle or something)

I really like this image. I'm not sure its necessary, but its very clean and dovetails nicely with the previous paragraph

I think it fits well here because it helps visualize where the boundary layer would detach on an object, and how this causes the difference in pressures that are responsible for drag

I think the picture is necessary. Overall, this section was a VERY technical read, and having some images and diagrams definitely appeals to different types of learners. Having the picture here doesn't hurt anyone so it should definitely be kept in.

Yeah, I personally really like this as well. It's definitely clean as you described.

I like the picture, however, could we have a real life example of this? Maybe we could talk about it or see it in class.

This reminds me of the golf ball question from the diagnostic... do the dimples increase boundary layer thickness making the drag lower?

So after we did our diagnostic, I decided to read into it, and found that there actually is a wikipedia article on golf balls:

http://en.wikipedia.org/wiki/Golf_ball

So it seems like golf balls are made with dimples to induce a boundary layer and reduce the drag force.

Helpful diagram.

Looks great and gets the point across perfectly. If it wasnt here I probably would've been trying to draw one in the margins.

That interpretation of the drag coefficient leads to an explanation of why the drag coefficient remains roughly constant as the Reynolds number goes to infinity – in other words, as the flow speed increases or the viscosity decreases. In that limit, the boundary-layer detachment point shifts as far forward as it goes, namely to the widest portion of the object. Then the drag coefficient is roughly 1.

This explanation mostly accounts for the high-Re data on drag coefficient versus Reynolds number. Here is the log–log plot from Section 5.6:

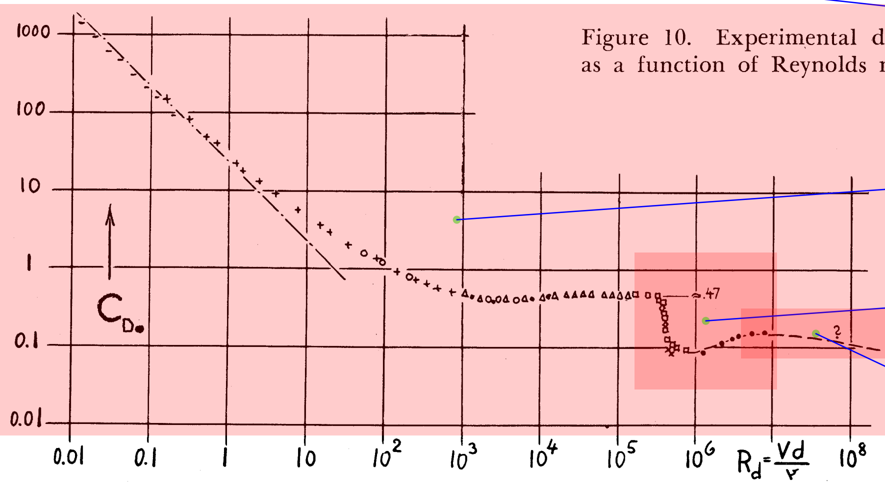

Figure 10. Experimental d
as a function of Reynolds r

The drag coefficient is roughly constant in the Reynolds-number range 10^3 to (almost) 10^6. But why does the drag coefficient drop sharply around $\mathrm{Re} \sim 10^6$? The boundary-layer picture can also help us understand this behavior. To do so, first compute Re_δ, the Reynolds number of the flow in the boundary layer. The Reynolds number is defined as

$$\mathrm{Re} = \frac{\text{typical flow speed} \times \text{distance over which the flow speed varies}}{\text{kinematic viscosity}}. \quad (7.13)$$

In the boundary layer, the flow speed varies from 0 to v, so it is comparable to v. The speed changes over the boundary-layer thickness δ. So

$$\mathrm{Re}_\delta \sim \frac{v\delta}{\nu}. \quad (7.14)$$

Because $\delta \sim r \times \mathrm{Re}^{-1/2}$,

Comments on page 4

where is the lumping?

This makes sense for a sphere. I'm not 100% sure about this being true for more complicated shapes.

I like that you show the plot again here instead of making the reader flip back and find it.

Agreed, definitely convenient for an ebook

What is the rationale for a log-log plot?

I guess this part answers my question earlier.

Why is the graph of Reynolds numbers versus distance supposed to have constant slope under the log-log scale?

So I've already been taught about what happens here physically in previous classes, but every time I see this graph I'm still perplexed by this little dip. I know why it does it, but it just doesn't seem "right" that this is just what happens naturally.

What did your previous classes say about this?

How do we explain this behavior?

I might be wrong, but it looks like it becomes constant again like it did at 10^3 to 10^6, except no fall again at an even higher Reynolds number. Anyone else have thoughts?

