

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: **how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.**

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

Another thought I had was... I do understand that dividing will eliminate errors.. but if you think about it... having many steps might exponentially increase the chances of error as well?

It seems like this should have been earlier - maybe right after divide and conquer was described

I agree this section would have helped reaffirm the power of divide and conquer and would have been helpful earlier in the course. That said, you'd probably have to move the probability section up with it, which may effect the flow of the course material.

**Read this section for Monday (memo due Monday at 9am). It applies the probability tools to analyze one of the other tools (divide and conquer).**

It doesn't seem like something that could be inaccurate – breaking up a big problem into smaller problems seems like such a classic problem-solving method, how could it be wrong?

I agree. Isn't this the basis of all problem solving techniques? Start with what you know in order to get to an unknown value?

Is there a reason why this section comes way after the introduction of divide-and-conquer is this comparing to the confidence of just throwing a random number out there vs breaking it down using divide and conquer?

i've always been kinda puzzled by this. in divide-and-conquer reasoning, we guess more numbers. if we guess a lot of individual numbers, as is the case for divide and conquer, how does that not decrease our confidence?

I think in general, breaking the problem down into many smaller pieces allows us to reduce our overall error. For example, we may have generated a lot more error in the bandwidth over the atlantic example if we had simply guessed at the number of CDs that could fit on the 747. Instead we break the estimation down into more manageable components like volume/mass that we have more confidence in.

I tend to purposely overestimate some numbers and underestimate others so that the error ends up balancing, I think he's mentioned this too.

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

isn't it kinda of the idea which diversifying your assets? you decreases the variance overall. Kinda like the idea that it's unlikely that all of the assets will have negative gains and so some things will offset others

How well does confidence correlate with accuracy? Sometimes I feel confident about an answer, but it turns out to be way off, and vice versa.

Although this is true. I think confidence is huge in approximating. Talking to the gut!

I concur, especially since this whole process is really just an extremely educated guess, being condifent in your answer, that is, increasing your certainty, is very important.

If this section goes on to discuss estimation error, etc., then I've been waiting for this section for a long time. I think it needs to come much earlier, particularly since we've been entering meaningless ranges into the homework solutions for the whole term.

I agree, it definitely would have been nice if it came earlier.

wait. why are they meaningless ranges? granted, i haven't always gotten the right answer, but when i was confident, i always thought my answers made sense.

Is this almost going to turn into a when to lump and when not to lump to increase accuracy?

I had the same thought. That's obviously a very important step which we have often asked Sanjoy about. Maybe this will lead us to a more quantitative version of following our gut.

I had no idea that telegraphic could mean concise.

I don't think it necessarily means concise but instead visual. This method gives us a way to break down the problem into more small problems that we can better understand.

That may be what the reading is about, but telegraphic does in fact mean concise or terse, and not visual. At least according to Merriam-Webster.

Here, I think it just means to say "in other words" or the "short answer"

I agree, the word seems very strange here

I was thinking this in the first paragraph, that it must be a factor of how many branches we have, but that connecting the branches makes up for many estimations! Good to know I am picking up some of the intuition.

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the **small penalty for combining many quantities**. To explain that telegraphic answer, let's **analyze in slow motion** a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1\text{ cm}^2$  and less than  $10^5\text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1\text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300\text{ cm}^2$  to  $3000\text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59\text{ cm} \times 27.94\text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

So the key idea is making sure that the small division calculation information is reliable. This is interesting because sometimes in divide and conquer we use quantities that are constants but have not really been proved.

That's what I was thinking. The use of "unproven" constants provides us some understanding towards a problem, even if we have little understanding of the overall concepts of the problem.

this is what i was talking about in my last comment! i guess i'd like to see more rigorous proof that the increased certainty in each of those pieces of info does indeed outweigh.

There might not even be a penalty, if we're lucky, since our estimates might balance themselves out.

True, although there are still uncertainties in all of our estimations. I'm hoping we can begin to quantify these errors that we've been minimizing through clever rounding

And the errors stack on top of each other, generally, especially if things get squared, etc.

Hold up, there could be two interpretations of the word "Penalty". He could either be saying that there's a small hassle to pay for having to go through the work of figuring out more quantities in order to be SURE about how we arrive at a number.

Or, he could be saying that there is a penalty in accuracy from introducing more quantities. Although, this second interpretation is what I thought when i read the sentence initially, I now believe the first interpretation to make more sense in this context.

I also read both interpretations when I started, but I think he means the second one since what we were talking about was accuracy

I agree. I think the important idea here is that although we may incur a small penalty in terms of the overall accuracy of our results, we gain in terms of understanding with more certainty the exact amount of inaccuracy we're dealing with. Therefore, we end up knowing how far off we are.

While this is true...isn't there a limit to the extent one could divide a problem into? From stats, we know that the variance in a term increases with the amount of terms we continue to add. Wouldn't the same apply if we divide a problem into many sub divisions?

lol

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer **done**.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

Is your goal to expose people to different types of vocabulary also? If not, I think dropping scarcely used words actually might distract the reader from your actual message. If you want people to increase their vocabulary then this is a fine way to do it.

Yep, and it works out even better when fellow readers define the terms right here!

I agree that it's a good idea to drop scarcely used words...unless you are going to include a dictionary of scarcely used words, most people won't take the time to look up what you mean & just gloss over it w/out knowing what you mean.

?

Maybe he meant to say 'alone'?

In lecture, you talked about how you hoped that there were 2 things that we were supposed to get out of the class: tools for estimating and a better understanding of the world. After all these examples about physics and drag, figuring out the area of a sheet of paper that isn't even really used in this country seems a lot less exciting/relevant.

I'm all for the easy examples! It's a nice break from the more complicated ones about drag.

Agreed! Easy examples are really helpful because we already know about what the example should be so we have something accurate to compare our estimation with.

The physics examples demonstrate how to apply the techniques we've been learning to harder problems like we'd encounter in the real world (scientifically), whereas the easier examples demonstrate how the techniques work and build up our confidence in what we're learning.

I like seeing a broad range of examples from all areas of life because I think it makes it slightly easier to apply approximations to something we haven't encountered before just by opening up our minds to something new. Not everything involves physics, thankfully :P

I agree, I enjoy seeing these different examples. Honestly, all the readings on drag are getting annoying. I like seeing how everything we're learning can be applied to many areas. Also, as a course 6 person, I'm never going to apply any of the information about drag in the real world, so it's just as useful in the real world as the sheet of paper is.

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

That's about 8.3"x 11.7". Isn't this also the legal-sized paper? You could refer to it as that, and people might be more familiar with it.

Whoops, I always thought A4 was 8.5 by 11"... turns out I was wrong :P

Why not just use the standard American paper, if you are teaching it in the US a lot of people will have no experience with European paper and don't know how close these estimations are... although if you said "standard 8 1/2 x 11 sheet of paper that would defeat the purpose. Maybe you could do something besides a piece of paper.

agreed. I have no idea what this size of paper would look like

Maybe because the European paper sizes are all related? A4 is half the size of A3 which is half the size of A2, etc. It is also the international standard of paper, so using 8.5x11 paper is like using imperial units when dealing with science that's done in SI. Also, the ISO standard papers have the nice property such that the ratios of side lengths are  $1:\sqrt{2}$ .

That's a good explanation.

Maybe because he's more familiar with European standards? I do agree that, since this is written for an American audience, the readings should be more targeted.

I like familiar things as examples, new for problem sets.

I think you should consider a diagram giving the size of A4 paper in comparison to paper that your reader is likely to be familiar with. I know I just went to go look it up.

The problem with using US paper is that I know the size exactly (because it's called "8.5 by 11"), and probably everyone else does too, so it's not good for illustrating how to combine uncertainties.

Also, since we know that 8.5x11 is the standard here, there isn't much reason to estimate. You have the dimensions.

I know others have commented on this, but I don't know what A4 is....even w/out saying it's actual area, but just comparing it to the size of an 8.5 x 11 sheet would be helpful.

I realize now you do this in a few paragraphs, maybe just a tad earlier?

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1\text{ cm}^2$  and less than  $10^5\text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1\text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300\text{ cm}^2$  to  $3000\text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59\text{ cm} \times 27.94\text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

Why? If you can estimate (or know off hand) the dimensions it's simple...

yeah i agree, you can just use your hand to get a rough estimate

i think what we can do is a. easily guesstimate the side lengths and b. easily multiply two numbers. but how many of us are actually getting an idea about the \*area\*?

i think what we can do is a. easily guesstimate the side lengths and b. easily multiply two numbers. but how many of us are actually getting an idea about the \*area\*?

I agree... the following sentences prove the difficulty in guessing the area without multiplying dimensions through. It is much harder to guess the number of square cm's than to break it into lengths and widths and multiply....the beauty of divide and conquer.

Haha, this exaggeration helps drive home the point. You could also forgo the exponent and just type out the zero's for added emphasis.

i dont think this is a very useful example as there are many more sensible ways to go about it.

He's just walking us through it. I think maybe something that we could find in our daily lives (volume of an average cup) would be more useful, but he's doing the best he can given that we don't know what A4 looks like

draw

or "I drew"

Is this drawing also on estimation or something that is precisely  $1\text{ cm}^2$

Does it really matter? Most people are familiar with the length of 1cm, so a ballpark drawing of a square does provide some insight on the A4 estimation.

This doesnt seem the most practical way to start

that would be cumbersome tho

that would be cumbersome tho

I agree. This seems like a very inefficient way of going about it.

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1\text{cm}^2$  and less than  $10^5\text{cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1\text{cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300\text{cm}^2$  to  $3000\text{cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59\text{cm} \times 27.94\text{cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

#### This seems pretty arbitrary

This sentence really doesn't tell you anything more than what you've already said. redundancy?

But it does tell you more. We have effectively reduced our range of  $1\text{-}10000\text{cm}^2$  to  $300\text{-}3000\text{cm}^2$ , which is substantially smaller (and more useful) than our first range.

I agree--this isn't exactly redundant. we just narrowed down our range so that now it spans 1 magnitude of 10 instead of 4

I think they're talking about the redundancy of explaining that a "few" = 3. I like it though, 300 to 3000 gives me a better picture in my head for some reason.

#### I like how we can narrow down our range so quickly through a quick visualization

Yeah, this is good and intuitive.

yeah this is a really cool method that I wouldn't have thought of!

#### even with just the above information. i begin thinking about a way to divide and conquer

#### I don't think this is the best word here.

Maybe accurate or...calculated? Useful?

I think it's alright. I've definitely heard British people use it this way before.

again. we come across the question of what the target audience for this book is :)

I often don't know any more which usages are American and which are British. That's maybe what happens when you mix two similar but not identical languages. "Two countries divided by a common language."

#### This sounds awkward, like I should be dividing the area into two parts, not breaking down the components of area.

#### I find this sentence confusing word-wise.

Yeah I think this is actually a grammatical mistake. There's definitely a better way to say this.

I read it a few times to try to get what's going on here.

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1\text{ cm}^2$  and less than  $10^5\text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1\text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300\text{ cm}^2$  to  $3000\text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59\text{ cm} \times 27.94\text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

This is pretty simple.

This is a good point.. I think when asked to estimate the area of something I would go directly to this approach and not even consider the approach discussed in the previous paragraph.

