

# Lecture 7

*LPset6 is due Thursday October 3*  
*Lab 3 is Due next Tuesday October 1*



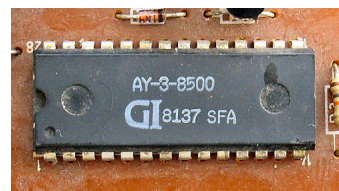
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1

## Pong in History:

- <http://www.pong-story.com/gi.htm>



*AY-3-8500 "Ball-and-Paddle" chip*

<https://commons.wikimedia.org/wiki/File:AY-3-8500.jpg>

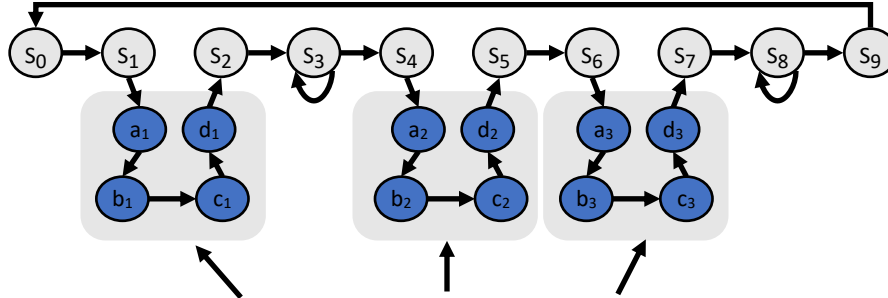
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2

## Toward FSM Modularity

- Consider the following abstract FSM:



- Suppose that each set of states  $a_x...d_x$  is a “sub-FSM” that produces exactly the same outputs.
- Can we simplify the FSM by removing equivalent states?  
*No! The outputs may be the same, but the next-state transitions are not.*
- This situation closely resembles a **procedure call** or **function call** in software...how can we apply this concept to FSMs?

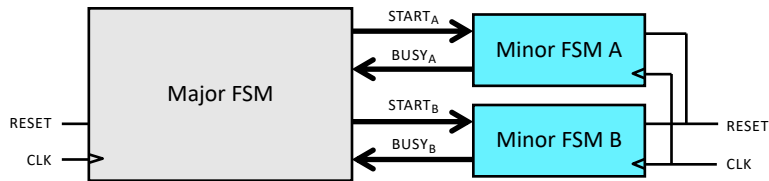
Acknowledgements: Rex Min

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3

## The Major/Minor FSM Abstraction



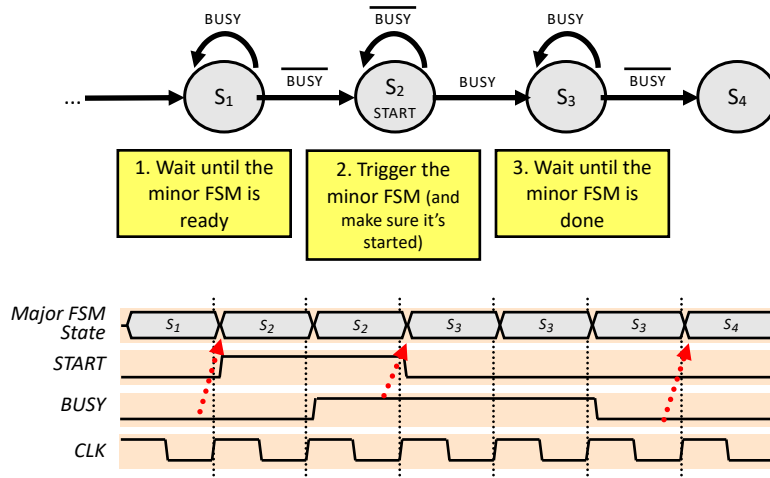
- Subtasks are encapsulated in **minor FSMs** with common reset and clock
- Simple communication abstraction:
  - START**: tells the minor FSM to begin operation (the call)
  - BUSY**: tells the major FSM whether the minor is done (the return)
- The major/minor abstraction is great for...
  - Modular designs (*always* a good thing)
  - Tasks that occur often but in different contexts
  - Tasks that require a variable/unknown period of time
  - Event-driven systems

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## Inside the Major FSM



Variations:

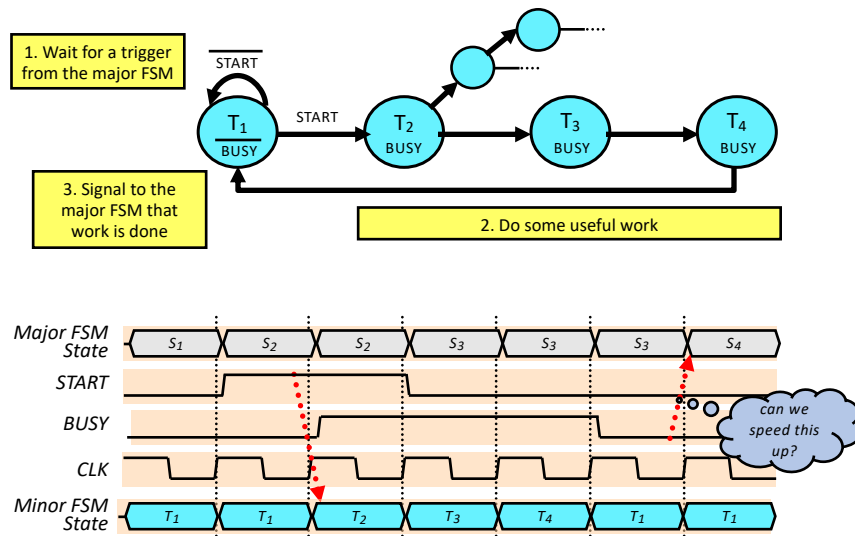
- Usually don't need both Step 1 and Step 3
- One cycle "done" signal instead of multi-cycle "busy"

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5

## Inside the Minor FSM



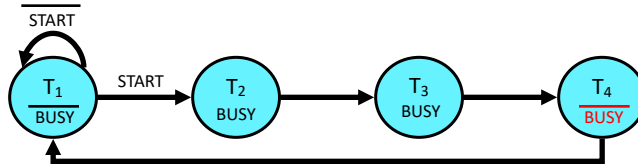
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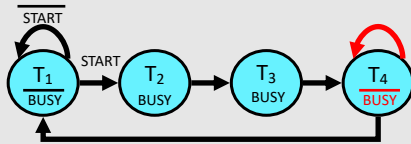
# Optimizing the Minor FSM

Good idea: de-assert BUSY one cycle early



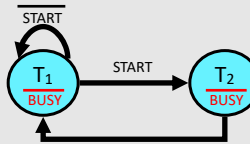
**Bad idea #1:**

T4 may not immediately return to T1



**Bad idea #2:**

BUSY never asserts!