I'd still like to see better axis labels here...put them actually outside of the graph.

I find this very frustrating. If the graph is actually a log-log plot then the high end is approximately 2*10^5 ... much closer to 10^5 than 10^6

that often bugs me too, but "closer" could be used here in a geometric (as opposed to arithmetic) sense.

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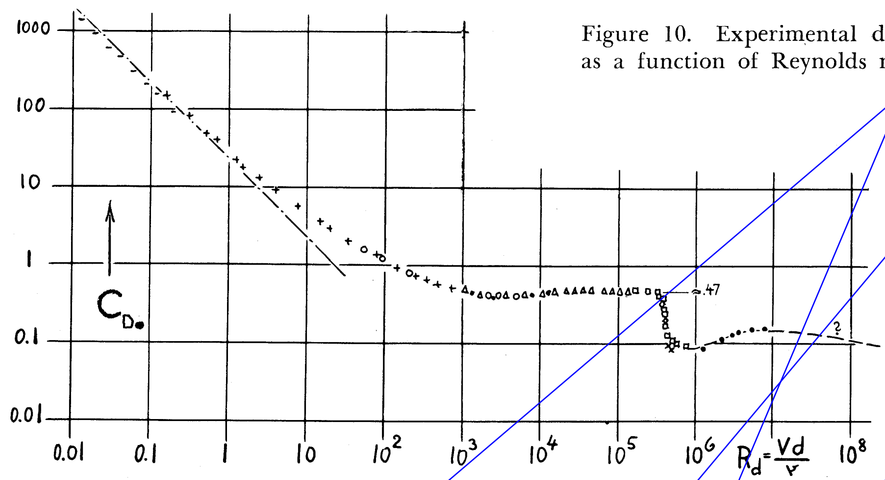

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$$\text{Re}_\delta \sim \frac{v\delta}{\nu}. \quad (7.14)$$

Because $\delta \sim r \times \text{Re}^{-1/2}$,

What does this "distance over which the flow speed varies" mean?

In other words, it's the characteristic length of the object. it would be the diameter of a pipe or golf ball.

Wait, I thought it was the thickness of the boundary layer, which may not be similar to the characteristic length of the object.

In this, are we assuming typical to be the average of the flow speed? would we used the approximated average value of the flow speed?

I like that this is redefined here, it makes for easy reading!

Same sometimes i get what variables mean mixed up because so many have been introduced.

This reintroduction is very helpful and explains the graph above.

If we were being more exact, would we just take the average (ie viscosity/2)? This would still be order of viscosity though.

what do you mean by comparable?

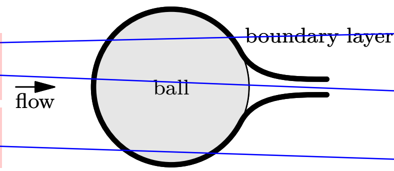
Same order of magnitude, I think.

$$Re_\delta \sim \frac{vr}{\nu} \times Re^{-1/2}. \quad (7.15)$$

The first fraction is just the regular Reynolds number Re, so

$$Re_\delta \sim Re \times Re^{-1/2} = Re^{1/2}. \quad (7.16)$$

Flows become turbulent when $Re \sim 10^3$, so the boundary layer becomes turbulent when $Re_\delta \sim 10^3$ or $Re \sim 10^6$. Hmm! Somehow, the boundary layer's becoming turbulent reduces the drag coefficient. Now recall the interpretation of the drag coefficient as the fraction of the cross-sectional area covered by the turbulent wake. When the boundary layer becomes turbulent, it sticks to the object much better, and detaches only near the back of the object. The result is less drag!



So, to get a low drag coefficient, make the object move fast enough that the Reynolds number is around 10^6. That high a Reynolds number is, however, difficult to achieve with a golf ball. That difficulty is the reason for the dimples on a golf ball. They trip the boundary layer into turbulence at a lower Reynolds number. The golf ball then travels with the benefit of this lower drag coefficient without needing to be hit at an unrealistically high speed.

Comments on page 5

Is this a general fact or did we derive it earlier in a reading? How should I know this?

Is this from the graph, since 10ˆ3 is where the curve becomes constant?

well, this is a general fact you learn in thermo, but it can also be seen in the graph. most of the flows you'll ever see are turbulent.

I feel like I've missed something here...where did this come from?

If the boundary layer becomes turbulent, is it still the boundary layer?

This seems intuitive, doesn't it?

Not to me... I'm pretty confused by it intuitively.

I think its because there is pushing and pulling from different directions, so instead of just leaving a pressure wake behind, now there is some push associated so the drag is reduced

I also don't understand this.