This sentence sounds awkward.

Yea, what?

I'm starting to see how this ties to probability. Sounds a lot like priors.

This definitely makes sense because if I look at a sheet of paper, it is much harder to guess the area of the paper than to just guess its length and/or width.

Is there a general rule for this? Is it cultural or in general will areas be easier to estimate than volumes?

I feel this depends on the situation. Volume deals with an extra dimension, so our accuracy will most likely drop with another estimation of a length. But say you examine a cylinder of water. Depending on size, you could compare the cylinder to 2L bottle of soda instead of trying to find the surface area.

Is it more experience? Or is eyeballing length just more intuitive anyways since area is a factor of lengths?

probably eyeballing lengths is more intuitive. if I think of area, I sort of imagine lengths and multiply them together to imagine the area; I can't just picture some arbitrary sq ft in my head.

I think the explanation and reference here to comfortable measurements is beneficial to understanding how your measurements were made.

I agree, we used this length measurement all semester to understand cross sectional area when looking at drag.

thats funny to think about- but makes so much sense



## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

Hmm..that is a really great way to prove this point!

I agree, it really highlights our intuition and experience.

yeah I agree, really good way to make the reader realize that they know more about lengths than areas.

I think this is not actually a great example, because in this case a person's height is a much more useful value than his cross-sectional area.

I think a better example (although perhaps way to close to the actual problem) would be the area of a computer screen. I know my screen has a 15" diagonal, but no immediate guess about its area (which is arguably the important quantity for content viewing).

Perhaps even better would be: what's the area of a computer desk, or a dining room table, or something similar. In these cases area is (usually) an important factor, but we (or I, at least) have a way better feel for the length scale.

Wouldn't it be funny if we actually measured people in cross sectional area. Growth charts would look a little different lol

agreed, area is not a familiar quantity

I like this - it is a good simple way to break down why divide and conquer seems easier.

what point are you trying to make here? i'm confused about where this is going.

He's trying to explain why divide and conquer makes the problem easier. It's easier to estimate height and width than area because we know those numbers better.

He's justifying why its easier to estimate a length as opposed to an area.

How we think (to find area using length and height rather than just guessing area) is more intuitive, so this is how we should go about estimating an unknown when the quantity we are looking for is area

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

Very true, although not something I think about too much. I like the strategy of using estimations we are accustomed to.

I think though, this strategy is hardly worth mentioning, since it is instinctive already for us to resort to familiar estimations than unfamiliar ones. Stating the obvious?

Obvious to some yes, but unlikely obvious to everyone.

It also makes sense to include it in our current discussion as it is a good example.

Yeah, I don't think it's the strategy he's emphasizing here—we learned divide and conquer in our first unit. here he's talking about why divide and conquer produces more accurate results than straight guessing—because we change the problem to use values we are more familiar with

Yeah I think it's really interesting to think about this. It wasn't so obvious to me before, but now it makes complete sense.

I agree it's instinctive, but that most likely means we thought about why we did it less.

I think this statement is interesting, but I'd like to see something to back it up. (I feel like it's intuitive, but it would still be nice to see just by how much the difference is, or how many people, etc)

I think it would be important to note the relative accuracy of knowing each of these quantities. Saying something along the lines of 'we are far more accurate estimating the length and the width than the area, granted the higher variance in our answer for estimating two quantities instead of one.'

Doesn't this fact kind of nullify the process of approximating? It's kinda like looking up an answer to a similar problem....

Yeah I saw this and I was like...huh? Isn't that the answer to our question?

No, this isn't saying that A4 IS letter pater, it's saying that it's the equivalent standard for them. They are different sizes.

Exactly - its in the same class of paper and therefore a good place to start.

I don't really understand the point of this paragraph. It gives too many facts that seem somewhat irrelevant to our point.

## 8.2 Why divide and conquer works

How does divide-and-conquer reasoning (Chapter 1) produce such accurate estimates? Alas, this problem is hard to analyze directly because we do not know the accuracy in advance. But we can analyze a related problem: how divide-and-conquer reasoning increases our confidence in an estimate or, more precisely, decreases our uncertainty.

The telegraphic answer is that it works by subdividing a quantity about which we know little into several quantities about which we know more. Even if we need many subdivisions before we reach reliable information, the increased certainty outweighs the small penalty for combining many quantities. To explain that telegraphic answer, let's analyze in slow motion a short estimation problem using divide-and-conquer done.

### 8.2.1 Area of a sheet of paper

The slow-motion problem is to estimate area of a sheet of A4 paper (A4 is the standard sheet size in Europe). On first thought, even looking at a sheet I have little idea about its area. On second thought, I know something. For example, the area is more than  $1 \text{ cm}^2$  and less than  $10^5 \text{ cm}^2$ . That wide range makes it hard to be wrong but is also too wide to be useful. To narrow the range, I'll draw a small square with an area of roughly  $1 \text{ cm}^2$  and guess how many squares fit on the sheet: probably at least a few hundred and at most a few thousand. Turning 'few' into 3, I offer  $300 \text{ cm}^2$  to  $3000 \text{ cm}^2$  as a plausible range for the area.

Now let's use divide-and-conquer and get a more studied range. Subdivide the area into the width and height; about two quantities my knowledge is more precise than it is about area itself. The extra precision has a general reason and a reason specific to this problem. The general reason is that we have more experience with lengths than areas: Which is the more familiar quantity, your height or your cross-sectional area? Therefore, our length estimates are usually more accurate than our area estimates.

The reason specific to this problem is that A4 paper is the European equivalent of standard American paper. American paper is known to computers and laser printers as 'letter' paper and known commonly in the United States as 'eight-and-a-half by eleven' (inches!). In metric units, those dimensions are  $21.59 \text{ cm} \times 27.94 \text{ cm}$ . If A4 paper were identical to letter paper, I could now compute its exact area. However, A4 paper is, I

my computer used to have a chinese windows os and run chinese word, and i basically had these numbers memorized so i could get my pages to print correctly. just a fun fact! :)

This sentence reads awkwardly

remember from living in England, slightly thinner and longer than letter paper. I forget the exact differences between the dimensions of A4 and letter paper, hence the remaining uncertainty: I'll guess that the width lies in the range 19...21 cm and the length lies in the range 28...32 cm.

The next problem is to combine the plausible ranges for the height and width into the plausible range for the area. A first guess, because the area is the product of the width and height, is to multiply the endpoints of the width and height ranges:

$$A_{\min} = 19 \text{ cm} \times 28 \text{ cm} = 532 \text{ cm}^2;$$

$$A_{\max} = 21 \text{ cm} \times 32 \text{ cm} = 672 \text{ cm}^2.$$

This method turns out to overestimate the range – a mistake that I correct later – but even the too-large range spans only a factor of 1.26 whereas the starting range of 300...3000 cm<sup>2</sup> spans a factor of 10. Divide and conquer has significantly narrowed the range by replacing quantities about which we have little knowledge, such as the area, with quantities about which we have more knowledge.

The second bonus, which I now quantify correctly, is that subdividing into many quantities carries only a small penalty, smaller than suggested by naively multiplying endpoints. The naive method overestimates the range because it assumes the worst. To see how, imagine an extreme case: estimating a quantity that is the product of ten factors, each that you know to within a factor of 2 (in other words, each plausible range is a factor of 4). Is your plausible range for the final quantity a factor of  $4^{10} \approx 10^6$ ?! That conclusion is terribly pessimistic. A more likely result is that a few of the ten estimates will be too large and a few too small, allow several errors to cancel.

To quantify and fix this pessimism, I will explain plausible ranges using probabilities. Probabilities are the tool for this purpose. As discussed in Section 8.1, probabilities reflect incomplete knowledge; they are *not* frequencies in a random experiment (Jaynes's *Probability Theory: The Logic of Science* [11] is an excellent, book-length discussion and application of this fundamental point).

To make a probabilistic description, start with the proposition or hypothesis

$$H \equiv \text{The area of A4 lies in the range } 300 \dots 3000 \text{ cm}^2.$$

**For someone who has never seen or heard of A4 paper before, though, I would have no idea how to go about estimating its size.**

Agreed. I think this description of the paper would be more useful when A4 paper is first introduced. While I still don't know what it looks like, I now have an idea of it, since its described as being slightly thinner and longer.

**Couldn't we just say these probably cancel out?**

It's *\_narrower\_*, not thinner, right?

Haha yeah, I started thinking that he was referring to thickness too and wondered why that was pertinent to anything

**is it like legal paper?**

**yeah when I first arrived in the US, I didn't realize that the paper was different but then I went home and brought my notebooks with me I noticed that they were really different!**

**seems like these two would cancel out...so you could just compute the area of letter paper and it would probably be the same as A4 area.**

**I used to live where A4 papers are used. It was weird when I moved here.**

**i really like what you're trying to do in this section, but it reads oddly to me because it sounds like sharing your own experiences. it's all based on what you happened to remember from living abroad (longer and thinner) and what you don't remember (the exact dimensions).**

While we certainly don't share these exact experiences, I think it's helpful to see how we can apply our experience and intuition, even if it's not specific, to the problem at hand.

**i like this use of ranges, I wish we had learned this earlier. I never used the range entry boxes in the psets**

**Also, you multiply the uncertainty of length and width, to get an even bigger uncertainty in the area.**

I think that the purpose of the range is to account for uncertainty. Instead of choosing 20 and 30 he chose numbers over a range to allow for the chance that he's off by a couple of centimeters.

remember from living in England, slightly thinner and longer than letter paper. I forget the exact differences between the dimensions of A4 and letter paper, hence the remaining uncertainty: I'll guess that the width lies in the range 19...21 cm and the length lies in the range 28...32 cm.

The next problem is to combine the plausible ranges for the height and width into the plausible range for the area. A first guess, because the area is the product of the width and height, is to multiply the endpoints of the width and height ranges:

$$A_{\min} = 19 \text{ cm} \times 28 \text{ cm} = 532 \text{ cm}^2;$$

$$A_{\max} = 21 \text{ cm} \times 32 \text{ cm} = 672 \text{ cm}^2.$$

This method turns out to overestimate the range – a mistake that I correct later – but even the too-large range spans only a factor of 1.26 whereas the starting range of 300...3000 cm<sup>2</sup> spans a factor of 10. Divide and conquer has significantly narrowed the range by replacing quantities about which we have little knowledge, such as the area, with quantities about which we have more knowledge.

The second bonus, which I now quantify correctly, is that subdividing into many quantities carries only a small penalty, smaller than suggested by naively multiplying endpoints. The naive method overestimates the range because it assumes the worst. To see how, imagine an extreme case: estimating a quantity that is the product of ten factors, each that you know to within a factor of 2 (in other words, each plausible range is a factor of 4). Is your plausible range for the final quantity a factor of 4<sup>10</sup> ≈ 10<sup>6</sup>?! That conclusion is terribly pessimistic. A more likely result is that a few of the ten estimates will be too large and a few too small, allow several errors to cancel.

To quantify and fix this pessimism, I will explain plausible ranges using probabilities. Probabilities are the tool for this purpose. As discussed in Section 8.1, probabilities reflect incomplete knowledge; they are *not* frequencies in a random experiment (Jaynes's *Probability Theory: The Logic of Science* [11] is an excellent, book-length discussion and application of this fundamental point).