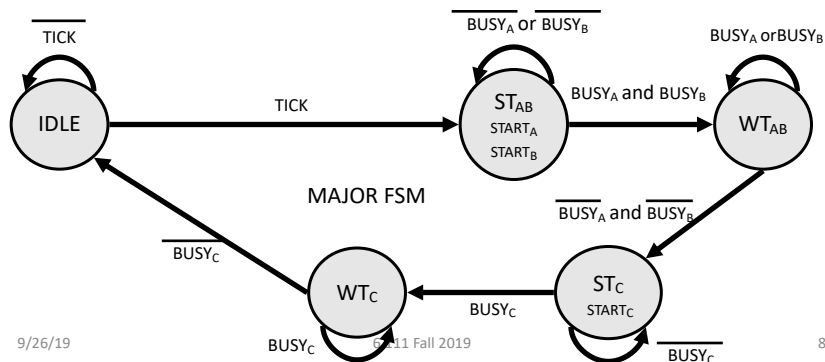
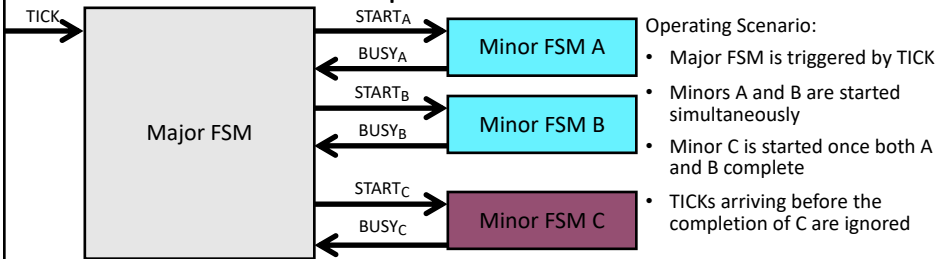


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7

# A Four-FSM Example

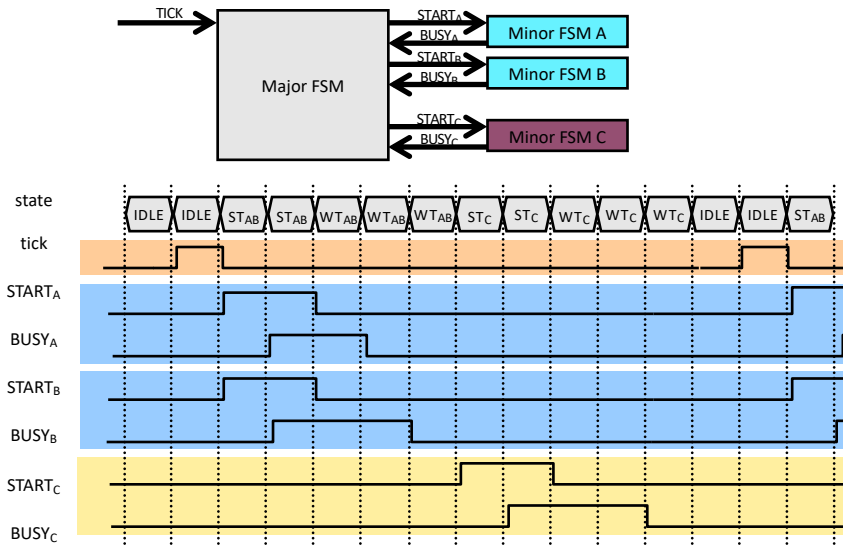


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8

## Four-FSM Sample Waveform



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Division (an example of an algorithm that takes an unknown amount of time)

```

1 def divider (dividend, divisor):
2     count = 0
3     while dividend > divisor:
4         dividend -= divisor
5         count += 1
6     return (count, dividend)

```

*Super efficient divider \s*

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10

## A Divider

- This is a Verilog FSM example of the algorithm on the previous page which will run an unknown number of times given a set of inputs
- This is how the functionality of a while loop could be developed in your modules
- Ugh has a few problems though I'm just seeing now.

```

1  module divider (input clk_in, input rst_in, input[15:0] dividend_in,
2                  input[15:0] divisor_in, input data_valid_in,
3                  output logic[15:0] quotient_out,
4                  output logic[15:0] remainder_out, output logic busy_out);
5      parameter RESTING = 0;
6      parameter DIVIDING = 1;
7
8      logic [15:0] quotient;
9      logic [15:0] dividend;
10     logic [15:0] divisor;
11
12     logic state;
13
14     assign busy_out = state;
15
16     always_ff @(posedge clk_in)begin
17         if (rst_in)begin
18             quotient <= 16'b0;
19             dividend <= 16'b0;
20             divisor <= 16'b0;
21             remainder_out <= 16'b0;
22             busy_out <= 0;
23             state <=
24         end else begin
25             case state
26                 RESTING: begin
27                     if (data_valid_in)begin
28                         state <= DIVIDING;
29                         quotient <= 16'b0;
30                         dividend <= dividend_in;
31                         divisor <= divisor_in;
32                     end
33                 end
34                 DIVIDING: begin
35                     if (dividend < divisor) begin
36                         state <= RESTING;
37                         remainder_out <= dividend;
38                         quotient_out <= dividend;
39                     end else begin
40                         quotient <= quotient + 1'b1;
41                         dividend <= dividend-divisor;
42                     end
43                 end
44             endcase
45         end
46     end
47 endmodule

