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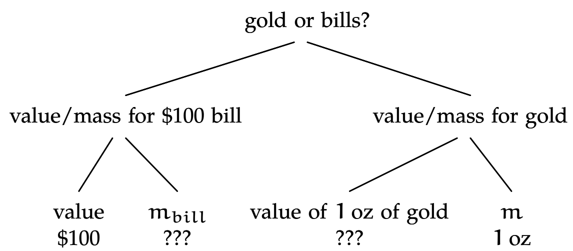

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but the value of gold is always changing...and so is the value of the dollar!

Yeah, I remember the last value of gold to be 1100 (it's now... 1165), but I check it very often. The value of the dollar is based on gold I believe, so I think the only important thing is the variations of the gold. At this rate, the 10s and the 20s are probably not as good!

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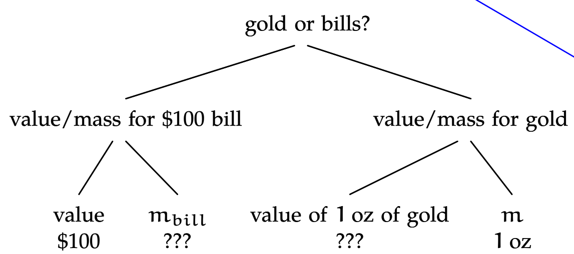

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Read this subsection for the memo due Tuesday at 10pm. It applies the plausible-range analysis to another divide-and-conquer problem (hopefully a hypothetical one).

what about forfeited?

This is one of my favorite readings from this class so far!

Wow. I'm excited. Hope the hype holds up.

Don't like this word. Could you replace it with "forewent" or something similar?

I agree. "Forgot" sounds strange here.

I think he means "forgo"

exactly what I was thinking. forgo, not forgot!

Does this include Sanjoy?

or do you perhaps mean bank-robbing?

This is a very exciting way to introduce the section because we have all thought about it once.

I hope we do this analysis before we break into the vault

I like this problem. It's something that we can all understand. On top of that, most of us, or at least myself, don't know the answer, so it makes it more interesting to read.

there's also this sort of, i don't really know how to say it... zing, to a problem that involves a bank hest :)

makes me want to read it more... or maybe i just seen to much oceans 11.

Nah, I think we're all intrigued. I'm just waiting for the news report about MIT students getting caught robbing a bank.

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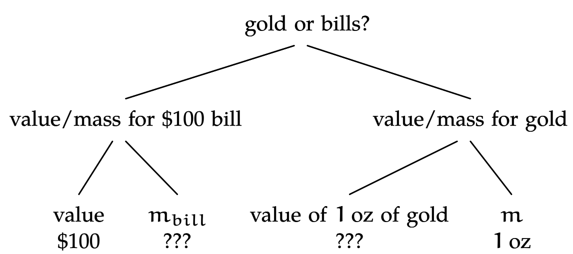

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Without reading further, I'm guessing take the gold. Per oz, it's worth more I think.

But you need to take ease of removal into consideration, gold is much more of a hassle to remove than bills

well considering that gold is worth over $1000 an ounce and a stack of printer papers weights 20lbs (320 oz.) I would think the gold is worth more than the bills.

You also might be less likely to get caught spending the gold since you could melt it or something.

The dollar is dying, gold is getting more valuable. Also you can wash the dye that's gonna explode when you open the bag off of gold but not a dollar bill. Take the gold.

There's also that whole issue of serial numbers on the paper cash

Well, that's the problem we're facing in this approximation example. ;) Though I personally would probably take the bills, assuming they were sufficiently large.

But sufficiently large bills are often marked and can be traced. And large amounts of cash are heavy and difficult to move, too.

Although, the same could be said for gold. Perhaps it is best if we just refrain from stealing from the vault?

... or just steal electronically...

Is this going to turn into a backpack problem? Cause honestly I think that would be really cool, I haven't done anything like that in two years

this is well...off topic, but for bills, can't they put tracers on it, or that purple stuff? i suppose since it is in a vault, maybe not...but who knows.

I guess it depends on your views of dollar and gold futures too...

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do they keep gold in bank vaults? just wondering

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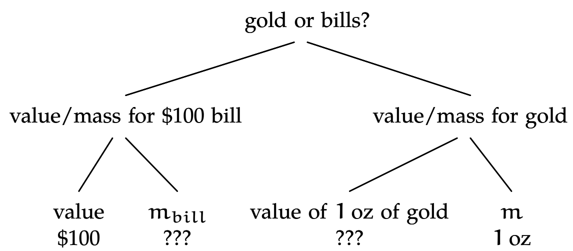

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i don't know what this word means in this context.

Perhaps 'taking'?

I believe it means getting rid of the money in a way that is not suspicious, i.e. so you can actually use the money to buy things instead of handing out stolen $100s everywhere.

You are correct: it simply refers to selling stolen goods.

In this context...getting rid of $100 is not hard. $100,000 bills would be harder. And how would you explain giant blocks of gold?

You can always melt gold! ;)

and there are people who would take it off your hands you deal in the business so a lesser then retail price - you would still make a killing

Selling, yes.

Just a British way of saying it!

I think it's common to all dialects of English (including the British and American ones).

Is that British for laundering?

Laundering is typically money, but fencing is for non-currency goods.

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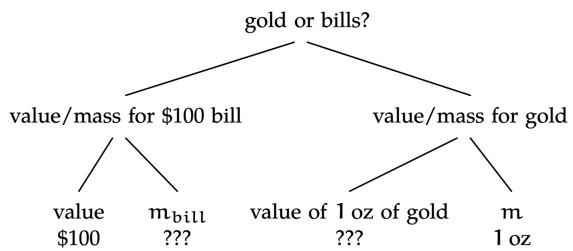

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This is what I was thinking about, although I don't the analysis would be too difficult, just a little more estimation in terms of space and weight

mmm, I think he is referring to things such as "washing" the money/gold, so that you would actually be able to sell the money/gold without being traced/caught.

For example, taking the money/gold and using it to acquire other goods (such as stamps - something constant in value and far less suspicious). This is often difficult or costs a lot to have someone else do it for you.

Couldn't you lump this cost? I feel like we could simply apply some "washing" fraction/percentage that will account for this.

no, it means which is easier to pawn, melt, purchase with, etc. or who you know and what they deal in. it doesn't have to do with weight/space at this point. and i agree, there's no way to estimate it from a pedagogical standpoint.

Probably beyond the scope of any real textbook!

Based on weight? I would guess the bills

Hmm with my gut instinct I'm really not sure...could they actually be quite comparable?

It depends on the price gold is going for, and what methods we have of extracting the gold or bills

gold keeps on increasing in value and will likely continue to do so while the american dollar has been decreasing in value in the global economy. it really is smarter to invest in gold.

while it is smart to invest in gold and that its value keeps on increasing, i don't think that you would just rob a bank to turn around and invest your money for a period of time (i.e. put it in another bank). You probably are gonna want to spend at least a large amount of it relatively soon.

That's a sure way to get caught. If you're working at burger king and you go out and buy a Bentley then people know that soimething's up and the government is gonna want to know how you bought it. It's smarter to put it away and spend slowly.

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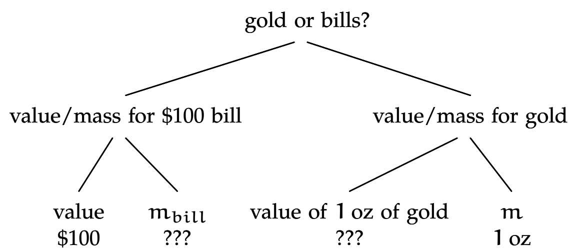

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wouldnt it really just be weight? either quantities get very heavy at reasonable carrying volumes

Possibly, but I feel like it's important to take it into consideration for the sake of determining probability with more evidence. It wouldn't cost us much more estimation time, either.

Nevermind, I just realized that we completely disregard it in the next sentence.

This reminds me of a certain knapsack problem from 6.00.

Hehe yup

I think shape matters too. The gold may be more (or less) awkward to carry, thus allowing less (or more) weight to be carried.

But in the knapsack problem you have many possible items to choose from. Here it seems like we're choosing between all gold or all bills - a binary choice. And we seem to have unlimited supplies of both.

do people really care about the weight or volume? they can just easily save the bills in the bank right?

"Carrying capacity." As in, how much can you carry out of the bank when you're robbing it? Weight and volume are the important factors there.

When you say you are going to forget volume, do you mean you are simply going to assume that the backpack is big enough for it? This seems wrong because I feel that there would be a big difference between 50 lbs of gold and dollar bills.

Maybe it's because you are more likely to be limited by weight before you would be limited by volume.

Yes, I feel like volume is the limiting factor... bills are lighter and easier to transport but not if you have to carry 20 backpacks' worth.

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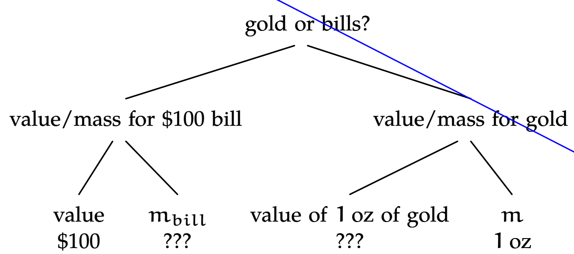

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isn't this a rather large assumption? i understood this for the 747 CDs problem, but not as much for this one.

I think the idea here is that you would max out the mass you can possibly carry before maxing out the volume you can take.

The weight probably isn't an issue for the cash - but for the bars of gold I think it would be the limiting factor for sure.

It would be nice to go back after finding the best choice according to mass and check that for a substantial haul it doesn't violate a reasonable volume (like a truck).

