

Logic Synthesis

Handouts

• | Pset #2

· lecture slides.

- Primitive logic gates, universal gates
- Truth tables and sum-of-products
- Logic simplification, Karnaugh Maps
- General implementation techniques: muxes and look-up tables (LUTs)
- Nexy4
- · Verilog basics

Reminder: Lab #1 due this Thu/Fri

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Late Policies

- Lab 1 check-offs (early) sign-up on checkoff queue in lab FIFO during staffed lab hours. Note bench number...
- · Please don't assume that you can wait until the last minute!
- No check-offs Saturday
- Checkoff: Lab 1: Thu 5p, Fri 1pm.
- Lab grade = Checkoff + Verilog grade (when needed)
- · Late labs:
 - 1 point/day late penalty (no penalty for Saturday)
 - · Late completed labs will receive 1 point.
 - 5 slack days available. This covers illness, interviews, overload, etc.
- A missing lab will result in a failing grade. We've learned that if you're struggling with the labs, the final project won't go very well.
- · Lpset no late submissions. Solutions at times presented in lecture.



Lab Hours

Lab hours: eds.mit.edu/labs Sun 1-11:45p, M-R 9-11:45p, F 9-5p



Checkoff Process

- May checkoff at any time prior to checkoff date.
- On checkoff date, checkoff will staff's be main priority
- Two checkoff dates: last name A-M (Thu), N-Z (Fri)
- Thu checkoff starts at 5pm, Fri 1pm
- · Schedule time on google doc



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Check for conflicting dates

https://unical.csail.mit.edu/fa19



Schematics & Wiring

- IC power supply connections generally not drawn. All integrated circuits need power!
- Use standard color coded wires to avoid confusion.
 - red: positive

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- -black: ground or common reference point
- Other colors: signals
- · Circuit flow, signal flow left to right
- Higher voltage on top, ground negative voltage on bottom
- Neat wiring helps in debugging!

Wire Gauge

• Wire gauge: diameter is inversely proportional to the wire gauge number. Diameter increases as the wire gauge decreases. 2, 1, 0, 00, 000(3/0) up to 7/0.

Resistance

- -22 gauge .0254 in 16 ohm/1000 feet
- 12 gauge .08 in 1.5 ohm/1000 feet
- High voltage AC used to reduce loss
- 1 cm cube of copper has a resistance of 1.68 micro ohm (resistance of copper wire scales linearly : length/area)



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Basic Boolean operators

- Bitwise operators perform bit-oriented operations on vectors
 ~(4'b0101) = {~0,~1,~0,~1} = 4'b1010
 - 4'b0101 & 4'b0011 = {0&0, 1&0, 0&1, 1&1} = 4'b0001
- Logical operators return one-bit (true/false) results
 - !(4'b0101) = 1'b0

~a	NOT
a&b	AND
a b	OR
a^b	XOR
a ~^ b a ^~ b	XNOR
	~a a & b a b a ^ b a ~^ b a ^~ b

Logical		
!a	NOT	
a && b	AND	
a b	OR	
a == b a != b	[in]equality returns x when x or z in bits. Else returns 0 or 1	
a === b a !== b	case [in]equality returns 0 or 1 based on bit by bit comparison	

Straightforward Synthesis

We can use

SUM-OF-PRODUCTS to implement any logic function.

Only need 3 gate types: INVERTER, AND, OR

Propagation delay:3 levels of logic



 No more than <u>3 gate delays</u> assuming gates with an arbitrary number of inputs. But, in general, we'll only be able to use gates with a bounded number of inputs (bound is ~4 for most logic families).

Note distinction between ~a and !a when operating on multi-bit values

ANDs and ORs with > 2 inputs



SOP w/ 2-input gates

Previous example restricted to 2-input gates:

3ps

3ps



Hint: to find overall t_{PD} we need to find max t_{PD} considering all paths from inputs to outputs.

More Building Blocks



NAND – NOR Internals

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Dual-In-Line Package

1ps

t_{CD}

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This device contains four independent gates each of which performs the logic NAND function.

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Universal Building Blocks

NANDs and NORs are universal:



Any logic function can be implemented using only NANDs (or, equivalently, NORs). Note that chaining/treeing technique doesn't work directly for creating wide fan-in NAND or NOR gates. But wide fan-in gates can be created with trees involving both NANDs, NORs and inverters.