```

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11

## Clocking Issues

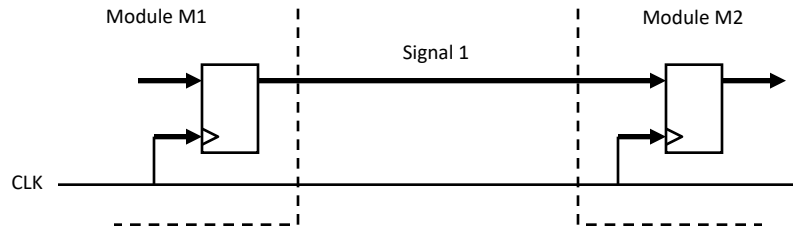
Thinking about a Few More Things Involving Clocks

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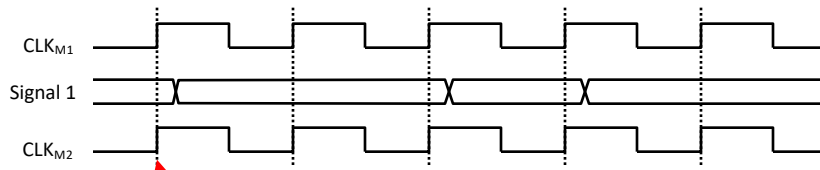
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12

# Clocking and Synchronous Communication



Ideal world:



M1 and M2 clock edges aligned in time

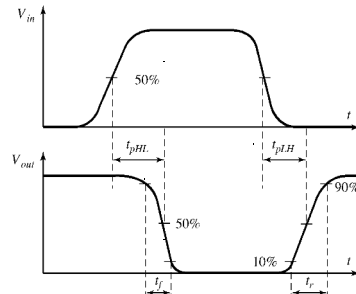
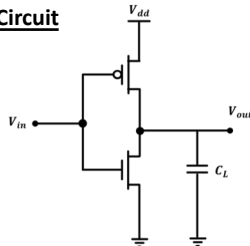
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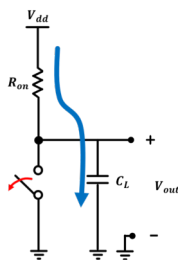
13

# Delay Estimation: Simple RC Networks

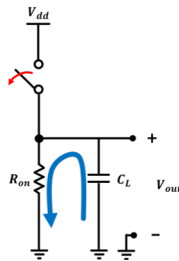
## Simple CMOS Circuit



## Low-to-High



## High-to-Low



review

$$v_{out}(t) = (1 - e^{-t/\tau}) V$$

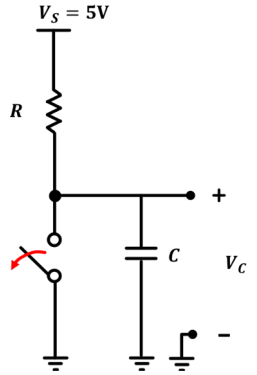
$$t_p = \ln(2) \tau = 0.69 RC$$

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14

# RC Equation



$$V_c = 5 \left( 1 - e^{-\frac{t}{RC}} \right)$$

$$V_s = 5 \text{ V}$$

Switch is closed  $t < 0$

Switch opens  $t > 0$

$$V_s = V_R + V_C$$

$$V_s = i_R R + V_C \quad i_R = C \frac{dV_c}{dt}$$

$$V_s = RC \frac{dV_c}{dt} + V_c$$

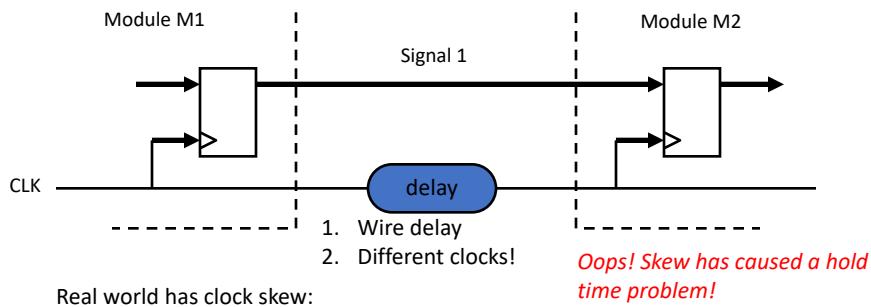
$$V_c = V_s \left( 1 - e^{-\frac{t}{RC}} \right)$$

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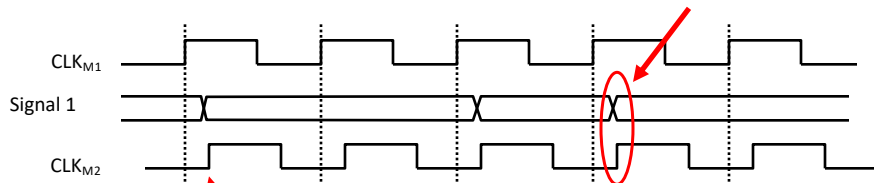
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15

# Clock Skew



Real world has clock skew:



M2 clock delayed with respect to M1 clock

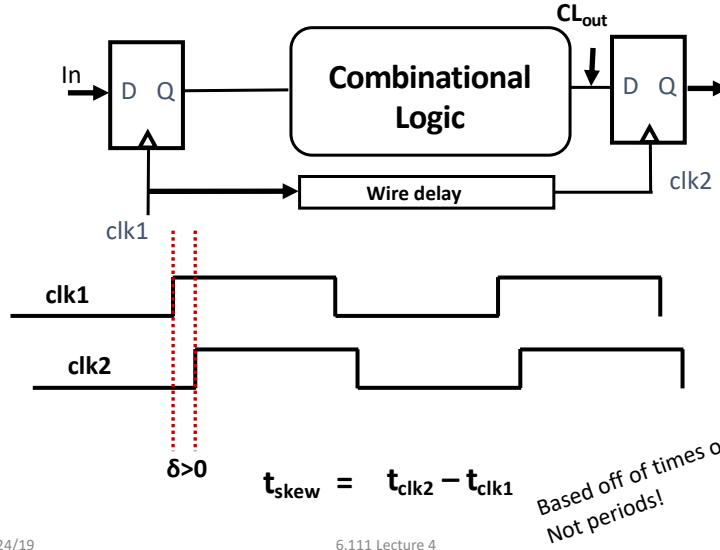
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16