So are we saying that there is some force that goes against the drag force here?

This isn't intuitive to me either. And understanding this would definitely help explain why there is less drag. I understand why chain of things that happen later, but with this missing piece of understanding in the beginning makes it all kind of blurry.

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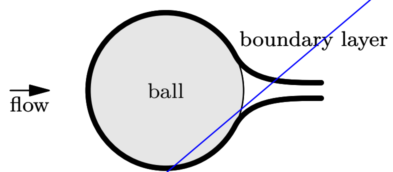

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So, to get a low drag coefficient, make the object move fast enough that the Reynolds number is around 10^6. That high a Reynolds number is, however, difficult to achieve with a golf ball. That difficulty is the reason for the dimples on a golf ball. They trip the boundary layer into turbulence at a lower Reynolds number. The golf ball then travels with the benefit of this lower drag coefficient without needing to be hit at an unrealistically high speed.

How do you know it sticks better and longer?

I was wondering the same thing. Instinctively, I would think that higher turbulence would make the boundary layer more bumpy and come out the back with a larger gap, increasing the fraction of the area covered by turbulent wake. Didn't we learn in middle school/high school that turbulent flow causes more drag than laminar flow?

Is this saying that at a certain point, more turbulence is better than less turbulence?

I am also confused to why this is intuitive. A turbulent boundary layer, would seem to dettach for the object in my mind. Could we define turbulence? Maybe that is where I am going wrong.

The explanation is due to the behavior of random walks and the interpretation of viscosity as diffusion of momentum. I realize now that I should swap this and the next unit, so that we can do random walks first and then apply it to viscosity. But in lecture I'll have a go at filling in the gap.

Why does it 'stick' better?

Yeah, I'm curious about that too.

I would think that more stick would increase drag?

I'm just confused as to how turbulence would stick but reduce drag at the same time. They seem to be somewhat opposite ideas.

Think about the streamlines he drew in class. If the air doesn't stick to the object (following the edge of the object down the backside as well), a lack of air behind the object induces a decrease in pressure behind the object. This pressure different between the front and the back of the ball is what generates air drag.

The picture kind of explains it. Since the fluid sticks, the wake area is much smaller. This is because the surroundings will come around behind the ball staying close to the surface and detaches much later than it would have with less surface turbulence.

As to why it sticks with more turbulence: the more turbulence the more the surroundings slush around a given area. Picture little protrusions along a smooth surface trapping whirlpools of air.

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The first fraction is just the regular Reynolds number Re, so

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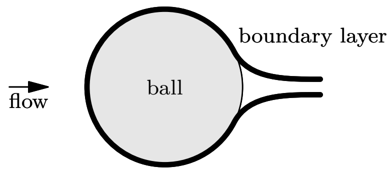

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Does this have anything to do with why throwing a baseball with backspin helps it carry longer in the air? the top and bottom of the ball have different turbulence due to the spin?

> Backspin rotates where the flow separates and generates a vortex around the ball, just as for a wing an angle of attack generates a vortex around the wing. This vortex is the origin of lift. So the backspin generates lift (and topspin generates negative lift).

These results make sense when they are explained this way. This is a cool application.

Because of such a small r, what about other kinds of sports balls, are any others so cleverly engineered? Probably golf is the only sport where you need the ball to go so far. But like, why does a volleyball have stripes while a soccer ball (roughly the same size) has hexagonal pieces?

I think this is an interesting question! Or why a baseball has horseshoe laces or weighs 9oz...

The laces on a baseball definitely have an effect on its flight. Soccer balls and volleyball can also do strange things in the air, usually when they're hit with little spin. I think the surface differences for those may be more due to history, controllability and construction methods.

There are soccer balls that have stripes as well, I think that the design of these balls are less engineered and more cosmetic because of less technical purpose they serve. They do not need to travel as far or be as precise as a golf ball.

Agreed, I think the designs on balls like the soccer ball has more of a historical influence. The golf ball is small and generally needs to travel great distances.

Nice end summary. I am still a little bit confused about the reading, but I think reading it a couple of more times might help.

I am a little confused as well. I feel like most times when we do examples in class, we get to an answer a different way than through normal means. It seems like this was taught very similarly in 2.005.

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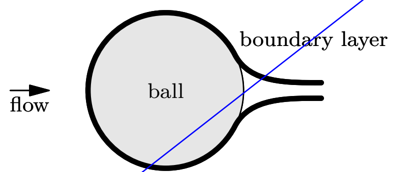

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Do the dimples affect the ball's ability to curve? (i.e. would a non-dimpled ball slice more or less?)

