9
Springs

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9.1 Why planets orbit in ellipses

For a planet moving around the sun (assumed to be infinitely massive), the planet's energy per mass is

$$\frac{E}{m} = -\frac{GM}{r} + \frac{1}{2}v^2, \tag{9.1}$$

where m is the mass of the planet, G is Newton's constant, M is the mass of the sun, r is the planet's distance from the sun, and v is the planet's speed.

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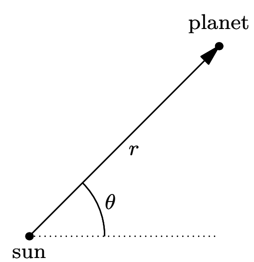

This energy contains two coordinates, which is one too many for an easy solution. We can get rid of a coordinate by incorporating a conservation law,

Global comments

There is way too much mathematical explanations in this section for me to fully grasp the subject. It's very non-intuitive for me and I am having trouble reading through the section.

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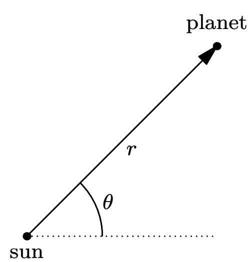

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Comments on page 1

Do you really mean contains, or is it more, "can be approximated as"

When I read the title, I was thinking, "there's no way this is going to only be about springs..."

really? " every "physical process?

yes. almost every physical process. it's not really that it literally contains a spring, just that they all act similarly ... the spring math can be used just about everywhere.

really, they "can be approximated as" a spring...

Contains or can be modeled as a spring?

I think he's treating those expressions as equivalent.

Ooh this promises to be interesting...

yeah, great opening sentnence

This is surprising. I look forward to reading more!

Yeah...im very curious to see how this is going to make sense

This sounds intriguing!

Sounds very 8.01!

Read this section for Monday (memo due Monday at 9am). It starts our final tool: the spring approximation.

Perhaps you could put a couple lines in the beginning to outline how we will go about addressing the topic (springs) with the example as you've done in previous readings. For me it makes the analysis easier to follow.

I really like this idea...i think it would have helped me get my head around the concept _a lot_ faster.

is there a reason that this isn't m[sun] >> m[planet]?

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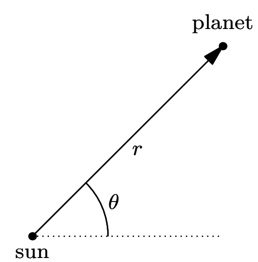

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is there a reason that this isn't m[sun] >> m[planet]?

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why? do we just take the gravitational constant for granted for that distance?

Would also like an answer to this. Also, it's a little unclear if it's the sun that's infinitely massive or the planet. Obviously it has to be the sun, but my first reaction was to think it's the planet and I was a little confused.

wouldn't the sun be moving around it?

He is referring to the sun here, not the planet

Is this a fair assumption? I mean, sure, the sun is VASTLY greater than a planet but if you compare say our sun and Jupiter, the difference is completely within several orders of magnitude and not near infinite.

the reason for the assumption is that we're assuming the sun stays in one place. if it weren't assumed infinitely massive, then we'd have to consider the (much smaller) orbit of the sun as well.

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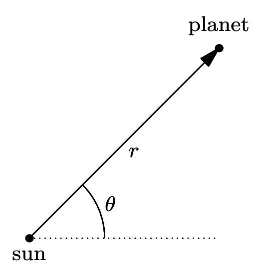

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Why do we have this condition?

I think because otherwise the planet would exert a gravitational force on the sun and it would mess up our calculations.

either the gravational force idea, or that we just want to assume the center of mass lies roughly inside the sun

Without assuming this, they would both essentially orbit around some point in space (like a binary star system). Making this assuming basically assumes that the sun doesn't move, and the center of the orbiting system lies at the center of the sun.

Additionally, the mass of the sun (or any star) is so many more times greater than the mass of its orbiting planet that this approximation is very accurate and turns out to basically make the math easier.

I agree. The sun's mass is so much larger that it makes the effects of the earth on the sun negligible and therefore you shouldn't have to calculate them

After reading this section it looks like we need to explain why we assume that the sun is infinitely massive as well as under what circumstances that assumption should be made and that it won't affect the equations you give later. There seems to be a lot of confusion in this section about this term.

As mentioned above by 08:18, the correct answer should be that an infinite mass will not move when orbited by another finite mass.

When people say it makes the "effect on the Earth" neglible, I hope you're NOT referring to force. Forces are equal and opposite in nature. You might be tempted to think that the sun exerts a larger force on the Earth, but there is no such thing as an object exerting its own force on another object. Forces are not possessive, but are the *NET* effect of pairs of objects interacting with each other. Therefore, each object in that pair will feel the same force, in opposite directions.

I understand why an assumption like this must be made to simplify the equations. however, is there a reason that the sun is "infinitely" massive as opposed to it's mass being 'much greater than' that of the planet, ie m[planet] << m[sun]

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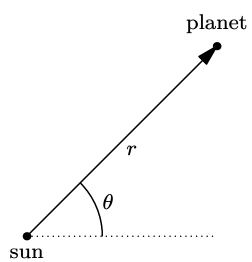

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After reading all the comments, I see that I stated it badly and partly incorrectly (if the sun were actually infinitely massive, then the planet would orbit at infinite speed!). A better statement is that the mass of the planet is very small compared to the mass of the sun (in E&M, that's the "test-charge condition"). Then the sun hardly moves, and the kinetic energy of the sun is small compared to the kinetic energy of the planet.

Here's a scaling argument to show that. The KE (kinetic energy) of the sun is roughly M*V^2 where M = mas sof sun and V =speed of sun. Conservation of momentum says M*V=m*v (where m=mass of planet, v = speed of planet). So the ratio of kinetic energies, M*V^2/(m*v^2), is m/M (after a few steps of algebra). So what I really mean is : m/M << 1.

How do you assume something as infinitely massive and then use that assumption to get energy per mass?

I think the suns mass is assume to be infinite, not the planets.

another assumption that is made but i havent been able to find so far is that the sun and planet are being treated as point masses.

is there a reason that this isn't m[sun] >> m[planet]?

why is this negative?

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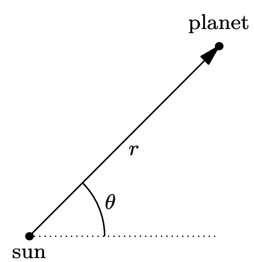

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I'm not familiar with planetary physics and the associated formulas. Where exactly does this come from? Perhaps defining E before you divide by m would make it more clear.

This is just the gravitational potential energy (-GMm/r) plus the kinetic energy of the plane moving about the sun (1/2 *m*v^2), then divided by m (the mass of the planet). M is the mass of the sun, and G is newton's Gravitation constant, and r is the distance from the sun.

This is a pretty standard representation of Energy (Just PE + KE), just in the planetary sense. This is fairly common for my major, but it may help if there was a basic explanation of how this is the potential + kinetic energies

I agree that its pretty recognizable but a quick debrief about energy at the intro of the section wouldn't be so bad.

Or maybe just starting a little less derived would help. As in showing where this equation came from would help clarify matters.

Even just below each quantity, just labeling it. Like "energy/mass = gravitational energy + kinetic energy"

So how does the sun's infinite mass affect this equation? since there are two masses, I'm assuming that one is the planet and one is the sun

I'm also not sure about this, maybe that assumption is what keeps the sun from moving as well as the earth, which would create a more dynamic system.

Yeah, the infinity makes things a bit strange here. The first term in E/m contains M, the sun's mass, so it should go to infinity.

oh so were going to say that E/m goes to zero so that we can solve the problem easier... it would be nice to see how you arrived at this

I'm not sure that we're going to have the energy go to zero. Also, this equation comes from the fact that the energy of the earth is the sum of the gravitational potential energy and the kinetic energy.

