



# Pipelining & Verilog

- Division
- Latency & Throughput
- Pipelining to increase throughput
- Verilog Math Functions
- Simulations

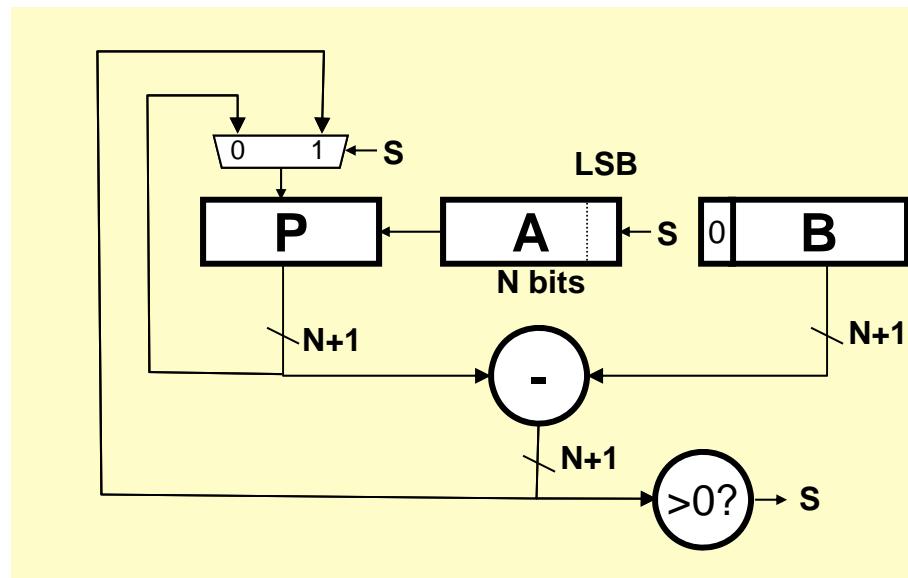
**Table 7-9: Supported Expressions**

Expression	Symbol	Status
Concatenation	{}	Supported
Replication	{()}	Supported
Arithmetic	+, -, *, **	Supported
Division	/	Supported only if the second operand is a power of 2, or both operands are constant.
Modulus	%	Supported only if second operand is a power of 2.
Addition	+	Supported
Subtraction	-	Supported
Multiplication	*	Supported
Power	**	Supported: <ul style="list-style-type: none"> <li>• Both operands are constants, with the second operand being non-negative.</li> <li>• If the first operand is a 2, then the second operand can be a variable.</li> <li>• Vivado synthesis does not support the real data type. Any combination of operands that results in a real type causes an error.</li> <li>• The values X (unknown) and Z (high impedance) are not allowed.</li> </ul>
Relational	>, <, >=, <=	Supported
Logical Negation	!	Supported
Logical AND	&&	Supported

[https://www.xilinx.com/support/documentation/sw\\_manuals/xilinx2019\\_1/ug901-vivado-synthesis.pdf](https://www.xilinx.com/support/documentation/sw_manuals/xilinx2019_1/ug901-vivado-synthesis.pdf)

# Sequential Divider

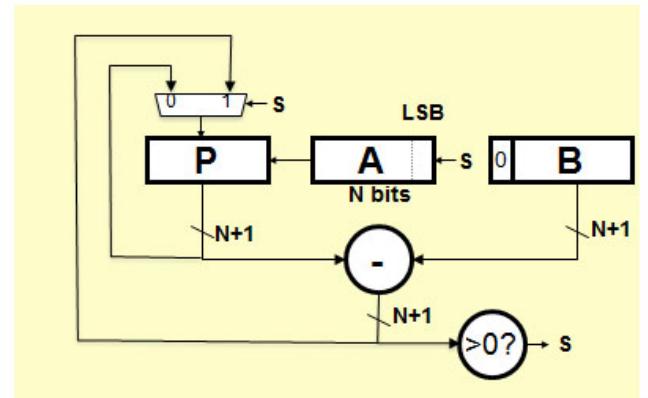
Assume the Dividend (A) and the divisor (B) have N bits. If we only want to invest in a single N-bit adder, we can build a sequential circuit that processes a single subtraction at a time and then cycle the circuit N times. This circuit works on unsigned operands; for signed operands one can remember the signs, make operands positive, then correct sign of result.



```
Init: P←0, load A and B  
Repeat N times {  
    shift P/A left one bit  
    temp = P-B  
    if (temp >= 0)  
        {P←temp, ALSB←1}  
    else ALSB←0  
}  
Done: Q in A, R in P
```