The Magnus effect (the turbulent boundary layer becoming asymmetric on a spinning ball) magnifies the curve of dimpled balls.

Thanks! Any chance you'd like to clarify on why? It seems intuitively that reducing drag would reduce the tendency to curve.

This isn't technical, but I think for the same reason the ball is able to travel farther _forward_ due to the dimples, the sideways motion/ spin is also amplified by the dimples. Curve balls in baseball are also made much more effective by the laces.

a very thorough explanation. makes sense.

Cool.

Hooray! Finally this question is answered!

I wonder, how does the size of the dimples affect the Reynolds number?

I assume that the current dimple size has been thoroughly researched and studied. But would bigger dimples decrease the effectiveness? What about smaller dimples?

Maybe golf balls with dimples that are too small create a boundary layer that is not "turbulent-enough".

Quite interesting. Is this the only reason?

This is the explanation I was looking forward to the most after taking the diagnostic, and I am glad I actually understand it! :D

This was a wonderful tie in to the original problem- which we all still remember. Definitely wish there was more of a follow up on how the dimples were calculated to minimize drag coefficient

This is really cool. I remember this question from the desert island diagnostic pretest and was really confused and curious when reading the question about the golf balls, and now it is finally answered. It's really interesting to see the engineering applications in sports.

I agree and really like this example because it was hardly intuitive when i first came across it. I'm glad we revisited it in this chapter.

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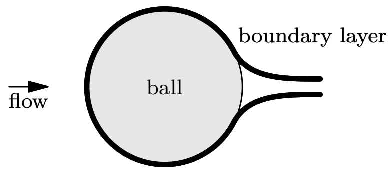

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I love how the questions (or variations of them) from the pre-test are answered in different sections!

How does it do this?

I think I need a bit more explanation as well. But this is really cool. I love the explanation of a nearly-everyday object!

The dimples cause the air around the golf ball to become more displaced. However when the air is interacting within a certain dimple, the Cd is low because of the high contact.

I like the explanation, but am left hanging here, so I also would like to hear a little more on this. Otherwise, good ending to the section.

In lecture I'll show pictures of the airflow with and without a turbulent boundary layer.

I wish you had talked more about this- you built it up so much in class all semester but then its all summed up in a few sentences? it's more interesting then the rest of this passage.. just an idea

Is this because it increases the distance over which the air travels

I think this is a perfect wrap up to things. I like how easily it can be placed in real world design factors.

I agree its always nice to see an actual application after the science behind it is explained.

Doesn't the turbulent flow destroy the boundary layer allowing less friction through mixing or something like that?

another diagram would be useful here (demonstrating changes in boundary layer caused by dimples?)

pretty neat.

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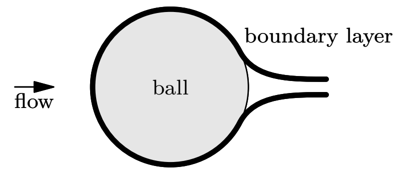

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Are there any other examples of things that fly farther by intentionally creating turbulence?

I think sharks can adjust the roughness of their skin in order to minimize drag – at least, that was one theory of what they were doing (and the basis of a project at Caltech to make low-drag submarines).

Is there any similar application in aviation? I know most fixed wing aircraft would stall in turbulent flow, but what about some rocket propulsion systems? Why don't rockets use this same technique?

Thats a good question. I feel like with more complex systems in aviation, this can be balanced using mechanical systems or electronics. Also the fact that most of those such systems will approach high speeds anyways, so its a little unnecessary.

this is a really nice and clear explanation of the benefits of turbulent flow and drag reduction. again the diagrams are crucial in conveying the idea.

I would just like to refer everyone to the Mythbusters episode where they took a car, covered it in clay, then carved out dimples. It had the same effect there and they showed the flow profile over it. pretty cool stuff, although they didnt go into Re #s for the episode.

I remember that episode. Didn't they not find a reasonable effect for having dimples on a car? They didn't deal with the effect of reynolds number at various speeds. For a golfball the size of a car, it should need less speed to get the same reynolds number since the size of the object increases the reynolds number already. I would think that this would make it easier for them to find an effect. I wonder if the shape of the car being different than that of a ball had an effect on their investigation.

Overall, while I understand that this section was the end of the chapter on lumping and was therefore going to be more advanced, I still found it to be an extremely technical read. It definitely took me a couple times to go over it, and I think it would be helpful for future use if it was less technical-heavy. Granted it's difficult to do that for a technical-oriented class, so I think having simpler explanations would help along the way, even if some of that information is redundant from earlier sections in the chapter.