^ Ditto.

using the real mass and not the infinite assumption of course. correct?

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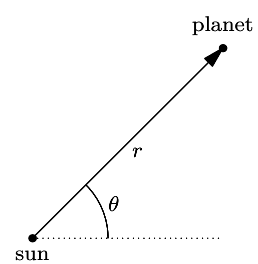

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If the sun's mass is assumed to be massive, why is the sun's mass in the equation. If we truly are making the assumption, wouldn't the equation go to -infinity?

Because the potential energy (essentially how much its gravity affects what is orbiting around it) is dependent upon the mass of that body.(Larger mass, more gravity) The sun is very massive compared to that of Earth, but it definitely isn't infinity.

But the line directly before this says "the sun (assumed to be infinitely massive)." That seems pretty straightforward to me. But how does that apply to this? A previous comment said something about center of mass, could it be just to make that assumption and nothing else?

I'm not sure why we made the original assumption, but it would be nice to have it spelled out. I had the same question here.

In order for this theory to be valid, it would require an infinite mass. However the equation used requires a real mass. I understand, but it would be more clear with further explanation.

would the theory still be valid if "m[sun] >> m[planet]"? I feel like this statement still allows us to make the assumption that the center of mass of the system is the center of mass of the sun and still accept that GM/r is not infinity.

Tangential velocity, I assume, not angular. Not that there really would be confusion at this point, but it might be worth pointing out.

I love polar coordinates for spherical motion (go figure...). In high school, our teacher wanted us to do circular motion in Cartesian coordinates "for instructive purposes and to make you appreciate polar coordinates."

I agree, it makes it much easier to understand a problem like this when it's in polar coordinates.

Wish my teacher did that, maybe I would like polar coordinate more.

my only problem with polar coordinates was that we were always forced to switch between them and cartesian

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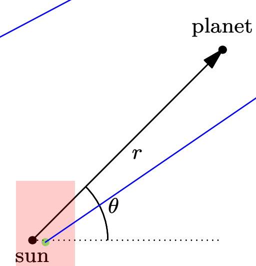

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i'm glad you distinguish between the potential and kinetic energy terms in the above equation.

agreed, but I feel like it should happen with the equation ... say in words before in the equation ... it would have helped me remember why this eq sooner.

how do you convert to polar coordinates? I forgot

Should this be an equation most of us should know? becasue i don't think I've ever seen it before

It's a transformation of the kinetic energy term in equation 9.2 to polar coordinates in 2 dimensions. dtheta/dt is the change in theta over time, which is a velocity. dr/dt is the change in radius over time, which is another velocity.

Interesting. Thank you for the explanation, although the next page also helps with the understanding.

Thanks, I was staring at this wondering what it was. I wonder if the general audience needs to be reminded of this?

even though I am aware of this equation...I think you should break it down and explain in greater detail

So, mass is missing in this equation because it's infinitely massive?

Because this equation is for the 'kinetic energy per mass'. The quantity E/m.

Wouldn't we be losing information by doing this? Why would we want to get rid of a coordinate?

technically, we're not just throwing it out. we're "getting rid of it" by canceling it out with another known equation. there is no loss.

Does the fact that the sun moves in an orbit itself complicate this significantly?

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angular momentum. The angular momentum per mass is

$$\ell = r^2 \frac{d\theta}{dt}, \quad (9.3)$$

The angular momentum per mass ℓ allow us to eliminate the θ coordinate by rewriting the $\frac{d\theta}{dt}$ term:

$$r^2 \left(\frac{d\theta}{dt}\right)^2 = \frac{\ell^2}{r^2}. \quad (9.4)$$

Therefore,

$$\frac{1}{2}v^2 = \frac{1}{2}\left[\frac{\ell^2}{r^2} + \left(\frac{dr}{dt}\right)^2\right]; \quad (9.5)$$

and

$$\frac{E}{m} = \underbrace{-\frac{GM}{r} + \frac{1}{2}\frac{\ell^2}{r^2}}_{V_{\text{eff}}} + \underbrace{\frac{1}{2}\left(\frac{dr}{dt}\right)^2}_{\text{KE per mass}}. \quad (9.6)$$

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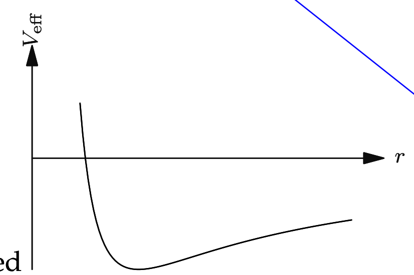
Now let's study the just the effective potential:

$$V_{\text{eff}} = -\frac{GM}{r} + \frac{1}{2}\frac{\ell^2}{r^2}. \quad (9.7)$$



The first term is the actual gravitational potential; the second term, which originated from the tangential motion, is called the centrifugal potential. To understand how they work together, let's make a sketch. For almost any sketch, the first tool to pull out is easy cases. Here, the easy cases are small and large r. At small r, the centrifugal potential is the important term because its

angular momentum isn't really a conservation law. There is a conservation law for it, but this makes it sound like you're saying the term itself is the law. It'd be like saying "energy is a conservation law".

Why r squared?

I would also like an answer to this question.

I think it's because of the definition of angular momentum L=r x p, where "r" is the moment arm and "p" is the linear momentum (which is mass x velocity). Since we only need tangential velocity for linear momentum, we use r*d_theta.

Of course we divide all this by mass since we want angular momentum per mass.

This makes a lot of sense. I was having trouble with the units for a second.

is this the symbol for angular momentum?it looks like an l

How does this help us if we don't know what the angular momentum is? We are basically substituting one variable for another

It helps because the angular momentum is constant, so you replace a changing quantity (d(theta)/dt) with a constant. it's the maxim from the invariance chapter: "When there is change, look for what does not change."

A constant, even an unknown one, is much easier to handle than a changing quantity. (For example, the constant has zero derivative.)

should be: allows

Hmm, quite clever. It took me a second to realize how we went from 9.3 to 9.4, but I see we just squared both sides and divided by r^2.

I agree. These simple substitutions are what always trips me up, because I am on the look out for something more complicated.

Ha, neat!

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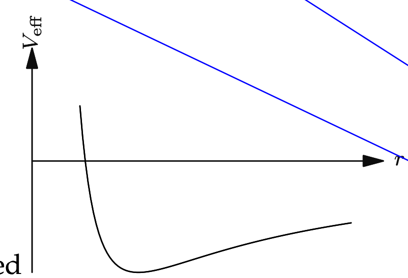

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does the radius change? it shouldn't so shouldn't dr/dt be zero?

I think that the point of this section is to explain why r does change. An ellipse is like a circle with varying radius.

I feel like it's a vote of confidence for my gut that I forgot to do this reading before lecture (oops! sorry!) and I still got the lecture demo right by intuition.

I guess I'm getting confused as to what velocity we're referring to. Above dTheta/dt was velocity in the momentum equation. But here dr/dt is velocity in the kinetic energy term.

If you look at the 9.2 equation, you will see that both dTheta/dt and dr/dt are components of velocity in the kinetic energy equation.

Took me a while to draw that conclusion. Lots to remember in a short period of time!

Be careful! dTheta/dt is an angular velocity, the actual component of velocity is r*dTheta/dt/

what is Veff?

I like it when you break down what the terms in the equation translate too

I agree, I think it could definitely help with the initial energy equation introduced on the first page as well.

I'm a big fan of how it has the little subtitles and the brackets throughout the chapters.