I think it would be weight for the gold and volume for the money. How would we be able to separate them this way?

the comments on this reading are hilarious – of course MIT students would approach a bank robbery this way

How does the final answer change if you choose volume instead?

That's interesting...so presumably, you are limited in volume or mass. Since the density of gold >> density of paper, then here it seems like the mass is the limiting factor, and not volume. Also, gold is typically valued per pound, not per m^3. I kind of feel like per volume is just awkward.

I would assume that the answer to this would be pretty obvious. Gold density >> paper/wood density, but estimating the order of magnitude of difference might be an interesting problem.

Is this assuming we'll be able to take the same amount (mass) of either the bills or the gold? i.e we'll be able to take a knapsack full of either or something?

no the "knapsack" would assume a volume limit. instead think of it as having infinitely many bags and filling them until you can't carry any more.

Yeah this is pretty clever. i mean it makes sense but i wouldn't have thought of this–if i was robbing a bank for some reason i'd probably just grab whatever i could take. but here he's saying that if you can only carry a certain amount (weight), which one would end up giving you more value for the same mass.

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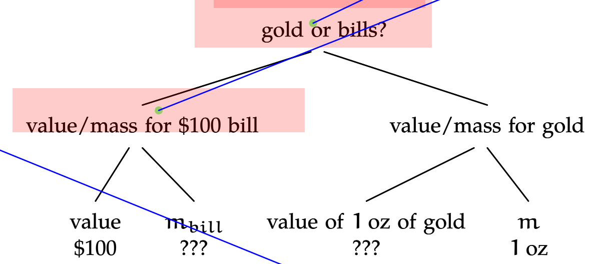

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My intuition would say that volume is the limiting factor.. if we are carrying the cash/gold out in a duffle bag, it seems that we could in theory fit as much weight as we want but the volume of the duffle is what is going to limit exactly how much we can take.

I like that this chart is here- it's very clear and the labels help to show exactly what we know and what we're looking for.

I think that you should specify before this diagram that we are talking about 100$ bills

Or maybe mention that we'll be determining what order to take bills versus gold (i.e., what order of $100 bills, $50 bills, $20 bills, gold, etc.), so we'll measure the value/mass of a N dollar bill with value $N since all bills have the same mass.

Solving for an arbitrary bill value does seem like a good idea for a more detailed analysis.

Yeah, might banks have greater denominations anyway?

Never mind - apparently they stopped printing anything above $100 in 1969!

Whoops – the previous version of this section did talk about $100 bills. But I just changed it to be more general (so that it better fits with the use of plausible ranges). But I forgot to remove from the tree the reference to $100 bills.

In 6.00 last semester we had to write an algorithm to solve a similar problem. It was more simplified, involving small point values instead of actual money, so I'm very interested in seeing how this works out.

This question is actually much different. In 6.00, we dealt with various items of weight and value and had to use power sets to decide with set was the best. In this situation, we are assuming unlimited amounts of gold and cash...the question being which one is the better value per weight or mass.

This is such a simple and nice example for the tree. You could introduce something like this when you first introduce divide and conquer.

It seems like it's been a while since we had a tree diagram, and coming back to them makes me realize that I really really like them as a way to organize information.

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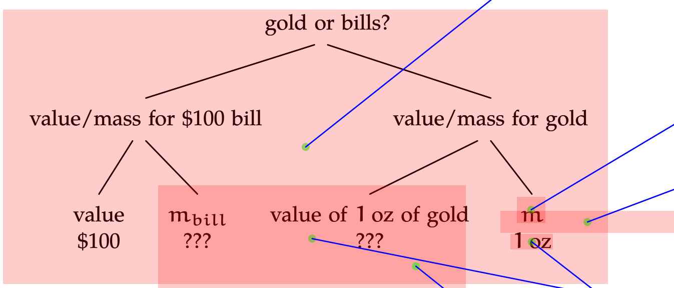

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At first glance, I couldn't imagine gold even coming close in value per mass. Now that I'm thinking about it though, small amounts of gold are worth a lot of money. This is really interesting.

This is an excellent d&a diagram for this problem.

agreed

Divide and conquer at its best!

agreed...if you throw in volume there you might even be able to figure out which is easier to get out of the bank!

probably want to label mgold since other one is labeled mbill.

i just realized why it only says m. regardless, it would nice to have consistency from both sides.

by m you mean weight, not mass correct?

I think it means mass - oz is a measure of mass.

an oz is actually a measure of weight, since 16 ounces make a pound, and a pound is a measurement of weight, technically.

But he later converts ounces into grams, which is a measure of mass. So we do in fact want to find the mass of the gold, so it's correct. He uses ounces here out of convenience because gold is priced in ounces.

I like that this tree immediately demonstrates that our unknown quantities are opposites (not literally) in each case, and that therefore we will have to do two distinct estimations. (Yeah, I know it's trivial, but I still liked it.)

Me too, it's funny how we know very well the opposite parameters for gold and paper money. I guess it speaks to the disconnection of paper currency from real material value.

yeah I agree, this tree is really helpful with the question marks to highlight what the unknown values are.

wouldn't grams work better for estimation? the whole powers of 10 thing....?

Perhaps, but the value of gold is most often referred to in terms of $/ounce.

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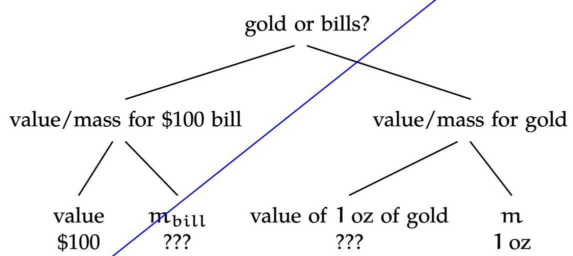

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perhaps branches?

What are these again?

I think I means making a best guess.

I think these are single-number estimates, instead of giving it a range.

that sounds right although i dont remember seeing this terminology before.

I agree, but didn't we use the plausible range method with a range? Maybe I am jumping to the second round of analysis too soon.

Yeah, that's what I would have chosen to do first too.

So do we set the probability of the plausible range to 2/3 again here?

seems low to me

well it's more like 1200/oz but i guess it's in the same order of magnitude.

are the fluctuations in the value of gold greater than that of bills?

well I would think that the fluctuations would be similar. If the value of gold went up then the dollar went down. In terms of goods gold is probably more stable

The fluctuation of gold is supposed to be zero. It's the bills that are supposed to fluctuate.

what if we don't remember what it is at all?

8.2.2 Gold or bills?

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▶ *Having broken into a bank vault, should we take the bills or the gold?*

The answer depends partly on the ease and costs of fencing the loot – an analysis beyond the scope of this book. Within our scope is the following question: Which choice lets us carry out the most money? Our carrying capacity is limited by weight and volume. For this analysis, let's assume that the more stringent limit comes from weight or mass. Then the decision divides into two subproblems: the value per mass for US bills and the value per mass for gold. In order to decide which to take, we'll compute both values per mass and their respective plausible ranges.

- gold or bills?
  - value/mass for \$100 bill
    - value \$100
    - m_{bill} ???
  - value/mass for gold
    - value of 1 oz of gold ???
    - m 1 oz

Two leaves have defined values: the value of a bill and the mass of 1 oz (1 ounce) of gold. The two other leaves need divide-and-conquer estimates. In the first round of analysis, make point estimates; then, in the second round, account for the uncertainty by using the plausible-range method of Section 8.2.

The value of gold is, I vaguely remember, around \$800/oz. As a rough check on the value – for example, should it be \$80/oz or \$8000/oz? – here is a historical method. In 1945, at the end of World War 2, the British empire had exhausted its resources while the United States became the world's leading economic power. The gold standard, which fell apart during the depression, was accordingly reinstituted in terms of the dollar: \$35 would be the value of 1 oz of gold. Since then, inflation has probably devalued the dollar by a factor of 10 or more, so gold should be worth around \$350/oz. My vague memory of \$800/oz therefore seems reasonable.

i feel like a lot of the problems in this book are done based on facts you "vaguely remember" but that i have no familiarity with whatsoever. the closest i could come to questimating this would be knowing how much some pieces of jewelry cost, but even then, i have no idea how much of it is for the gold and how much is for labor/advertising, etc.?

The price of gold is often mentioned in news and business reports on the radio and in newspapers, so it is something that people can at least know to an order of magnitude. Your idea about jewelry is a good one for bounding the price/oz. if you had to estimate.

Except that I know that carat matters when discussing gold, and I have no idea what that means for either the gold bars or the price of gold jewelery, as I've never seen the price for 24-carat samples of either.

So, when you do the calculation for this problem, how do you incorporate the price elasticity of gold?

I personally have no background in this area, I probably would have made this estimating using what I know about the price of gold jewelry (estimating the cost of a ring, the mass of that ring, and then making the proper conversions). With this method I actually got \$1000 which is pretty close!

I saw that you know a lot of little stories (historical evidence or every day life evidence) that help you tremendously in your estimation

these stories are probably also key in helping to actually remembering the actual values of the estimates

Even if I knew the historical evidence, i still wouldnt know if it was correct or not to guess \$800/oz. Would it have been a problem if I would have guessed anything from \$300 - \$1200?

I think that if you took the average value of your range and used it per oz, you would be fine.

Yes, it is quite entertaining when he launches into detailed anecdotes but not quite applicable to my problem solving process...

I think that we will accumulate more stories like these throughout our lifetimes too.