To make a probabilistic description, start with the proposition or hypothesis

$$H \equiv \text{The area of A4 lies in the range } 300 \dots 3000 \text{ cm}^2.$$

are you saying both Amin and Amax overestimate the range or just Amax

I think it means overestimate the range in the full answer 532cm<sup>2</sup> - 672cm<sup>2</sup>. Each individual answer is an underestimate or overestimate respectively, and together they overestimate the range.

That's right – the range is too wide on both ends. In other words, you know more than you think you do based on that wide range.

Why would it overestimate the range?

Because we multiplied the smallest width and height to get the bottom range and the largest width and height to get the upper.

Yup. Realistically its not going to be super-skinny and super-tall, it would either be super-skinny medium tall, or super-tally medium skinny.

Nice way of explaining the reason for overestimation... thanks!

It makes sense that dividing will decrease chances of error.

I think this is a powerful statement with everyday problems. Estimations require order of magnitude guesses but usually our intuition is much better

that really is an amazing improvement

was this really divide and conquer? in both cases you 'divided' the problem into something you knew.

i thought that was the idea of divide and conquer.

That's the definition of divide and conquer though, isn't it? You divide something into something you know how to deal with.

Is narrowing the range so that it does not include the actual answer really beneficial though?

Yes we have knowledge about the hieght and length, but I had no knowledge about the relationship between A4 and 8X11

remember from living in England, slightly thinner and longer than letter paper. I forget the exact differences between the dimensions of A4 and letter paper, hence the remaining uncertainty: I'll guess that the width lies in the range 19...21 cm and the length lies in the range 28...32 cm.

The next problem is to combine the plausible ranges for the height and width into the plausible range for the area. A first guess, because the area is the product of the width and height, is to multiply the endpoints of the width and height ranges:

$$A_{\min} = 19 \text{ cm} \times 28 \text{ cm} = 532 \text{ cm}^2;$$

$$A_{\max} = 21 \text{ cm} \times 32 \text{ cm} = 672 \text{ cm}^2.$$

This method turns out to overestimate the range – a mistake that I correct later – but even the too-large range spans only a factor of 1.26 whereas the starting range of 300...3000 cm<sup>2</sup> spans a factor of 10. Divide and conquer has significantly narrowed the range by replacing quantities about which we have little knowledge, such as the area, with quantities about which we have more knowledge.

The second bonus, which I now quantify correctly, is that subdividing into many quantities carries only a small penalty, smaller than suggested by naively multiplying endpoints. The naive method overestimates the range because it assumes the worst. To see how, imagine an extreme case: estimating a quantity that is the product of ten factors, each that you know to within a factor of 2 (in other words, each plausible range is a factor of 4). Is your plausible range for the final quantity a factor of  $4^{10} \approx 10^6$ ?! That conclusion is terribly pessimistic. A more likely result is that a few of the ten estimates will be too large and a few too small, allow several errors to cancel.

To quantify and fix this pessimism, I will explain plausible ranges using probabilities. Probabilities are the tool for this purpose. As discussed in Section 8.1, probabilities reflect incomplete knowledge; they are *not* frequencies in a random experiment (Jaynes's *Probability Theory: The Logic of Science* [11] is an excellent, book-length discussion and application of this fundamental point).

To make a probabilistic description, start with the proposition or hypothesis

$$H \equiv \text{The area of A4 lies in the range } 300 \dots 3000 \text{ cm}^2.$$

**So divide and conquer is really divide until you know a quantity then combine!**

the combining is the conquering part

**This is the evidence of our probability unit, isn't it? This is slowly making a lot of sense in terms of the new unit we're starting.**

**If each of the smaller sections contains smaller penalties, wouldn't they combine to result in a similar level of penalty?**

**I like this example...after rereading...it helps quantify why more estimates should reduce the certainty.**

**Typo: allowing.**

remember from living in England, slightly thinner and longer than letter paper. I forget the exact differences between the dimensions of A4 and letter paper, hence the remaining uncertainty: I'll guess that the width lies in the range 19...21 cm and the length lies in the range 28...32 cm.

The next problem is to combine the plausible ranges for the height and width into the plausible range for the area. A first guess, because the area is the product of the width and height, is to multiply the endpoints of the width and height ranges:

$$A_{\min} = 19 \text{ cm} \times 28 \text{ cm} = 532 \text{ cm}^2;$$

$$A_{\max} = 21 \text{ cm} \times 32 \text{ cm} = 672 \text{ cm}^2.$$

This method turns out to overestimate the range – a mistake that I correct later – but even the too-large range spans only a factor of 1.26 whereas the starting range of 300...3000 cm<sup>2</sup> spans a factor of 10. Divide and conquer has significantly narrowed the range by replacing quantities about which we have little knowledge, such as the area, with quantities about which we have more knowledge.

The second bonus, which I now quantify correctly, is that subdividing into many quantities carries only a small penalty, smaller than suggested by naively multiplying endpoints. The naive method overestimates the range because it assumes the worst. To see how, imagine an extreme case: estimating a quantity that is the product of ten factors, each that you know to within a factor of 2 (in other words, each plausible range is a factor of 4). Is your plausible range for the final quantity a factor of  $4^{10} \approx 10^6$ ?! That conclusion is terribly pessimistic. A more likely result is that a few of the ten estimates will be too large and a few too small, allow several errors to cancel.

To quantify and fix this pessimism, I will explain plausible ranges using probabilities. Probabilities are the tool for this purpose. As discussed in Section 8.1, probabilities reflect incomplete knowledge; they are *not* frequencies in a random experiment (Jaynes's *Probability Theory: The Logic of Science* [11] is an excellent, book-length discussion and application of this fundamental point).

To make a probabilistic description, start with the proposition or hypothesis

$$H \equiv \text{The area of A4 lies in the range } 300 \dots 3000 \text{ cm}^2.$$

**Is this something we have been relying on all semester? I really like this analysis**

I remember using this for the very first pset...for example with the imported oil in the US, we knew that if we over estimated cars, we might want to fudge another factor. Though this reading brings up the great point that when you have more than a few factors that are estimated, there is no need to consciously fudge! It is more likely that it will just work out!

Yeah I feel like if you go with what you think is right everything tends to work out in the end :-). Of course, if your errors keep stacking, that's not a great place to be...

If you knew you overestimated cars then why wouldn't you lower your estimate instead of fudging another factor?

I completely agree with 5:07. This allowing uncertainties to cancel only works if the errors are, in fact, unknown. If we know that an error is off by a factor of 2 in a particular direction, we can't expect the other errors to cancel it out. (This is vaguely reminiscent of martingales, I feel, since we can approximate each estimation as having an expected error factor of 1.)

The variance increases, no doubt. But this assumption relies heavily on probabilities and expected values.

In response to 5:07: If I remember correctly, we used a high number for the number of cars because it was a round number and our estimated number of cars wasn't round. We then fudged the next calculation to a lower round number to make up for it. It makes the calculations easier if you use round numbers so you fudge up and down and balance it so that they cancel.

Answered my earlier question, I feel like this is something that should have been introduced much earlier, it is really valuable

**I was wondering when this was going to come into play!**

**ooh nice transition into this unit**

remember from living in England, slightly thinner and longer than letter paper. I forget the exact differences between the dimensions of A4 and letter paper, hence the remaining uncertainty: I'll guess that the width lies in the range 19...21 cm and the length lies in the range 28...32 cm.

The next problem is to combine the plausible ranges for the height and width into the plausible range for the area. A first guess, because the area is the product of the width and height, is to multiply the endpoints of the width and height ranges:

$$A_{\min} = 19 \text{ cm} \times 28 \text{ cm} = 532 \text{ cm}^2;$$

$$A_{\max} = 21 \text{ cm} \times 32 \text{ cm} = 672 \text{ cm}^2.$$

This method turns out to overestimate the range – a mistake that I correct later – but even the too-large range spans only a factor of 1.26 whereas the starting range of 300...3000 cm<sup>2</sup> spans a factor of 10. Divide and conquer has significantly narrowed the range by replacing quantities about which we have little knowledge, such as the area, with quantities about which we have more knowledge.

The second bonus, which I now quantify correctly, is that subdividing into many quantities carries only a small penalty, smaller than suggested by naively multiplying endpoints. The naive method overestimates the range because it assumes the worst. To see how, imagine an extreme case: estimating a quantity that is the product of ten factors, each that you know to within a factor of 2 (in other words, each plausible range is a factor of 4). Is your plausible range for the final quantity a factor of  $4^{10} \approx 10^6$ ?! That conclusion is terribly pessimistic. A more likely result is that a few of the ten estimates will be too large and a few too small, allow several errors to cancel.

To quantify and fix this pessimism, I will explain plausible ranges using probabilities. Probabilities are the tool for this purpose. As discussed in Section 8.1, probabilities reflect incomplete knowledge; they are *not* frequencies in a random experiment (Jaynes's *Probability Theory: The Logic of Science* [11] is an excellent, book-length discussion and application of this fundamental point).

To make a probabilistic description, start with the proposition or hypothesis

$$H \equiv \text{The area of A4 lies in the range } 300 \dots 3000 \text{ cm}^2.$$

**I feel like I often make this mistake. I guess because there is a fine line between what we know and what has been done experimentally. Also, the way we're taught probability focuses on random experiment.**

This is a hard concept to grasp; Thanks for the book note here; I think I need to read it, because like the above comment, I am consistently stuck in the random experiment mindset.

Here's an example, which I'll discuss more in lecture, that shows why the random-experiment interpretation is lacking. What is the probability that the trillionth (or ten trillionth) digit of pi is a 7? There's only one pi, so there's no way to do the experiment, and it isn't random at all. But you can still give a reasonable answer for the probability.

**i don't get this phrase**

I think he's saying that probability doesn't just describe the frequency of some measured value in an experiment, but rather describes our state of knowledge. In other words, probabilities aren't just about the number of times a coin lands on heads if you flip it x number of times. But I agree that the use of the term frequencies here threw me off a bit at first.

yeah, I was confused by it also but your explanation really helped.

**This needs more explaining**

**an**

nice catch, i just glanced over it assuming it was already "an"



and information (or evidence)

$I \equiv$  What I know about the area *before* using divide and conquer.

Now I want to find the conditional probability  $P(H|I)$  – namely, the probability of  $H$  given my knowledge before trying divide and conquer. There is no known algorithm for computing a probability in such a complicated problem situation. How, for example, does one represent my state of knowledge? In these cases, the best we can do is to introspect or, in plain English, to talk to our gut.

My gut is the organ with the most access to my intuitive knowledge and its incompleteness, and it tells me that I would feel mild surprise but not shock if I learned that the true area lay outside the range  $300 \dots 3000 \text{ cm}^2$ . The surprise suggests that  $P(H|I)$  is larger than  $1/2$ . The mildness of the surprise suggests that  $P(H|I)$  is not much larger than  $1/2$ . I'll quantify it as  $P(H|I) = 2/3$ : I would give 2-to-1 odds that the true area is within the plausible range.