Oh ok,s o that's why we substituted earlier for a seemingly unknown L. However, I'm not sure if I would have fully understood this without the comment right here.

Eliminating the dependency on its angle by using angular momentum, and then realizing angular momentum is constant is a clever trick. I don't think I would think about making this move if I wasn't shown how to do it.

because they're perpendicular, right?

wait what? i don't follow....

Is there a particular reason they have that name?

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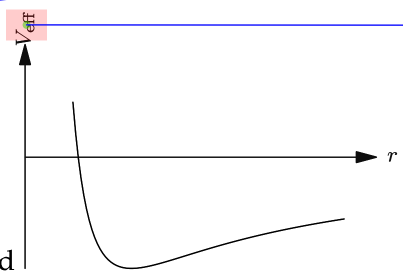

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We need an alphabet with more letters... When I first saw Veff in the equation I thought it had something to do with velocity since the other term was kinetic energy.

Yeah I was confused by that notion initially as well. It would be nice to see either a different notion or to see this current one (V_eff) defined earlier.

Right, I always thought potential was U.

I think you're think of potential energy – which this is not, it's potential energy per mass. It's similar but different enough that the V reminds us that we're not dealing with units of J but J/kg.

at least it's capitalized.

What is the difference between regular potential and effective potential. Does one take into account the mass of both of the planet and the sun?

Right here I started to wonder where springs or estimation were going to come in at all. Of course, I'll keep reading, it but it would be kind of nice to have had a preview before here. So far it's read like a physics textbook.

define effective?? I know you described this in the previous paragraph, but what deliniates it from other potential measurements?

It's different from other potential measurements because it's not simply gravitational/electrical/chem potential energy. What it is, however, is a measure of energy which depends only on distance (not, for example, on speed), so it *behaves* like a potential energy.

I think this is a typo?

I suspect that it may have been difficult to label the graph this way, but I think it's actual perfectly clear the if you rotate it 90 degrees to the right.

Why would we cant to rotate the axis? We are looking at the potential as a function of radius not the other way around. I think it would be much more confusing if we flipped it

not rotate the axis, rotate the axis label. and i agree that it would have been a lit easier to read without turning my head or the page

I agree, and I tried to rotate them. But I couldn't convince the figure-drawing program ("asymptote") to do it, but I'll try again for the next version.

angular momentum. The angular momentum per mass is

$$\ell = r^2 \frac{d\theta}{dt}, \tag{9.3}$$

The angular momentum per mass ℓ allow us to eliminate the θ coordinate by rewriting the $\frac{d\theta}{dt}$ term:

$$r^2 \left(\frac{d\theta}{dt}\right)^2 = \frac{\ell^2}{r^2}. \tag{9.4}$$

Therefore,

$$\frac{1}{2}v^2 = \frac{1}{2}\left[\frac{\ell^2}{r^2} + \left(\frac{dr}{dt}\right)^2\right]; \tag{9.5}$$

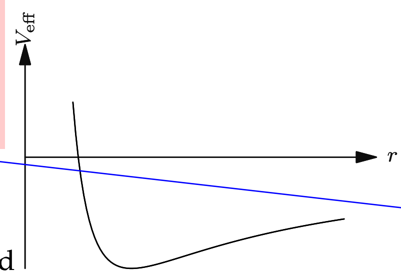
and

$$\frac{E}{m} = \underbrace{-\frac{GM}{r} + \frac{1}{2}\frac{\ell^2}{r^2}}_{V_{\text{eff}}} + \underbrace{\frac{1}{2}\left(\frac{dr}{dt}\right)^2}_{\text{KE per mass}}. \tag{9.6}$$

Because the gravitational force is central (toward the sun), the planet's angular momentum, when computed about the sun, is constant. Therefore, the only variable in E/m is r. This energy per mass describes the motion of a particle in one dimension (r). The first two terms are the potential (the potential energy per mass); they are called the effective potential V_{eff}. The final term is the particle's kinetic energy per mass.

Now let's study the just the effective potential:

$$V_{\text{eff}} = -\frac{GM}{r} + \frac{1}{2}\frac{\ell^2}{r^2}. \tag{9.7}$$



The first term is the actual gravitational potential; the second term, which originated from the tangential motion, is called the centrifugal potential. To understand how they work together, let's make a sketch. For almost any sketch, the first tool to pull out is easy cases. Here, the easy cases are small and large r. At small r, the centrifugal potential is the important term because its

I think graphs are always helpful to help with our understanding of the material.

You might want to explain intuitively what "effective potential" means; otherwise, it's just another term being thrown out.

Since I have no knowledge as to what "effective potential" is referring to and how it differs from a normal "potential", I agree.

"effective" just means we take the normal potential and adjust it somehow, so it seems like its potential is something else.

Agreed. For all practical purposes you can just consider the effective potential to be like any other potential. The only difference is that the effective potential includes some other quantity (angular momentum here) that isn't necessarily a "potential" as we might usually think of one. It's really just a way of lumping the terms in our equation to make it easier to handle.

I agree with the fourth person, there's really not much to the term effective and I think for this case it's explained in the line above so it isn't too confusing.

I think it would be nice just to have a short sentence that explains what effective potential is. It would be a nice refresher for people who already know and a helpful piece of information for people who are unfamiliar.. plus it wouldn't take up too much space.

I don't see why we need the term effective potential at all- right after we lumped everything together and called it the "effective potential", we take it apart again and analyze its parts separately anyway

alternatively, explain what the potential is and how the effective potential differs. It'd be nice to see the base case here.

I agree. I dont know why we really need to rename it. I understood the first time that you wrote this equation above, that it consisted of the potential and kinetic energy components. If you are going to name this term or clarify its components for readers, I would recommend doing it earlier in the reading.

If we're assuming the sun is infinitely massive, why doesn't this term drop out or just make the expression infinite?

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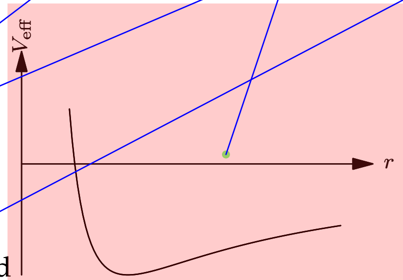

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I like this sketch.

This looks like a lot of other things we've seen in class. It might be interesting at the end to figure otu which graphs look like they relate to each other and find out why....nature seems to make things work in certain ways, and just keeps repeating them!

I definitely agree with this I think it would be really cool to look at relations that have come out of the work we've done in this class that we might never see otherwise.

How was this graph compiled? Easy cases?

I'm curious about this too - was it compiled with easy cases or was it simply plotted with software?

Actually, this is exactly the same plot we saw when estimating the size of an atom using electrostatic potential and Heisenberg uncertainty.

as opposed to the fake potential?

I think it's the "actual" because we're used to thinking about potential as gravitational potential. The other term isn't part of the "actual" gravitational potential that we usually see.

I think I see where this is going already, and it's very cool!

these potentials are kinda confusing

I always like to see a familiar technique come into play for a new problem!!

I like to see when easy cases are used. I think easy cases are the basis for solving a lot of these problems and it almost could have been taught earlier.

I agree–easy cases are easily applied since everything can have easy cases, and the nature of easy cases is quite intuitive as well, so it's always a good starting point.

yeah I agree.. I feel like easy cases should have been the first or second topic, since it's a simply technique that most people use without realizing it.

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If this analysis and sketch look familiar, that's because they are. The effective potential has the same form as the energy in hydrogen (Section 7.3 or r26-lumping-hydrogen.pdf); therefore, our conclusions about planetary motion will generalize to hydrogen.