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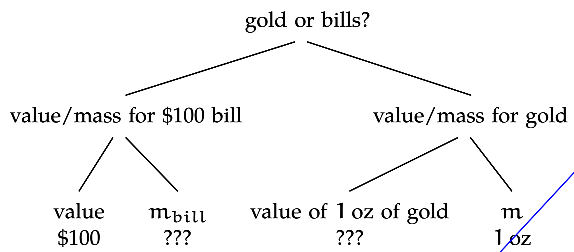

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I enjoy reading annodotes like this because I am able to relate things unrelated to mathematics, physics to approximation. It makes everything seem more real and useful.

I agree. It makes the process feel more intuitive.

kind of a random piece of knoweldge to know

excellent anecdote, but is it expected we will develop such a large repertoire of anecdotes to help ourselves out?

I just saw this yesterday on The Pacific. I was wondering why it seemed so cheap

I'm amazed that you know this bit of history! And yet, not the price of a current-day gold piece

Right, if I had to look this up - might as well look up price of gold currently. I would try to estimate how much my last purchase of gold weighed and how much it cost instead.

Where did you get $35? Or is that a historical fact that you used because you knew it. If we weren't so gifted historically, how would we get an estimate?

Oh nevermind. I guess no matter what the estimate, if we could figure it out to be around $50 or so, it puts you in the ballpark of your $800 estimate.

But I still do feel that we have to rely a lot on previous knowledge or fact.

I'm way less likely to know anything about this than the current value of gold, which I also do not know. I'd be in trouble on this problem.

I don't think he is suggesting this as the "right" or only way of going about estimating this. Since he seems to have a great memory of history then this works for him. I personally think trying to remember the weight and price of a recent gold purchase is another fine approach.

As someone that has never researched the price of gold or purchased gold in the recent past I wouldn't have any clue where to start. I guess after doing the reading I could start with a guess of $800/oz though

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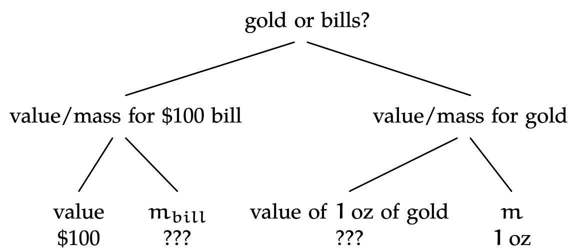

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Hence all the commercials saying "trade your cash for gold!"

Not sure that this value is accurate anymore...I thought gold had been trading at more than $1000/oz for some time? $1000/oz is a nicer number to work with anyway.

Well like anything the price of gold goes up with demand, so all the commercials saying cash for gold are trying to cash in on the huge swing towards gold.

Yup, it's been trading at around $1000/oz. Since I haven't read till the end yet, I'm assuming there's a reason he's using 80/800/8000?

I agree - if I were doing this problem I would have chosen 100 - more accurate and easier to deal with

You mean $1000, not $100 right? Gold is almost at $1200 to be exact.

Yep, I looked it up too, just to be sure. It's pretty close to 1200$/oz.

strangely enough, if you say there is 4% inflation per year 35*(1.04)^80 = $806.74

so it's totally reasonable, but isn't 350 closer to 80 than 800?

I feel like I wouldn't know this kind of information off hand. I guess if I really wanted to make an accurate guess, I would google the value of gold before hand.

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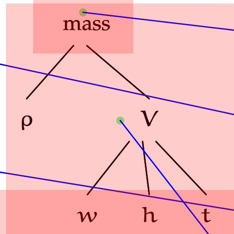

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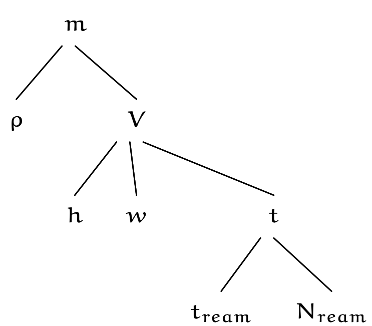

What is the estimate for ρ, the density of a bill?

Even though the information in this tree is obvious, I still like that it's here because it adds clarity.

If we are going to use the volume to calculate the mass, why don't we also consider the volume as part of the bigger problem.

what is a good way to estimate the thickness? since its so small

I don't think you would estimate the thickness of an individual bill. But what if you estimated the thickness of a stack of bills? You know that a stack of 500 pages of paper is 2-3 inches (thanks to Athena paper!). I would say the thickness of a bill is similar to the thickness of a sheet of paper. Does someone have a better idea?

I like how you created an additional tree instead of trying to cram this stuff onto the previous tree. It makes everything look more simple and clear.

If we could use a ruler on the bills, couldn't we have done the same for the piece of paper...

I don't think he's saying out should measure it, just to get an idea in your mind of how big it might be. The paper would be bigger then a standard ruler.

handy trick... a dollar bill is about 6" long. i've been known to use it as a ruler before.

This would be a more interesting question with a different currency, with different sized bills, like the Euro

Is it necessary to eyeball when you could use your body to measure. For instance, your thumb is probably about 1 inch from the tip to the closest knuckle. Why not measure the bill using that.

This reminds me of the reading yesterday, however, if we did all of that approximation, it would get pretty complicated.

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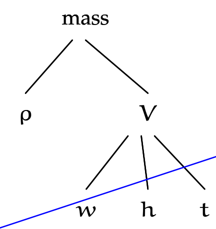

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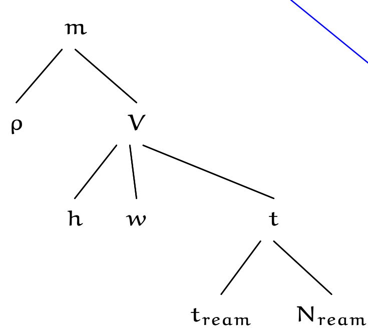


▶ *What is the estimate for ρ, the density of a bill?*

My favorite technique is to use various ’segments’ of my fingers for measuring. I know the last segment of my index finger is an inch, the second to last 1.25", and various others. I also know that my hand spread is 8.5", for larger measurements.

This comes in handy since I think most people (myself included) tend to misjudge how big or small an inch actually is.

yeah, I think this is a great tool....just so long as you have stopped growing. Don’t have a 13 year old try to memorize his dimensions. Although it might be easier to find a segment that is close to standard (1in, 2in, 10 cm, etc.) since 1.25 although its exact, might not be the easiest length to work with .

I was thinking it’d be easier to estimate a stack of bills, like you see in the money suitcases of movies! That’s closer to what you’d pick up anyway.

who has experience with a stack of bills in a suitcase??

Maybe just guessing from seeing them in movies... I think the height and width are easy to estimate, but the depth is harder and it would probably be easier to do in stacks. Its hard to estimate when you get as small as mm.

Yeah i agree. Also, we could assume cash weighs about the same as printer paper. We know roughly how much a ream of paper weighs then we can divide that by about 6 to get a stack of cash.

This is an awesome estimation idea, I would have never thought of it myself-thanks!

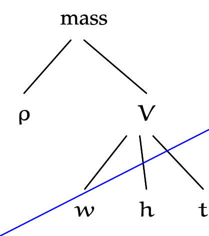
good life skill!

Should we make guesses within a range first? I has been very helpful to me to make an estimate since some ranges just seem foolish

I feel liek this should have been said earlier in the book .. reiterating it is good, but i feel like this particular verbage would have been more useful earlier.

redundant

I like this suggestion.



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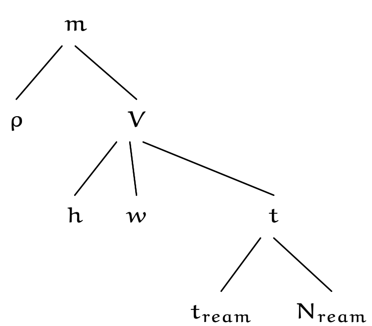

▶ *What is the estimate for ρ, the density of a bill?*

This actually is true. I can usually accurately estimate length of up to 3 meters with 4-5 cm accuracy. Not too bad...

This reminds me about a lecture a couple weeks ago talking about "perfect practice" (the lecture about chess masters).

Would it be easier to measure the thickness of a stack of bills, and then divide by the number of bills in the stack? It seems like that would be a lot easier, unless there's some issue with that method that I'm not seeing.

I would think that unless you had a stack of freshly printed bills this method wouldn't work because the wrinkles and creases in used bills are probably much larger than the thickness of a bill. So a stack of bills would probably be 3 or 4 times the thickness it should be.

For the Mech E students, recall that a sheet of paper is about .003". You use this to touch off the tool tip w/o breaking it.

let me just whip out my calipers... (yeah, course 2!)

SHould this have despite?

Should this say "having no experience with such small lengths, my eye does not help much"?

Agreed.

you have an extra "not" in here

minor quibble– i have no idea what section 8.2 is. can you start naming the files with their section number rather than reading number so we can go back and reread them?

If he renamed them I think the text would get really wordy- since this is meant for people who will own this book, it will be really easy to flip back to section 8.2.

I really appreciate the reference and reiteration of this concept.

I agree. The succinctness was also great such that if you remember what was discussed in 8.2, you're not bombarded with a long review.

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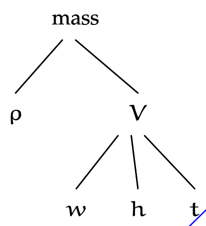

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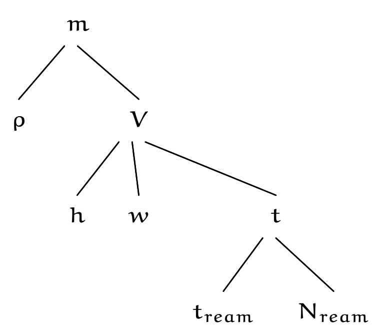

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hmm, so I thought they were made from cloth. I looked it up and it turns out they are part paper, and part cloth (cotton, silk and linen).