# Clocks are Not Perfect: Clock Skew

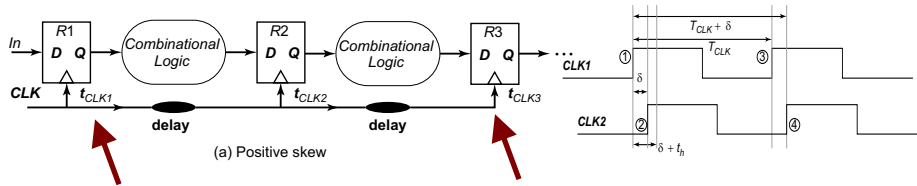


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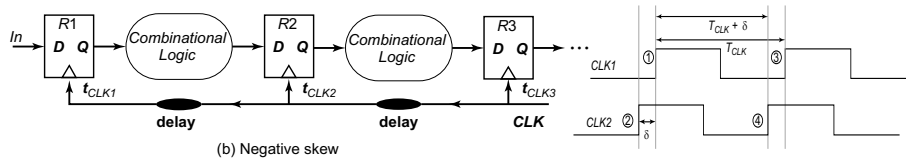
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17

# Positive and Negative Skew



Launching edge arrives before the receiving edge (positive skew)



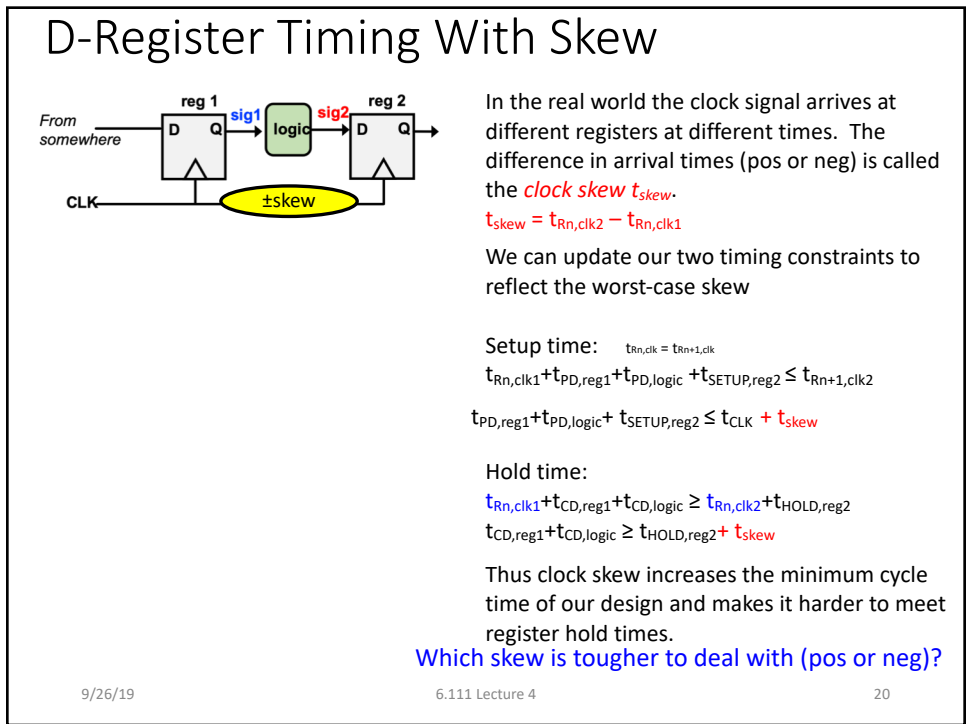
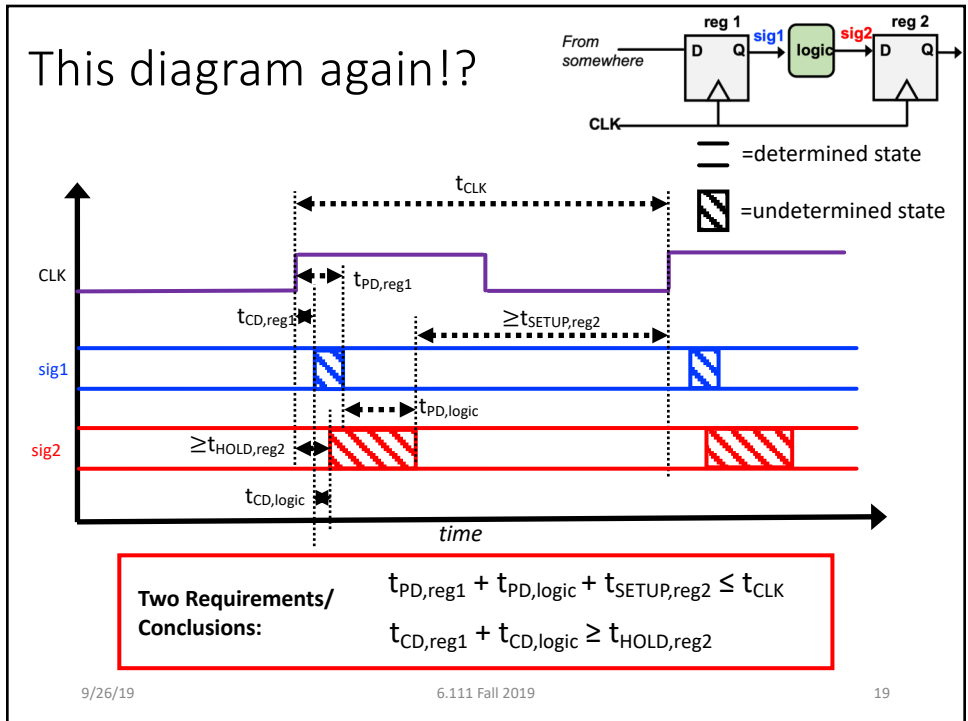
Receiving edge arrives before the launching edge (negative skew)

> Adapted from J. Rabaey, A. Chandrakasan, B. Nikolic,  
"Digital Integrated Circuits: A Design Perspective" Copyright 2003 Prentice Hall/Pearson.