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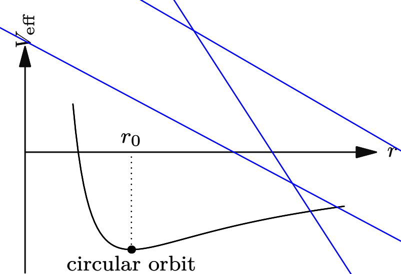

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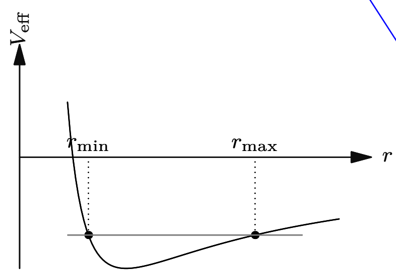

This paragraph is a great and concise explanation.

It also makes a lot of intuitive sense in terms of how planetary orbits work. I like this paragraph and the diagram a lot.

This is one of my favorite quick sketch techniques that we use in this class that has been incredibly useful in my other classes

I agree, I love this strategy.

I agree. I've found this very useful as well.

same...this is really helpful!

Yeah I've also been using this for my other classes.

I agree as well, I had a feeling that this model would emulate the other hydrogen curve from when we first saw the equation.

This is asymptotic approximation. right?

it's like this kind of graph is everywhere, especially when we calculate energy!

I like that you point this out because I'm not sure I would have noticed this myself and it's a really interesting piece of information!

Me too, I especially like the reference in the next line to the exact pages we can find it.

This sort of thing has popped up so many times in my physics courses.

agreed. i like the very specific reference instead of something general

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quantum physics is so interesting...very small and very large objects follow similar rules

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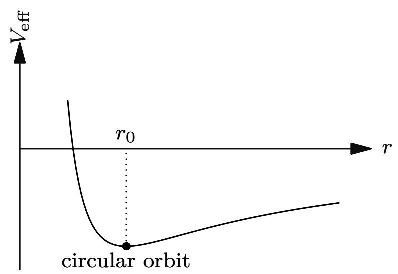

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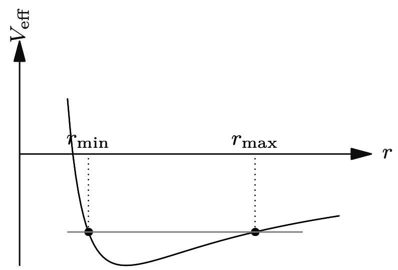

I like how this connected the readings, and how there's a direct reference to it, making it easier to find.

yeah I agree...I feel like in earlier sections when you wanted to reference a previous example you just mentioned what part of the book it was from, which was not as helpful. Instead here you give the exact section number AND the pdf which is really useful in the context of our class!

thinking along the lines of 1:22 ... when you publish the book, i think that including page numbers with your section references would be very helpful

This is pretty cool - but it makes sense when you just think about how similar the atom and planetary models are.

That's true, but it's still surprising given that at the atomic scale we have to worry about quantum effects and at the planetary scale quantum definitely does not apply.

I was thinking about the reason for the similarity a bit more. In both cases it's due to angular momentum. For the planet, the 1/r^2 term arises from using angular-momentum conservation to replace the d(theta)/dt term. For hydrogen, the 1/r^2 term arises from the uncertainty principle (Delta-p)*(Delta-x) = hbar. And hbar is the quantum of angular momentum (the size of a packet of angular momentum).

It does, which is weird when you think about how big planets are in relation to atoms but the physics are the same for the two situations, but then apparently when you go from atoms to the quantum level the physics changes.

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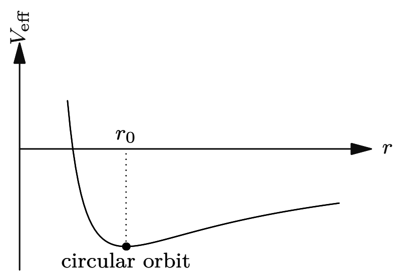

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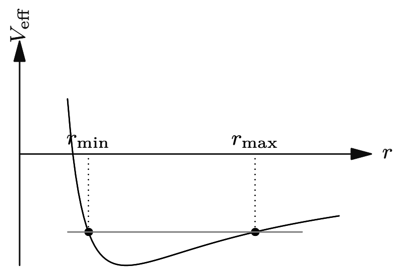

Wow, I am surprised by this conclusion.

Exactly! similar charts. It also reminds me a bit of what the graphs from the plane drag looked like when we added them to find the ideal height or velocity of the plane

Again I'd love to maybe spend the last day or two looking at the similarities we've derived in different areas of the physical world, it's crazy how much behaves in a certain way

Is this saying that planets orbit the sun in a similar way that an electron "orbits" the nucleus of a hydrogen atom? If not, in what way does this problem generalize to hydrogen?

For hydrogen, you also would make an effective potential The difference is that we can't treat hydrogen classically because it is so small.

The coloumb potential and the gravitational potential have the same form though, so it's not surprising that the graph is the same.

Yeah, this is very interesting. At first it seems quite intuitive that these two models would be similar, but then when you think about it, it's pretty crazy that things on the scale of hydrogen and things on the scale of planets, which use obey laws and forces, still can be generalized

For the last day, I've planned (following the tradition I learnt from Donald Knuth's courses) a short summary and then mostly "your turn", i.e. ask any question and I'll do my best. Explaining similarities would be a fun question.

I love how we can connect things from opposite ends of the spectrum- hydrogen atoms and planets behaving in the same way

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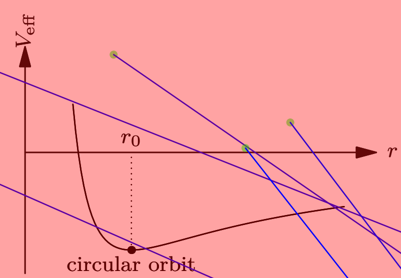

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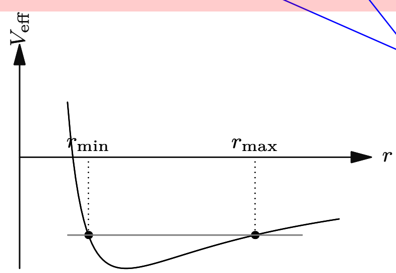

In response to a similar observation in another thread, I posted the following (which I'll just repeat here):

I was thinking about the reason for the similarity a bit more. In both cases it's due to angular momentum. For the planet, the 1/r^2 term arises from using angular-momentum conservation to replace the d(theta)/dt term. For hydrogen, the 1/r^2 term arises from the uncertainty principle (Delta-p)*(Delta-x) = hbar. And hbar is the quantum of angular momentum (the size of a packet of angular momentum).

[My double posting of the response reminds me of a NB project that could make a good UROP (or I might work on it myself at Olin next year): discussions that can be merged, modified, and otherwise improved for the benefit of all the readers.

A model for that is stackoverflow.com, which I learnt about from Micah Siegel (my roomate in graduate school, whose college roommate created stackoverflow). In the FAQ at http://stackoverflow.com/faq look at the section on "Reputation" for an interesting list of the possibilities that could happen if students could be NB moderators.]

so we are trying to minimize the energy, just like the hydrogen

ok, sweet, my previous comment has been answered

It's nice seeing the results visualized this way.

It would be interesting to see a graph of the circularity of the orbit compared to r.

Hmm, I guess that's true, but it's not really relevant here. You can look that up whenever, this is specifically about elliptical orbits.