Bills really have silk in them? wow. Explains why the CAN go through the washer...

Lol, maybe to be more exact, we can say 2 bills is as thick as 3 sheets of paper. I'm not sure if this is necessary in an approximation class though.

wouild it not be easier to conform a number of bills to 1mm and count the number and deduce the thickness that way?

Is this the same ream that our paper comes in?

Yeah–obviously the bill paper isn't the exact same as our athena cluster printer paper, but for the sake of estimation it will give us a pretty close estimate.

Yeah, a ream is actually an official unit of measurement for paper - 500 sheets.

I wonder what the range of thicknesses of paper is- I bet their actually is a lot of variety

I was just thinking of this as a plausible way to figure it out

I enjoy the way you estimate using daily items, however, in cases like this, when we would have to go out to search for a ream of paper, it would seem that a better example might be possible.

As others have suggested I thought this was a bit involved and would've been easier if you stacked 1's to make 1mm and then figured out the thickness from that.

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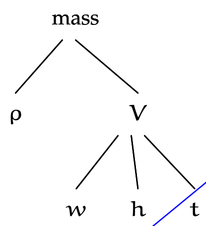

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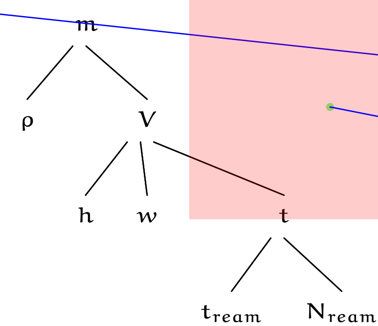


▶ *What is the estimate for ρ, the density of a bill?*

Above, I assumed we would figure out the thickness by stacking enough bills to measure a millimeter or two. I'm happy to see a version of my solution was eventually used.

Yes, me too. I just made a comment above asking if it would be easier to measure a stack of bills instead of just one. Looks like I should have just read a few paragraphs ahead!

Ha, I didn't think of this at all... I wanted to take it from the other approach of thinking of something comparably small (i.e., the width of a hair), but this makes more sense.

This is a good approach; one I didn't think about until I got here. For the above comment on stacking bills; bills get wrinkled and unless you ironed 100 bills and stacked them, compressing slightly to take out air, you won't get an accurate result. The paper in a ream is compressed and all nice for your ease of estimation.

It seems like most used bills take up significantly more space vertically when stacked due to wrinkles that develop through use. I don't think this approach would take this into account.

I feel like we have already learned this from d&a questions in the past. When we estimate things such as the fuel efficiency of a plane, we "multiply" by relating to a long plane ride in which we know the price of a ticket, # people on board and time of flight.

Clever. A really simple idea, but I would have never thought of putting a ream of bills together, measure their thickness, and divide by the number of bills.

I feel like this paragraph would also go better in the D&C section ... it's really well written

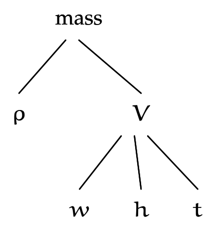
This is such an incredibly useful thing to know.

It's nice to point it out explicitly too, even if we have done in before.

Yea, I'd never really thought about it like that but its what we try to do all the time.

typo: repeated "and"

Since this is the same chart as at the top of this page, maybe for space efficiency you only need this one?



For the bill, its mass breaks into density (ρ) times volume (V), and volume breaks into width (w) times height (h) times thickness (t). To estimate the height and width, I could lay down a ruler or just find any bill – all US bills are the same size – and eyeball its dimensions. A $1 bill seems to be few inches high and 6 in wide. In metric units, those dimensions are $h \sim 6\,\mathrm{cm}$ and $w \sim 15\,\mathrm{cm}$. [To improve your judgment for sizes, first make guesses; then, if you feel unsure, check the guess using a ruler to check. With practice, your need for the ruler will decrease and your confidence and accuracy will increase.]

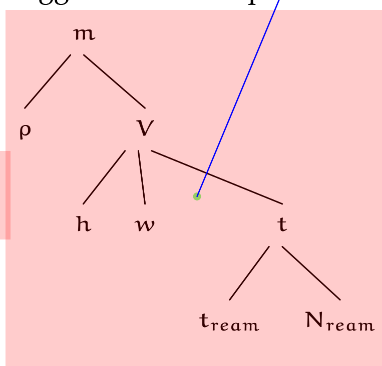
The thickness, alas, is not easy to estimate by eye or with a ruler. Is the thickness 1 mm or 0.1 mm or 0.01 mm? Having experience with such small lengths, my eye does not not help much. My ruler is calibrated in steps of 1 mm, from which I see that a piece of paper is significantly smaller than 1 mm, but I cannot easily see how much smaller.

An accurate divide-and-conquer estimate, we learned in Section 8.2, depends on replacing a vaguely understood quantity with accurately known quantities. Therefore to estimate the thickness accurately, I connect it to familiar quantities. Bills are made from paper, a ubiquitous substance (despite hype about the paperless office). Indeed, a ream of printer paper is just around the corner. The thickness of the ream and the number of sheets that it contains determines the thickness of one sheet:

$$t = \frac{t_{\text{ream}}}{N_{\text{ream}}}.$$

You might call this approach ‘multiply and conquer’. The general lesson for accurate estimation is to magnify values much smaller than our experience, and and to shrink values much bigger than our experience.

The magnification argument adds one level to the tree and replaces one leaf with two leaves on the new level. Two of the five leaf nodes are already estimated. A ream contains 500 sheets ($N_{\text{ream}} = 500$) and has a thickness of roughly 2 in or 5 cm.



▶ *What is the estimate for ρ, the density of a bill?*

You might want to do the division out for the thickness of an individual sheet of paper here as it feels like you kind of just leave us hanging.

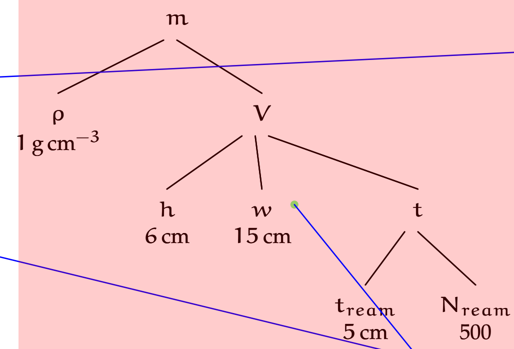
I don’t think it is necessary as he has written the formula for it above.

So we just assumed that the thickness of printer paper is the same as cash?

this gets you an answer surprisingly close to the actual thickness of paper - as someone who does a fair amount of machining, I know regular printer paper is about .003" thick.

I find this chart more confusing than the others.

The only missing leaf value is ρ, the density of a bill. Connect this value to what you already know such as the densities of familiar substances. Bills are made of paper, whose density is hard to guess directly. However, paper is made of wood, whose density is easy to guess! Wood barely floats so its density is roughly that of water: $1\,\mathrm{g\,cm^{-3}}$. Therefore the density of a bill is roughly $1\,\mathrm{g\,cm^{-3}}$.



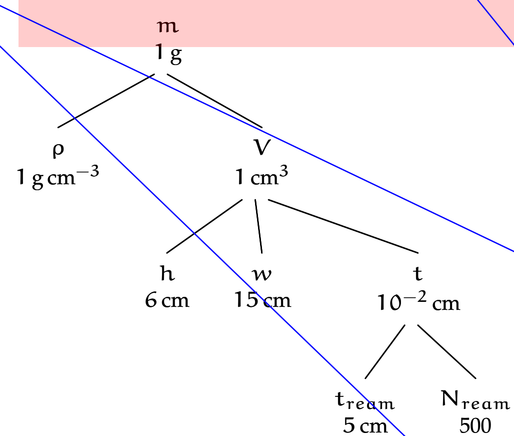
Now propagate the leaf values upward. The thickness of a bill is roughly 10^{-2} cm, so the volume of a bill is roughly

$$V \sim 6\,\mathrm{cm} \times 15\,\mathrm{cm} \times 10^{-2}\,\mathrm{cm} \sim 1\,\mathrm{cm}^3.$$

The mass is therefore

$$m \sim 1\,\mathrm{cm}^3 \times 1\,\mathrm{g\,cm^{-3}} \sim 1\,\mathrm{g}.$$

and the value per mass of an \$N bill is therefore \$N/g. How simple!



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Obviously bills are different than paper, how much of a change would that create in the approx answer versus the actual?

I agree, I think the breakdown here is informative but when carrying out as many dollar bills as possible, a small variation in density could be crucial to the answer.

Are cotton and silk really that much denser than paper? I don't think they necessarily are, so it wouldn't be that bad to estimate them as wood. However, I don't know the density of gold off the top of my head and that might be harder!

Agreed – cotton and silk seem like the would also both barely float, giving them densities similar to that of wood.

Dollar bills are actually made from a mixture of cotton and linen. But this estimate should be pretty close still–all plant material.

and wood pulp/paper. http://www.ehow.com/how-does_4613179_what-dollar-bill-made.html

don't believe the first yahoo! answers that comes up when you search "what are bills made of"

Yep, I had the same thought; bills are not made of wood.

I like these figures. They make the writing very clear.

While it makes sense, I'm not that comfortable using this method. I don't really like it.

I agree - I think paper is pretty removed from wood in my mind...

If you don't want to trust this, weight a ream of paper and divide by its volume. I haven't done it, by my guess is that it checks out.

another way to think of it is that the weight of a ream of paper is about the same weight as a similar sized piece of wood. so a ream of paper is pretty heavy, but if you think about it, the same volume of pure water would probably be heavier than a ream of printer paper.