SOP with NAND/NOR

When designing with NANDs and NORs one often makes use of De Morgan's laws: De Morgan-ized NAND symbol				
NAND form: A	$\overline{A \cdot B} = \overline{A} + \overline{B}$	A		
NOR form: \overline{A}	$+B = \overline{A} \cdot \overline{B}$	A B B Z	z B De Morgan-ized NOR s	ymbol
So the following "SOP" circuits are all equivalent (note the use of De Morgan-ized symbols to make the inversions less confusing):				
		≥-z⊃-z	De Morgan-i Inverter B c	zed
AND/OR form	NAND/NAN This will be handy you'll be able to us implement your cir	D form in Lab 1 since se just 7400's to rcuit!	NOR/NOR form All these "extra" inverters may seem less than ideal but often the buffering they provide will reduce the capacitive load on the inputs and increase the output drive.	
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Boolean Minimization:				

Logic Simplification

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- Can we implement the same function with fewer gates? Before trying we'll add a few more tricks in our bag.
- BOOLEAN ALGEBRA:

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OR rules:	a + 1 = 1 $a + 0 = a$ $a + a = a$
AND rules:	$a \cdot 1 = a a \cdot 0 = 0 a \cdot a = a$
Commutative:	$a+b=b+a$ $a \cdot b=b \cdot a$
Associative:	$(a+b)+c = a+(b+c) (a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive:	$a \cdot (b+c) = a \cdot b + a \cdot c$ $a+b \cdot c = (a+b) \cdot (a+c)$
Complements:	$a + \overline{a} = 1$ $a \cdot \overline{a} = 0$
Absorption:	$a + a \cdot b = a$ $a + \overline{a} \cdot b = a + b$ $a \cdot (a + b) = a$ $a \cdot (\overline{a} + b) = a \cdot b$
De Morgan's Law:	$\overline{a \cdot b} = \overline{a} + \overline{b}$ $\overline{a + b} = \overline{a} \cdot \overline{b}$
Reduction:	$\overline{a \cdot b + \overline{a} \cdot b} = b$ $(a+b) \cdot (\overline{a}+b) = b$

Key to simplification: equations that match the pattern of the LHS (where "b" might be any expression) tell us that when "b" is true, the value of "a" doesn't matter. So "a" can be eliminated from the equation, getting rid of two 2-input ANDs and one 2-input OR.

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An Algebraic Approach

Lets simplify the equation from slide #3:

 $Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot C + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$

Using the identity

 $\alpha A + \alpha \overline{A} = \alpha$

For any expression α and variable A:

$$Y = \overline{A} \cdot B \cdot C + A \cdot \overline{B} \cdot C + A \cdot B \cdot \overline{C} + A \cdot B \cdot C$$
$$Y = B \cdot C + A \cdot C + A \cdot B$$

The tricky part: some terms participate in more than one reduction so can't do the algebraic steps one at a time! 6.111 Fall 2019

Karnaugh Maps: A Geometric Approach

K-Map: a truth table arranged so that terms which differ by exactly one variable are adjacent to one another so we can see potential reductions easily.

А	В	С	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Here's the layout of a 3-variable K-map filled in with the values from our truth table:



000

Why did he

shade that

row Gray?

It's cyclic. The left edge is adjacent to the right edge. It's really just a flattened out cube.

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Finding Subcubes

We can identify clusters of "irrelevent" variables by circling adjacent subcubes of 1s. A subcube is just a lower dimensional cube.



On to Hyperspace

Here's a 4-variable K-map:





Again it's cyclic. The left edge is adjacent to the right edge, and the top is adjacent to the bottom.

We run out of steam at 4 variables – K-maps are hard to draw and use in three dimensions (5 or 6 variables) and we're not equipped to use higher dimensions (> 6 variables)!

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Write Down Equations

Write down a product term for the portion of each cluster/subcube that is invariant. You only need to include enough terms so that all the 1's are covered. Result: a minimal sum of products expression for the truth table.



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Two-Level Boolean Minimization

Two-level Boolean minimization is used to find a sum-of-products representation for a multiple-output Boolean function that is optimum according to a given cost function. The typical cost functions used are the number of product terms in a two-level realization, the number of literals, or a combination of both. The two steps in two-level Boolean minimization are:

•Generation of the set of prime product-terms for a given function.

•<u>Selection</u> of a minimum set of prime terms to implement the function.

We will briefly describe the Quine-McCluskey method which was the first algorithmic method proposed for two-level minimization and which follows the two steps outlined above. State-of-the-art logic minimization algorithms are all based on the Quine-McCluskey method and also follow the two steps above.