Furthermore, I'll use this probability or odds to define a plausible range: It is the range for which I think 2-to-1 odds is fair. I could have used a 1-to-1 odds range instead, but the 2-to-1 odds range will later help give plausible ranges an intuitive interpretation (as a region on a log-normal distribution). That interpretation will then help quantify how to combine plausible ranges.

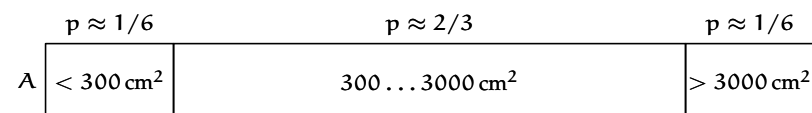
For the moment, I need only the idea that the plausible range contains roughly  $2/3$  of the probability. With a further assumption of symmetry, the plausible range  $300 \dots 3000 \text{ cm}^2$  represents the following probabilities:

$$P(A < 300 \text{ cm}^2) = 1/6;$$

$$P(300 \text{ cm}^2 \leq A \leq 3000 \text{ cm}^2) = 2/3;$$

$$P(A > 3000 \text{ cm}^2) = 1/6.$$

Here is the corresponding picture with width proportional to probability:



For the height  $h$  and width  $w$ , after doing divide and conquer and using the similarity between A4 and letter paper, the plausible ranges are

**Love how a simple notion can be so easily represented by variables!**

**Clever. I really like and understand how you have phrased this conditional probability**

I like that you show how different estimating methods are somewhat related. Of course, this would explain why this is in Section 8 instead of Section 1.

**I think that this section should have come before the last one. I think it is an easier introduction to probability.**

I agree.

**Yeah i was wondering about this too. We don't necessarily have finite states that we have probabilities about. I guess we could use continuous random variables?...**

I wasn't quite sure about this, but read on and you'll see how he approximates it.

**so does this really help us out in the end?**

But can introspection give us an actual probability? I guess its almost what we've been doing on the problem sets with our ranges, but even then thats almost just guessing at how confident we are arbitrarily.

I think he is saying that probability is a way of explaining (in an objective and explicit manner) what we know and what we don't know. Talking to our gut (introspecting) is supposed to help us gain a better understanding of what we don't know.

**This was my first worry at the start of this unit. I didn't think there could possibly be ways to quantify some of the reasoning we've been using, and thus, probability seemed like a waste of time. But now I see that we're using probability in a different and more useful way to enhance our methods.**

**my gut is confused and quiet! :(**

and information (or evidence)

$I \equiv$  What I know about the area *before* using divide and conquer.

Now I want to find the conditional probability  $P(H|I)$  – namely, the probability of  $H$  given my knowledge before trying divide and conquer. There is no known algorithm for computing a probability in such a complicated problem situation. How, for example, does one represent my state of knowledge? In these cases, the best we can do is to introspect or, in plain English, to talk to our gut.

My gut is the organ with the most access to my intuitive knowledge and its incompleteness, and it tells me that I would feel mild surprise but not shock if I learned that the true area lay outside the range  $300 \dots 3000 \text{ cm}^2$ . The surprise suggests that  $P(H|I)$  is larger than  $1/2$ . The mildness of the surprise suggests that  $P(H|I)$  is not much larger than  $1/2$ . I'll quantify it as  $P(H|I) = 2/3$ : I would give 2-to-1 odds that the true area is within the plausible range.

Furthermore, I'll use this probability or odds to define a plausible range: It is the range for which I think 2-to-1 odds is fair. I could have used a 1-to-1 odds range instead, but the 2-to-1 odds range will later help give plausible ranges an intuitive interpretation (as a region on a log-normal distribution). That interpretation will then help quantify how to combine plausible ranges.

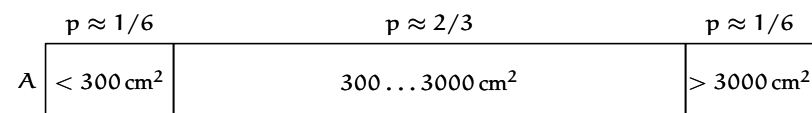
For the moment, I need only the idea that the plausible range contains roughly  $2/3$  of the probability. With a further assumption of symmetry, the plausible range  $300 \dots 3000 \text{ cm}^2$  represents the following probabilities:

$$P(A < 300 \text{ cm}^2) = 1/6;$$

$$P(300 \text{ cm} \leq A \leq 3000 \text{ cm}^2) = 2/3;$$

$$P(A > 3000 \text{ cm}^2) = 1/6.$$

Here is the corresponding picture with width proportional to probability:



For the height  $h$  and width  $w$ , after doing divide and conquer and using the similarity between A4 and letter paper, the plausible ranges are

**I really enjoyed the class lecture about gut feelings and intuition, it would be awesome to see a chapter in this book dedicated to that idea.**

I agree, I have found that having confidence in my gut and looking at things the right way is the most important part of figuring out many of these problems.

I too agree, but sometimes I wonder if I'm trusting my gut too much in some of our examples where I don't have a lot of natural intuition. Having some of these ideas as a chapter/section of the book would be helpful.

Regardless of whether it gets a whole chapter or just a mention such as this, I feel like it would be beneficial to the reader if it was mentioned earlier on, as early as you mentioned it in class.

**This seems a little ironic. In this method, you are going to find the information based on what you know, but in the probabilistic description, you are going with your gut. This doesn't seem consistent.**

**This seems like kind of a ridiculous argument**

I think it's a pretty interesting way to think about probability—how surprised would you be if your guess was wrong? as opposed to—on a scale of 0 to 1 how right do you think you are...hm interesting

for "ridiculous": Are you talking about the use of the metaphor of a guessing intestine or complaining about something more substantial? Having confidence intervals are important for gauging how to proceed, and this range is already based on some reasoning.

This is a nice paragraph. I like the idea of actually having to 'talk to the gut' in order to get an good estimate. There are times where you can just simply say "I know it's at least this" even with no basis because of the experiences I had, hence the training my 'gut' has.

Don't think of it as an argument. That is, don't think of it as a proof for the value of  $P(H|I)$ . Rather, it's a measurement of my own (subjective) value of  $P(H|I)$ . You can use the same instrument to measure your own  $P(H|I)$ .

**Sounds like a handy way to think about probability when you need to problem solve without references.**

and information (or evidence)

$I \equiv$  What I know about the area *before* using divide and conquer.

Now I want to find the conditional probability  $P(H|I)$  – namely, the probability of  $H$  given my knowledge before trying divide and conquer. There is no known algorithm for computing a probability in such a complicated problem situation. How, for example, does one represent my state of knowledge? In these cases, the best we can do is to introspect or, in plain English, to talk to our gut.

My gut is the organ with the most access to my intuitive knowledge and its incompleteness, and it tells me that I would feel mild surprise but not shock if I learned that the true area lay outside the range  $300 \dots 3000 \text{ cm}^2$ . The surprise suggests that  $P(H|I)$  is larger than  $1/2$ . The mildness of the surprise suggests that  $P(H|I)$  is not much larger than  $1/2$ . I'll quantify it as  $P(H|I) = 2/3$ : I would give 2-to-1 odds that the true area is within the plausible range.

Furthermore, I'll use this probability or odds to define a plausible range: It is the range for which I think 2-to-1 odds is fair. I could have used a 1-to-1 odds range instead, but the 2-to-1 odds range will later help give plausible ranges an intuitive interpretation (as a region on a log-normal distribution). That interpretation will then help quantify how to combine plausible ranges.

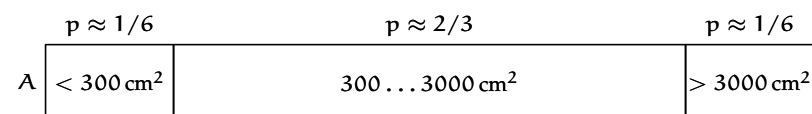
For the moment, I need only the idea that the plausible range contains roughly  $2/3$  of the probability. With a further assumption of symmetry, the plausible range  $300 \dots 3000 \text{ cm}^2$  represents the following probabilities:

$$P(A < 300 \text{ cm}^2) = 1/6;$$

$$P(300 \text{ cm} \leq A \leq 3000 \text{ cm}^2) = 2/3;$$

$$P(A > 3000 \text{ cm}^2) = 1/6.$$

Here is the corresponding picture with width proportional to probability:



For the height  $h$  and width  $w$ , after doing divide and conquer and using the similarity between A4 and letter paper, the plausible ranges are

This sounds like what I do when assigning weighted numbers for guessing on our homework sets...

That IS what we do in our homework!

I understand what you're saying here, but "talking to your gut" regarding probability can quickly lead you down a slippery slope.

I agree, in probability classes they usually tell you to not go with your gut or first instinct.

True, but you are quantifying your gut in this case, so you are going by how much you believe in your gut.

This probabilistic reasoning makes me uneasy. Many people are unlikely to intuitively understand probabilities unless they've seen them many times before. Not sure I would count on my gut in these situations.

Yeah. I also don't see how that leads to a probability of  $1/2$ . Is this a general rule of thumb to use a half as a probability for a range that we're pretty sure about.

Yeah, I'm also feeling uneasy about using your gut for probability. I'm sure the professor can do that just fine and be reasonably correct since he's done this for so long, but we've only started doing this, and I don't feel like my "gut" is developed enough to be able to approximate probabilities well.

I agree, I'm not sure where the  $1/2$  came from and I think saying 50% is more intuitive for most people than a probability of  $1/2$

this is interesting and useful

This section sums up a nice confidence interval for how reasonable we think our estimates are, but I don't see how this is going to improve our approximations

I'm still not as comfortable with the "odds" notation as I am with normal fractions or percentages to represent probabilities so I like that you wrote it both ways here.

I find this odds ratio a bit harder to use intuitively than something like 1:1 or 9:1.

wait, I am confused as to where this  $2/3$  came from

I think we basically just decided that it felt about right.

and information (or evidence)

$I \equiv$  What I know about the area *before* using divide and conquer.

Now I want to find the conditional probability  $P(H|I)$  – namely, the probability of  $H$  given my knowledge before trying divide and conquer. There is no known algorithm for computing a probability in such a complicated problem situation. How, for example, does one represent my state of knowledge? In these cases, the best we can do is to introspect or, in plain English, to talk to our gut.

My gut is the organ with the most access to my intuitive knowledge and its incompleteness, and it tells me that I would feel mild surprise but not shock if I learned that the true area lay outside the range  $300 \dots 3000 \text{ cm}^2$ . The surprise suggests that  $P(H|I)$  is larger than  $1/2$ . The mildness of the surprise suggests that  $P(H|I)$  is not much larger than  $1/2$ . I'll quantify it as  $P(H|I) = 2/3$ : I would give 2-to-1 odds that the true area is within the plausible range.

Furthermore, I'll use this probability or odds to define a plausible range: It is the range for which I think 2-to-1 odds is fair. I could have used a 1-to-1 odds range instead, but the 2-to-1 odds range will later help give plausible ranges an intuitive interpretation (as a region on a log-normal distribution). That interpretation will then help quantify how to combine plausible ranges.