Aren't American bills made from cotton fiber paper? I'm not sure if this type of paper has the same density as wood.

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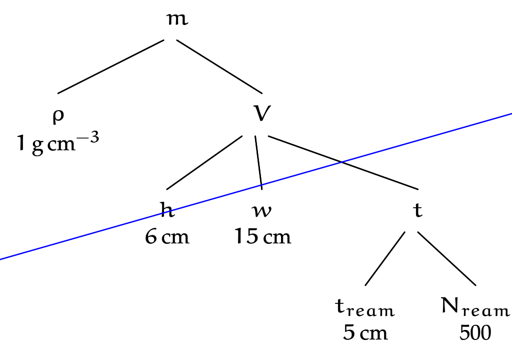

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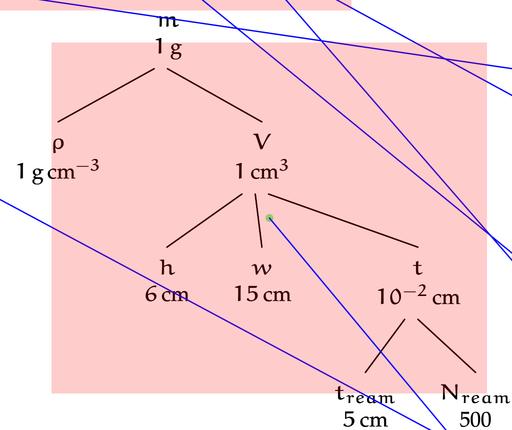

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What also floats in water? A duck. Exactly! So, logically... If... she.. weighs the same as a duck, she's made of wood. And therefore–? A witch

Monty Python would disagree with you here.!

Sadly, I had the same thought when I read that phrase. Also, not all wood floats, for those amateur boat-builders out there.

you could also get this guess by noticing that paper floats in water and thus is less than or equal to the density of water.

True. I guess we could reason that loggers use rivers to transport large pieces of wood, so the wood used in paper floats..

barely? I always thought wood floated pretty well. I mean, it can float with extra weight ontop of it even. (ie: a raft)

really interesting and cool way of approximating the density of a bill by relating paper to wood and considering the fact that it floats on water.

I understand this is a close approx. but why not use something like .9? It will likely play out in the long run as a significant mass when we multiply by thousands of bills.

It will still only be 10% different, and like we have been talking about in class, a factor of 1.1 is mostly going to disappear in combining uncertainties. Are you really sure it's closer to 0.9 than 0.98?

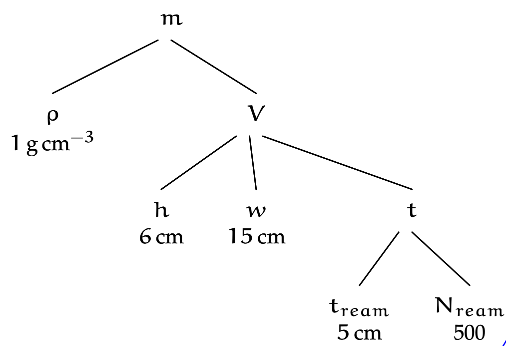
This should stick with the formatting of the rest of the paragraph, imo.

This is really cool. Though I feel that paper floats more easily than wood.

Another fun way to do this would be to fold a bill into a tiny box, and estimate its volume. Mine (it was a flat box) was 3mm*15mm*22mm=1.4cm^3. Pretty close, especially considering it's not perfectly compressed.

That's really clever! I wouldn't have thought to try that.

these figures really help me understand what is going on



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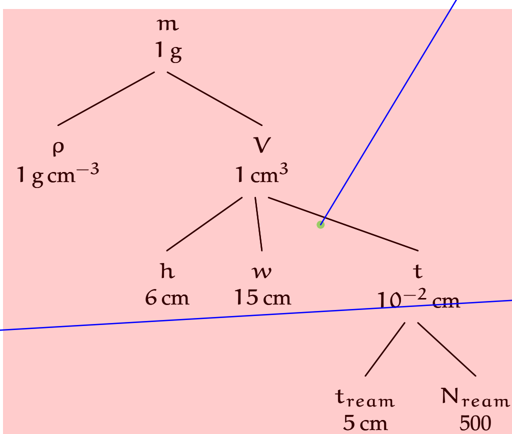

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This might be a personal thing, but I feel these trees are a bit skewed and stretched out of place with respect to the text. Any way of reformatting them?

They do seem to be a bit funny since the branches are rather one sided, but I don't think it's problematic to the point of being detracting.

They could be more compact, but I like how there is an illustration for this problem considering we just dealt with a lot of known values.

I thought we were going to change to having sideways trees with the method of acting on them written on the branch? (multiplication, addition, all that stuff)

I plan on trying that. I still need to rewrite the tree-language-to-pdf compiler to generate trees that grow horizontally. Once that happens, all trees will grow sideways (and I'll probably need to check that they aren't too wide).

I agree. The diagram is useful in keeping track of all the values we've just estimated. Also it's nice to come full circle back to material from the beginning of the class, and the diagram helps to reinforce that idea.

i just tried to guess the mass of of a bill and came to this number. i feel like we can guess this, from things like knowing the mass of paperclips, pretty well. at least personally, i'm better equipped to guess the mass of a dollar bill than it's density and thickness.

I probably would have guessed that too from the very beginning. But since this is such an easy number to remember, I'll never have to do the calculations!

I think I'd personally hesitate when guessing something with a small mass just because I'm not really attuned to what a gram feels like (even versus a paperclip, cuz it has a higher density).

My first intuition was that it would be smaller than a gram, so I would have been off by at least a factor of 2 here if we hadn't done out all the calculations.

Having held a ream of paper, it seems odd to me that it would float, even though I looked up the density of paper and found that it is quite close to this estimate.

Try thinking about picking up a log of wood. It also floats but is pretty heavy....

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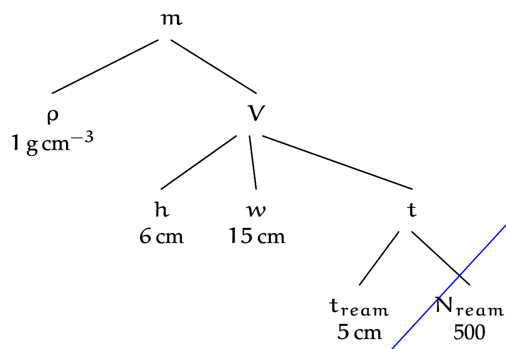

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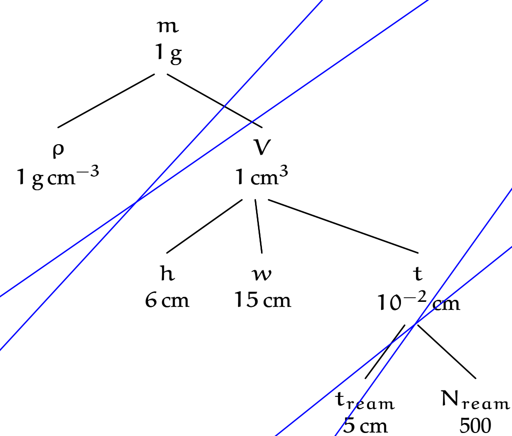

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Grams and ounces in the same problem? i see comparison problems here.

I think the problem is that the price of gold is often specified in terms of USD/oz instead of USD/g (since the US uses imperial units). Converting shouldn't be too much of a problem, just another step or so.

Amazingly yes! This problem was a nice break from the drag concepts and a good refresher on dividing and conquering. Especially since so many of the unkowns were tricky, at least for a bit, to relate to a known fact.

vague/unclear what it refers to. Compare the value of per mass of dollars to the value per mass of gold.

I don't really see how this is vague or unclear... I thought it read fine.

This might be a slightly annoying, trivial comment, but I personally find arithmetic done in the paragraphs, even if it is really simple, harder to read than if it had just been split off.

Not something I can say is in my immediate memory.

I was trying to figure out how i would get this not knowing it... I think I would use measuring cups in my kitchen. I know they have both cups and liters on them, and I know how many ounces in a cup and how to convert from liters to grams for water.

I always find it easier to remember that 3.5 oz = 100g. And I know that from working with yarn...

Good to know. It does depend on the material tho right? Ounce is a volume while grams are mass?

Nice fudging to make the calculation easier.

You say that the value of gold is about \$28/g, and the value of a \$20 bill is \$20/g, and yet you still say to take the \$20 bills before the gold. Contradiction?

He says take the gold before the \$20 bills...

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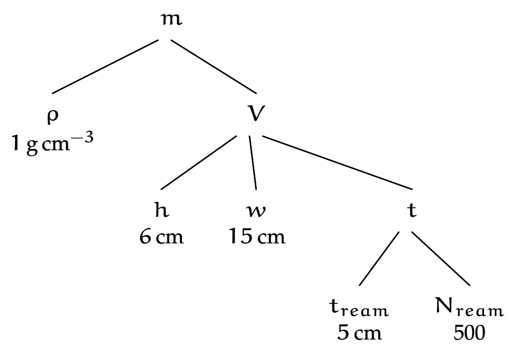

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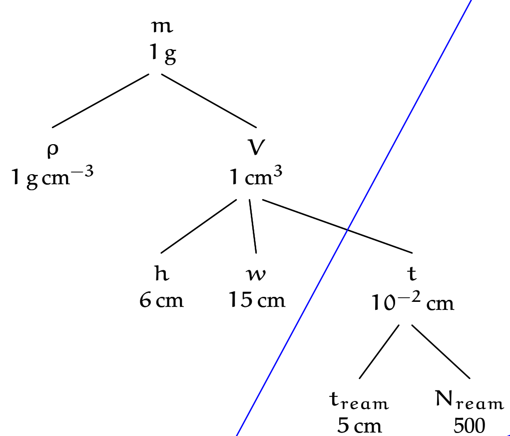

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Good to keep in mind..