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Prime Term Generation

F = f(W,X,Y,Z) $W X Y Z \quad label$

0

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Start by expressing your Boolean function using 0-terms	W	Χ
(product terms with no don't care care entries). For	0	0
compostness the table for example 4 input 4 output	0	1
compactness the table for example 4-input, 1-output	0	1
function F(w,x,y,z) shown to the right includes only entries	1	0
where the output of the function is 1 and we've labeled	1	0
and arthur with it's desired against	1	0
each entry with it's decimal equivalent.	1	0

0 1 1 1 7 1 0 0 0 8 1 0 1 9 1 0 1 0 10 1 0 1 1 11 1 1 1 0 14 1 1 1 1 1

0 0

0 1

Look for pairs of 0-terms that differ in only one bit position and merge them in a 1-term (i.e., a term that has exactly one '--' entry). Next 1-terms are examined in pairs to see if the can be merged into 2-terms, etc. Mark k-terms that get merged into (k+1) terms so we can discard them later.

1-terms	0, 8	-000 [A]	2-terms: 8, 9, 10, 11, 10 [P]
	5, 7	01-1 [B]	10,11,14,15 1-1-[E]
	7,15	-111 [C]	
	8, 9	100-	0 /
	8,10	10-0	3-terms: none!
Example due to	9,11	10-1	
Srini Devadas	10,11	101-	Label unmerged terms:
Shin Deviduo	10,14	1-10	these terms are prime!
	11,15	1-11	
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S

0 0

1 0

0

0 1

0

1

1 0

1 0

1

1

A B

0 1 1 0

1

1

0 0

0 1

1 0

1

0 0

0 0

1

1

Prime Term Table

An "X" in the prime term table in row R and column K signifies that the 0-term corresponding to row R is contained by the prime corresponding to column K.

	1 ABCDE
Goal: select the minimum	0000 X
set of primes (columns)	0101 . X →B is essential 01-1
such that there is at least	0111 . X X
one "X" in every row. This	1000 XX.
	1001 X . —>D is essential 10
is the classical minimum	1010 X X
covering problem.	1011 X X
51	1110 X → E is essential 1-1-
	1111 X . X

Each row with a single X signifies an essential prime term since any prime implementation will have to include that prime term because the corresponding 0-term is not contained in any other prime.

In this example the essential primes "cover" all the 0-terms.

$$F = f(W,X,Y,Z) = \overline{XYZ} + \overline{W}XZ + W\overline{X} + W^{2}$$

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$S = \overline{A} \cdot B \cdot \overline{C} + A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{B} \cdot C + A \cdot B \cdot C = A \oplus B \oplus C_i$ $C_o = A \cdot C + B \cdot C + A \cdot B$

Logic that defies SOP simplification

Full Adder

A B

FA

S

The sum S doesn't have a simple sum-of-products implementation even though it can be implemented using only two 2-input XOR gates.

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Virtex II Slice Schematic



Virtex II Sum-of-products



Figures from Xilinx Virtex II datasheet

The Need for HDLs

A specification is an engineering contract that lists all the goals for a project:

• goals include area, power, throughput, latency, functionality, test coverage, costs (NREs and piece costs), ... Helps you figure out when you're done and how to make engineering tradeoffs. Later on, goals help remind everyone (especially management) what was agreed to at the outset!

• top-down design: partition the project into modules with well-defined interfaces so that each module can be worked on by a separate team. Gives the SW types a head start too! (Hardware/software codesign is currently all the rage...)

• Example – a well defined Instruction Set Architecture (ISA) can last for generations ...

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The Need for HDLs (cont'd.)

A <u>behavioral model</u> serves as an executable functional specification that documents the exact behavior of all the individual modules and their interfaces. Since one can run tests, this model can be refined and finally verified through simulation.

We need a way to talk about what hardware should do without actually designing the hardware itself, i.e., we need to separate behavior from implementation. We need a

Hardware Description Language

If we were then able to <u>synthesize</u> an implementation directly from the behavioral model, we'd be in good shape!

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Using an HDL description

So, we have an executable functional specification that

- documents exact behavior of all the modules and their interfaces
- · can be tested & refined until it does what we want

An HDL description is the first step in a mostly automated process to build an implementation directly from the behavioral model



A Tale of Two HDLs

<u>VHDL</u>

ADA-like verbose syntax, lots of redundancy (which can be good!)