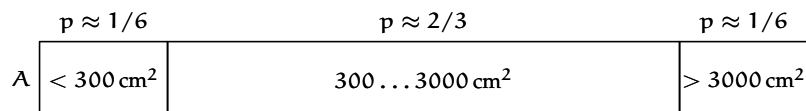
For the moment, I need only the idea that the plausible range contains roughly  $2/3$  of the probability. With a further assumption of symmetry, the plausible range  $300 \dots 3000 \text{ cm}^2$  represents the following probabilities:

$$P(A < 300 \text{ cm}^2) = 1/6;$$

$$P(300 \text{ cm} \leq A \leq 3000 \text{ cm}^2) = 2/3;$$

$$P(A > 3000 \text{ cm}^2) = 1/6.$$

Here is the corresponding picture with width proportional to probability:



For the height  $h$  and width  $w$ , after doing divide and conquer and using the similarity between A4 and letter paper, the plausible ranges are

what do you mean about these odd's? 2-1 vs 1-1?

yeah, like I said earlier, i think speaking in terms of percentages would make it much easier to follow the probabilities that you are talking about.

Why?

This paragraph is a bit dense. Maybe you could just say that Choosing 2-1 instead of another odds ratio leads to easier interpretation...

I agree. I think that breaking down the second sentence would help a lot, it's currently kind of long and a bit haphazard

I agree with the other comment about hand-waving. While you know it will help later, if we were working on this problem, we wouldn't...

This seems a bit like hand waving to me.

i agree. it makes sense but i'm not so easily convinced to subscribe to this. if it's going to be a chapter, i want to be convinced more.

Yeah, that is true. Perhaps some more on this topic owuld be nice.

I don't entirely understand this section but it seems weirdly analytical for an estimate based on your gut...

Yeah, this section makes it seem like you knew that 2 to 1 odds is useful for plotting purposes (and has the name 'plausible range'), and you therefore found a range that fit this certainty, rather than finding a range and fitting a certainty as the prior section suggests. It's all just jumbled enough to have forced me to read it a couple of times.

I fell like this paragraph takes a lot away from your explanation...you're all but saying it really doesn't matter what's right, so i'm going to use this. it's frustrating.

assuming its equally likely to be bigger or smaller. or is that inherent by defining the range in the center of your likelihood distribution?

I really like this example and it is helping me understand the concept pretty well.

and information (or evidence)

$I \equiv$  What I know about the area *before* using divide and conquer.

Now I want to find the conditional probability  $P(H|I)$  – namely, the probability of  $H$  given my knowledge before trying divide and conquer. There is no known algorithm for computing a probability in such a complicated problem situation. How, for example, does one represent my state of knowledge? In these cases, the best we can do is to introspect or, in plain English, to talk to our gut.

My gut is the organ with the most access to my intuitive knowledge and its incompleteness, and it tells me that I would feel mild surprise but not shock if I learned that the true area lay outside the range  $300 \dots 3000 \text{ cm}^2$ . The surprise suggests that  $P(H|I)$  is larger than  $1/2$ . The mildness of the surprise suggests that  $P(H|I)$  is not much larger than  $1/2$ . I'll quantify it as  $P(H|I) = 2/3$ : I would give 2-to-1 odds that the true area is within the plausible range.

Furthermore, I'll use this probability or odds to define a plausible range: It is the range for which I think 2-to-1 odds is fair. I could have used a 1-to-1 odds range instead, but the 2-to-1 odds range will later help give plausible ranges an intuitive interpretation (as a region on a log-normal distribution). That interpretation will then help quantify how to combine plausible ranges.

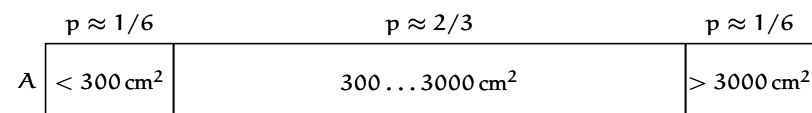
For the moment, I need only the idea that the plausible range contains roughly  $2/3$  of the probability. With a further assumption of symmetry, the plausible range  $300 \dots 3000 \text{ cm}^2$  represents the following probabilities:

$$P(A < 300 \text{ cm}^2) = 1/6;$$

$$P(300 \text{ cm} \leq A \leq 3000 \text{ cm}^2) = 2/3;$$

$$P(A > 3000 \text{ cm}^2) = 1/6.$$

Here is the corresponding picture with width proportional to probability:



For the height  $h$  and width  $w$ , after doing divide and conquer and using the similarity between A4 and letter paper, the plausible ranges are

**This method for decision making makes a lot of sense.**

I agree, I think the whole book would flow very nicely if there were many charts and tables like this.

I agree - the preceding paragraphs were somewhat confusing, but this set of equations and the chart below helped a lot.

I agree - the preceding paragraphs were somewhat confusing, but this set of equations and the chart below helped a lot.

**i think the picture is much easier to comprehend than this table and the picture could stand alone.**

**Okay this is notation much more familiar to me from taking probability. So I guess without even knowing it, we're using continuous random variables. Which makes sense for this problem.**

Well the reason we're using continuous random variables and not discrete random variables is because we're estimating the Area to be within a certain range, and not assigning it a perfect value. In the course we've always had our approximations within a range of values, so using continuous random variables is the clear choice.

**I really like that this is here... It might seem kind of obvious to some but I think it's useful in defining our parameters.**

Agreed. This serves as a good illustration of gut-thinking.

and information (or evidence)

$I \equiv$  What I know about the area *before* using divide and conquer.

Now I want to find the conditional probability  $P(H|I)$  – namely, the probability of  $H$  given my knowledge before trying divide and conquer. There is no known algorithm for computing a probability in such a complicated problem situation. How, for example, does one represent my state of knowledge? In these cases, the best we can do is to introspect or, in plain English, to talk to our gut.

My gut is the organ with the most access to my intuitive knowledge and its incompleteness, and it tells me that I would feel mild surprise but not shock if I learned that the true area lay outside the range  $300 \dots 3000 \text{ cm}^2$ . The surprise suggests that  $P(H|I)$  is larger than  $1/2$ . The mildness of the surprise suggests that  $P(H|I)$  is not much larger than  $1/2$ . I'll quantify it as  $P(H|I) = 2/3$ : I would give 2-to-1 odds that the true area is within the plausible range.

Furthermore, I'll use this probability or odds to define a plausible range: It is the range for which I think 2-to-1 odds is fair. I could have used a 1-to-1 odds range instead, but the 2-to-1 odds range will later help give plausible ranges an intuitive interpretation (as a region on a log-normal distribution). That interpretation will then help quantify how to combine plausible ranges.

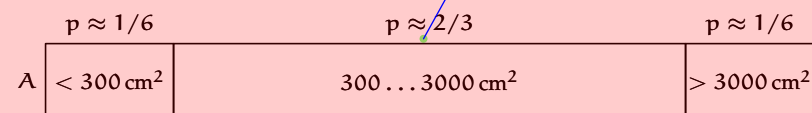
For the moment, I need only the idea that the plausible range contains roughly  $2/3$  of the probability. With a further assumption of symmetry, the plausible range  $300 \dots 3000 \text{ cm}^2$  represents the following probabilities:

$$P(A < 300 \text{ cm}^2) = 1/6;$$

$$P(300 \text{ cm} \leq A \leq 3000 \text{ cm}^2) = 2/3;$$

$$P(A > 3000 \text{ cm}^2) = 1/6.$$

Here is the corresponding picture with width proportional to probability:



For the height  $h$  and width  $w$ , after doing divide and conquer and using the similarity between A4 and letter paper, the plausible ranges are

**This is really useful to those of us who haven't had much probability, I think some diagrams like this in the previous section would be helpful to its understanding**

Although the jumping between "odds" and "probability" is kind of confusing. I think I'd prefer it if you stuck with just one (probability, preferably)

The "jump" between odds and probability really shouldn't be a jump at all. Here's a quick explanation that should simplify things:

Let  $p$  the probability that an event occurs. Then the probability that the event does not occur =  $1 - p$ . The odds of an event occurring refers to the ratio of the probability that the event occurs divided by the probability that the event does not occur. That is, the odds =  $p/(1-p)$ .

This is a good explanation from 8:55. Thanks.

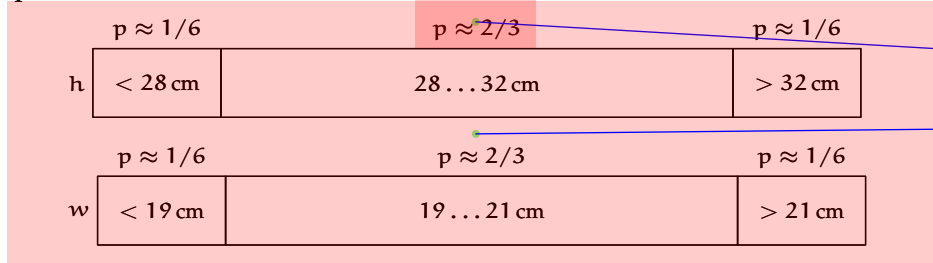
I agree. Odds are simply a more everyday way of talking about a probability. They are directly related to each other so it shouldn't be too much to switch between the two.

I think that using both odds & probability is a good thing...I totally get the probability better, but odds are used more in everyday life and we need to get a better understanding of them anyway

I agree. If such diagrams were available in the previous section, I think I might have absorbed much more.

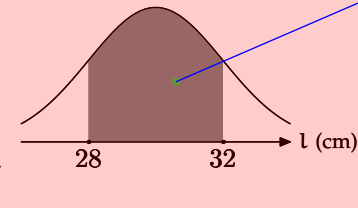
**I'd expect a picture to be more like a graph or histogram, but this is useful!**

28...32 cm and 19...21 cm respectively. Here are their probability interpretations:



Computing the plausible range for the area requires a complete probabilistic description of a plausible range. There is an answer to this question that depends on the information available to the person giving the range. But no one knows the exact recipe to deduce probabilities from the complex, diffuse, seemingly contradictory information lodged in a human mind.

The best that we can do for now is to guess a reasonable and convenient probability distribution. I will use a log-normal distribution, meaning that the uncertainty in the quantity's logarithm has a normal (or Gaussian) distribution. As an example, the figure shows the probability distribution for the length of A4 length (after taking into account the similarity to letter paper). The shaded range is the so-called one-sigma range  $\mu - \sigma$  to  $\mu + \sigma$ . It contains 68% of the probability – a figure conveniently close to 2/3. So to convert a plausible range to a log-normal distribution, use the lower and upper endpoints of the plausible range as  $\mu - \sigma$  to  $\mu + \sigma$ . The peak of the distribution – the most likely value – occurs midway between the endpoints. Since 'midway' is on a logarithmic scale, the midpoint is at  $\sqrt{28 \times 32}$  cm or approximately 29.93 cm.



The log-normal distribution supplies the missing information required to combine plausible ranges. When adding independent quantities, you add their means and their variances. So when multiplying independent quantities, add the means and variances in the logarithmic space.

Here is the resulting recipe. Let the plausible range for  $h$  be  $l_1 \dots u_1$  and the plausible range for  $w$  be  $l_2 \dots u_2$ . First compute the geometric mean (midpoint) of each range:

So every time we guess a range, we use the plausible range probability distribution (1/6, 2/3, 1/6), because all of our guts are probably 2/3 right?

why is the probability the same?

I like these charts, very clear!

I agree - it really makes the point about how the probabilities interrelate.