This problem is really counter-intuitive, most of us (including myself) would have initially said take the gold...quite an interesting and useful conclusion though!!

I wonder which is actually easier to launder. I wouldn't have the slightest idea how to launder gold bricks, but I feel like they could track the serials on 100s.

I was just thinking the same thing..the police can definitely track the serials fairly easily, whereas it think it's pretty easy to sell gold on the black market. You'd probably be in with that crowd if you were robbing a bank.

you could melt it down.

also...hahaha i had totally not thought about the different value amounts of the bill....hahaha

I think if you're stealing that many \$100 bills, it wouldn't be difficult for you to disperse it overseas, or to use it as "credit" without anyone actually spending it publicly.

Especially if you don't go and spend all of the bills on one big thing.

This is in interesting question with unexpected results. Thanks for sharing...although hopefully I will never need it.

So even if gold is valued at \$1000/oz as someone said before, then it doesn't change our answer because 1000/28 <50.

the current price of gold is 1100 per ounce.

So it still makes more sense to grab \$100 bills. Interesting! I wonder if this changes if we know someone who can turn the gold into jewelry, because per ounce i'm sure that is far more valuable, although there are time-hours in there too

Yeah, this is an interesting result. I wonder if the result would be the same if we chose to consider volume as the limiting factor, and not the mass. I'm guessing the higher bills would still win out since a gold brick is so large.

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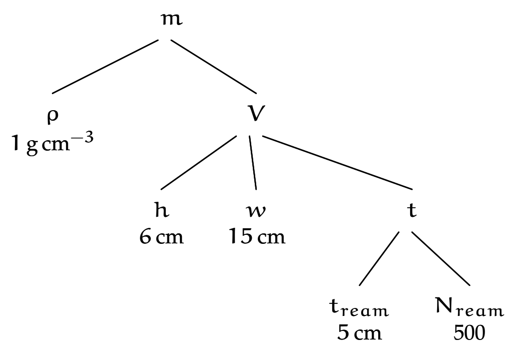

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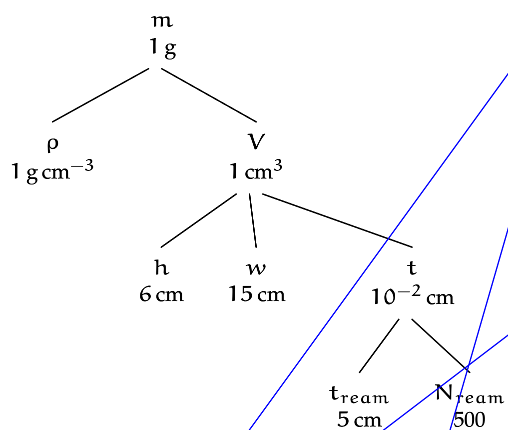

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Is this assuming we are only grabbing bills? Because otherwise wouldn't it make more sense to grab the 100's, 50's, and then some amount of gold?

also do banks store \$1000 dollar bills? or would these be way too easy to track?

That's what he's doing here–grabbing the 100's then the 50's then the gold then the 20's. the 20's have the least value per mass as shown by his calculations for bill values/mass

I'm surprised that you would even take the \$50 bills before the gold!

one thing to think about is that 20 dollar bills are far more widely circualted than 100 dollar bills. as a result depending on the weight that is determined that one can carry it may be more advisable to steal the 20 dollar bills first as they are harder to trace. just a thought..

That's adding in the problem of what to do with the stolen goods once we have them, which is outside the scope of this class and would probably get Sanjoy in trouble if he published it...

This seems really verbose. The word accurate/accuracy/inaccurate in used in these three sentences 4 times.

and propagate them upward – thereby obtaining plausible ranges for the value per mass of bills and gold.

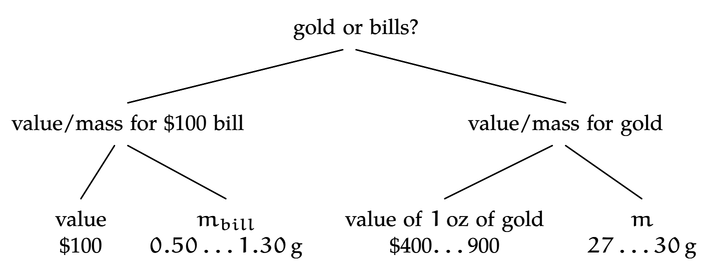
Problem 8.2 Your plausible ranges
What are your plausible ranges for the five leaf quantities t_{ream}, N_{ream}, w, h, and ρ? Propagate them upward to get plausible ranges for the interior nodes including for the root node m.

Here are my ranges along with a few notes on how I estimated a few of them:

1. thickness of a ream, t_{ream}: $4\ldots6$ cm.
2. number of sheets in a ream, N_{ream}: 500. I'm almost certain that I remember this value correctly, but to be certain I confirmed it by looking at a label on a fresh ream.
3. width of a bill, w: $10\ldots20$ cm. A reasonable length estimate seemed to be 6 in but I could give or take a couple inches. In metric units, $4\ldots8$ in becomes (roughly) $10\ldots20$ cm.
4. height of a bill, h: $5\ldots7$ cm.
5. density of a bill, ρ: $0.8\ldots1.2\,\mathrm{g\,cm^{-3}}$. The argument for $\rho = 1\,\mathrm{g\,cm^{-3}}$ – that a bill is made from paper and paper is made from wood – seems reasonable. However, the many steps required to process wood into paper may reduce or increase the density slightly.

Now propagate these ranges upward. The plausible range for the thickness t becomes $0.8\ldots1.2\cdot10^{-2}$ cm. The plausible range for the volume V becomes $0.53\ldots1.27\,\mathrm{cm^3}$. The plausible range for the mass m becomes $0.50\ldots1.30$ g. The plausible range for the value per mass is \$79...189/g (with a midpoint of \$122/g).

The next estimate is the value per mass of gold. I can be as accurate as I want in converting from ounces to grams. But I'll be lazy



and try to remember the value while including uncertainty to reflect the

Comments on page 4

I really like that this is here!! It allows for your above conclusion to be so much more concise, but for the people who want to see how you came to it, there is this section to look over. Really great idea!! I think this set-up would do well in other sections too.

How do we determine how much weight to assign to volume as opposed to mass?

why do you not take into account that \$\$ paper is about twice as thick as standard printer paper??

Thats just as easy as looking up a value on the internet for something you don't know, so is there a rule like "if its on the label it doesn't count" for estimating stuff?

> I was wondering about this too. I know he made the comment about paper reams being everywhere, but still it seems like cheating. How do you make the distinction between types of easy-of-access information?
>
> > Well, a ream is an official unit of measurement for paper - it doesn't change from company to company or label to label.

that is a super long bill

I'm a little confused here. It says the width of a bill is 10-20cm, as well as the length. This is a mistake right?

I feel like this is a particularly huge range for a bill that you (could) have right in front of you.

(To make my estimate I folded it in half and it appeared to be very close to 3 inches.)

> Very smart, folding the bill in half would increase our accuracy.

would it be better to say length and width of a bill? since bills don't stand up?

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Here are my ranges along with a few notes on how I estimated a few of them:

1. thickness of a ream, t_{ream}: $4\ldots6$ cm.
2. number of sheets in a ream, N_{ream}: 500. I'm almost certain that I remember this value correctly, but to be certain I confirmed it by looking at a label on a fresh ream.
3. width of a bill, w: $10\ldots20$ cm. A reasonable length estimate seemed to be 6 in but I could give or take a couple inches. In metric units, $4\ldots8$ in becomes (roughly) $10\ldots20$ cm.
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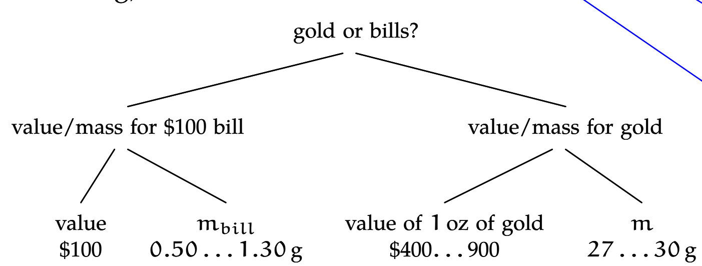
Now propagate these ranges upward. The plausible range for the thickness t becomes $0.8\ldots1.2\cdot10^{-2}$ cm. The plausible range for the volume V becomes $0.53\ldots1.27\,\text{cm}^3$. The plausible range for the mass m becomes $0.50\ldots1.30$ g. The plausible range for the value per mass is \$79…189/g (with a midpoint of \$122/g).

The next estimate is the value per mass of gold. I can be as accurate as I want in converting from ounces to grams. But I'll be lazy and try to remember the value while including uncertainty to reflect the



is this what we should be putting the answer boxes on our psets?

My answers are not quite so in depth, but this would be good. Alternately, you can outline your logic or assumptions or write your explanation as if you were giving hints to do the problem.

My answers are rarely this in depth - but I think he suggested only a few sentences for that purpose.

I like how this is laid out though. If gives a little more insight into how all the values came about.

This seems a lot like Sanjoy's explanations on the pset answers. I don't think ours need to be so in depth, but I guess it would be really useful for him to read exactly how our minds work!

From the grading policy, Sanjoy said, "For a reasonable effort on the homeworks, give a coherent one- or two-sentence explanation or reasoning for each problem... Write down what you could tell your earlier self in order to make finding the solution smooth sailing."

