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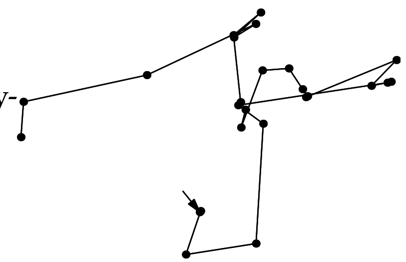

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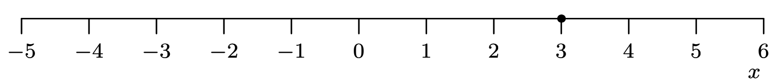

This was a rather lengthy reading, at least compared to the other ones. I feel like you tried to give me too much information at one time.

That's probably true. I'll aim for better planning next time...

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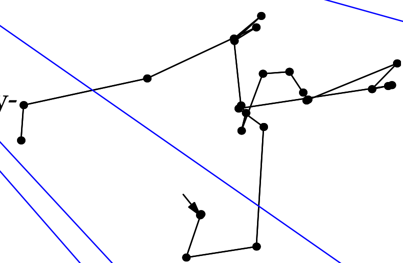

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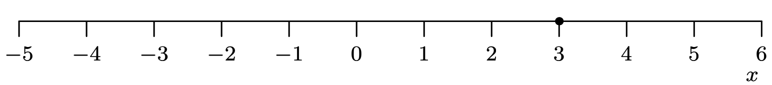

Read this section for Friday (memo due Friday at 9am).

maybe it's better to give some context because not everyone knows about the game

I think the rest of the examples convey the idea of something moving through a field toward an eventual destination.

I agree. It's okay that this reference might not reach everyone because there are so many others. But I do think this example is pretty well known.

what is a random walk? maybe your going to define it later

that's all i did in 4th grade

that was what we played after we were "too big" for train.

What is train?

War!!! Yes!!! Wonderful.

I though train was still more fun. War got boring =D

I think it would be more effective to remove this sentence, and then just start with the questions. The questions make an easy transition to the next paragraph.

I agree, I was in the mindset of 6.00 graphs and nodes. And then I got confused for a bit.

Yeah, I too agree. This sentence seems somewhat out of place compared the rest of the content.

I have been introduced to this concept before in my probability class but I am really interested in learning it in the context of 6.055.

I never thought about that...

huh? the walk or the card game? this probs should get reworded.

I think it's pretty clear it's referring to the card game.

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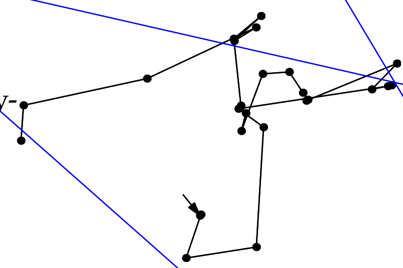

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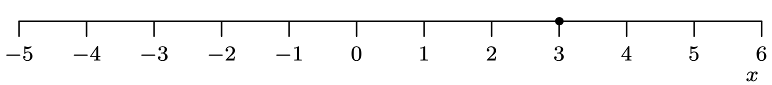

It would work better if this were another question the way the War example is.

i agree. this is a strange transition between thoughts.

Really random transition! But I am excited that we might talk about neurons and signal transduction

FOREVER and I felt like I always lost and for someone who really likes to win that brings back some bad memories

This paragraph seems very disjointed. You immediately go from talking about random walks to card games to neurotransmitters. The switchover to neurotransmitters is the worst. The sentence looks like you've been talking about neurotransmitters for a paragraph, when you really just started it. You should start this paragraph with the first sentence. Then define a random walk. Then say that some examples are card game length, neurotransmitters, etc.

yeah I agree...this list of unrelated examples at the beginning without a definition of random walks is pretty confusing.

Be careful here. You're implying that the synapse is at the neuromuscular junction. If it is, there is no "random walk"–it is guaranteed that your muscle will twitch. You don't want to rely on probability when a lion is chasing you, etc.

I feel like the bigger problem here is that he left out all the stuff in between. He goes very detailed about the neurotransmitter, then jumps to a leg twitching. Why not just say "and a signal is passed to the next neuron" or something more accurate.

more than you wanted to know about vesicles: http://en.wikipedia.org/wiki/Vesicle_%28Biology_and

Every time I've learned about random walks they mention the "drunken sailor." Perhaps it should be included with these examples?

I agree with this - its always presented as a man walking left or right with the question of will he get home. A quick mention of this could jog some people's memories

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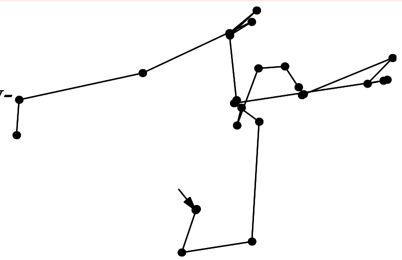

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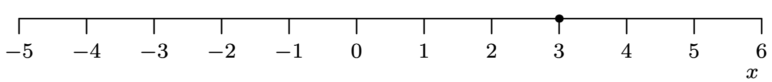

Um... what?? This paragraph is worded VERY strangely, with no transitions whatsoever between thoughts. I don't know what the exact definition of a random walk is, and this opening paragraph just confused me. It would be better to put the formal definition first.

I agree

I actually liked this paragraph. It gave a lot of examples demonstrating that random walks are everywhere, giving me some insight into what a random walk may be, before telling me the definition.

I agree with the original comment, that the paragraph was worded strangely. Perhaps more obvious transitions between examples would have helped the flow and help us understand what a random walk is before the formal definition is given.

I agree with the second comment. I liked the paragraph and although it did explicitly define what a random walk was, I was able to kinda get the idea from the multiple examples given. Plus I kinda assumed that a random walk was kinda self explanatory: the path that results from someone or something walking randomly with no logic....completely arbitrary. The wording doesn't bother me because it sounds very conversational

Yeah I would rather be given something to think about then be told a formal definition. It helps me make the idea my own and really understand it.

I like the idea of this paragraph as an opener, but I agree it's really choppy and kind of hard to follow.

Talking about random walks reminds me of the example from the beginning of the semester when we questioned whether a person climbing up and down a mountain would ever be at the same place at the same time, I really liked that example so I am hoping this discussion is just as interesting!

This would be a good example to mention again in the intro paragraph to kind of tie this section w/ examples from the past.

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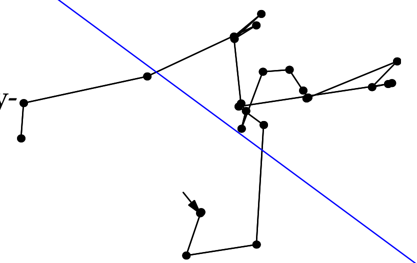

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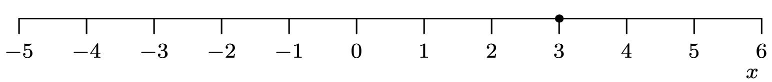

I don't see how this example has to do with random walks...

Yeah I definitely did a double-take...does this actually relate?

Just guessing...the vesicle bobs around at the terminal bouton before releasing so how fast it releases is (to some extent) dependent on random walk (how long it takes to make it to the membrane)...cold would be random motion of colder/slower particles and how often they contact your skin and slow down skin molecules...

but the vesicle is directed by receptors on the post-synaptic end...so its not really random movement

This box was around the "winter day" example, I believe. In this case, my guess is that the random walk determines how long it takes for a cold air molecule to get to your skin. (Well, perhaps how long it'd take molecules to hit a large enough area of skin to make you feel "cold".)

Referring back to the neurotransmitter, the molecule is directed but that doesn't mean that it has to necessarily make the same path from point A to B everytime. It just has to get to B.

Maybe I don't understand "random walks" yet, but I don't think this one fits in. It doesn't seem random at all. Every time you don't wear enough clothing, you'll always feel cold.

I remember the first year I was at MIT. Every time my friend walked outside during the winter he would say "aww it's not that bad" we'd hardly walk 50 yards before he'd be freezing.

I think it's referring to the motion of molecules, since all temperature is, is how fast the molecules are moving. Since it's cold they aren't moving as fast and are probably easier to approximate a random walk for.

That's the beauty of randomness: that it can be understood. The molecules move randomly, but there are so many of them that you can predict what they do in the aggregate – and they always make you feel cold.

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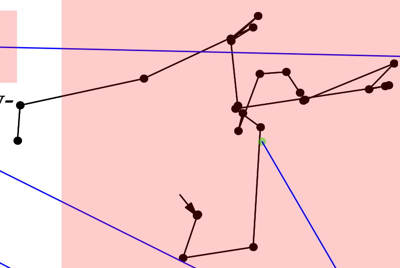

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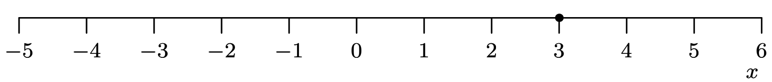

Does War fall into this category? It makes the point in a way that makes sense and is easy to relate to, but I"m not sure I think of War as a physical situation the same way as I do the movement of molecules

I think the comparison is that in War, every play is random. You don't know if person A or person B will win.