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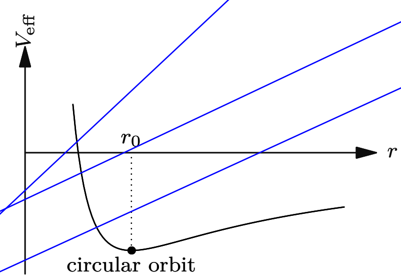

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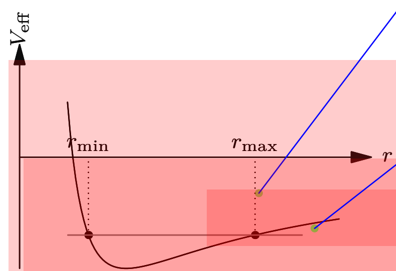

I'm confused as to if this 'kick' is one that knocks the planet into a different type of orbit or if it just shifts the orbit by a certain distance....

I think it just means to disturb it from its original orbit.

Think of a comet

What gives the planet its initial perturbation? That it is nearly impossible to form a planet with 0 initial radial velocity?

does this mean that the orbits were circular until they were disturbed?

Good point.

Can r_min and r_max be anywhere just as long as r_min is less that r0 and r_max is greater than r0?

so are we saying that the dtheta/dt is always constant during this? wasnt sure with he wording

I think this is a little easier to visualize if you say something like, "imaging a marble resting on the line we have drawn and you thump it. It will slide up one side and then up the other. Then it will eventually roll back to the minimum energy position. The farthest points it rolls to form the max and min." This gives a visual and intuitive representation of the conversion from kinetic to potential energy as the ball rolls up the hill.

Thanks for this point, it makes me visualize this situation a lot better.

That's a nice picture. I had talked myself out of using it for the following reason. In the hill picture, the marble's velocity along the hill (not the horizontal component of that velocity) is determined by the kinetic energy (the difference between the total energy and the potential-energy curve). For the planet, the kinetic energy translates directly into a horizontal component (dr/dt). But that flaw may be small compared to the benefit in helping one's intuition.

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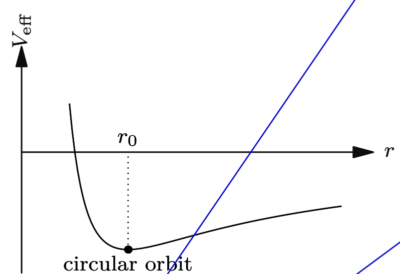

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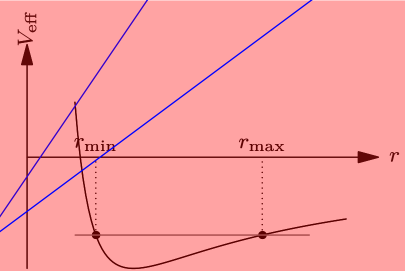


This is a lot of information presented very quickly and for those who have forgotten their 8.01, it's extremely confusing. You might want to expand on this section a bit.

Yeah, agreed. I'm getting a little lost in all the math here that isn't really explained.

Yeah, I am one of those people who has forgotten 8.01 so a longer explanation would definitely be helpful to me.. However, I wonder if a too in-depth would take away from the point of this section... maybe for people who are unfamiliar with this concept there could be an appendix in the back

I'm a bit lost too – where are the springs!?

At first, I expected that after perturbing the orbit, it would return to its lowest energy state, but then I remembered that there is conservation of momentum.

This is really interesting- what type of "kick" could have caused this phenomenon in real life?

I'm not sure you this type of "kick" would ever actually occur.

Maybe collision with a smaller object?

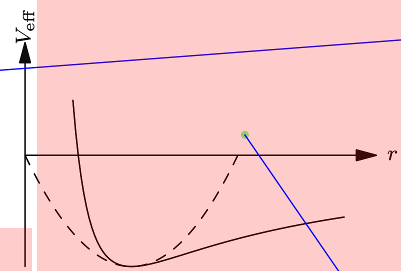
Maybe a large meteor hitting the planet and altering its path a bit? Seems like a kick...

This was the only kick I could think of...would seem difficult to get a significant man-made kick.

What the kick does is to make the angular momentum no longer match with the total energy for a circular orbit. So, although it's hard to imagine the kick directly, one can imagine other ways in which the angular momentum does not match with the total energy for a circular orbit. For example, if the planet is going too fast for a circular orbit, it will move in an ellipse (or even a hyperbola).

can we see this last part in a picture/image of what it means?

But what shape is that orbit? Finding the shape seemingly requires solving the differential equation for r, using the conservation of angular momentum to find $d\theta/dt$, and then integrating to find $\theta(t)$.



An alternative approach is to use the observation that, near its minimum, the effective potential is shaped like a parabola. We can find that parabola by expanding V_{eff} in a Taylor series about the minimum. To do so, first find the the minimum-energy radius r_0 (the radius of the circular orbit). By setting $dV_{\text{eff}}/dr = 0$, we find

$$r_0 = \frac{\ell^2}{GM}. \tag{9.9}$$

The Taylor series about r_0 is

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Comments on page 4

Isn't this why calculus was invented?

But we don't like using calculus in this class!

I'm confused. How are r_min and r_max not just the minor and major axis of the ellipse?

Aren't they though? Though I think Sanjoy is not assuming we know the shape is an ellipse and instead is leaving it open to be any given shape.

it is kind of weird that we are assuming something that we don't already know something

We're not assuming that it's an ellipse. We have, however, deduced that, whatever the shape, it will have an r_min and r_max (set by conservation of energy). Later, once we find out how fast the planet oscillates between r_min and r_max, we'll also see that it's an ellipse with the sun not at the center.

interesting approach to looking at the orbit of a planet..

when should we use each approach? should we just use this one when we want to see the shape?

The effective potential shape is the actual shape of the orbit?

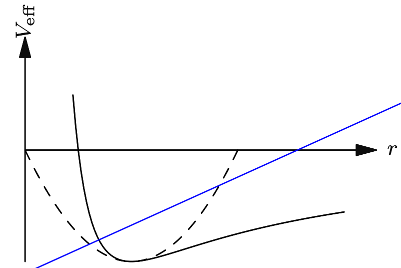
I believe it should be due to the fact that the potential relies on the distance.

The potential here is a graph that tells you the effective potential depending on how far you are from r=0.

it isn't the shape of hte orbit, but it's shape determines the characteristics of the orbit, such as where it is and what its frequency is.

thank you

if only i could remember how to use taylor series...



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it would help me if there was a line above this with the equation where this came from

I believe there is, and it's "By setting dV/dr=0..."

All he did was differentiate equation (9.7) with respect to r and set the derivative equal to 0. The value of r that solved that equation he called r_0

is there another method we can use besides Taylor seies? I thought using series is to make things simpler.

I would not/do not remember how to do this. Oops.

Nor do I, but the frequency that Taylor series pop up in my classes probably means I should look at them again...

It may be useful to include an appendix in the book of useful math tools for people like me who forgot about Taylor series...

I agree, a reference of common equations/expansions would be really useful. On a side note, there could also be a related section with all the handy approximation formulas we've derived over the course of the text.

This is a lot to remember. I'm glad we have this written here.

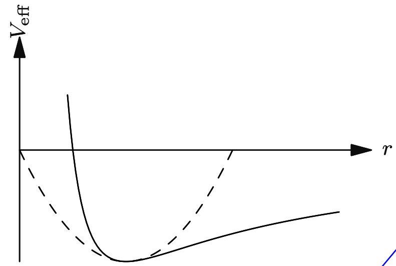
How many terms should we evaluate?

yeah i definitely would not have remembered how to do this :S

For Taylor series and other normally useless math formulas (like Euler), I did what Sanjoy suggested for common variables and wrote them all down on a piece of paper. Not only do I have a reference now, it helps me remember them too.

Having things like Taylor series scares me...

But what shape is that orbit? Finding the shape seemingly requires solving the differential equation for r, using the conservation of angular momentum to find $d\theta/dt$, and then integrating to find $\theta(t)$.