I think these long explanations are just to make sure anyone who was confused can understand it.

I think it would have been really helpful to have more explanations of this back at the beginning of the course when we were first learning how to get a feel for ranges using our gut.

We already established that it floats in water, so why couldn't we minimize this to 0.8...1g/cm^3?

Because it is so thin, my thought was that it might float due to surface tension rather than density. Like some bugs...denser than water but still float because of how their feet are arranged

wood floats, but does money?

I feel more comfortable dealing with this range, than using your original estimate of rho=1.

I say that a bill would float, so wouldn't that cut off the upper part?

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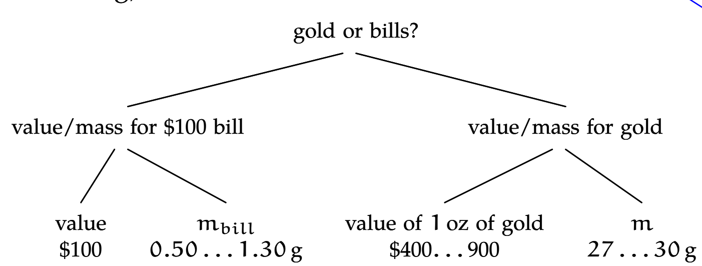

The next estimate is the value per mass of gold. I can be as accurate as I want in converting from ounces to grams. But I'll be lazy and try to remember the value while including uncertainty to reflect the

Maybe you could be slightly more explicit since this is a new paragraph and say upward through the branches in the tree.

This is an important concept that I think could have used more explanation. We went over it in class a bit and it seems crucial to getting a final range but the details were not clearly mapped out.

I'm still a little confused on the details actually...

Agreed. It's a neat concept, but the idea or purpose of this technique isn't really understood until after the example is finished. It would've been nice to explain the concept first in theory before doing an example on it.

I'm still not sure how you get these ranges? Did you just do out all the math of the previous section, or is there some shortcut?

for this propogation are you doing the same squaring/ square rooting as before?

Do you mean adding in quadrature? It's interesting to note that you can't always add errors that way, eg. if the variable in question does not go linearly or goes as a logarithm.

I would like this paragraph more if it were written with the math drawn out ... I feel like it would actually reinforce the lesson better ... rather than being an exercise in page flipping (i may be biased by the fact that going back here is _a lot_ harder than flipping pages) ... but it would be a lot better to see the numbers drawn out.

How are these endpoints arrived at? They are not merely the upper and lower bounds of the density and volume multiplied respectively.

I think he just gave a range of values that seemed likely to him. These values are much cleaner than if he merely used lower/upper bounds.

I feel like this is what we were doing the 1st couple weeks of class, just adding a probablistic element to it to make it more accurate.

How do we get these numbers from the \$100 value and the mass range of 0.5g ... 1.3g? The values of \$79 per gram and \$189 per gram are not (100/1.3) and (100/0.5). I'm confused here.

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3. width of a bill, w: 10 . . . 20 cm. A reasonable length estimate seemed to be 6 in but I could give or take a couple inches. In metric units, 4 . . . 8 in becomes (roughly) 10 . . . 20 cm.
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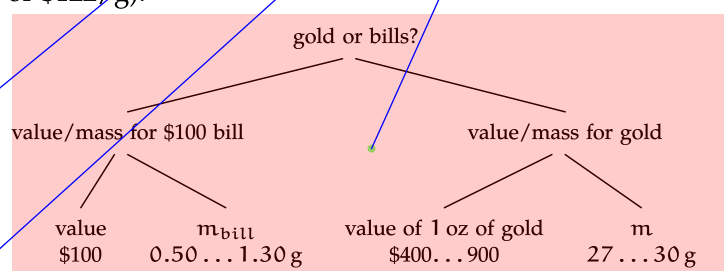


The next estimate is the value per mass of gold. I can be as accurate as I want in converting from ounces to grams. But I'll be lazy and try to remember the value while including uncertainty to reflect the

Even now, at the end of the class, I think these trees will be the most tangible thing I've learned in this class. They lay everything out so simply, a true epitome of the organization of the problem at hand.

They're a really nice formalism for something that most people try to do messily in their heads in all the time.

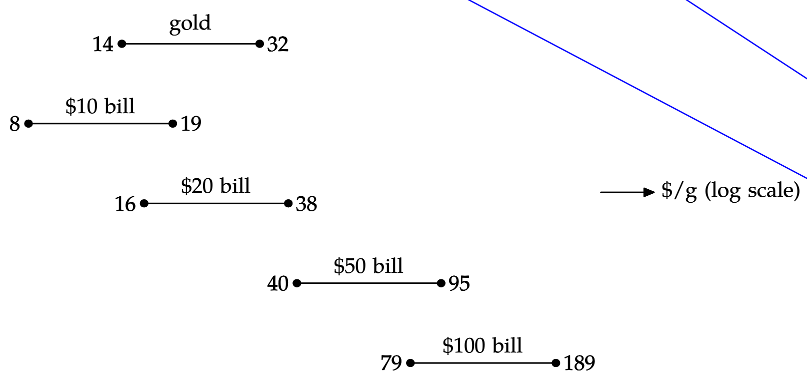
Yeah, the trees are especially helpful for me because when I try to work out problems in my head I tend to forget one important facet of the problem.. with the trees I can visualize the important pieces of information before I begin my estimations and calculations.

You say you'll be lazy instead and just try to do it from memory? What would be the other option? (I.e., would you just use more accurate estimation methods to convert it?)

Haha I just laughed a little. I like your bits of humor everywhere.

fallibility of memory; let's say that $1oz = 27\ldots30\,\mathrm{g}$. This range spans only a factor of 1.1, but the value of an ounce of gold will have a wider plausible range (except for those who often deal with financial markets). My range is $400...900. The mass and value ranges combine to give $14...32/g as the range for gold.

Here is a picture comparing the range for gold with the ranges for US currency denominations:



Looking at the locations of these ranges and overlaps among them, I am confident that the $100 bills are worth more (per mass) than gold. I am reasonably confident that $50 bills are worth more than gold, undecided about $20 bills, and reasonably confident that $10 bills are worth less than gold.

Comments on page 5

I am not sure if I understand the connection between this and probability? This memo seems more like a review of divide and conquer and some other methods we learned in the past.

> It uses probability to analyze divide and conquer. Perhaps I should make this example, without rangs, the vey first example in the book. Then return to it to illustrate ranges.

my understanding of everything so far is that the point of probability is to calculate the error ranges.

the placement or wording of this phrase sounds weird to me. But I understand the point you are making with it.

You mean in terms of guessing, right?

that is a very large range

A large range would reflect how certain I am about my estimate of the price of gold (not certain at all).

fallibility of memory; let's say that $1oz = 27 \ldots 30\,$g. This range spans only a factor of 1.1, but the value of an ounce of gold will have a wider plausible range (except for those who often deal with financial markets). My range is \$400. . . 900. The mass and value ranges combine to give \$14. . . 32/g as the range for gold.

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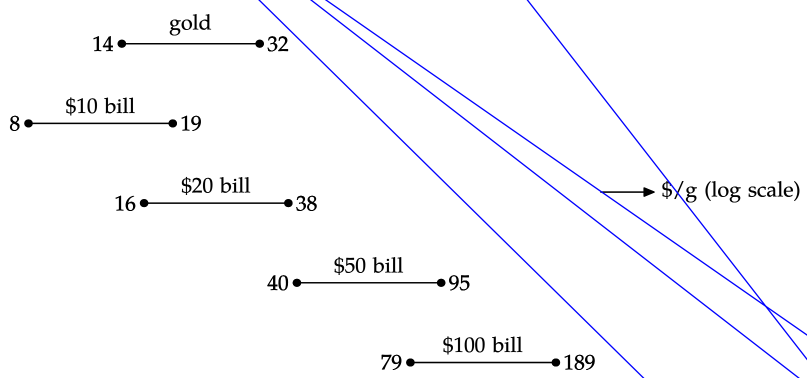

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This still misses the actual price by a decent margin.

It's interesting that Sanjoy's lower bound is 400\$ lower than his original estimate, but his upper bound is only 100\$ higher, especially given that the actual number is 400\$ higher than the original. Specifically, it's interesting that he was more sure it'd be lower than his estimate than higher, even given the current state of the economy.

even though he misses the actual price of gold, the guess is still close enough to tell us that if we're robbing a bank we should grab all the 100's (and probably the 50's). ie even if the price of gold was higher than what we used here, the value per mass of the 100's is still definitely larger than that of gold.

There are two answers to that. First, it's good if some of the ranges don't include the actual value. Otherwise I'd wonder if my ranges were too wide. I do want to be surprised once in a while (roughly one-third of the time).

Second, I wrote the main text in 2008 before the financial collapse. And financial collapses usually increase the value of precious metals. So if I had included that idea when I estimated the price, I would have used a higher range – maybe raising all the prices by 20 or 30%.

I just looked up the average price of gold in 2008 and it was \$871.

I would have expected this too be much higher.

I would have expected this too be much higher considering how much we value gold.

It seems to me that probabilistic divide and conquer in this case simply adds more of a range for uncertainty, a little bit more robust way of producing our ranges on our answers to hw. Graphic below really helps.

interesting visual representation of the situation.

fallibility of memory; let's say that $1oz = 27 \ldots 30\,\mathrm{g}$. This range spans only a factor of 1.1, but the value of an ounce of gold will have a wider plausible range (except for those who often deal with financial markets). My range is \$400...900. The mass and value ranges combine to give \$14...32/g as the range for gold.