And the physical situation is the actual cards themselves moving from one player's hands to the other's.

After giving examples, perhaps it would be a good idea to state a formal definition of random walks. I've never heard of it before and would want to make sure that I am coming to the right conclusions.

That might be a good idea. Although formal definitions are, I've realized, alien to my way of thinking. I'm not sure what the formal definition of a random walk is, but I "know it when I see it." The picture I use in my own head is, "it's what a gas molecule does as it zooms around, bouncing off other gas molecules." The main point is that the motion after a collision is in a completely random direction (i.e. no information about the previous path remains).

These physical situations are examples of random walks, but I only have a vague, but not definite idea, of what a random walk is. I think it would be very helpful to be given a definition.

aw, your e-mail made me hope that you'd be using an example of an aimless stroll.

i'm just now figuring out that that's not what you meant...

This is an effective diagram documenting randomness.

I actually think this is somewhat unnecessary, though I guess it serves it's purpose.

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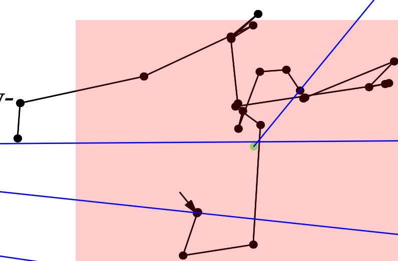

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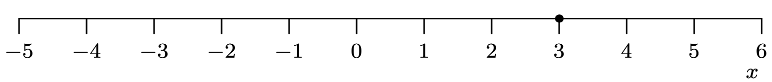

are you going to reference this diagram? it's different than the "regular walk" described below.

Agreed, it seemed a bit "random" without any sort of explanation/ reference to it.

I think it's okay if you don't reference it. It adds to the paper and it is pretty clear what this figure suggests.

Haha maybe it's a random figure showing a random walk. Clever

Regardless of the fact that it seems loosely related to the subject at hand, I still think some explanation would be helpful.

When I think of a random walk, I imagine motion that is undirected. So I was initially confused by the idea that war is random walking. But, the cards are played against one another in a somewhat random order.

Once again, I don't really get the random walks. I feel like random walks should not behave in a pattern that we can figure out.

A definition would be good in here.

I agree. I can get an idea of what a random walk is from your examples, but a formal definition is the only way outside of a lot of examples or reading through the whole section that I could really be sure of the definition and I think it would be better to know what you're talking about before reading the section.

don't they behave "randomly"?

I think their behavior can be predicted (ie brownian motion).

Again with the cannot uncheck a question thingie! Yah, i think he means how they behave so we can model it. "random" isnt exactly a model

is there a diffusion equation for the game of war?

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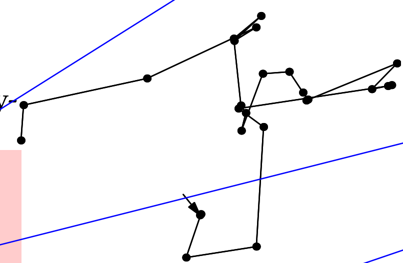

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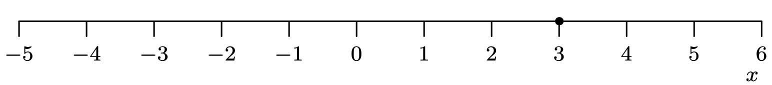

I like this description of the process we're going to take.

I agree. For me, it's really useful to have our plan-of-attack spelled out beforehand, so that we can better understand what we're supposed to be taking away from each part of the problem.

I've been learning from the previous reading memos! One of the main themes is that roadmaps are greatly valued, so I've been trying to add them to subsequent readings.

Of all the examples, this is the only one that I know what we are going figure out from this randomness

Cool! I was looking forward to this when we talked about it in one of the first lectures.

Sounds like it.

Wait why is it so complicated?

Because they can move any direction for any length they want. There's no real way to easily predict where they'll end up without using any tricks.

It's complicated in terms of evaluating the behavior. The elements are too variable, making the situation very complicated.

I feel like I haven't gotten an explicit definition of a random walk, so I'm a little confused (particularly because the situation being complex makes me wonder if there are rules or something)

The paragraph right after your highlighted box starts defining what a random walk is. Although, it doesn't explicitly make that clear.

????

This sentence seeems somewhat awkwardly worded. I'm not really sure what it means here.

yea i just don't get it

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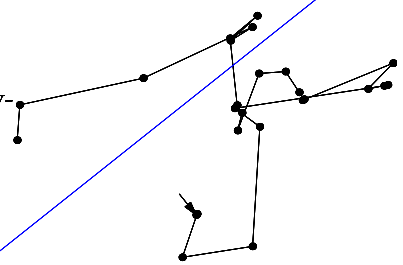

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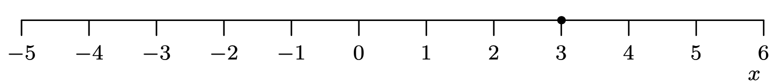

The first example of a random walk was the game war. I thought that was a great example of what we are trying to model. This first part of the section seemed a little hard to follow as I had the idea of 'war' stuck in my head. Would it be easier to understand for the reader to model the example on 'war'?

Agreed. Also, assuming that most us have played war before, it would be easier to picture the cases.

I've been thinking about "war" a bit more. The randomness is that each person turns up a card, and you don't know whose will show the bigger number. But, as you win more cards, you are likely to be getting smaller into your pack, so your probability of winning depends on how many cards you have won. Thus, history matters – which is not how a true random walk works. So "war" may not be a 100% valid example. More thinking required...

or that they have a fixed random chance of turning in a new direction after a certain distance.

I agree with your explanation. Because he's only using collision to refer to the particles example named earlier.

Yeah, this makes a lot more sense - like a person walking down the street and deciding to continue or turn around at each intersection.

I'd like an explanation for why we made the assumptions we did.

I am a little confused on what were talking about here? what is this random walk?

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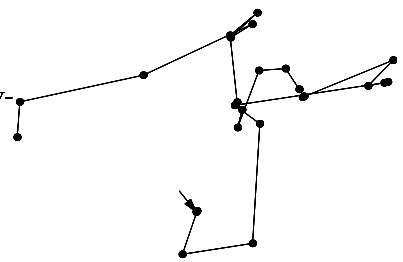
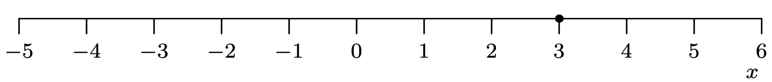
These physical situations are examples of random walks – for example, a gas molecule moving and colliding. The analysis in this section is in three parts. First, we figure out how random walks behave. Then we use that knowledge to derive the diffusion equation, which is a reusable idea (an abstraction). Finally, we apply the diffusion equation to heat flows (keeping warm on a cold day).



8.3.1 Behavior of regular walks

In a general random walk, the walker can move a variable distance and in any direction. This general situation is complicated. Fortunately, the essential features of the random walk do not depend on these complicated details. Let's simplify. The complexity arises from the generality – namely, because the direction and the distance between collisions are continuous. To simplify, lump the possible distances: Assume that the particle can travel only a fixed distance between collisions. In addition, lump the possible directions: Assume that the particle can travel only along coordinate axes. Further specialize by analyzing the special case of one-dimensional motion before going to the more general cases of two- and three-dimensional motion.

In this lumped one-dimensional model, a particle starts at the origin and moves along a line. At each tick it moves left or right with probability 1/2 in each direction. Here it is at $x = 3$:



This seems like it is making the movement much less random..?

I think it is keeping things random but taking some of the generality out of it. Basically, look at a very simple easy to analyze case of a random walk. (it is indeed possible for a walk to be random and follow all of these assumption, although its probably very unlikely). After we look at the simple case then we can try to generalize. Sort of like the solitare example from several weeks ago.

sort of, but we can always build off of this. Say we solve it for one axis, then 2, then 3... we could begin to extrapolate to infinite dimensions, and this would again return us to any direction is possible. We could also do something similar for distance. but we have to find somewhere to start.

I agree this is a little bit confusing but I think I'm starting to understand what he's getting at.

This is confusing. Is random one-dimensional motion moving back and forth on a line?

So easy cases and lumping! I ilke how the methods were addressed.

only by one unit (I believe the following implies)

Thank you for the clarification - I was a little confused what our fixed distance was.

any particular readon its at 3? are we assuming is been walking for a while?