This wording of this sentence is kind of confusing to me.

This is confusing to me - I'm not sure why we have to relate talking to our gut to probability if it is really impossible to come up with a good probability

Yet another nice diagram!!

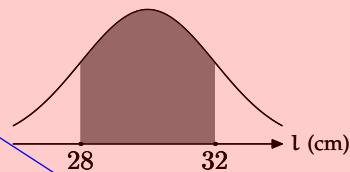
The diagrams from this reading have been very helpful.

28...32 cm and 19...21 cm respectively. Here are their probability interpretations:

	$p \approx 1/6$	$p \approx 2/3$	$p \approx 1/6$
h	< 28 cm	28...32 cm	> 32 cm
	$p \approx 1/6$	$p \approx 2/3$	$p \approx 1/6$
w	< 19 cm	19...21 cm	> 21 cm

Computing the plausible range for the area requires a complete probabilistic description of a plausible range. There is an answer to this question that depends on the information available to the person giving the range. But no one knows the exact recipe to deduce probabilities from the complex, diffuse, seemingly contradictory information lodged in a human mind.

The best that we can do for now is to guess a reasonable and convenient probability distribution. I will use a **log-normal** distribution, meaning that the uncertainty in the quantity's logarithm has a normal (or Gaussian) distribution. As an example, the figure shows the probability distribution for the length of A4 length (after taking into account the similarity to letter paper). The shaded range is the so-called one-sigma range  $\mu - \sigma$  to  $\mu + \sigma$ . It contains 68% of the probability – a figure conveniently close to 2/3. So to convert a plausible range to a log-normal distribution, use the lower and upper endpoints of the plausible range as  $\mu - \sigma$  to  $\mu + \sigma$ . The peak of the distribution – the most likely value – occurs midway between the endpoints. Since 'midway' is on a logarithmic scale, the midpoint is at  $\sqrt{28 \times 32}$  cm or approximately 29.93 cm.



The log-normal distribution supplies the missing information required to combine plausible ranges. When adding independent quantities, you add their means and their variances. So when multiplying independent quantities, add the means and variances in the logarithmic space.

Here is the resulting recipe. Let the plausible range for  $h$  be  $l_1 \dots u_1$  and the plausible range for  $w$  be  $l_2 \dots u_2$ . First compute the geometric mean (midpoint) of each range:

**why would we use this over just a normal one?**

It seems like this method keeps the error from doubling when we multiply the length and the area? Log-normal graphs are used a lot in probability.

also, since each length is clearly bounded below by 0, the log-normal is more appropriate.

A log normal also centers the range about the geometric mean, not arithmetic mean, which is much more appropriate for this type of problem.

It also doesn't look particularly different from a normal distribution when the range is concentrated away from 0.

Those are indeed the reasons. To really see the difference between normal and log normal, I should use a range like  $10^2 \dots 10^4$  (i.e.  $10^3$  give or take a factor of 10).

What we mean intuitively is that 1000 is the most likely value, and 300 is about as likely as 3333 (=10000/3). That is the log-normal interpretation.

In the normal interpretation, the most likely value would be 5050, which is already very strange. And there would be a significant probability of the value being less than 0 (because 0 is only about 1 sigma below the mean of 5050).

It would be nice if log-normal was defined at some point, for those of us who may not have encountered the term before.

Nevermind. It is... I just assumed because that was the second time I'd seen it. Should have read further.

No, I agree – it's not really that well defined even later on the page.

**This seems a little far to go just to estimate the area of a piece of paper**

True, but it will be really useful in real life estimations, if we don't know values and it becomes important to estimate them exactly

It's a simple example of a two-parameter problem.

I'd still rather examine an example where I wouldn't just guess length and width and error.

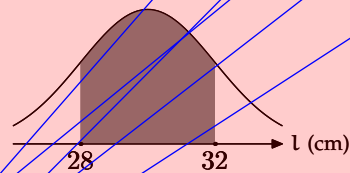


28...32 cm and 19...21 cm respectively. Here are their probability interpretations:

	$p \approx 1/6$	$p \approx 2/3$	$p \approx 1/6$
h	< 28 cm	28...32 cm	> 32 cm
	$p \approx 1/6$	$p \approx 2/3$	$p \approx 1/6$
w	< 19 cm	19...21 cm	> 21 cm

Computing the plausible range for the area requires a complete probabilistic description of a plausible range. There is an answer to this question that depends on the information available to the person giving the range. But no one knows the exact recipe to deduce probabilities from the complex, diffuse, seemingly contradictory information lodged in a human mind.

The best that we can do for now is to guess a reasonable and convenient probability distribution. I will use a log-normal distribution, meaning that the uncertainty in the quantity's logarithm has a normal (or Gaussian) distribution. As an example, the figure shows the probability distribution for the length of A4 length (after taking into account the similarity to letter paper). The shaded range is the so-called one-sigma range  $\mu - \sigma$  to  $\mu + \sigma$ . It contains 68% of the probability – a figure conveniently close to 2/3. So to convert a plausible range to a log-normal distribution, use the lower and upper endpoints of the plausible range as  $\mu - \sigma$  to  $\mu + \sigma$ . The peak of the distribution – the most likely value – occurs midway between the endpoints. Since 'midway' is on a logarithmic scale, the midpoint is at  $\sqrt{28 \times 32}$  cm or approximately 29.93 cm.



The log-normal distribution supplies the missing information required to combine plausible ranges. When adding independent quantities, you add their means and their variances. So when multiplying independent quantities, add the means and variances in the logarithmic space.

Here is the resulting recipe. Let the plausible range for  $h$  be  $l_1 \dots u_1$  and the plausible range for  $w$  be  $l_2 \dots u_2$ . First compute the geometric mean (midpoint) of each range:

I wish my high school stats class had taught me cool cases like the ones you use as an example.

This is a very interesting idea and easily applicable to many real life examples.

I agree, the analysis here is very clear and well described.

you should just mention that mu is the mean before putting this out there

It might be helpful if you stated that mu was the mean and sigma the standard deviation.

I think that you should introduce the 68-95-99 rule here.

Ah its all starting to come together. A standard deviation away from the mean on both sides is 2/3 which is what he set to be the probability of our range. So I guess in general, that's what we should always set the probability to.

While I understand his motivation for setting the probability to 2/3 in this case since he wanted to give 2-to-1 odds and it also was very close to the 68% confidence interval encompassed by being one standard deviation (or alternatively, one z-score) away from the mean, he also did it because 2-to-1 odds was the appropriate probability in this problem. If the odds really aren't 2-to-1, it would be unwise to "always set the probability to 2/3" as you mentioned.

what do you mean it is on a log scale?

I think it's also fair to add that sometimes you will guess too low and sometimes you will guess to high. This, over time, will even out.

I feel like using a log scale is unnecessary and complicates things.

I was also unsure of why we use a log-normal scale. I understand the need for a normal / Gaussian graph, but why a logarithmic one? Is there something in this problem that helps us identify when to use which?

Why not just use the average of your two ranges and multiply those to get your guess?

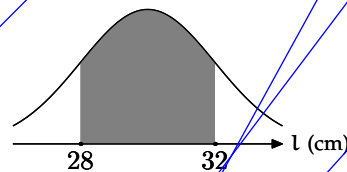
Is this an approximation or a definition of the midpoint of a log graph? I'm not really up on statistics.

28...32 cm and 19...21 cm respectively. Here are their probability interpretations:

	$p \approx 1/6$	$p \approx 2/3$	$p \approx 1/6$
h	< 28 cm	28...32 cm	> 32 cm
w	$p \approx 1/6$	$p \approx 2/3$	$p \approx 1/6$
	< 19 cm	19...21 cm	> 21 cm

Computing the plausible range for the area requires a complete probabilistic description of a plausible range. There is an answer to this question that depends on the information available to the person giving the range. But no one knows the exact recipe to deduce probabilities from the complex, diffuse, seemingly contradictory information lodged in a human mind.

The best that we can do for now is to guess a reasonable and convenient probability distribution. I will use a log-normal distribution, meaning that the uncertainty in the quantity's logarithm has a normal (or Gaussian) distribution. As an example, the figure shows the probability distribution for the length of A4 length (after taking into account the similarity to letter paper). The shaded range is the so-called one-sigma range  $\mu - \sigma$  to  $\mu + \sigma$ . It contains 68% of the probability – a figure conveniently close to 2/3. So to convert a plausible range to a log-normal distribution, use the lower and upper endpoints of the plausible range as  $\mu - \sigma$  to  $\mu + \sigma$ . The peak of the distribution – the most likely value – occurs midway between the endpoints. Since 'midway' is on a logarithmic scale, the midpoint is at  $\sqrt{28 \times 32}$  cm or approximately 29.93 cm.



The log-normal distribution supplies the missing information required to combine plausible ranges. When adding independent quantities, you add their means and their variances. So when multiplying independent quantities, add the means and variances in the logarithmic space.

Here is the resulting recipe. Let the plausible range for  $h$  be  $l_1 \dots u_1$  and the plausible range for  $w$  be  $l_2 \dots u_2$ . First compute the geometric mean (midpoint) of each range:

I understand why you would use this number in approximating, but doesn't this method complicate the overall estimating process? Also, why couldn't we have just used  $(28+32)/2 = 30$ cm ?

I'm in 18.440 and we just went over this concept, it's cool to see it applied here!

So I was following the reading until now.... Any 18.440 students wish to explain this further?

What exactly do you mean by variances here?

I'm glad we finally got to this point. It means that the level of uncertainty continues to increase as we continue to multiply quantities, assuming that there is no new information into the system.

I hope we go over this in class

I'm confused as to why  $h$  and  $w$  are described in both  $l$  and  $u$ ? Why isn't  $h$  described as  $l_1 \dots l_2$  and  $w$  as  $u_1 \dots u_2$ ? The combination of variables is confusing and unintuitive to me.

$l$  and  $u$  refer to lower and upper bounds, I assume.

I agree, the use of variables here is a bit strange

$$\mu_1 = \sqrt{l_1 u_1};$$

$$\mu_2 = \sqrt{l_2 u_2}.$$

The midpoint of the range for  $A = hw$  is the product of the two midpoints:

$$\mu = \mu_1 \mu_2. \tag{8.9}$$

To compute the plausible range, first compute the ratios measuring the width of the ranges:

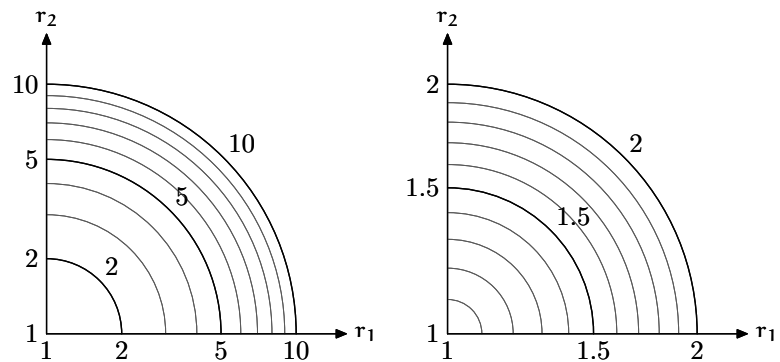
$$r_1 = u_1/l_1;$$

$$r_2 = u_2/l_2.$$

These ratios measure the width of the ranges. The combined ratio – that is, the ratio of endpoints for the combined plausible range – is

$$r = \exp\left(\sqrt{(\ln r_1)^2 + (\ln r_2)^2}\right).$$

For approximate range calculations, the following contour graphs often provide enough accuracy:



After finding the range, choose the lower and upper endpoints  $l$  and  $u$  to make  $u/l = r$  and  $\sqrt{lu} = \mu$ . In other words, the plausible range is

$$\frac{\mu}{\sqrt{r}} \dots \mu \sqrt{r}.$$

**Problem 8.1 Deriving the ratio**

Use Bayes theorem to confirm this method for combining plausible ranges.