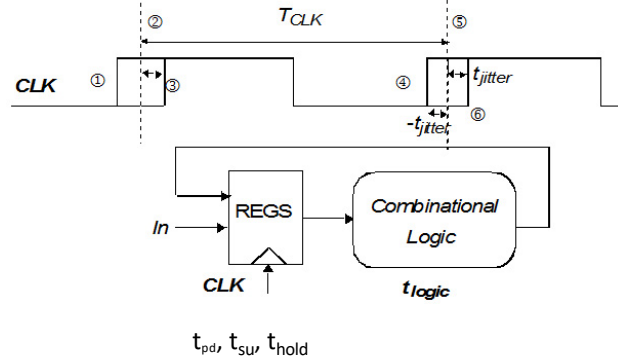
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18



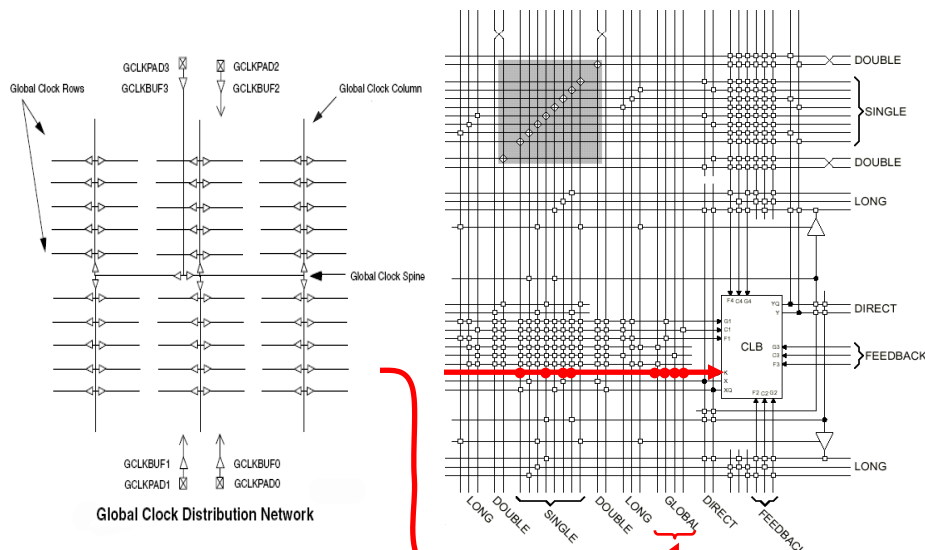
# Clocks Are Not Perfectly Periodic either Ugggh: Jitter



$$t_{clk} - 2t_{jitter} > t_{pd} + t_{su} + t_{logic}$$

Typical crystal oscillator  
100mhz (10ns)  
Jitter: 1ps

# Low-skew Clocking in FPGAs



Figures from Xilinx App Notes

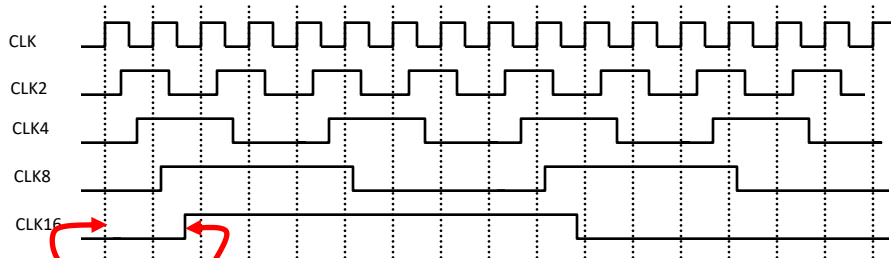
## Goal: use as few clock domains as possible

Suppose we wanted clocks at  $f/2$ ,  $f/4$ ,  $f/8$ , etc.:

```
reg clk2,clk4,clk8,clk16;
always @(posedge clk) clk2 <= ~clk2;
always @(posedge clk2) clk4 <= ~clk4;
always @(posedge clk4) clk8 <= ~clk16;
always @(posedge clk8) clk16 <= ~clk16;
```

No! don't do it this way

No vsync!



Very hard to have synchronous communication between clk and clk16 domains

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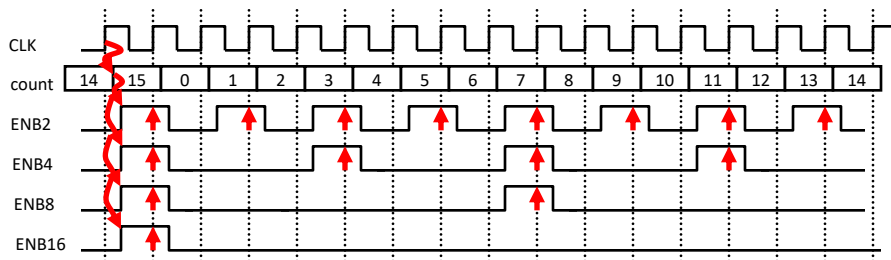
23

## Solution: One clock, Many enables

Use one (high speed) clock, but create enable signals to select a subset of the edges to use for a particular piece of sequential logic

```
logic [3:0] count;
always_ff @(posedge clk) count <= count + 1; // counts 0..15
logic enb2, enb4, enb8, enb16;
assign enb2 = (count[0] == 1'b1);
assign enb4 = (count[1:0] == 2'b11);
assign enb8 = (count[2:0] == 3'b111);
assign enb16 = (count[3:0] == 4'b1111);
```

```
always_ff @(posedge clk)
if (enb2) begin
// get here every 2nd cycle
end
```



= clock edge selected by enable signal

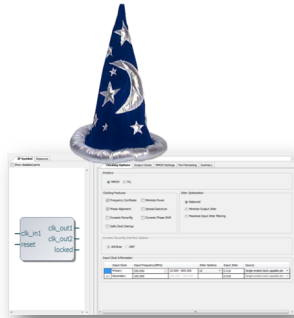
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24

## Generating Other Clock Frequencies (again)

The Nexys4 board has a 100MHz crystal (10ns period). Use “clock wizard” to generate other frequencies e.g., 65MHz to generate 1024x768 VGA video.