I don't think there's a reason, its random. He might have been going for a while and ended up at 3 or he might have happened to go right 6 times in a row as soon as it started.

I think it's just clarifying the starting condition.

I just picked that spot randomly, but I should explain and use that information in the paragraph.

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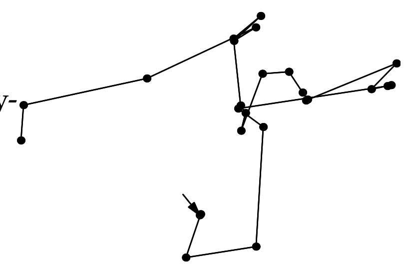

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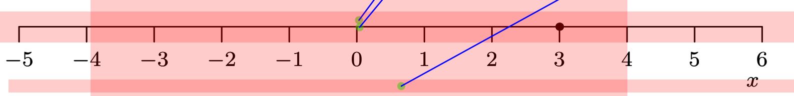

So is this number line a simplification of the random walk? why did we limit the distance it can travel and then say it can only move in one dimension? I don't really see where we're going here

This is a special case of the general random walk (where the particle can move a random distance and in a random direction). Here, it's been restricted to move a fixed distance (a unit distance) and in one of only two directions (left or right). The analysis is simpler but the main results transfer to the more general case.

It may be helpful if you had several of these diagrams...this will simulate the movement. perhaps a actual simulation

What am I supposed to be getting out of this diagram? Its just a number line with a dot at 3. I'm usually all for diagrams, but this one wasn't particularly helpful

It's just so you can visualize moving left or right at every step.

The diagram explains the above situation. You start at x = 0. Then, after the first time step, you're at x = 1 with probability 1/2 or at x = -1 with probability 1/2. That is, at each time step, you move to the left or to the right by one with probability 1/2 each. The diagram is supposed to help you graphically understand the random walk described above.

specifically it is the set of positions that a particle moving in a one-dimensional random walk can follow. our model assumes discrete, integer steps. also, no hops and each time interval corresponds to one step with equal probabilities in available directions.

That's exactly what it's supposed to be! This just lets you visualize two directions, left and right with equal proability.

Does this imply that the model is discrete?

Right, after using lumping, the continuous model turns into a discrete model.

Let the position after n steps be x_n, and the expected position after n steps be $\langle x_n \rangle$. The expected is position is the average of all its possible positions, weighted by their probabilities. Because the random walk is unbiased – motion in each direction is equally likely – the expected position cannot change (that's a symmetry argument).

$$\langle x_n \rangle = \langle x_{n-1} \rangle .$$

Therefore, $\langle x \rangle$, the first moment of the position, is an invariant. Alas, it is not a fascinating invariant because it does not tell us anything that we did not already understand.

Let's try the next-most-complicated moment: the second moment $\langle x^2 \rangle$. Its analysis is easiest in special cases. Suppose that, after wandering a while, the particle has arrived at 7, i.e. $x = 7$. At the next tick it will be at either $x = 6$ or $x = 8$. Its expected squared position – *not* its squared expected position! – has become

$$\langle x^2 \rangle = \frac{1}{2}\left(6^2 + 8^2\right) = 50.$$

The expected squared position increased by 1.

Let's check this pattern with a second example. Suppose that the particle is at $x = 10$, so $\langle x^2 \rangle = 100$. After one tick, the new expected squared position is

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Yet again $\langle x^2 \rangle$ has increased by 1! Based on those two examples, the conclusion is that

$$\langle x_{n+1}^2 \rangle = \langle x_n^2 \rangle + 1.$$

In other words,

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$$\langle x_n^2 \rangle \propto t.$$

Typo. Should read: "The expected position is the average..."

I vaguely remember this notation, but you might want to point it out a little more strongly...that one means average versus actual

I get what this word means in this context but I think it could be potentially confusing if one didn't know the concept.

But isn't it clarified by the aside after the em dash?

what do you mean by this? that the expected position is the origin? or that it's the same for each walk?

That the expected position is to return to baseline (which in this case is not the origin, its at 3)

Stating this more explicitly as in the comment at 1:25 would be helpful.

He means that the expected position is the same for each walk for which you start at the same point. The starting point (also referred to the baseline in the above comment) for this problem was the origin, x = 0. So in this problem, the expected position is in fact the origin. The above comment at 1:25 is incorrect by saying the baseline is x = 3, because that is not what the problem states above. It simply shows the particle at x = 3 after a certain amount of ticks.

That makes much more sense than the previous comments.

What about the variance?

You might want to just say expected value of the position or explain what the first moment is.

yea what is the "moment?"

I think he means that since the average/expected value is constant, you can just throw out the 'n', since it doesn't chance. So instead of <xn> you get just <x>;.. By "first moment" i think he just means where it starts, since that is where the average will lie.

This might help: http://en.wikipedia.org/wiki/Moment_%28mathematics%29

Different moments provide different characteristics of a population. The first moment is the population mean.

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is that what the angled brackets mean?

I don't believe so. I believe the angled brackets are simply used to distinguish the action position from the expected position.

Also, why is it an invariant? I thought it was random...?

I think he's talking about the expected value of x i.e. <x> and that is zero and since it is zero (constant) it is invariant

It's good to see invariants making a comeback here.

But I don't understand... what's the first moment? I get that its invariant but I'm lost from there.

Ah. I have just been informed that the first moment is simply the expected value of x, so just the average. If this is the case, just take a couple words to mention that terminology.

I agree. Introducing the term "first moment of the position" is kind of unnecessary (since we already have several other terms for this quantity) and vaguely confusing. If it's really important that it be kept, it would be helpful to define it a little more explicitly.

Moments are a specific definition in probability. Is it worth specifying what a moment means here?

He means moment as in the next physical movement in time right, but that doesn't seem right, what is it?

Ya, this term is ambiguous to me in this context. I think you should clarify what you mean.

I should do that. The first moment is the expected value of x^1. The second moment is the expected value of x^2, etc.

I don't understand why the 2 here is a superscript (as if to indicate a squaring) rather than a subscript (which would indicate a second walk)?

because it is squared.

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Why is it squared?

Should that be a subscript ?

It shouldn't be a subscript but I'm still confused why we are looking at squares?

Why are we interested in expected square position- are we trying to calculate variance and expected value?

I think its because it has already moved once, <X> and now it is moving again, the same distance but we don't know the direction, so distance <Xˆ2>

The second moment is equal to the variance, which I think equals the position squared.

The second moment will give us different information than the first moment, and in this case more useful information, since we already knew the information provided by the first moment. And yes, the second moment is equal to the variance which is equal to the average value of position squared, as the notation suggests.

What does he mean by second moment? I see us simply moving in increments of one. It doesn't make sense to me that we use xˆ2. What is the reason to calculate variance?

Let the position after n steps be x_n, and the expected position after n steps be $\langle x_n \rangle$. The expected is position is the average of all its possible positions, weighted by their probabilities. Because the random walk is unbiased – motion in each direction is equally likely – the expected position cannot change (that's a symmetry argument).

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the n-th moment is defined as:

<x^n>

or "the expected value of the nth power of the variable"

like people said previously, the 1st moment happens to be the mean, the 2nd moment happens to be the variance.

we are calculating mean and variance because it tells us information about the nature of a random walk. Since there is equal probability of going in any direction, the mean is wherever the particle starts at. The variance, on the other hand, is the square of the standard deviation, which tells us that 67% of the time, the particle will fall within one standard deviation after moving for a time t. From the formula, we see that one standard deviation is the square root of t.

The primary reason we use x^2, as you asked, is because it will take into account movement in the negative direction. Let me put it this way:

The particle is equally likely to move in either direction, so if we run this experiment for many trials, the mean, or <x>, will average out to wherever the particle started at (e.g. x=0). But to get an idea of what the average "variation" in distance from the mean is in those trials (hence the term variance), we need to take how far the particle has gotten from the mean in each trial and average all those distances. Trouble is, the particle is equally likely to have negative distances as positive distances, and with equal probability of going in either direction, such an average would come out to zero. Surely, you wouldn't believe taht the average distance traveled in each trial is zero. Thus, mathematically, what we do is square the distance traveled in each trial to get rid of negative distances. Then we average that to get the average squared distance traveled. Finally, all we have to do is take the square root to get a more practical interpretation: the standard deviation, which tells us about how far each particle travels in time t from the mean.

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I think the important part here is linking what we're doing to what people reading will understand more intuitively. Saying that the square root of <x^2>=stdev would help a lot since most people reading this would understand standard deviations.

We are looking for the expected value of x^2, i.e. <x^2> (not the square of <x>). Since x^2 is always nonnegative, it's expected value is also nonnegative (and is in fact positive for n>0).