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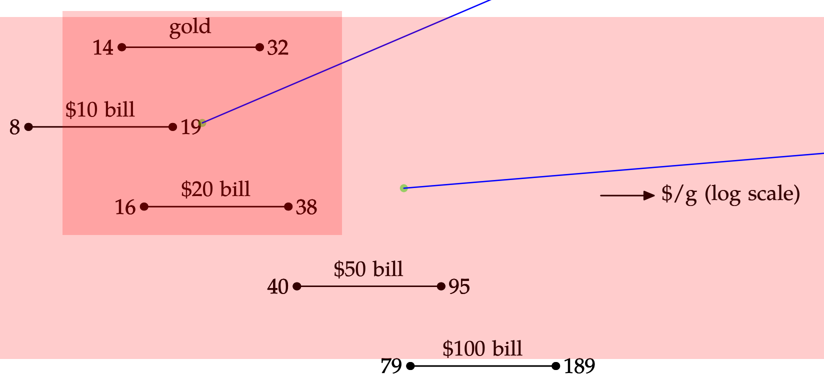

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Earlier you say to take the gold before the 20s. This seems inconsistent.

That was when we made just one estimation. Wit this range it seems "likely" that the gold comes before the 20's.

His point here is that with just a single data point, he estimated that you should take the gold before the 20s. With a more careful calculation figuring in his relative uncertainties he realizes he's not actually so sure that taking the gold is better than taking the 20s.

I really like the picture it just might be easier to see the meaning right away if there was some kind of axes framing the data, just an x-axis.

i really like this figure

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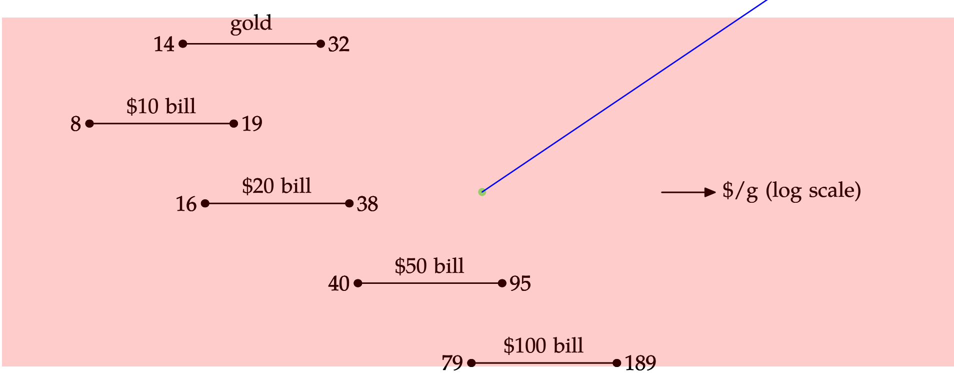

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is this the new method we are learning in this chapter? Everything else seemed like stuff we learned a while ago

From class it seems that this chapter is focusing on ranges and creating an accurate range in order to determine the best answer. This chart is very easy to read and makes comparison much easier.

I really like this chart. It very exactly answers the question we constantly ask about how much our wrong approximations could affect the final answer.

I like the chart as well. Graphical demonstration of exactly what we just went through!

This is a beautiful graphic. It really helps to show what you were doing previously, my one question would be why you have said earlier to take the gold as opposed to the 20's? Looking at this, I would take the 20's.

I absolutely agree, the chart enables me to conceptualize the ranges and how they would effect my approximation...which through the +'s and -'s of each estimation, is usually amazingly close!

Also, will we ever quantify uncertainty in other ways than ranges?

I agree, this diagram really helps illustrate the uncertainty

I really appreciate the chart as well. It makes more sense than explaning it in words and is pictured better as well.

I agree that this diagram is really helpful, but it misses one point, which is that the value/mass for each of the bills is not independent – they're all the same size and (roughly) the same material, so knowing where in the range the \$100 bill lies will tell you where the \$50 and \$20 and so on lie. This chart, however, makes it seem like there is some change that a \$100 bill is worth less than a \$50, which is obviously false.

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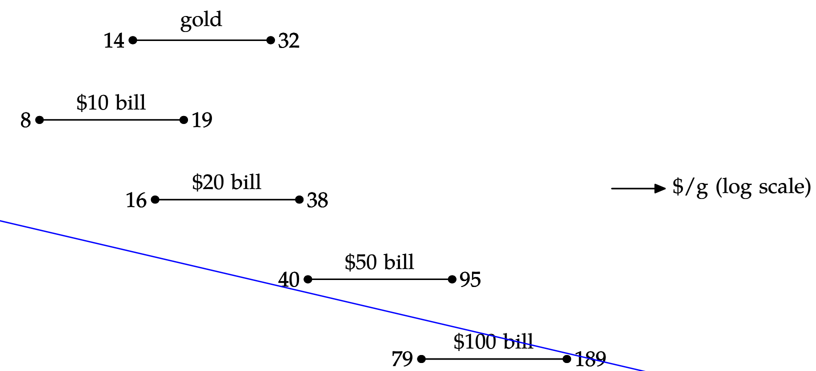

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That's a good point. It's hard to display joint probability distributions, and this diagram "solves" the problem by ignoring all the correlations. But it should be sufficient information for the original question about what to take when you are in the bank vault.

The correlations are useful once you find out more information. For example, suppose that you later find out the true mass of a bill. That would shift all the ranges for the bills. But that information would probably come only after you leave the bank vault (unless you bring a scale with you!), so the correlations wouldn't affect your decision about what to take.

I really like this picture. It sums up the entire problem in an easy to understand way.

I think what could make this picture a little better woud be to put what we got and then the errors on both sides. I think it's called a candlestick or something in that respect, but otherwise, a very helpful picture, and I wish we had had mroe of these.

yep I agree, it is a great way to visualize the ranges.

This is a good, clear way to show us how you analyzed the results you got.

why did you decide to put this on a log scale?

I think before calculating, I kind of guessed this answer, but it was cool to follow through the 6.055 method.

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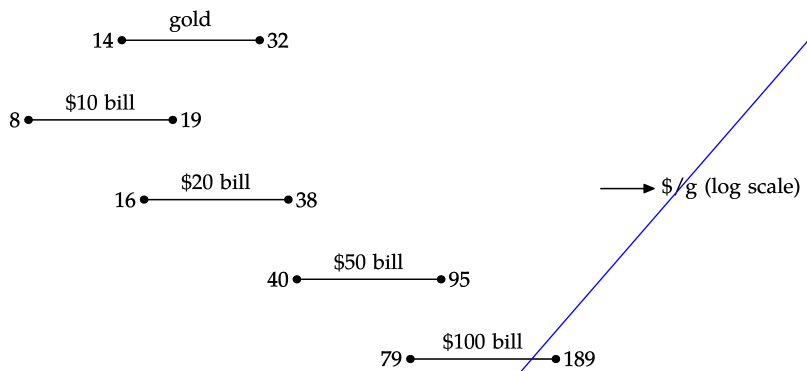

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I guess I was wrong. I'll remember to take the bills instead of the gold next time I rob a bank.

Huh, interesting, never would have expected the difference (even accounting for uncertainty) to be so big. Value of gold per gram is roughly the same as that of a \$20 bill? No wonder the price of gold has been going up...

I guess banks should carry mainly gold bricks as opposed to bills if they want to lose less money in the event of a robbery.

there are, however, the biggest thing that this doesn't take into account is that banks dont keep gold bricks. they have safety deposit boxes – full of jewelry (and other things they're less likely to report missing) ... this means that (1) they don't necessarily know the "size of your score" and (2) what you're stealing is not generally just gold...it includes all of those gems that are set in that gold ... which is generally worth _much_ more than the gold its self.

I like how we're using probability to double-check our approximations.

This use of probability really makes me more confident in using "everyday values" for divide-and-conquer methods.

I don't feel like we're using probability though. These are just ranges, not probabilities. They are somewhat similar, but insofar as the calculations go, no explicit probabilities were used.

So by "using probability", does that just mean establishing a range of values that you're reasonably sure are correct (like 2/3 probability you are correct, and 1/3 probability you are incorrect)? Personally, that doesn't feel like applying probability. That just feels like the basic estimating skills we were taught from the beginning.

this is an awesome problem- I really really liked it

this section could theoretically be right after divide and conquer and allow for you to calculate errors for al of the chapters afterward

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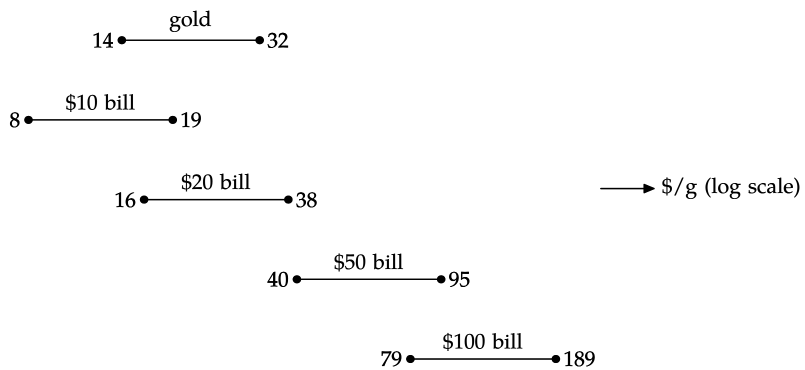

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I really appreciate this example as a stepping-stone from the previous reading. I feel like it added just enough additional complexity so it didn't feel like we were overworking the problem, while at the same time wasn't very difficult following the simple example from the previous reading.

I also thought that this reading was excellent. I really enjoyed it, and it was very well-explained and tied the probabalistic methods and divide-and-conquer methods together. Well done.