Clock Wizard can also synthesize certain multiples/fractions of the CLKIN frequency (100 MHz):

$$f_{CLKFX} = \left( \frac{M}{D} \right) f_{CLKIN}$$

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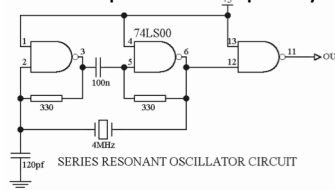
25

## Where do we get frequencies?



16MHz Crystal

- Most frequencies come from Crystal Oscillators made of quartz
- Equivalent to very High-Q LRC tank circuits
- [https://en.wikipedia.org/wiki/Crystal\\_oscillator\\_frequencies](https://en.wikipedia.org/wiki/Crystal_oscillator_frequencies)
- Incorporate into circuit like that below and boom, you’ve got a square wave of some specified frequency dependent largely on the crystal



<http://www.z80.info/uexosc.htm>  
[https://en.wikipedia.org/wiki/Crystal\\_oscillator](https://en.wikipedia.org/wiki/Crystal_oscillator)

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26

## High Frequencies

- Very hard to get a crystal oscillator to operate above ~200 MHz (7<sup>th</sup> harmonic of resonance of crystal itself, which usually is limited to about 30 MHz due to fabrication limitations)
- Where does the 2.33 GHz clock of my iPhone come from then?
- Frequency Multipliers!
- Talk about Phase Locked Loops along the way!

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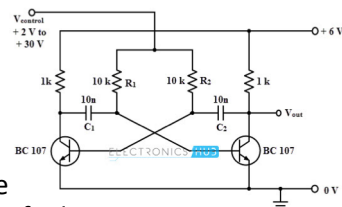
27

## Voltage Controlled Oscillator

- It is very easy to make voltage-controlled oscillators that run up to 1GHz or more.
- Why don't we just:



- Pick the voltage  $V_i$  that is needed to get the frequency we want  $f_o$ ? That's gotta be specified right?
- Same reason we don't see op amps in open loop out in the wild...they are too unstable...gotta place them in negative feedback



A simple VCO (not type found in FPGA)

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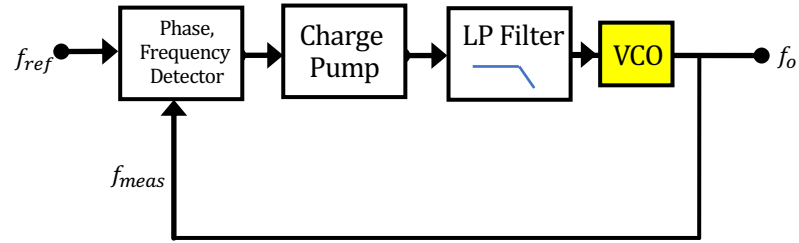
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28

<http://www.electronicshub.org/voltage-controlled-oscillators-vco/>

## Phase Locked Loop

- Place the unstable, but capable VCO in a feedback loop.
- This type of circuit is a phase-locked loop variant



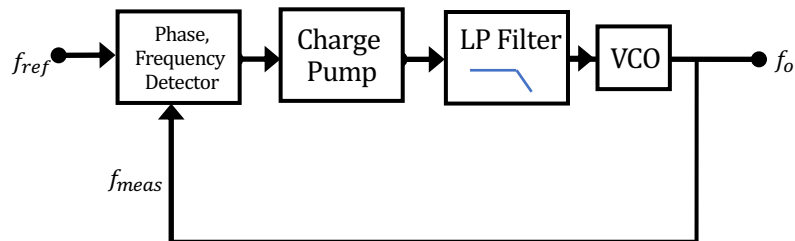
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29

## Phase Locked Loop

- Circuit that can track an input phase of a system and reproduce it at the output

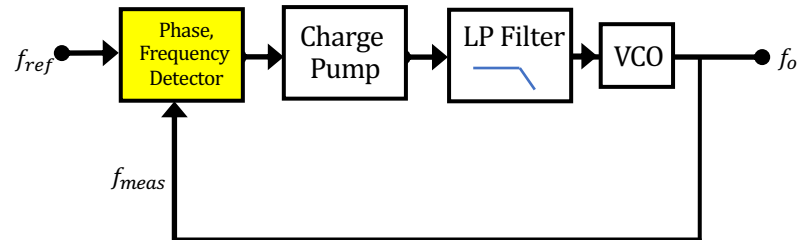


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30

## Phase, Frequency Detector



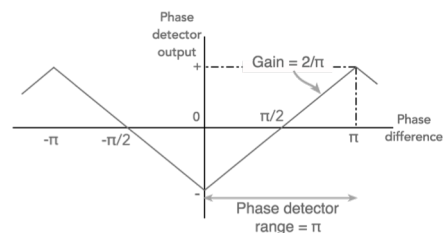
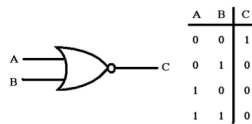
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31

## Phase Detector

- Can be a simple XOR gate



- If near the desired frequency already this can work...if it is too far out, it won't and can be very unreliable since phase and frequency are related but not quite the same thing, it will lock onto harmonics, etc...

- For frequency we instead use a PFD:
  - Phase/Frequency Detector:

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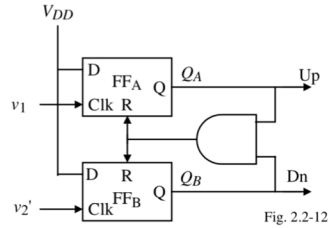
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32



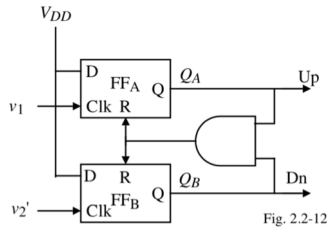
# Phase-Frequency Detection

- Detects both change and which clock signal is consistently leading the other one
- Using MOSFETs you charge/discharge a capacitor accordingly which also with some resistors low-pass filter's the signal
- The output voltage is then roughly proportional to the frequency error!