To find <x^2> by brute force, we'd need to find the probability of x^2 being 0, of it being 1, of it being 2, etc. and weight 0, 1, and 2, etc. by those respective probabilities. I should definitely give more details in the book, and then say, "But we're going to do it by easy cases and guessing instead of by brute force." But at least then everyone would know what we are trying to guess!

At this point it became pretty confusing for me. The <x^2> thing didn't really make sense when you started adding in the subscripts.

Maybe you could explain what <x^2> means in the reading.

i think adding <x^2> = 49 here would be good.

What is the difference between expected square position and squared expected position (in terms of their purpose)? I get that the former is a way of finding averages. Is the latter significant in any way?

This is actually a very subtle but really important difference. I feel like there should be a short explanation about it.

Yeah, I was a bit confused at first.

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It is important you explain this difference because the number 1 confusion in probablity is differentiating <x>^2 from <x^2>

I'm confused what you mean by this - can you elaborate? (relatedly, I'm also confused about this portion of the reading and what the x^2 is supposed to mean.)

The difference here seems to be:

<x^2> = 50 <x>^2 = 49

This is because <x> = 7, and squaring it (or finding the squared expected position) is equal to 49.

This would be a really great additional way to write this sentence!

I'm confused...how is the number 50 useful? it just means you are in the same position 7.

why wouldn't this be 1/2(7^2)=24.5. why are we using these two quantities.

Not really sure where the half came from?

im lost on how we got here. the average of the squares?

yes, what is the basis for this operation?

Yeah, I'm very confused on this line, on the whole x^2 thing, and on the bracket <> notation. I think a review of probability formulas and terminology would be appropriate before diving into this section.

So he's trying to find the expectation of the squared position. If we're at x = 7, then we'll move to x = 6 with probability 1/2, and move to x = 8 with probability 1/2. Remember, the value x is the position. So, E[x^2] = (probability x = 6)*(6^2) + (probability x = 8)*(8^2) = 1/2*(6^2) + 1/2*(8^2) = 1/2*(6^2 + 8^2) = 1/2*(36 + 64) = 1/2*(100) = 50. The bracket <> simply denotes expected value.

I understand this, but I don't understand how multiplying the probability by the square of the next possible position is helpful t determining anything. In a one dimension scenario does that sort of regularization account for anything?

I don't understand why we have an x^2.

how does finding this help us? What does 50 have to do with anything?

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I'm SO confused what's going on here, I feel like this section isn't as well explained as other sections with tougher information.

I agree - i'm not really sure where this is going either...

I get the concept mathematically, but not conceptually.It seems like you are trying to make a point that everything is relative. I'll keep reading.

Had the same feeling - was a bit lost but want to see where this is going...hopefully i get the destination

so what does this mean? i don't know much about probability or what we're looking for here.

I'm wondering the same.

Hm yea I forget this stuff, but I think a big reason to use squared anything is to get rid of the differences between positive and negative changes. So my guess about what this means is that the absolute value of the expected position increased by one. Or something like that maybe.

Keep reading. You'll see that he's trying to find a pattern for the invariant. We literally just wanted to find out that the expected squared position increased by 1.

I understand that the squared position increased by one. But why does this matter?

Well, if you read above, it clarifies that this is easy cases - the squared (or second moment) is the easiest to analyze "in special cases"

how is it increased by 1

You should mention about how 7ˆ2 was 49. it took me a bit to figure out what you meant by plus one

I am glad you wrote this note, because I was rather confused myself.

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oops, sorry there are multiples of this comment, my firefox malfunctioned...

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$$\langle x_{n+1}^2 \rangle = \langle x_n^2 \rangle + 1.$$

In other words,

$$\langle x_n^2 \rangle = n.$$

Because each step takes the same time (the particle moves at constant speed),

$$\langle x_n^2 \rangle \propto t.$$

can someone please define what the expected squared value means?

It means exactly what its algebraic formalism gives it as, and it's a stepping stone to finding the variance and standard deviation of a distribution.

I just realized that you were taking the difference of X^2 and <x^2>. But I dont know whY?

I know that this is the equation for variance in probability, but I don't know why either. I think this is a different way of proving that variance in the difference between these two magnitudes, but I'm still confused by his method.

that's actually really cool – it makes sense in retrospect that (n+1)^2+(n-1)^2 = n^2 + 1 but i never thought about it like that

Why is the expected squared position important, why no the expected cubes position, or some other function of the expected position.

Why do we take the avg of the squares?

I feel like just using numerical examples is not convincing - what would be better is an algebraic argument. You can show x^2+1 = .5(x-1)^2+.5(x+1)^2, which might be more intuitive for MIT students.

Oh, I really like your explanation here. I didn't get why it kept increasing by one everytime, but I understand now that you say x^2+1 = .5(x-1)^2+.5(x+1)^2. I would definitely consider adding this explanation to the text.

I thought the explanation was alright, though I was lost until the equations were put down. After they were put down it was a "Oh, wow, cool" moment.

Let the position after n steps be x_n, and the expected position after n steps be $\langle x_n \rangle$. The expected is position is the average of all its possible positions, weighted by their probabilities. Because the random walk is unbiased – motion in each direction is equally likely – the expected position cannot change (that's a symmetry argument).

$$\langle x_n \rangle = \langle x_{n-1} \rangle .$$

Therefore, $\langle x \rangle$, the first moment of the position, is an invariant. Alas, it is not a fascinating invariant because it does not tell us anything that we did not already understand.

Let's try the next-most-complicated moment: the second moment $\langle x^2 \rangle$. Its analysis is easiest in special cases. Suppose that, after wandering a while, the particle has arrived at 7, i.e. $x = 7$. At the next tick it will be at either $x = 6$ or $x = 8$. Its expected squared position – *not* its squared expected position! – has become

$$\langle x^2 \rangle = \frac{1}{2}\left(6^2 + 8^2\right) = 50.$$

The expected squared position increased by 1.

Let's check this pattern with a second example. Suppose that the particle is at $x = 10$, so $\langle x^2 \rangle = 100$. After one tick, the new expected squared position is

$$\langle x^2 \rangle = \frac{1}{2}\left(9^2 + 11^2\right) = 101.$$

Yet again $\langle x^2 \rangle$ has increased by 1! Based on those two examples, the conclusion is that

$$\langle x_{n+1}^2 \rangle = \langle x_n^2 \rangle + 1.$$

In other words,

$$\langle x_n^2 \rangle = n.$$

Because each step takes the same time (the particle moves at constant speed),

$$\langle x_n^2 \rangle \propto t.$$

This can be just as easily be seen by thinking of x_n as the sum of n random variables y_1...y_n, each y_i=1 or -1.

Since each var(y_i)=1, var(x_n) = var(y_1+...+y_n) = n.

(And since <x_n>=0, var(x_n) = <(x_n)^2>.)

p.s., this approach would also apply to more general walks, for which we only need to know that <y_i>=0, and the variance of each y_i.

This is more applicable when the distance at each step is some continuous distribution (like in the log of our errors during estimation, for example, where each var(y_i) log(r_i)^2).

I don't make this connection. n is the number of moves?

Yes, I think so. so x(n) describes a sequence of events, and x(n) is the position at time n.But i still don't understand how this works...what is n in this case? (from the example of x=10, <x^2>=100?)

So if we start at the origin, is the <xn> –> x=0 at n=0 or n=1?

but <7^2> was 50 ....? how is that n?

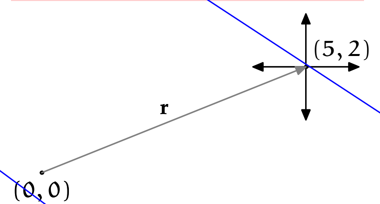
I think you may be confused about the notation. n is not where you start, it's the number of tick marks that you move away from a starting position...so in the examples above, we moved once and found that the expected value increased by 1.

So simple yet hard to realize.

I agree, it is rather fascinating.

Yeah I think it's crazy once you think about it. In a good way of course.

The result that $\langle x^2 \rangle$ is proportional to time applies not only to the one-dimensional random walk. Here's an example in two dimensions. Suppose that the particle's position is $(5, 2)$, so $\langle x^2 \rangle = 29$. After one step, it has four equally likely positions:



Rather than compute the new expected squared distance using all four positions, be lazy and just look at the two horizontal motions. The two possibilities are $(6, 2)$ and $(4, 2)$. The expected squared distance is

$$\langle x^2 \rangle = \frac{1}{2}(40 + 20) = 30,$$

which is one more than the previous value of $\langle x^2 \rangle$. Since nothing is special about horizontal motion compared to vertical motion – symmetry! – the same result holds for vertical motion. So, averaging over the four possible locations produces an expected squared distance of 30.

For two dimensions, the pattern is:

$$\langle x_{n+1}^2 \rangle = \langle x_n^2 \rangle + 1.$$

No step in the analysis depended on being in only two dimensions. In fancy words, the derivation and the result are invariant to change of dimensionality. In plain English, this result also works in three dimensions.