# Sequential Divider

P	A	P-B	7/3 0111/11 B=0011
0000	0111		Initial value
0000	1110		Shift
0000		-3	Subtract
0000	1110		Restore, set $A_{LSB} = 0$
0001	1100		Shift
0001		-2	Subtract
0001	1100		Restore, set $A_{LSB} = 0$
0011	1000		Shift
0011		0	Subtract
0000	1001		Subtract, set $A_{LSB} = 1$
0001	0010		Shift
0001		-2	Subtract
0001	0010		Restore, set $A_{LSB} = 0$
R	Q		



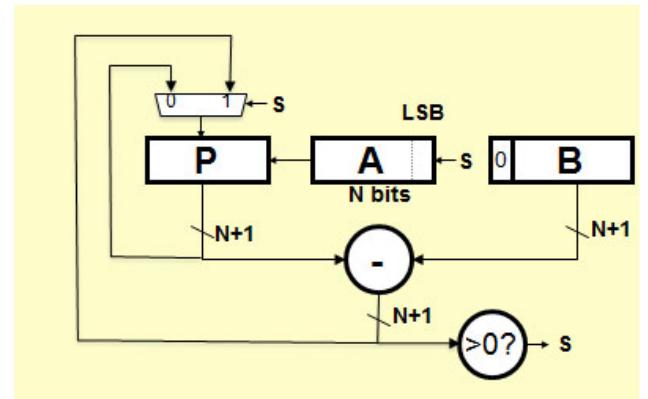
```

Init: P←0, load A and B
Repeat N times {
    shift P/A left one bit
    temp = P-B
    if (temp >= 0)
        {P←temp, ALSB←1}
    else ALSB←0
}
Done: Q in A, R in P

```

# Sequential Divider

P	A	P-B	0001/0000
0000	0001		Initial value
0000	0010		Shift
0000		0	Subtract
0000	0011		Subtract, set $A_{lsb} = 1$
0000	0110		Shift
0000		0	Subtract
0000	0111		Subtract, set $A_{lsb} = 1$
0000	1110		Shift
0000		0	Subtract
0000	1111		Subtract, set $A_{lsb} = 1$
0000	1110		Shift
0000		0	Subtract
0000	1111		Subtract, set $A_{lsb} = 1$
R	Q		



```

Init: P←0, load A and B
Repeat N times {
    shift P/A left one bit
    temp = P-B
    if (temp >= 0)
        {P←temp, ALSB←1}
    else ALSB←0
}
Done: Q in A, R in P
  
```

# Verilog divider.v

```

// The divider module divides one number by another. It
// produces a signal named "ready" when the quotient output
// is ready, and takes a signal named "start" to indicate
// the the input dividend and divider is ready.
// sign -- 0 for unsigned, 1 for twos complement

// It uses a simple restoring divide algorithm.
// http://en.wikipedia.org/wiki/Division_(digital)#Restoring_division

module divider #(parameter WIDTH = 8)
    (input clk, sign, start,
     input [WIDTH-1:0] dividend,
     input [WIDTH-1:0] divider,
     output reg [WIDTH-1:0] quotient,
     output [WIDTH-1:0] remainder;
     output ready);

    reg [WIDTH-1:0] quotient_temp;
    reg [WIDTH*2-1:0] dividend_copy, divider_copy, diff;
    reg negative_output;

    wire [WIDTH-1:0] remainder = (!negative_output) ?
        dividend_copy[WIDTH-1:0] : ~dividend_copy[WIDTH-1:0] + 1'b1;

    reg [5:0] bit;
    reg del_ready = 1;
    wire ready = (!bit) & ~del_ready;

    wire [WIDTH-2:0] zeros = 0;
    initial bit = 0;
    initial negative_output = 0;

    always @(*)
    begin
        if( start )
            begin
                bit = WIDTH;
                quotient = 0;
                quotient_temp = 0;
                dividend_copy = (!sign || !dividend[WIDTH-1]) ?
                    {1'b0,zeros,dividend} :
                    {1'b0,zeros,~dividend + 1'b1};
                divider_copy = (!sign || !divider[WIDTH-1]) ?
                    {1'b0,divider,zeros} :
                    {1'b0,~divider + 1'b1,zeros};

                negative_output = sign &&
                    ((divider[WIDTH-1] && !dividend[WIDTH-1])
                     || (!divider[WIDTH-1] && dividend[WIDTH-1]));
            end
        else if ( bit > 0 )
            begin
                diff = dividend_copy - divider_copy;
                quotient_temp = quotient_temp << 1;
                if( !diff[WIDTH*2-1] ) begin
                    dividend_copy = diff;
                    quotient_temp[0] = 1'd1;
                end
                quotient = (!negative_output) ?
                    quotient_temp :
                    ~quotient_temp + 1'b1;
                divider_copy = divider_copy >> 1;
                bit = bit - 1'b1;
            end
        end
    end
endmodule