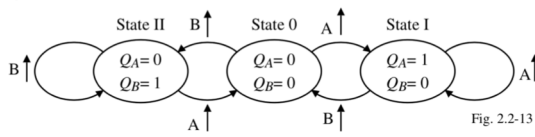
<http://www.globalspec.com/reference/72819/203279/2-7-phase-detectors-with-charge-pump-output>

# Phase Frequency Detection



- Clock 1 and clock 2 are constantly competing with one another to generate up and down signals
- The up signals charge up a capacitors through a pair of transistors...the down signal discharges the capacitor

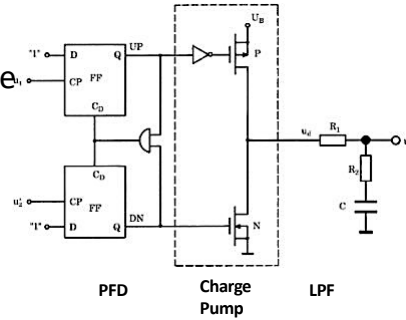
PFD State Diagram:



[1.pallen.ece.gatech.edu/Academic/ECE\\_6440/Summer\\_2003/L070-DPLL\(2UP\).pdf](http://1.pallen.ece.gatech.edu/Academic/ECE_6440/Summer_2003/L070-DPLL(2UP).pdf)

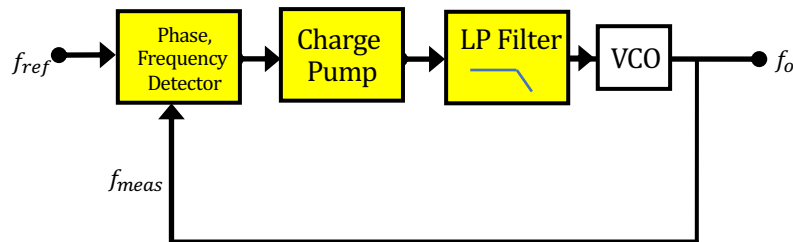
# Phase-Frequency Detection

- Detects both change and which clock signal is consistently leading the other one
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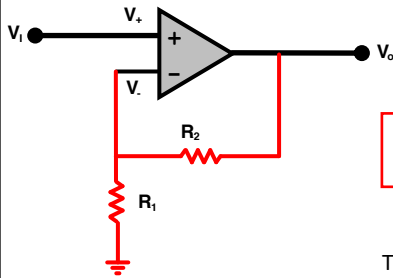
<http://www.globalspec.com/reference/72819/203279/2-7-phase-detectors-with-charge-pump-output>

# PFD, Charge Pump, LP Filter



- So this circuit can make  $f_0 = f_{ref}$  That doesn't help us!
- How can we make a higher frequency?

### Use Resistors in Voltage Divider in Feedback Path!



- A voltage divider in feedback path gives us voltage gain!

$$K = \frac{1}{1 - p + G} \quad p \approx 0.9999 \text{ means} \quad K = \frac{1}{G}$$

$$G = \frac{R_1}{R_1 + R_2}$$

The gain  $A_v$  of this circuit is therefore:

$$A_v = \frac{R_1 + R_2}{R_1}$$

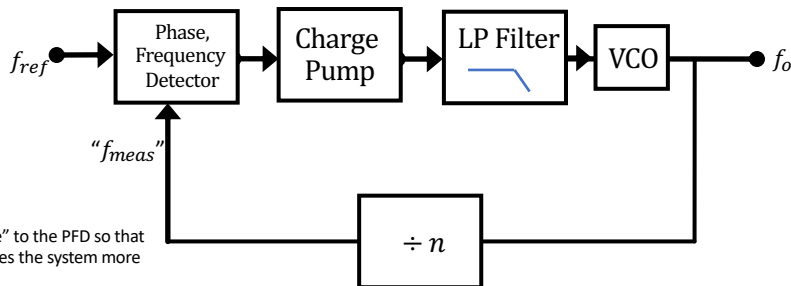
The gain of a "non-inverting amplifier"

$$V_- = V_o \frac{R_1}{R_1 + R_2}$$

**Same Idea with Phase Locked Loops!**

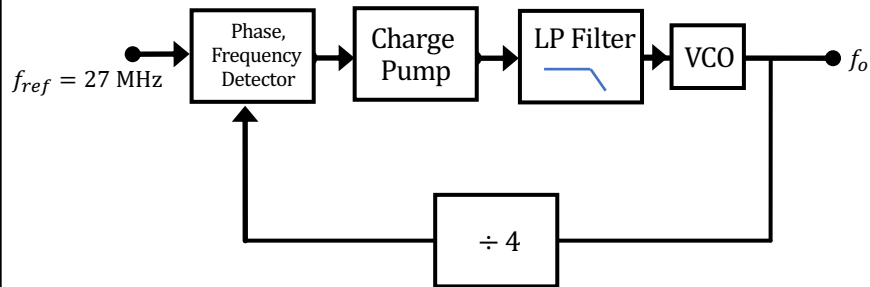
### Use a Clock Divider in Feedback Path!

- A clock divider in feedback path gives us clock gain!



We "lie" to the PFD so that it pushes the system more

## Use a Clock Divider in Feedback Path!



```
reg clk2,clk4,clk8,clk16;
always @(posedge clk) clk2 <= ~clk2;
always @(posedge clk2) clk4 <= ~clk4;
always @(posedge clk4) clk8 <= ~clk16;
always @(posedge clk8) clk16 <= ~clk16;
```

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39

## Number Representation

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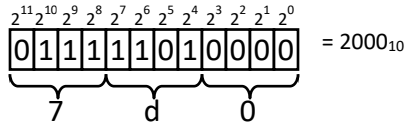
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40

# Encoding numbers

It is straightforward to encode positive integers as a sequence of bits. Each bit is assigned a weight. Ordered from right to left, these weights are increasing powers of 2. The value of an n-bit number encoded in this fashion is given by the following formula:

$$v = \sum_{i=0}^{n-1} 2^i b_i$$



Oftentimes we will find it convenient to cluster groups of bits together for a more compact notation. Two popular groupings are clusters of 4 bits and ever so rarely, 3 bits.