Let's check this method in a simple example where the width and height ranges are 1...2 m. What is the plausible range for the area? The naive

I'm pretty lost as well. Also not sure why we choose to use two approximations and then combine them.

I got very lost here...and I still am. I get what you're saying on the next page, but (even after sleeping on it) I just can't seem to follow what you're doing here.

I got kind of lost in here. I think it might be the l, u notations. They don't seem very intuitive.

I think l stands for lower and u for upper. But I had to read it a few times before I saw that.

Good call! thanks that really helped me make sense of it when I was reading it

While that does make sense, the mu used for midpoint is very similar to the u. I think it would be less confusing if the u and l were capitalized.

you lost me from here down...

I am confused on what you are talking about

Could this be explained graphically as well?

$$\mu_1 = \sqrt{l_1 u_1};$$

$$\mu_2 = \sqrt{l_2 u_2}.$$

The midpoint of the range for  $A = hw$  is the product of the two midpoints:

$$\mu = \mu_1 \mu_2. \tag{8.9}$$

To compute the plausible range, first compute the ratios measuring the width of the ranges:

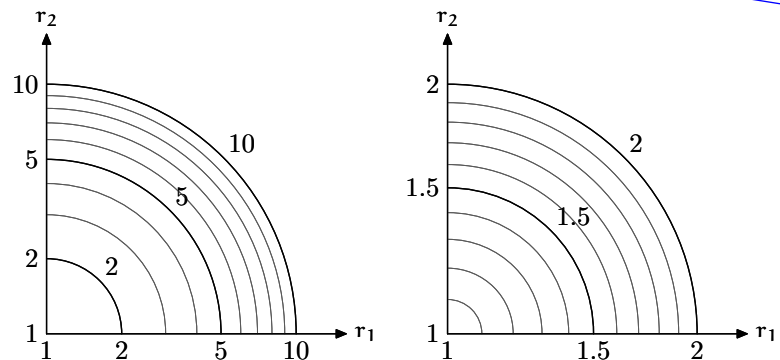
$$r_1 = u_1/l_1;$$

$$r_2 = u_2/l_2.$$

These ratios measure the width of the ranges. The combined ratio – that is, the ratio of endpoints for the combined plausible range – is

$$r = \exp(\sqrt{(\ln r_1)^2 + (\ln r_2)^2}).$$

For approximate range calculations, the following contour graphs often provide enough accuracy:



After finding the range, choose the lower and upper endpoints  $l$  and  $u$  to make  $u/l = r$  and  $\sqrt{lu} = \mu$ . In other words, the plausible range is

$$\frac{\mu}{\sqrt{r}} \dots \mu \sqrt{r}.$$

**Problem 8.1 Deriving the ratio**

Use Bayes theorem to confirm this method for combining plausible ranges.

Let's check this method in a simple example where the width and height ranges are 1...2 m. What is the plausible range for the area? The naive

**This is the most confusing part. I don't know where this came from.**

Yeah I feel like all of this suddenly comes out of nowhere - more exposition would have been nice.

i agree.

Yes, agreed. A bit more explanation for those of us who haven't taken a probability class would be great.

I would add that an overview on log's as they relate to this class would help me with a lot of this stuff. I learned the change of base formula and basic manipulations back in freshmen year, but it seems to me we use them a bit more extensively in this class.

This follows from the fact that variances of the logs add for log-normal distributions.

If we had assumed these were normal distributions, we would simply have  $s = \sqrt{s_1^2 + s_2^2}$ , where  $s$  = standard deviation instead of endpoints ratio.

Unfortunately, I feel like most of this came out of nowhere. Like why we can add the means and variances on the log space or something.

**So, on a log-normal plot, the variance is the square of the log of the standard deviation of the original data? I think that makes sense, but it would probably be worth explaining.**

**I'm not familiar with these plots...what are they showing/proving exactly? Does the normal distribution above not suffice?**

Yeah...what is this? More explanation is needed.

I think it's just an easy way to find  $r$  from  $r_1$  and  $r_2$ . The magnitude of the contour at  $(r_1, r_2)$  is  $r$ .

I'm also confused as to what these are showing. More explanation would probably be helpful.

**I haven't really encountered these a lot... a little more explanation might be nice?**

I think that this problem just has a few really complicated concepts. The first was about multiplying independent quantities, and now the contour graphs and the ratio analysis.

$$\mu_1 = \sqrt{l_1 u_1};$$

$$\mu_2 = \sqrt{l_2 u_2}.$$

The midpoint of the range for  $A = hw$  is the product of the two midpoints:

$$\mu = \mu_1 \mu_2. \tag{8.9}$$

To compute the plausible range, first compute the ratios measuring the width of the ranges:

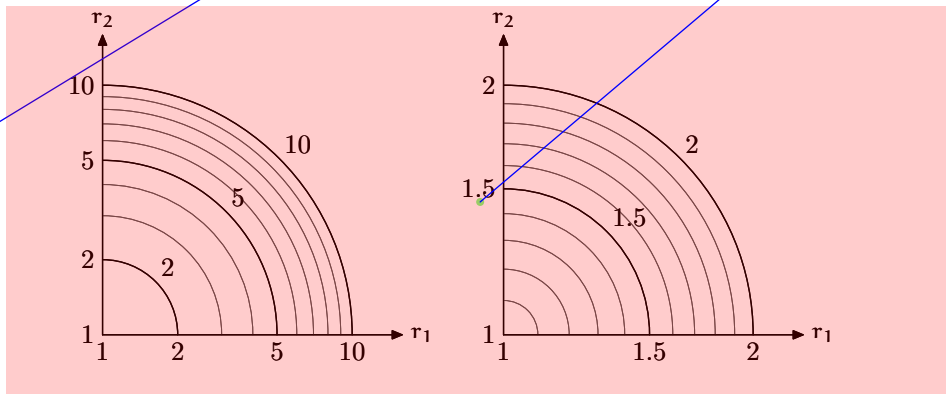
$$r_1 = u_1/l_1;$$

$$r_2 = u_2/l_2.$$

These ratios measure the width of the ranges. The combined ratio – that is, the ratio of endpoints for the combined plausible range – is

$$r = \exp(\sqrt{(\ln r_1)^2 + (\ln r_2)^2}).$$

For approximate range calculations, the following contour graphs often provide enough accuracy:



After finding the range, choose the lower and upper endpoints  $l$  and  $u$  to make  $u/l = r$  and  $\sqrt{lu} = \mu$ . In other words, the plausible range is

$$\frac{\mu}{\sqrt{r}} \dots \mu \sqrt{r}.$$

**Problem 8.1 Deriving the ratio**

Use Bayes theorem to confirm this method for combining plausible ranges.

Let's check this method in a simple example where the width and height ranges are 1...2 m. What is the plausible range for the area? The naive

I'm pretty confused, as I think others are too...this seems like a lot of work to do – when should we actually use this method? I'm also afraid I would not be able to implement it correctly.

agreed. i also don't know what to make of these plots. kind of confusing.

I agree. I don't really understand what these graphs are supposed to be used for.

Is this necessary just to prove why divide and conquer works??

A lot of people, including myself, seem to struggle to understand what exactly the meaning of these graphs are. I'm hoping he'll explain it further in class.

Yeah, I'm also confused here. Are these graphs necessary? What exactly do they represent, and what are you trying to show with them?

These graphs are not necessary, but I personally actually like them. They are just a demonstration of how to find  $r$  from  $r_1$  and  $r_2$  within a reasonable accuracy.

I think this is helpful for the more theoretically oriented people who want an explanation. As an engineer, I don't care how things work, as long as they work. Why worry about reinventing the wheel. I wouldn't bother reading this chapter if I bought this book, but I think it's necessary for a class to include this explanation

$$\mu_1 = \sqrt{l_1 u_1};$$

$$\mu_2 = \sqrt{l_2 u_2}.$$

The midpoint of the range for  $A = hw$  is the product of the two midpoints:

$$\mu = \mu_1 \mu_2. \tag{8.9}$$

To compute the plausible range, first compute the ratios measuring the width of the ranges:

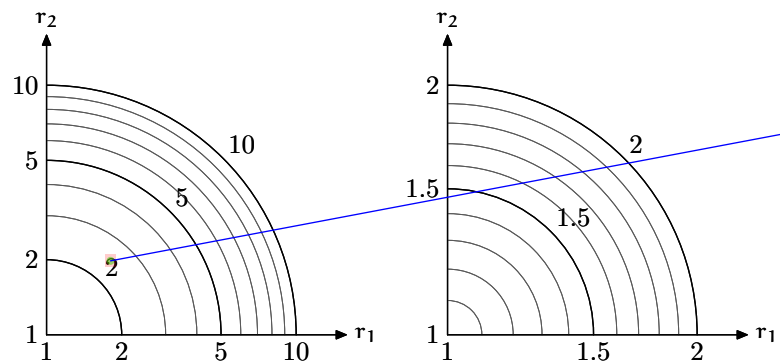
$$r_1 = u_1/l_1;$$

$$r_2 = u_2/l_2.$$

These ratios measure the width of the ranges. The combined ratio – that is, the ratio of endpoints for the combined plausible range – is

$$r = \exp\left(\sqrt{(\ln r_1)^2 + (\ln r_2)^2}\right).$$

For approximate range calculations, the following contour graphs often provide enough accuracy:



After finding the range, choose the lower and upper endpoints  $l$  and  $u$  to make  $u/l = r$  and  $\sqrt{lu} = \mu$ . In other words, the plausible range is

$$\frac{\mu}{\sqrt{r}} \dots \mu \sqrt{r}.$$

**Problem 8.1 Deriving the ratio**

Use Bayes theorem to confirm this method for combining plausible ranges.

Let's check this method in a simple example where the width and height ranges are 1 ... 2 m. What is the plausible range for the area? The naive

What is the difference between these two graphs? (I see the numbers are different, but I'm really confused as to what this shows)

Yeah, I'm pretty lost here as well—what do these two show? the upper and lower ranges?

I also don't quite get what point the graphs are supposed to show.

I think they are illustrations for two different ranges so you can see what a log plot looks like in this contour form. The second plot is the lower left quarter-circle enlarged. I think you take the two endpoints of the range, then it would give you a sense of the likeliest value. So if  $r_1 = 2$  and  $r_2 = 5$ , that point on the graph is somewhere just outside the 5 contour line, so  $r \approx 5.5$ ? I am still pretty confused about the use of these graphs though.

I'm confused about these graphs as well.

that explanation, that one a blow-up of the other helps, but it still doesn't really make the point very clear.