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I don't see how the third term is interesting but the 2nd term is not. At minimum energy (r = r0) doesn't the 3rd term and all subsequent terms go to zero as well?

So, all the (r-r0)s are not evaluated at r0, but just the derivatives of Veff. The second term vanishes because the derivative of Veff is 0.

the second derivative is not necessarily 0. if you think about the curve, the first derivative is 0 at the min, but then it starts increasing, so the second derivative is positive.

but the second derivative at r0 is 0, which is where it's being evaluated

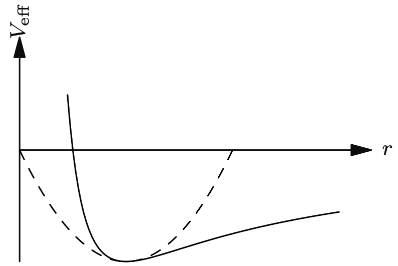
if we were just going to use calculus, why do we need to go through all these steps?

Because this is very simple calculus on a problem that was reduced in complexity greatly. I don't think it's uncommon to still use techniques such as calculus, etc. to help us solve problems, but it's about reducing the complexity of these problems so they are easy to solve with simple "second derivatives" as demonstrated here.

what about the other terms in the taylor series? aren't they important? if not why do we ignore them?

As long as the perturbations are small (as long as the orbit is close to a circle), the other terms are even smaller than the quadratic term.

But what shape is that orbit? Finding the shape seemingly requires solving the differential equation for r, using the conservation of angular momentum to find $d\theta/dt$, and then integrating to find $\theta(t)$.



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Just out of curiosity- what is this irrelevant constant?

Maybe the value of the constant is much smaller compared to the value of the second term?

Why do you bother to make that constant irrelevant but then still have the 1/2 in the equation?

The constant is irrelevant because the only thing that matters is the difference between two potential energies, not the absolute value itself (i.e. it doesn't matter what you set to be 0).

This is the case in most types of potential energy we've seen. Take electrical potential for instance – it doesn't matter if you consider ground to be 0, 10, or 100 Volts, as long as the relative voltages are correct.

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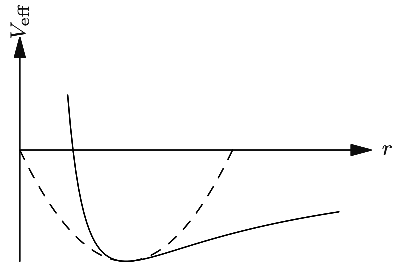
it does seem like a little more explanation is needed here.

Looks kinda similar...

but i dont remember what the potential energy per mass of a mass on a spring looks like...

But what shape is that orbit? Finding the shape seemingly requires solving the differential equation for r, using the conservation of angular momentum to find $d\theta/dt$, and then integrating to find $\theta(t)$.



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I was wondering when we would finally tie this in with springs... maybe you could put a brief outline of how you approach this problem in the beginning of the reading, because so far, I've just been reading this and wondering what this has to do with springs.

Yeah, the title was deceiving.

Really? I just assumed from the moment we started talking about energy we'd have some system acting like a spring whereby PE is converted to KE, and so forth... I think the diagram also sort of hints at that.

I think we can assume those connections, but the lead in is just way too long.

true, but it would still be nice to have a brief outline of the approach before diving in to the very complicated problem

$$V = \frac{1}{2}\frac{k}{m}(x-a)^2, \tag{9.14}$$

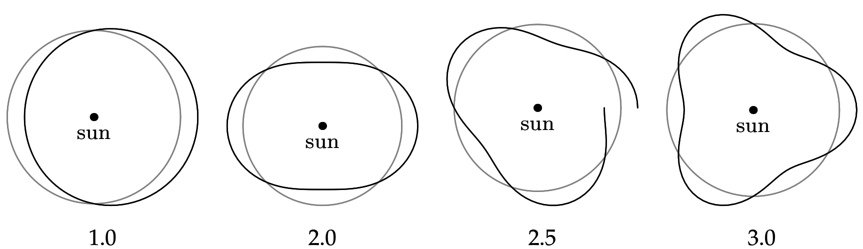
where a is the spring's equilibrium length. The spring and the planet have the same form for the potential energy per mass. One needs only the following mapping:

$$x \leftrightarrow r.$$
$$a \leftrightarrow r_0. \tag{9.15}$$
$$k/m \leftrightarrow GM/r_0^3.$$

Therefore, like the x coordinate for the mass on the spring, the planet's distance to the sun (the r coordinate) oscillates in simple harmonic motion! For the mass on a spring, the angular frequency of oscillation is $\omega = \sqrt{k/m}$. Therefore, using the preceding mappings, we find that the planet's radial distance oscillates about r_0 with angular frequency

$$\omega_{\text{perturb}} = \sqrt{\frac{GM}{r_0^3}}. \tag{9.16}$$

The planet's motion is therefore described by two frequencies. The first is ω_{perturb}, the just-computed oscillation frequency of the r coordinate. The second is ω_{orbit}, the oscillation frequency of the orbital motion around the sun (the tangential frequency). Their dimensionless ratio $\omega_{\text{perturb}}/\omega_{\text{orbit}}$ determines the shape of the orbit. Here are the orbit shapes marked with the ratio $\omega_{\text{perturb}}/\omega_{\text{orbit}}$, with each orbit drawn against the unperturbed circular orbit.



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$$\omega_{\text{orbit}} = \frac{v}{r_0}, \tag{9.17}$$

where v is the orbital velocity (the tangential velocity). To solve for v/r_0, equate the centripetal acceleration to the gravitational acceleration:

would help to put "+ irrelevant constant" here, I think...

Is our familiar formula for the potential energy of a spring an approximation?

We just calculated V_eff in general, and now we want to compare it to potential energy per mass for a spring. They're two different calculations, and this one doesn't have the extra constant.

why is there a comma?

Proper sentence punctuation!

that's so interesting. it makes sense, but i never would have guessed it/believed it/asserted it

The use of the word "one" in this sentence is confusing- I understand that it's trying to say "one" as in a person, but it could also come off as "one" referring to either the spring or the planet and in that case the sentence is extremely confusing.

I often hear the term "one needs only" and immediately understood what he was referring to. In either way I think the sentence conveys the point appropriately.

I like this mapping it makes the analogy really clear

yep I agree, even though I feel like the arrow shouldn't be double sided.

Yeah I kind of stopped on the formula for a little bit trying to think this through only to realize it was explained later. It may be better to have it say something like : If you take these steps and convert the following variables, then you have this equation.

I feel like this is a little overly obvious from the comparison above and doesn't need to be so explicitly stated...

I actually like this. It's nice to have everything all in one place and it's a good summary.

Yeah, I also feel like it's good to have this comparison here. It definitely doesn't hurt, so what motive is there to take it out. I say keep it.

Definitely keep it. I know I'm not alone from the other comments that these couple lines helps a lot in clarifying these relationships.

I love the clarifications in any sort of graphical form!

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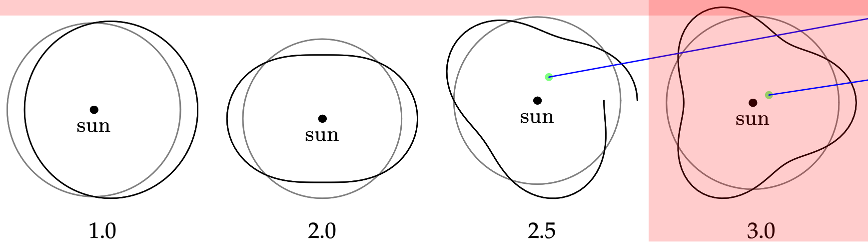

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This is a good presentation of the material.

this two terms look really different - are they actually comparable?