<b>03720</b>	<b>0x7d0</b>
Octal - base 8	Hexadecimal - base 16
000 - 0	0000 - 0    1000 - 8
001 - 1	0001 - 1    1001 - 9
010 - 2	0010 - 2    1010 - a
011 - 3	0011 - 3    1011 - b
100 - 4	0100 - 4    1100 - c
101 - 5	0101 - 5    1101 - d
110 - 6	0110 - 6    1110 - e
111 - 7	0111 - 7    1111 - f

# Binary Representation of Numbers

How to represent negative numbers?

- Three common schemes:
  - sign-magnitude, ones complement, twos complement
- Sign-magnitude: MSB = 0 for positive, 1 for negative
  - Range:  $-(2^{N-1} - 1)$  to  $+(2^{N-1} - 1)$
  - Two representations for zero: 0000... & 1000...
  - Simple multiplication but complicated addition/subtraction
- Ones complement: if N is positive then its negative is  $\bar{N}$ 
  - Example: 0111 = 7, 1000 = -7
  - Range:  $-(2^{N-1} - 1)$  to  $+(2^{N-1} - 1)$
  - Two representations for zero: 0000... & 1111...
  - Subtraction is addition followed by end-around carry (subtraction is different from addition unit)

Basically flip every bit of the number to negate it

## Representing negative integers

To keep our arithmetic circuits simple, we'd like to find a representation for negative numbers so that we can use a single operation (binary addition) when we wish to find the sum of two integers, independent of whether they are positive or negative.

We certainly want  $A + (-A) = 0$ . Consider the following 8-bit binary addition where we only keep 8 bits of the result:

$$\begin{array}{r} 11111111 \\ + 00000001 \\ \hline 00000000 \end{array}$$

which implies that the 8-bit representation of -1 is 11111111. More generally

**Negation:**  
**Complement**  
**and add 1**

$$\left\{ \begin{array}{l} -A = 0 - A \\ = (-1 + 1) - A \\ = (-1 - A) + 1 \\ = \sim A + 1 \end{array} \right.$$

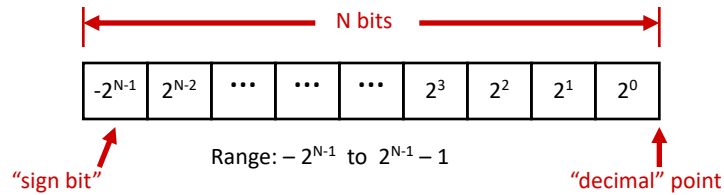
$\sim$  means bit-wise complement

9/26/19

6.111 Fall 2019

43

## Signed integers: 2's complement



8-bit 2's complement example:

$$11010110 = -2^7 + 2^6 + 2^4 + 2^2 + 2^1 = -128 + 64 + 16 + 4 + 2 = -42$$

If we use a two's complement representation for signed integers, the same binary addition mod  $2^n$  procedure will work for adding positive and negative numbers (don't need separate subtraction rules). The same procedure will also handle unsigned numbers!

By moving the implicit location of "decimal" point, we can represent fractions too:

$$1101.0110 = -2^3 + 2^2 + 2^0 + 2^{-2} + 2^{-3} = -8 + 4 + 1 + 0.25 + 0.125 = -2.625$$

9/26/19

6.111 Fall 2019

44

## Sign extension

Consider the 8-bit 2's complement representation of:

$$\begin{aligned}
 42 &= 00101010 & -5 &= \sim 00000101 + 1 \\
 & & &= 11111010 + 1 \\
 & & &= 11111011
 \end{aligned}$$

What is their **16-bit** 2's complement representation?

$$42 = 00000000000101010$$

$$-5 = 1111111111111011$$



Extend the MSB (aka the "sign bit") into the higher-order bit positions

9/26/19

6.111 Fall 2019

45

## Using Signed Arithmetic in Verilog

"<<<" and ">>>" tokens result in arithmetic (signed) left and right shifts: multiply by 2 and divide by 2.

Right shifts will maintain the sign by filling in with sign bit values during shift

```
wire signed [3:0] value = 4'b1000; // -8
```

```
value >> 2 // results in 0010 or 2
```

```
value >>> 2 // results in 1110 or -2
```

9/26/19

6.111 Fall 2019

46

## Using Signed Arithmetic in Verilog

ALL OF THE FOLLOWING ARE TREATED AS **UNSIGNED** IN VERILOG!!!

- Any operation on two operands, unless **both operands are signed**
- Based numbers (e.g. 12'd10), unless the explicit "s" modifier is used)
- Bit-select results a[5]
- Part-select results a[4:2]
- Concatenations

```
logic [15:0] a; // Unsigned
logic signed [15:0] b;
logic signed [16:0] signed_a;
logic signed [31:0] a_mult_b;
```

```
assign signed_a = a; // Convert to signed
assign a_mult_b = signed_a * b
```

*Example of multiplying signed by unsigned*

<http://billauer.co.il/blog/2012/10/signed-arithmetics-verilog/>

9/26/19

6.111 Fall 2019

47

## For example:

```
module test_one;
  logic signed [3:0] x;
  logic [3:0] y;
  logic signed [8:0] z;
  initial begin
    x = -2;
    y=3;
    z = x*y;
    $display(x, y, z);
    $finish;
  end
endmodule
```

**Result:**

-2 3 42

```
module test_two;
  logic signed [3:0] x;
  logic signed [3:0] y;
  logic signed [8:0] z;
  initial begin
    x = -2;
    y=3;
    z = x*y;
    $display(x, y, z);
    $finish;
  end
endmodule
```

**Result:**

-2 3 -6

*Not really synthesizable here (\$finish, \$display, etc)...but shows what Verilog is thinking*

9/26/19

6.111 Fall 2019

48



## Signed Numbers

- Once you start using signed Verilog, just make everything you're using signed. If you do that, you should be ok.
- Make sure everything upstream of a calculation has been done in only a signed environment (held in signed logics and used with signed logics).
- Signed/Unsigned bugs are some of the hardest to find so be cautious