In a standard walk in a straight line, $\langle x \rangle \propto$ time. Note the single power of x. The interesting quantity in a regular walk is not x itself, since it can grow without limit and is not invariant, but the ratio x/t, which is invariant to changes in t. This invariant is known as the speed.

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This qualitative difference between a random and a regular walk makes intuitive sense. A random walker, for example a gas molecule or a very

will this work even when the particle moved at fractions of tick marks?

I still dont understnad why we are loking at xˆ2

It's essentially (glossing over some technicalities) the variance of your location after n steps. Variance is sort of a measure of spread, and its square root is standard deviation. Since this is a random walk, you don't know exactly where you'll end up, but the variance/standard deviation gives you an idea about how far you might be from your starting point.

Well put.

Very helpful. I could have really used such an explanation in the text somewhere.

at this point I realized that following your x's is really hard sometimes...you should use subscripts for all of them, exp when the eq for obtaining them is different .. it took me _way_ too long to figure out why this wasn't 14.5

Note, and remember for later, that x is a vector magnitude here (unless you mean r instead of x)

Maybe show these on the diagram below?

It looks as though that is what the 4 arrows are meant to show. They just don't have the coordinates associated with them.

I think he should have also mentioned he's using lumping here since really you could move in more than for directions.

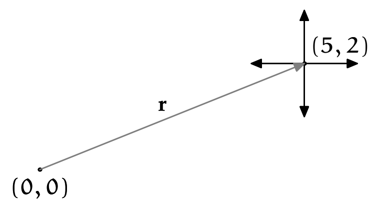
has the particle always been 2 dimensional? initially i thought everything was (x,0)

For the particular example where you placed your comment, it is two dimensional. Previous examples were 1D though.

I don't really understand what is going on...this is not really making sense to me. I guess I don't understand what an expected value is.

Squared distance makes a little more sense to use in 2 dimensions than 1.

The result that $\langle x^2 \rangle$ is proportional to time applies not only to the one-dimensional random walk. Here's an example in two dimensions. Suppose that the particle's position is $(5, 2)$, so $\langle x^2 \rangle = 29$. After one step, it has four equally likely positions:



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This qualitative difference between a random and a regular walk makes intuitive sense. A random walker, for example a gas molecule or a very

So essenially on average for a 2 dimensional random walk, the most likely probability is a ring that is slowly moving outwards?

Actually the most likely position is the origin! The distribution is roughly Gaussian (a normal distribution) with variance <x^2>. So the distribution has a peak at the origin, and it spreads farther and farther with time.

I like this simplification- I think it will help me to understand the example in a more simple way while still being able to apply the rules to a more complex example

so we limited it to horizontal but later we don't really go back and zoom out to include vertical –> shouldn't this change the answer?

i'm confused as to where these numbers are coming from.

From the vector coordinates: 40=6^2 + 2^2 20=4^2 + 2^2

Thanks! This is helpful.

I guessed this but I think it would be helpful to include somewhere before here how <x^2> will be calculated in 2D.

This is pretty cool that it works out like this again. The notation confused me a bit but this helps clear things up.

I like symmetry–makes things easy.

And it seems to be much easier to apply when you're talking about probabilities and can simply take the average.

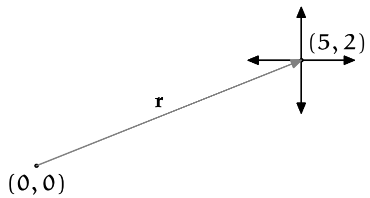
I'm not sure why, but when you mention symmetry this example seems more effective than the previous.

now

I like this paragraph! even if the 1st sentence is a little awkward. it's great.

This paragraph reads weird when you understand all three sentences, but it is a thorough explanation.

The result that $\langle x^2 \rangle$ is proportional to time applies not only to the one-dimensional random walk. Here's an example in two dimensions. Suppose that the particle's position is $(5, 2)$, so $\langle x^2 \rangle = 29$. After one step, it has four equally likely positions:



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This qualitative difference between a random and a regular walk makes intuitive sense. A random walker, for example a gas molecule or a very

This is a great sentence. Very clear

I think saying this 2 different ways is unnecessary

Interesting to see speed as an invariant.

I am not sure that I understand why, unless we made the assumption of constant motion previously.

Agreed, I've always seen it as an easy and common variable to change.

How is this invariant?

I had the same question, a proof of this would be nice.

it's invariant because we're looking at distance/speed = velocity, and one of our assumptions was that the molecule moves at a constant speed.

x/t is invariant but x is variant because if we look at x, it depends on where we just were. but the rate theoretically is always the same. like if i walked 3 steps, i could walk forward forward forward or forward backward forward, and my x would be different. but the pace at which i walk would still be the same

i didn't catch this distinction here.

This section is kind of confusing. I don't see how <x^2> is variant with different t but the ratio is invariant.

The <x^2> is variant because it always changes where you were previously before the step. However, the rate/speed at which you move is always the same. I like to think that I'm walking one-step/second.

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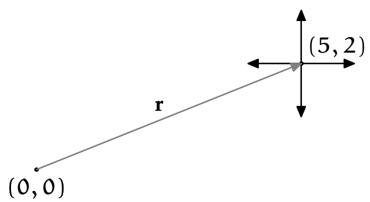
If x = 2*t, then x is not invariant to changes in t (t goes up, x goes up). However, x/t is always 2 - an invariant.

It feels like this paragraph is really dense...

The result that $\langle x^2 \rangle$ is proportional to time applies not only to the one-dimensional random walk. Here's an example in two dimensions. Suppose that the particle's position is $(5, 2)$, so $\langle x^2 \rangle = 29$. After one step, it has four equally likely positions:



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This qualitative difference between a random and a regular walk makes intuitive sense. A random walker, for example a gas molecule or a very

Oh wow, even though I should have known we were going here eventually, this caught me completely off-guard by being perfectly intuitive. x/t –> x^2/t. Diffusion, makes sense.

You might note, more explicitly, that this doesn't depend on whether we are in the 1D, 2D, or higher-dimensional case. (I had to go back and double check the math to convince myself.)

Agreed - you might want to make the conclusion more explicit.

He says above that the result is invariant to change of dimensionality.

I feel like this point was made pretty clear shortly before this.

This is pretty cool, I've never seen any of this before and it's definitely not something intuitive

this has units of length * speeed. but i thought the invariant WAS speed. could you explain please?

The invariant is speed in the case of the normal walk, but for the random walk the diffusionconstant is the invariant.

We've just changed cases and selected the next appropriate invariant. From x/t to x^2/t

I've definitely seen easier ways to explain the difference between regular and random walks; however, i think your explanation is more relevant to the course.

I like seeing what would classify a random walker, it helps to get a feel for what we are really looking at here.

I have a feeling it's anything that could be classified in terms of the motion described in this section.

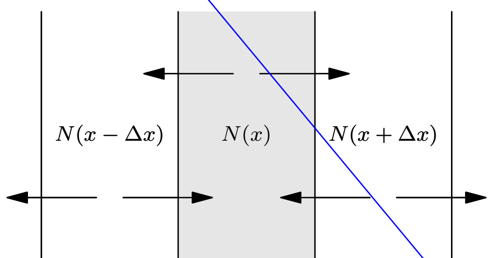
I like the two completely different examples reinforcing the same point.

drunk person, moves back and forth, sometimes making progress in one direction, and other times undoing that progress. So, in order to travel the same distance, a random walker should require longer than a regular walker requires. The relation $\langle x^2 \rangle / t \sim D$ confirms and sharpens this intuition. The time for a random walker to travel a distance l is $t \sim l^2/D$, which grows quadratically rather than linearly with distance.

8.3.2 Diffusion equation

The preceding conclusion about random walks is sufficient to derive the diffusion equation, which describes how charge (electrons) move in a wire, how heat conducts through solid objects, and how gas molecules travel. Imagine then a gas of particles with each particle doing a random walk in one dimension. What is the equation that describes how the concentration, or number density, varies with time?

Divide the one-dimensional world into slices of width Δx, where Δx is the mean free path. Then look at the slices at $x - \Delta x$, x, and $x + \Delta x$. In every time step, one-half the molecules in each slice move left, and one-half move right. So the number of molecules in the x slice changes from $N(x)$ to



$$\frac{1}{2}(N(x - \Delta x) + N(x + \Delta x)).$$

The change in N is

$$\Delta N = \frac{1}{2}(N(x - \Delta x) + N(x + \Delta x)) - N(x)$$
$$= \frac{1}{2}(N(x - \Delta x) - 2N(x) + N(x + \Delta x)).$$

This last relation can be rewritten as

$$\Delta N \sim (N(x + \Delta x) - N(x)) - (N(x) - N(x + \Delta x)).$$

In terms of derivatives, it is

$$\Delta N \sim (\Delta x)^2 \frac{\partial^2 N}{\partial x^2}.$$

nice example :)

I was wondering where this would come in when it was missing from that intro paragraph...

haha

do you mean distance or displacement?