They do look really different, but when you simplify the dimensions, they are comparable: k/m = (F/d)/m = (ma/d)/m = a/d = 1/t^2. GM/d^3 = (md^3/mt^2)/d^3 = 1/t^2.

That's really cool.

And it ties back in with dimensional analysis! You should mention that here, it seems like a good place to apply our dimensional analysis skills.

you mean if you considered the orbit in 2d or something? a circular orbit would leave the planet at nearly the same distance the entire time...?

This is very interesting - it took a lot of work to get us here though, so I'm still not sure how applicable springs are for generally approximation problems.

I like this connection even though it did take a while to get here. At first I expected we would only be doing approximations that could happen very quickly but after seeing cases like this it makes me understand that you can solve an extremely long problem in a decent amount of time using approximations.

can you explain a little back round about what kind of perturbations are occurring

but I thought that planets orbited in ellipse but not centered around the sun

why isn't this orbit closed?

Are there any planets that actually orbit like this?

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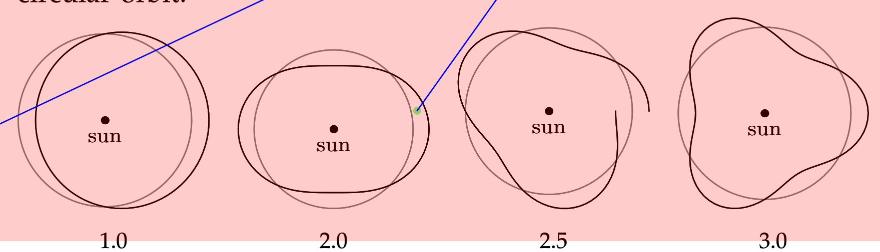

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This is really cool

While the picture is cool, I think it might be useful to provide a little more background on the ratios...

Agreed, this diagram is awesome. It's really cool to think of an orbit like that. Could you maybe add more to this diagram for higher rations of w_perturb / w_orbit. I think it would be interesting to see this at many different values.

I agree that this is a cool diagram, but as said above, I think it'd be nice to see some more extreme cases of what really high and really low frequencies can look like.

Its interesting how the ratio of frequencies must be a whole number in order for the orbit to be closed

How do you get from the ratio to these shapes? Also, why is 2.5 an open loop? Would a ratio of 4 have 4 lobes?

I agree. I'd like to see more explanation of why these diagrams take the shapes they do.

Yeah, I'm pretty lost about where these pictures came from, and also how you drew these conclusions by applying springs. I think a little more explanation is necessary here.

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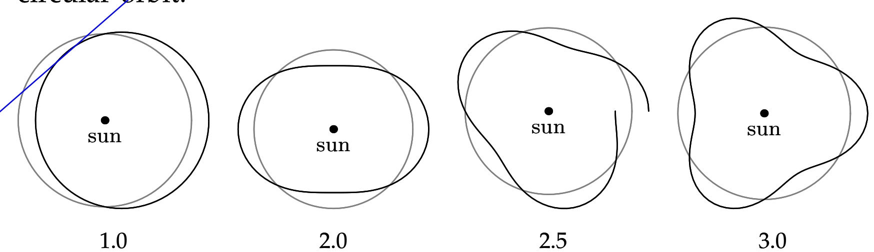

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Yeah, it might be helpful to see a ratio of 4 if it's possible to fit it within this space... But I do think the ratio is directly correlated to the number of lobes.

I'm confused too. These need more explanation, if they're really necessary.

yeah i'm also curious about 2.5. after a certain number of orbits does the path eventually start repeating itself?

I believe 2.5 is open because it is not a whole number. After a whole number, a rotation is finished, and it can line up again to complete the circuit. However, after only half a rotation, the the orbit misaligns.

you get from the ratio to these shapes because the ratios are how many periods are there in the period of a normal, circiular orbit.

While I don't feel like these kinds of shapes exist in real life, it is essentially saying this:

Take your normal circular orbit. One full circle is one period. If we stretch that length out in a straight line, that is a smooth, undisturbed movement.

Then, we add a disturbance in the form of a sinusoidal oscillation (this is modeled after springs after all). In the case that the period of the disturbance is exactly the same as the period of the circular orbit, half the sinusoidal disturbance will deviate downwards from the [straight line, that represents the stretched out circular orbit] and half will be upwards. If we overlap this new sinusoidal path on top of the straight line of the straightened out circular path, and then reconnect the straight line path back into a circle, we will get the first shape.

(This is kind of like the concept of making mobius strips out of paper. It would've been more intuitive, in my opinion, if he stretched out the paths first and then rejoined them)

Another way of looking at this problem is to think of the sinusoidal disturbance as a *radial* force that varies sinusoidally. This means that at any given point along the circle, the disturbance can either point inwards towards the center, or outward parallel to the radius. In this way, you can imagine that only disturbance frequencies that are whole number multiples of the frequency of the original circular orbit will line up where it started. As a simple check: In the first shape, the disturbance frequency is exactly the same (multiple of 1) as the circular orbit's frequency. Therefore, a sinusoidal disturbance will spend the first half of the orbit pushing inwards, and the second half pushing radially outwards. The net effect is to shift the entire circle to the right.

$$V = \frac{1}{2}\frac{k}{m}(x-a)^2, \tag{9.14}$$

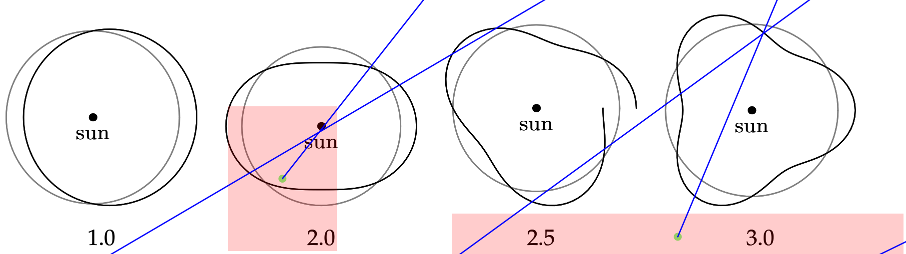
where a is the spring's equilibrium length. The spring and the planet have the same form for the potential energy per mass. One needs only the following mapping:

$$\begin{aligned} x &\leftrightarrow r. \\ a &\leftrightarrow r_0. \\ k/m &\leftrightarrow GM/r_0^3. \end{aligned} \tag{9.15}$$

Therefore, like the x coordinate for the mass on the spring, the planet's distance to the sun (the r coordinate) oscillates in simple harmonic motion! For the mass on a spring, the angular frequency of oscillation is $\omega = \sqrt{k/m}$. Therefore, using the preceding mappings, we find that the planet's radial distance oscillates about r_0 with angular frequency

$$\omega_{\text{perturb}} = \sqrt{\frac{GM}{r_0^3}}. \tag{9.16}$$

The planet's motion is therefore described by two frequencies. The first is ω_{perturb}, the just-computed oscillation frequency of the r coordinate. The second is ω_{orbit}, the oscillation frequency of the orbital motion around the sun (the tangential frequency). Their dimensionless ratio $\omega_{\text{perturb}}/\omega_{\text{orbit}}$ determines the shape of the orbit. Here are the orbit shapes marked with the ratio $\omega_{\text{perturb}}/\omega_{\text{orbit}}$, with each orbit drawn against the unperturbed circular orbit.



The orbital frequency of the circular orbit is

$$\omega_{\text{orbit}} = \frac{v}{r_0}, \tag{9.17}$$

where v is the orbital velocity (the tangential velocity). To solve for v/r_0, equate the centripetal acceleration to the gravitational acceleration:

The diagram at the end looks more like a circle while I would consider this to resemble an ellipse better. How does the bigger w(perturb) make the orbit shape more ellipse-y?

are there actually any orbits that look like this?