Clearly I need to explain this better. I'll have a go in lecture and hopefully that will result in a better explanation for the book too.

So for the problem below, the range is somewhere like here? I guess that looks like 2.67

What do the varying contours mean? Don't really understand the significance of these diagrams..

is this really a process you would undertake to simply estimate the area of a sheet of paper?

It's definitely overkill for the sheet of paper. But the process is general, and if you want to know how accurate an estimate is (i.e. how confident you can be about an estimate), now you know how to do it. In lecture we'll do a more substantial example, and maybe I should also put that in the reading.

I got confused here that we were no longer still talking about the paper since these numbers seemed too big for that. But after reading a bit more, I see that this was just an example of easy cases to test the theory. You could mention that to tie this back more clearly to earlier sections

approach of simply multiplying endpoints produces a plausible range of  $1 \dots 4 \text{ m}^2$  – a width of a factor of 4. However, this range is too pessimistic and the correct range should be narrower. Using the log-normal distribution, the range spans a factor of

$$\exp(\sqrt{2 \times (\ln 2)^2}) \approx 2.67.$$

This span and the midpoint determine the range. The area midpoint is the product of the width and height midpoints, each of which is  $\sqrt{2} \text{ m}$ . So the midpoint is  $2 \text{ m}^2$ . The correct endpoints of the plausible range are therefore

$$\frac{2 \text{ m}^2}{\sqrt{2.67}} \dots 2 \text{ m}^2 \times \sqrt{2.67}$$

or  $1.23 \dots 3.27 \text{ m}^2$ . In other words, I assign roughly a  $1/6$  probability that the area is less than  $1.23 \text{ m}^2$  and roughly a  $1/6$  probability that it is greater than  $3.27 \text{ m}^2$ . Those conclusions seem reasonable when using such uncertain knowledge of length and width.

Having checked that the method is reasonable, it is time to test it in the original illustrative problem: the plausible area range for an A4 sheet. The naive plausible range was  $532 \dots 672 \text{ cm}^2$ , and the correct plausible range will be narrower. Indeed, the log-normal method gives the narrower area range of  $550 \dots 650 \text{ cm}^2$  with a best guess (most likely value) of  $598 \text{ cm}^2$ . How did we do? The true area is exactly  $2^{-4} \text{ m}^2$  or  $625 \text{ cm}^2$  because – I remembered only after doing this calculation! – **A $n$  paper is constructed to have one-half the area of A $(n-1)$  paper, with A0 paper having an area of  $1 \text{ m}^2$ . The true area is only 5% larger than the best guess, suggesting that we used accurate information about the length and width; and it falls within the plausible range but not right at the center, suggesting that the method for computing the plausible range is neither too daring nor too conservative.**

The analysis of combining ranges illustrates the two crucial points about divide-and-conquer reasoning. First, the main benefit comes from subdividing vague knowledge (such as the area itself) into pieces about which our knowledge is accurate (the length and the width). Second, this benefit swamps the small penalty in accuracy that results from combining many quantities together.

Thanks for giving us another example!

Why is the extra square root of 2.67 taken here? I thought the span was just  $2.67^{\wedge} 1$ .

I was a little confused at times about when we stopped talking about this, and it seemed to take a while to get back to this, although it brings the points together very clearly when it is brought up again, I think that it could be sooner/ more integrated into the preceding examples.

Is there an example that's just as simple and illustrative but maybe a little more relevant or ..interesting than the area of a sheet of European paper? (no offense to European paper or anything)

I am really confused on why we are doing this probability calculations? what are they actually telling us?

I think this reading overall was very helpful and informative. I do always get a little intimidated when a lot of equations and numbers are thrown in.

Given that this represents a change of about 3% from the original endpoints, it seems sort of silly to go through all the log-normal calculations. Wouldn't the original range be "good enough" for most practices?

This notation was confusing at first, maybe because I'm not used to referring to paper size like this. I'm not sure how to make it clearer though, maybe with an example?

I was also confused as to why you were saying "An paper" ... breaking up this sentence would probably help a lot...or include a chart with A0, A1, A2, ...

Very interesting fact. explains why each size is double the previous, although i had no idea A0 was  $1 \text{ m}^2$ .

Huh...really? I wasn't even aware that there was A2 or A3 paper. That's pretty cool.

Yeah, this seems to make a bit more sense to me now.

This is a really cool piece of semi useful knowledge.

there is so much rhyme and reason to metric.

approach of simply multiplying endpoints produces a plausible range of  $1 \dots 4 \text{ m}^2$  – a width of a factor of 4. However, this range is too pessimistic and the correct range should be narrower. Using the log-normal distribution, the range spans a factor of

$$\exp(\sqrt{2 \times (\ln 2)^2}) \approx 2.67.$$

This span and the midpoint determine the range. The area midpoint is the product of the width and height midpoints, each of which is  $\sqrt{2} \text{ m}$ . So the midpoint is  $2 \text{ m}^2$ . The correct endpoints of the plausible range are therefore

$$\frac{2 \text{ m}^2}{\sqrt{2.67}} \dots 2 \text{ m}^2 \times \sqrt{2.67}$$

or  $1.23 \dots 3.27 \text{ m}^2$ . In other words, I assign roughly a  $1/6$  probability that the area is less than  $1.23 \text{ m}^2$  and roughly a  $1/6$  probability that it is greater than  $3.27 \text{ m}^2$ . Those conclusions seem reasonable when using such uncertain knowledge of length and width.

Having checked that the method is reasonable, it is time to test it in the original illustrative problem: the plausible area range for an A4 sheet. The naive plausible range was  $532 \dots 672 \text{ cm}^2$ , and the correct plausible range will be narrower. Indeed, the log-normal method gives the narrower area range of  $550 \dots 650 \text{ cm}^2$  with a best guess (most likely value) of  $598 \text{ cm}^2$ . How did we do? The true area is exactly  $2^{-4} \text{ m}^2$  or  $625 \text{ cm}^2$  because – I remembered only after doing this calculation! – A $n$  paper is constructed to have one-half the area of A $(n-1)$  paper, with A0 paper having an area of  $1 \text{ m}^2$ . The true area is only 5% larger than the best guess, suggesting that we used accurate information about the length and width; and it falls within the plausible range but not right at the center, suggesting that the method for computing the plausible range is neither too daring nor too conservative.

The analysis of combining ranges illustrates the two crucial points about divide-and-conquer reasoning. First, the main benefit comes from subdividing vague knowledge (such as the area itself) into pieces about which our knowledge is accurate (the length and the width). Second, this benefit swamps the small penalty in accuracy that results from combining many quantities together.

**I thought this method was overkill, applied here to estimating paper area, but it served as an easy to follow example of how to apply it, even if the example itself wasn't necessarily the most interesting one to use it on.**

I think this was the point of doing it in 'slow motion' hopefully we will also see it applied to more exciting examples later in the book.

It was nice to have an easy example, but at certain points the "slow motion" ran a little too slow so I lost site of the motivation for much of the example. As the original commenter suggested, maybe a slightly more relatable example would have been helpful.

**It seems like we've been doing this without realizing it for a while, is the extra time needed with this method ever worth it over the speed of trusting one's gut? (assuming of course we have a reasonable gut intuition)**

Well, all semester people have been complaining that they are not sure of their answers and their guts are giving them really bit ranges...so here is a way to narrow it down and explain exactly what you should do when asked to make a gut reasoning.

I was wondering the same. This seems like an excessive amount of work for an approximation.

I agree. This is making some of the approximations that we did at the beginning of the term make mathematical sense.

I think that most of the work in this section has been to look at the errors in what you're estimating...these are not actually calculations that you would be doing, unless you need to know how far off your approximations are.

I really like this conclusion. Great summary!

I really like this conclusion. Great summary!

**Im not really sure i got this from the reading...**



approach of simply multiplying endpoints produces a plausible range of  $1 \dots 4 \text{ m}^2$  – a width of a factor of 4. However, this range is too pessimistic and the correct range should be narrower. Using the log-normal distribution, the range spans a factor of

$$\exp(\sqrt{2 \times (\ln 2)^2}) \approx 2.67.$$

This span and the midpoint determine the range. The area midpoint is the product of the width and height midpoints, each of which is  $\sqrt{2} \text{ m}$ . So the midpoint is  $2 \text{ m}^2$ . The correct endpoints of the plausible range are therefore

$$\frac{2 \text{ m}^2}{\sqrt{2.67}} \dots 2 \text{ m}^2 \times \sqrt{2.67}$$

or  $1.23 \dots 3.27 \text{ m}^2$ . In other words, I assign roughly a  $1/6$  probability that the area is less than  $1.23 \text{ m}^2$  and roughly a  $1/6$  probability that it is greater than  $3.27 \text{ m}^2$ . Those conclusions seem reasonable when using such uncertain knowledge of length and width.

Having checked that the method is reasonable, it is time to test it in the original illustrative problem: the plausible area range for an A4 sheet. The naive plausible range was  $532 \dots 672 \text{ cm}^2$ , and the correct plausible range will be narrower. Indeed, the log-normal method gives the narrower area range of  $550 \dots 650 \text{ cm}^2$  with a best guess (most likely value) of  $598 \text{ cm}^2$ . How did we do? The true area is exactly  $2^{-4} \text{ m}^2$  or  $625 \text{ cm}^2$  because – I remembered only after doing this calculation! – A $n$  paper is constructed to have one-half the area of A $(n-1)$  paper, with A0 paper having an area of  $1 \text{ m}^2$ . The true area is only 5% larger than the best guess, suggesting that we used accurate information about the length and width; and it falls within the plausible range but not right at the center, suggesting that the method for computing the plausible range is neither too daring nor too conservative.

The analysis of combining ranges illustrates the two crucial points about divide-and-conquer reasoning. First, the main benefit comes from subdividing vague knowledge (such as the area itself) into pieces about which our knowledge is accurate (the length and the width). Second, this benefit swamps the small penalty in accuracy that results from combining many quantities together.

**I wonder if this section would be better placed near the divide and conquer section.**

perhaps but the divide and conquer is supposed to be really easy, and this is not so easy to start off with. I think it makes sense here too.

I think it's better suited here. At first, I think it's best to accept divide-and-conquer as an approximation method that works without us having to see proof of it. In order to understand how it works with a proof in probability, you would need to be familiar with a little probability and I like having it in this section. Plus, having this at the beginning of the course might scare some people with no previous exposure to probability away.

I think with the probability usage, it's rightly placed here.

**I think this sentence was a very clear conclusion.**

**This makes a lot of sense.**

**wasn't quite sure how this was proved in the reading above.**

His original guess was  $300 \dots 3000$  - a very broad guess, because our knowledge is vague. By subdividing the unknown into things we know more about (lengths), we gain some inaccuracy because we're multiplying unknowns, but less inaccuracy than simply multiplying our uncertainty (because our uncertainty in one number will to some extent be canceled by our uncertainty in the other number). And this inaccuracy is much less than what we gained by going to numbers we know more about - hence why we can get to a range of  $550-650$  using divide and conquer (as compared to his original  $300-3000$ ). Hence the small penalty in accuracy from using many numbers is swamped by the benefit of using those many numbers. Thus divide and conquer works.