Except in the (unlikely) case that the drunk and regular walker make the same moves.

This is why he uses the word 'should'....but yes this is possible as well.

is this valid only when l is very small

I don't think so. It still makes sense over longer periods of time.

This a pretty interesting statement. However, it intuively doesn't make sense because on average, the drunk person would have still traveled zero distance.

Yes, but the expected value of the square is different, since in the case where the mean is zero, that is the variance, which would be nonzero in this case.

well also the case that he moved exactly nowhere at any given point seems intuitively unlikely, although expected value of his overall movement may still be zero.

This of it this way. On average, he would make zero progress, but randomly he'll one way make his way home. Now if he's so drunk he doesn't recognize his own house, he might go all the way back and cancel out that distance, but we assume that once he somehow reaches his house, he goes inside.

I really like these explanations using intuition.

I think these intuitive explanations solidify my understanding.

This section was very interesting - it made a lot of other things I've seen before about moments make a lot more sense.

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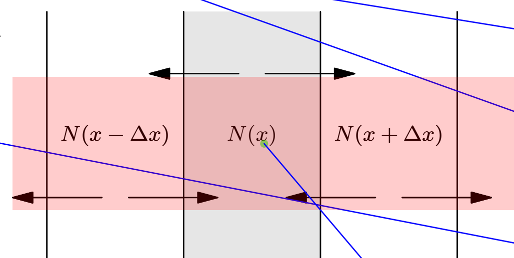

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$$\begin{aligned} \Delta N &= \frac{1}{2}(N(x - \Delta x) + N(x + \Delta x)) - N(x) \\ &= \frac{1}{2}(N(x - \Delta x) - 2N(x) + N(x + \Delta x)). \end{aligned}$$

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In terms of derivatives, it is

$$\Delta N \sim (\Delta x)^2 \frac{\partial^2 N}{\partial x^2}.$$

Would this change if we were observing in three dimensions. For example if we were discussing an airplane's movements or a spacecraft.

> It's the same conclusion in any number of dimensions: the variance grows linearly with time. I'll give another argument in class to show it by using dot products. The argument is more general simpler than the one in the text, but somehow less convincing (at least to me) because it is algebraic and not numerical. I think I'll put both arguments in the text, so that people of different persuasions can be persuaded by whatever resonates.

Again, it's nice to see examples of what we are going to be talking about before it is introduced.

Instead of introducing this section like this, I think it might be helpful to make the transition by explaining that random walks are simple mathematical models of diffusion.

I like how we eased into this section with simple explanations before diving into more complex equations.

I really like how this complex example relates back to what we just did with random walks- I'm glad to see the two separated into 2 examples rather than 1 example that crams both in.

what does this mean?

= how far on average the particle travels before it hits something or is forced to change its trajectory

Thanks for the clarification, I actually initially interpreted this phrase differently but that makes more sense.

ah...this diagram reminds me of 8.02 using Gauss' Law.

Haha, it does for me too. This I think is a fantastic diagram!

this diagram is a little confusing? what are the three layers?

This is a much simpler but nicer explanation of what was covered in another class (which did not, unfortunately, let us calculate using estimation!)

drunk person, moves back and forth, sometimes making progress in one direction, and other times undoing that progress. So, in order to travel the same distance, a random walker should require longer than a regular walker requires. The relation $\langle x^2 \rangle / t \sim D$ confirms and sharpens this intuition. The time for a random walker to travel a distance l is $t \sim l^2/D$, which grows quadratically rather than linearly with distance.

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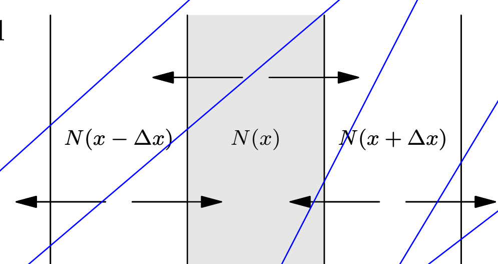

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In terms of derivatives, it is

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I feel as though this concept in particular is best explained in class.

Is this always true in every dimension?

I think this is true in every direction because as long as we assume similar conditions, diffusion can happen in any direction.

Tracing this particular jump in logic was a little hard.

Is this supposed to be -N(x- delta x)?

Otherwise, the expression reduces to 0...

Well, no, it doesn't, it reduces to 2N(x+dx)-2N(x), but I'm pretty sure it is supposed to be +N(x-dx).

Whoops, thanks for catching that mistake.

Thank you for explaining how you got this. I would have had to take it at face value if you didn't.

Where does he explain this?

If I saw this equation before you broke it down, I would be terrified. Thanks for making it seem simple

I agree; the explanations prior to introducing the equation make it a lot clearer.

I think most of us have seen the technical expression for a derivative as a limit, but probably fewer have seen the connection of a second derivative to a limit. This is expression was not an obvious result of the previous line, and it took me some time to convince myself that it was reasonable...

You're right, I should explain that (and will do so in class).

The slices are separated by a distance such that most of the molecules travel from one piece to the neighboring piece in the time step τ. If τ is the time between collisions – the mean free time – then the distance is the mean free path λ. Thus

$$\frac{\Delta N}{\tau} \sim \frac{\lambda^2}{\tau}\frac{\partial^2 N}{\partial x^2},$$

or

$$\dot{N} \sim D\frac{\partial^2 N}{\partial x^2}$$

where $D \sim \lambda^2/\tau$ is a diffusion constant.

This partial-differential equation has interesting properties. The second spatial derivative means that a linear spatial concentration gradient remains unchanged. Its second derivative is zero so its time derivative must be zero. Diffusion fights curvature – roughly speaking, the second derivative – and does not try to change the gradient directly.

8.3.3 Keeping warm

One consequence of the diffusion equation is an analysis of how to keep warm on a cold day. To quantify keeping warm, or feeling cold, we need to calculate the heat flux: the energy flowing per unit area per unit time. Start with the definition of flux. Flux (of anything) is defined as

$$\text{flux of stuff} = \frac{\text{stuff}}{\text{area} \times \text{time}}.$$

The flux depends on the density of stuff and on how fast the stuff travels:

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how does this correlate to the mean free path?

This may be intuitive to all of the course 2 majors out there but as a course 6 major, I find myself getting very confused about all this terminology I haven't seen before.

It helps to look at it a bit more mathematically and work yourself through the problem moving units around as they should be.

Thanks! Thinking about the units did help clarify where all these quantities come from.

that "dot" is kind of hard to see. I thought it was a piece of dust on my screen at first. Maybe use dN/dt?

Good point.

David Hogg, a friend who teaches at NYU, said he doesn't even allow his students to use the dot notation because it obscures the dimensions, whereas dN/dt makes the dimensions clear. So he recommended that I take out all uses of the dot notation from _Street-Fighting Mathematics_. Which I did, and I'll do the same for this book.

I feel like I've seen this in a microelectronics class. Is this the same principle used in calculating thermal equilibrium states in pn junctions?

Oh, that makes sense now.

Oh man, this looks suspiciously like the stuff we did above with the second moment. I like it.

what are some values of D to give us a feel for what it is

For gas molecules diffusing around (e.g. air molecules), D is about 10^-5 m^2/s – which is the kinematic viscosity of air.

Is it really a constant if tau and gamma are variable for each problem? I think of a constant as something like Avogadro's number which never changes..

This paragraph goes through a lot of information very quickly, it might be worth it to explain it more.

The slices are separated by a distance such that most of the molecules travel from one piece to the neighboring piece in the time step τ. If τ is the time between collisions – the mean free time – then the distance is the mean free path λ. Thus

$$\frac{\Delta N}{\tau} \sim \frac{\lambda^2}{\tau}\frac{\partial^2 N}{\partial x^2},$$

or

$$\dot{N} \sim D\frac{\partial^2 N}{\partial x^2}$$

where $D \sim \lambda^2/\tau$ is a diffusion constant.

This partial-differential equation has interesting properties. The second spatial derivative means that a linear spatial concentration gradient remains unchanged. Its second derivative is zero so its time derivative must be zero. Diffusion fights curvature – roughly speaking, the second derivative – and does not try to change the gradient directly.

8.3.3 Keeping warm

One consequence of the diffusion equation is an analysis of how to keep warm on a cold day. To quantify keeping warm, or feeling cold, we need to calculate the heat flux: the energy flowing per unit area per unit time. Start with the definition of flux. Flux (of anything) is defined as

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I find that this is the easiest way for me to remember how (thermal, in my context) diffusivity acts. It might be worth elaborating on.

i.e. If a long thin rod has a heat source at one end set to 50 degrees, and a heat sink at the other end set to 0 degrees, diffusion will act to make the temperature profile a straight line (from 50 at one end to 0 at the other) rather than some quadratic curve, etc.

thank you for the explanation! i was just about to ask what "diffusion fights curvature" means.