If you're asking has this happened, then yes. If you're asking is there a body that has a stable orbit like this, no, because these are perturbances in the orbit, not forces constantly acting on them during every period.

Out of curiosity, when has it happened before?

I'm sure tons of space debris has been subject to high perturbation to orbit ratio. However I doubt they stay in it very long.

so is this just turbulence in the orbit that happens occasionally until it stabilizes?

If you keep reading, it says that the ratio is one, so it would seem that the answer is 'no'.

It would be nicer to have the definition of omega(orbit) before the diagrams where you are using the ratio

somewhat related question: in elliptical orbits the velocity is greater at certain points around the ellipse...is this velocity that much greater and at what point does it happen for Earth?

It remember it has to do with sweeping out equal areas in equal time. So when it is farther away from the sun, the velocity is slower and when it is closer it is faster. I don't have quantitative numbers though.

That's right. The velocity is only much greater if the orbit is highly elliptical...so for a comet it might make a difference, but not for a planet.

thanks!

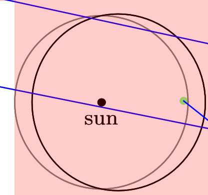
this probably should have brought up a lot earlier.

$$\frac{v^2}{r_0} = \frac{GM}{r_0^2}. \tag{9.18}$$

To manufacture v/r_0 on the left side, divide both sides by r_0 and take the square root:

$$\underbrace{\frac{v}{r_0}}_{\omega_{\text{orbit}}} = \sqrt{\frac{GM}{r_0^3}}. \tag{9.19}$$

This expression is also ω_{perturb}. Therefore, $\omega_{\text{orbit}} = \omega_{\text{perturb}}$.



This result explains the elliptical (Kepler) orbits. First, the ratio is an integer, so the orbit is closed (compare the orbit with the ratio 2.5) – as it should be. Second, the ratio is 1, so the orbit's center is slightly away from the sun – as it should for a planetary orbit (the sun is at one focus of the ellipse, not at the center). The surprising conclusion of all the analysis is that a planetary orbit contains a spring; once this fact is appreciated, a spring analysis allows us to understand the complicated orbital motion without solving complicated differential equations.

As a bonus, the effective potential has the same form as the energy in hydrogen. Therefore, that energy also looks like a parabola near its minimum. The consequence is that a chemical bond acts like a spring (for small extensions). This second application is not a mere coincidence. Near a minimum, almost every function looks a parabola, so almost every physical system contains a spring. Springs are everywhere!

is this the right term? it makes it sound like you're making things up.

I agree this sounds funny. But I think the message get through that you're trying to get V/r0 on the left.

yeah I feel like a better word might be "obtain"

The distortion of space-time causes the orbital and angular frequencies to be different, so the orbit of Mercury doesn't perfectly close on itself in a noticeable way.

This is also exactly why the period for a person falling through a frictionless tunnel all the way through the earth is the same as the orbital period of someone just above the surface of the earth, which is a rather random fact we proved in 8.012.

I like this derivation and the explanation that follows. It's pretty cool.

I like you put the visual representation first so the reader gets an idea where the proof is going

I think it would be really cool if you had an experiment of physical phenomena that shows how this orbit is a spring

Either an experiment or some more concrete example that we could reference in the future. It seems like those really help to drive home the points made in class

I'm hoping to see those in the rest of this chapter!

This looks like a circle, not an ellipse to me. Did I miss something?

whether this is a circle or an ellipse, I don't know for sure.

But, circles are a subset of ellipses (they are the special case where the two foci of the ellipse lines up). So saying that this is an ellipse cannot be wrong. But whether it is a circle, i'm not sure.

I think that here it is drawn as a circle but it wouldn't be a circle in reality because the gravity would be a little weaker as the earth gets further from the sun so the orbit would be more of an elongated ellipse. Maybe the circle was just used because it's a simplification that shows where the offset of the center of the orbit.

Oh, I see. Good explanation.

$$\frac{v^2}{r_0} = \frac{GM}{r_0^2}. \tag{9.18}$$

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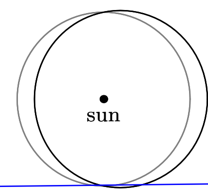

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I don't see why this is necessarily true...

The sun is located at one of the focii of the ellipse that describes the orbit of a planet. The focus of an ellipse is not in the geometric center but on the major axis of the ellipse.

Yes, but why does it make sense that it is this way rather than being a circular orbit with the sun at the center?

The focus of the sun-earth ellipse is very close to the center of the sun because it's so much more massive than the earth (The focus is within the radius of the sun if I recall correctly). This is why we generally think of us orbiting around the sun, but actually its around this focii point.

I understand all the parts individually, but I don't completely understand it when we put it all together... Like, I know that the orbit of the earth is elliptical, but I don't understand quite how we got that conclusion... The diagrams helpa bit but I don't understand where the diagrams came from.

is there a term for the other focus of the ellipse then? does this hold any significance?

stll dont fully get the spring thing- I think this is hard to read and understand for me

after this whole thing i still dont really understand the connection. maybe an example using just springs and not the complicated stuff would help to explain. i suspect that's coming in future sections though.

While I think that the lead in was a bit heavy, the diagrams and simple matching techniques made it clear how a spring and its properties could be applied here.

well it wasnt the easiest set of equations to get to that point either. at least to people unaccustomed to planetary motion

it's so cool how these things are all interconnected

That is awesome.

Agreed.

It's really interesting that planetary orbits, hydrogen, and mass on spring all relate so well. I guess this goes to show that a few physical laws can go very far in explaining the universe

$$\frac{v^2}{r_0} = \frac{GM}{r_0^2}. \quad (9.18)$$

To manufacture v/r_0 on the left side, divide both sides by r_0 and take the square root:

$$\underbrace{\frac{v}{r_0}}_{\omega_{\text{orbit}}} = \sqrt{\frac{GM}{r_0^3}}. \quad (9.19)$$

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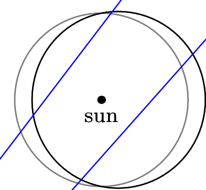

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I have heard this before. I guess I never really thought about this before, but now I can see better why.

Yeah, I love this analogy. It is really helpful in understanding small, medium, and large scale processes!

This is interesting, but I'm not seeing the benefits of using the spring analysis instead of lumping for the hydrogen problem.

Well, we did end up using the lumping analysis instead of a using the SHO for hydrogen.

I agree that this is interesting. I'm also interested to see just how far this statement holds.

Is there a relevant example of a minimum where modeling as a spring fails horribly?

I suspect that as long as your effective potential is roughly parabolic near it's minimum, then it's fair game to approximate as a spring, and for most potentials this is probably fair near the minimum, but if you had some strange potential that wasn't smooth at it's minimum or something the model might break down.

Yeah I'd say as long as your function is smooth at the minimum you should be able to approximate it this way

I like the analysis that I read somewhere about how basically all atomic bonds can be considered as springs, and heat is just those bonds flexing. I think deffinitly this statement explains a lot.

This was an interesting lecture and pretty easy to follow.

I agree that this was a fairly easy reading to follow, but it wasn't as good as the section on probability. I guess that's my personal preference though, which is neither here nor there.

I totally disagree. it might be that i've done little-to-no work on any of this kind of stuff since 8.01 in 2004, but i found this section quite hard to follow. the outline was good and easy to follow, but I would have liked a lot more on were a lot of this stuff came from.

Interesting (though presented in most basic physics classes, I believe) and simple. I wonder where we're going from here with the spring method.