Extensible types and simulation engine. Logic representations are not built in and have evolved with time (IEEE-1164).

Design is composed of <u>entities</u> each of which can have multiple <u>architectures</u>. A <u>configuration</u> chooses what architecture is used for a given instance of an entity.

Behavioral, dataflow and structural modeling. Synthesizable subset...

Harder to learn and use, not technology-specific, DoD mandate

Verilog

C-like concise syntax

Built-in types and logic representations. Oddly, this led to slightly incompatible simulators from different vendors.

Design is composed of modules.

Behavioral, dataflow and structural modeling. Synthesizable subset...

Easy to learn and use, fast simulation, good for hardware design



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Constraint File

• Text file (.XDC) containing the mapping from a device independent HDL circuit net to the physical I/O pin. This allows Verilog (HDL) to be device independent.

set_property -dict { PACKAGE_PIN R12 IOSTANDARD LVCMOS33 }
 [get_ports { led16_b }]; #IO_L5P_T0_D06_14 Sch=led16_b

- led16_b is physically tied to IC package R pin 12
- Voltage spec based on low voltage CMOS 3.3
- Schematic name is led16_b #IO_L5P_T0_D06_14

· All signals defined in XDC but commented out.

SystemVerilog logic values

Since we're describing hardware, we'll need to represent the values that can appear on wires. SystemVerilog uses a 4-valued logic:

When using a tri-state bus, we'll need to represent the values that can appear on bus and need to use Verilog with a 4-valued logic:

Value	Meaning
0	Logic zero, "low"
1	Logic one, "high"
Z or ?	High impedance (tri-state buses)
Х	Unknown value (simulation)

"X" is used by simulators when a wire hasn't been initialized to a known value or when the predicted value is an illegitimate logic value (e.g., due to contention on a tri-state bus).

Numeric Constants

Constant values can be specified with a specific width and radix:

123	<pre>// default: decimal radix, unspecified width</pre>
'd123	// 'd = decimal radix
ʻh7B	<pre>// 'h = hex radix</pre>
'o173	// 'o = octal radix
ʻb111_1011	<pre>// 'b = binary radix, "_" are ignored</pre>
'hxx	<pre>// can include X, Z or ? in non-decimal constants</pre>
16'd5	// 16-bit constant 'b0000_0000_0000_0101
11'h1X?	<pre>// 11-bit constant 'b001_XXXX_ZZZZ</pre>

By default constants are unsigned and will be extended with 0's on left if need be (if high-order bit is X or Z, the extended bits will be X or Z too). You can specify a signed constant as follows:

8'shFF // 8-bit twos-complement representation of -1

To be absolutely clear in your intent it's usually best to explicitly specify the width and radix.

Logic (SystemVerilog) Wires (Verilog)

We have to provide declarations* for all our named wires (aka "nets"). We can create buses - indexed collections of wires - by specifying the allowable range of indices in the declaration:

logic a,b,z; logic [31:0] memdata: logic [7:0] b1,b2,b3,b4; // four 8-bit buses logic [W-1:0] input;

// three 1-bit wires // a 32-bit bus // parameterized bus

Note that [0:7] and [7:0] are both legitimate but it pays to develop a convention and stick with it. Common usage is [MSB:LSB] where MSB > LSB; usually LSB is 0. Note that we can use an expression in our index declaration but the expression's value must be able to be determined at compile time. We can also build unnamed buses via concatenation:

{b1,b2,b3,b4} // 32-bit bus, b1 is [31:24], b2 is [23:16], ... {4{b1[3:0]},16'h0000} // 32-bit bus, 4 copies of b1[3:0], 16 0's

* Actually by default undeclared identifiers refer to a 1-bit wire, but this means typos get you into trouble. Specify "`default_nettype none" at the top of your source files to avoid this bogus behavior.

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Verilog Syntax

Bit selected allowed on a wire but not sum

```
logic [2:0] sum;
sum = sw[1:0] + sw[3:2];
assign led_r = sum[1];
assign led_r = (sw[1:0] + sw[3:2])[2];
```

Assign not allowed in always block

Gesture Controlled Drone Fall 2014



Track hands with a camera and determine x,y coordinates

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- Based on movement of the coordinates, recognize gestures.
- Generate real time digital signals and convert to analog format for transmission to drone - controlling pitch. roll. hover
- Innovation: using hand motion and recognition of gestures to control fliaht

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