Haha well explained! Again, had someone in my course 20 class explained it like that it would have made life a lot easier...This explanation might actually be worthwhile to add into the book

yes, this was great! Even to a non-engineering or science major, I was able to clearly understand this property with the above example.

I had almost exactly that explanation but without a diagram. Instead of making a diagram (it was late at night) I decided to take out the example – clearly the wrong choice!

it keeps my attention when you relate it to a real world example the whole way through instead of briefly in introductory sentences.

so this section actually doesn't have to do with random walks. it just stems from the diffusion equation. you should make that more clear in the intro.

He mentions that after he concludes on the diffusion constant.

The diffusion equation is the macroscopic view of heat flow. Under the hood, it is a random walk that produces the diffusion equation.

is there a way to quantify how some people get colder easier than others?

I now understand the sentence in the very first paragraph about keeping warm, I didn't realize then that you were talking about diffusion. I now understand why "keeping warm" is part of random walks.

the day i learned about fins in 2.005 was the day i understood why my family (all impossibly tall and lanky) is perennially cold.

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I would have loved to use approaches this elegant and stream-lined in 2.005

I don't think this needs to be simplified to the point of using the word "stuff" - I actually find that more confusing

i think he means to generalize the def of flux, so it's the flux of ANYTHING. therefore "stuff"

I agree with the first comment. I find the word "stuff" confusing. We've covered much more confusing information in this class, so I think making it a bit more complicated than "stuff" wouldn't be a problem.

What are the dimensions of stuff? M?

"Stuff" refers to idea that many things can represent that variable. For example, [stuff]=M if we were looking at mass-flux but it could also [stuff]=energy in energy-flux and [stuff]=L^3 for volumetric-flux.

I think this is helpful because it is a general formula for flux–stuff can mean anything. the general idea is how much of something flows through a given area per time

I was trying to make it more general. In solar flux, for example, the stuff is energy; and flux of stuff is energy/area/time or power/area. There's also momentum flux, where the stuff is momentum; and number (or partcile) flux, where the stuff is particles. All of those definitions of flux have the same structure (an abstraction!), and I wanted to make that explicit.

Perhaps in the text I should give one or two other examples of using the abstraction.

I like this very simplified explanation.

Yeah it's simple, but the best part is that it actually does apply to a variety of things and makes sense at the same time.

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i find this easier to understand as [(stuff per area) per time].

For me it's just the opposite. I find (stuff per time, per area) slightly more intuitive. I guess this is a good compromise.

I like both of these definitions because they both describe this equation better than the reading.

This makes a lot of sense to me. I think it is because flux is such an important concept in classes like 8.02.

I like this ... I think it's very well written.

volume of stuff? or volume stuff is flowing into?

Per unit volume... stuff is flowing into.

This note is going to a direction that I had not imagined; I didn't think we would start with random walk and come here.

I'm not sure I follow how we got this.

That's a neat, simple, and convincing justification.

It makes sense that over a long period of time, little diffusion occurs. Taking for instance heat. When you apply the same amount of heat to an object, the change in temperature at the beginning is more than the change in temperature at a later time in the same time interval

kappa/l ?

yes. he definitely has it inverted.

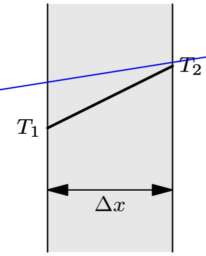
You're right. Thanks – I should check the dimensions of everything that I write, not only teach other people to do the same.

Combine the thermal energy per volume with the diffusion speed:

$$\text{thermal flux} = \rho c_p T \times \frac{\kappa}{l}.$$

The product $\rho c_p \kappa$ occurs so frequently that it is given a name: the thermal conductivity K. The ratio T/l is a lumped version of the temperature gradient $\Delta T/\Delta x$. With those substitutions, the thermal flux is

$$F = K\frac{\Delta T}{\Delta x}.$$



With one side held at T_1 and the other at T_2, the temperature gradient is $(T_2 - T_1)/\Delta x$.

To estimate how much heat one loses on a cold day, we need to estimate $K = \rho c_p \kappa$. To do so, put all the pieces together:

$$\rho \sim 1\,\text{kg}\,\text{m}^{-3},$$
$$c_p \sim 10^3\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1},$$
$$\kappa \sim 1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1},$$

where we are guessing that $\kappa = \nu$ (because both κ and ν are diffusion constants). Then

$$K = \rho c_p \kappa \sim 0.02\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}.$$

Using this value we can estimate the heat loss on a cold day. Let's say that your skin is at $T_2 = 30\,°\text{C}$ and the air outside is $T_1 = 0\,°\text{C}$, making $\Delta T = 30\,\text{K}$. A thin T-shirt may have thickness 2 mm, so

$$F = K\frac{\Delta T}{\Delta x} \sim 0.02\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1} \times \frac{30\,\text{K}}{2 \cdot 10^{-3}\,\text{m}} \sim 300\,\text{W}\,\text{m}^{-2}.$$

Damn, we want a power rather than a power per area. Ah, flux is power per area, so just multiply by a person's surface area: roughly 2 m tall and 0.5 m wide, with a front and a back. So the surface area is about $2\,\text{m}^2$. Thus, the power lost is

$$P \sim FA = 300\,\text{W}\,\text{m}^{-2} \times 2\,\text{m}^2 = 600\,\text{W}.$$

No wonder a winter day wearing only thin pants and shirt feels so cold: 600 W is large compared to human power levels. Sitting around, a person produces 100 W of heat (the basal metabolic rate). When 600 W escapes,

It would be easier to have this equation on the last page, so we don't flip from page to page to see how this equation was derived.

I still find it amazing in science and math in general that so many formulas can condense down into one simple equation, with lots of lumped "black boxes" in them (even if in this case we know what the constant is hiding)

I like the explanation. I found it easy to follow the path taken.

Agreed. It was much easier to comprehend than the previous sections.

I found the last section pretty comprehensible, but I agree that this one was very elegant and well written.

That's a benefit of reading memos. When I first taught a version of this class (in IAP 2006), the class did reading memos on paper, which they turned in the day we had the lecture on the topic. So, I couldn't use the reading memos to fix that particular chapter in time to give students (since they had already read it).

But I did use the previous memos to figure out how students were thinking and what explanations would help. With this particular subsection, I remember using what I learnt from the previous memos and then spending 8 hours straight rewriting the explanation of flux and diffusion until I was happy with it.

how can we make this assumption? what is k again?

K is the thermal diffusivity, nu is the kinematic viscosity (momentum diffusivity constant).

I think that the model you are using for heat loss needs to be more clearly explained before diving into all of these quantities. For instance, it appears you are calculating the K for air, but you don't say so (or why) before jumping right in. I think that'd be useful and would make this analysis clearer.

I agree, some of us have not taken that much thermo-stuff, and some background would definitly help.

How does wind affect this calculation? Do you just insert the wind-chill temperature in for T1?

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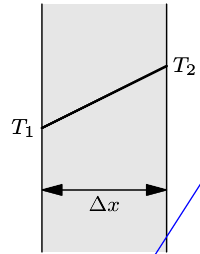

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How does wind affect this calculation? Do you just insert the wind-chill temperature in for T1?

I'd assume so, since when there's wind chill it "feels like" some lower temperature.

> Wind chill is a very interesting application of all the previous ideas. The wind chill affects your exposed skin (the wind doesn't matter to the areas protected by a jacket). On the exposed areas, a fast wind means a thin boundary layer (shorter time for momentum to diffuse), so the temperature gradient DT/Dx from skin to air is large – hence there is more heat flowing from your skin, just as it would on a windless but colder day.

Good question, I'd be curious to know the answer of this.

is this supposed to be the thermal conductivity of air? the standard values for that are 10-100 W/mK.

> Typical solids are about 1 W/(m*K) and metals (due to electrons being good movers of heat) are around 100 W/(m*K) but air is a lot lower than all of the above. Wikipedia gives 0.025 W/(m*K).

(To the original poster) Could you be thinking about convection over a surface (often h=10 to 100 W/m^2K) rather than conduction?

It's pretty cool deriving and using the equations we derived months ago in 2.005 in a completely different way!

Seriously, I wonder if there are equations that physics hasn't been able to simplify but these types of methods might be able bring us to some kind of close approximation.

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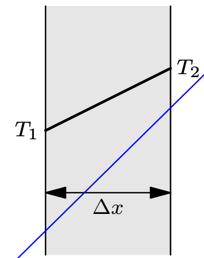

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I'm not sure this is appropriate for a textbook....

It is Sanjoy's textbook...

yeah, Sanjoy knows we are all adults anyway

I think this situation doesn't warranty that extreme of a response. It seems like a minor annoyance as opposed to something you would curse about. (irregardless of whether it is appropriate or not)

lol I'm surprised anybody actually commented on this

I think it's totally appropriate – it makes it seem like a person is speaking to you instead of some black and white text on a page is proclaiming something.

HAHAAHAHAHAHAHAHA. Love it

I mean I think it's fine... This whole textbook is pretty conversational.

I feel like this math was really well done. thanks

Hmm, I think at 600W power loss, your internal temperature drop would be significant, no? (i.e. your skin temperature would be significantly low). This would lower our result, maybe a significant amount (perhaps expand the example to include skin and flesh as thermal insulators?)

Right, if you didn't do anything about it. Hence your body starts shivering, and it's almost an unconscious action – to burn enough fuel to keep your core temperature constant. Your body is willing to let the extremities get too cold, if it has to (hence frostbite) in order to keep the core temperature from dropping too much. Enzymes have very tightly optimized shapes and even small changes in temperature can significantly lower reaction rates.

wow...

I've always heard that your head and feet lose the most heat...is that true and how does that fact play into this equation?

Yeah that's true. I wonder if that plays a role here, or if it all averages out to something close to our estimate

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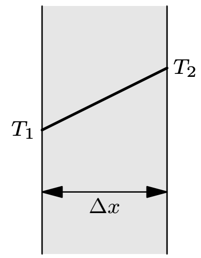

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Which are...?

one is losing far more than the basal metabolic rate. Eventually, one's core body temperature falls. Then chemical reactions slow down. This happens for two reasons. First, almost all reactions go slower at lower temperature. Second, enzymes lose their optimized shape, so they become less efficient. Eventually you die.

One solution is jogging to generate extra heat. That solution indicates that the estimate of 600 W is plausible. Cycling hard, which generates hundreds of watts of heat, is vigorous enough exercise to keep one warm, even on a winter day in thin clothing.

Another simple solution, as parents repeat to their children: Dress warmly by putting on thick layers. Let's recalculate the power loss if you put on a fleece that is 2 cm thick. You could redo the whole calculation from scratch, but it is simpler is to notice that the thickness has gone up by a factor of 10 but nothing else changed. Because $F \propto 1/\Delta x$, the flux and the power drop by a factor of 10. So, wearing the fleece makes

$$P \sim 60\,\text{W}.$$

That heat loss is smaller than the basal metabolic rate, which indicates that one would not feel too cold. Indeed, when wearing a thick fleece, only the exposed areas (hands and face) feel cold. Those regions are exposed to the air, and are protected by only a thin layer of still air (the boundary layer). Because a large Δx means a small heat flux, the moral is (speaking as a parent): Listen to your parents and bundle up!

Only 7 Cal/minute. Not a very efficient or fun calorie burning technique.

Wait. I forgot the difference between Cal and cal.

1 Cal = 1 kcal = 1000 cal.

I love seeing these explanations (biology, and all the other things I'd never expect to see in a course 2 or 6 class) that really give us the whole story of what we're looking at.

I agree. In general, it would be nice to hear even more of this stuff although you do show us a lot. These pieces of information are what I enjoy most.

it would also be cool to comment on how long that would be

Finally an explanation in my major!! I'm no longer left out! :D

Tell it like it is, Sanjoy.

I love it!

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I've lost how we're using random walks to solve this last problem

Was just going to bring that up myself - where is the random walk here?

there wasn't one here. i mentioned this in an earlier note. we're only using the result of one of the random walks–the diffusion constant.

Yup, diffusion constant and an interesting note on why you should bundle up in the winter!

I'm confused about the connection to probability. It was much more explicit in the last sections, but how is it being used here? In a section on how to use probability, we seemed to use it briefly in the beginning with 1/2 in either direction/expected positions.

I feel like this last part of the section got slightly off track– maybe introduce that you will be doing this at the beginning of the section?

We used probabilistic analysis of a random walk to determine the Diffusion Equation and then applied this equation in cases where random walks occur (i.e. losing heat on a cold day).

yeah, the connection from probability to random walks to diffusion constant to heat loss was amazingly informative. i am impressed by how much we can figure out by just combining some premises from differing fields of study.

I actually agree with this comment. Although I like the example, I'd rather see an example that uses random walks than something it came up with (if that is even possible).

typo, delete

i really like this ... good example & well written

proportional reasoning!!!

proportional reasoning!!!

one is losing far more than the basal metabolic rate. Eventually, one's core body temperature falls. Then chemical reactions slow down. This happens for two reasons. First, almost all reactions go slower at lower temperature. Second, enzymes lose their optimized shape, so they become less efficient. Eventually you die.

One solution is jogging to generate extra heat. That solution indicates that the estimate of 600 W is plausible. Cycling hard, which generates hundreds of watts of heat, is vigorous enough exercise to keep one warm, even on a winter day in thin clothing.

Another simple solution, as parents repeat to their children: Dress warmly by putting on thick layers. Let's recalculate the power loss if you put on a fleece that is 2 cm thick. You could redo the whole calculation from scratch, but it is simpler is to notice that the thickness has gone up by a factor of 10 but nothing else changed. Because $F \propto 1/\Delta x$, the flux and the power drop by a factor of 10. So, wearing the fleece makes

$$P \sim 60\,\mathrm{W}.$$

That heat loss is smaller than the basal metabolic rate, which indicates that one would not feel too cold. Indeed, when wearing a thick fleece, only the exposed areas (hands and face) feel cold. Those regions are exposed to the air, and are protected by only a thin layer of still air (the boundary layer). Because a large Δx means a small heat flux, the moral is (speaking as a parent): Listen to your parents and bundle up!

This is cool though - it's interesting to know that the amount of power lost is inversely proportional to the thickness of my outerwear...sometimes parents do know best!

This answer is cool, but I have trouble visualizing 60W of power.

Well, you don't have to visualize that quantity. But if you want to, just think of what the text says with respect to clothing.

does your metabolic rate increase when it is cold?

That's why you shiver!

Is it a matter of thickness or density of material? There seem to be very thin, yet warm jackets these days...

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if your body produces 100, and you dissipate 60, then you're netting 40. do you build up heat and get too warm?

I think this is really simplistic, but at some point yes you would get warm, but not if you were just sitting there in the cold in your jacket

Not all of your body is covered. So the uncovered part would lose faster.

I have a feeling overall, although perhaps in theory you're generating enough heat, our estimations mean that you would definitely tell you were somewhere cold, not chillin on a beach in florida.

Don't take the 60 W too literally, it could easily be off by a factor of 2. But, the general point you make remains – if you are generating more heat than you lose, you'll feel warm (and probably decide to take off a jacket or find a cooling breeze).

only a parent would end a chapter with that proclamation.

great conclusion. I love it.

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I feel like this conclusion might have exterior motivations, given that you are a parent and all.

> Definitely ulterior motives for it as I try to convince my daughter that she needs to put on her coat when it is cold (or her sun hat when it is very sunny). The next baby is arriving in a week or so, but she'll be less able to argue, at least for a couple years.

I've also heard (and seen on television) that you can use mental "toughness" to raise your temperature enough to prevent hypothermia and death. Navy SEALS are trained to do this, I believe.

Woah, that sounds pretty awesome. I wonder what kind of physiological reactions are going on exactly?

Well you know how when you get really angry your body temperature rises..maybe they think unhappy thoughts?

Its interesting to see how 'mind over matter' actually works.

> My own experience staying up all night doing physics problem sets or 24-hour take-home exams was that I had to eat a huge amount, e.g. a whole pizza, and then I was fine. I think that's because I was burning lots of calories doing so much thinking.
>
> Consistent with that experience, I was told that Garry Kasparov, when he plays world-championship matches, eats 8000 kcal/day (instead of the usual 2500 kcal/day) and loses weight.

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I don't know if this is coming, but it'd be great to relate random walks to errors in estimation. If we are using divide and conquer, our log(error factor) is a variation of the random walk discussed here, where each step itself has some (unknown but approximable, e.g. by the log normal curve) distribution.

I think this would tie together several different topics, and would reinforce the point that our error's variance grows (linearly?) with the number of divisions we make, and therefore the standard deviation grows at a lesser rate.

> Yes, that would work well. For error estimations, it's still a one-dimensional walk. But the step sizes are now not fixed; instead, each factor contributes one step, and its size is normally distributed with mean 0 and standard deviation equal to the number of log units of plus/minus (e.g. if it's 10^(a +/- b) then the standard deviation is b).

How about using the random walk analysis to use Brownian motion to figure out the size of molecules?

That would be an interesting section and would also allow you to firm up your 3 cubic angstrom assertion.

That would be interesting to see. I'd like to see that worked out.