```

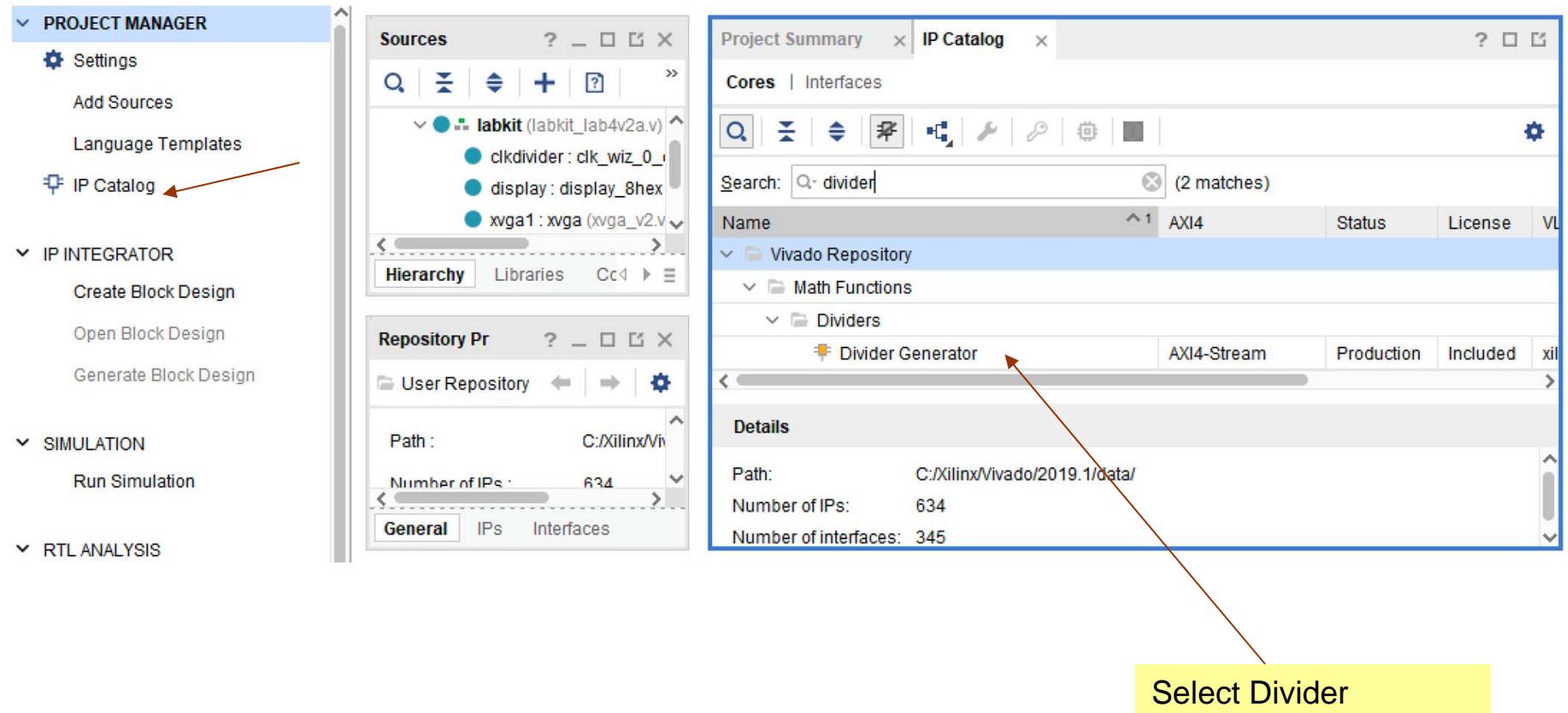
L. Williams MIT '13

# Math & Other Functions in IP Catalog

Wide selection of math functions available

The screenshot shows the Vivado Project Manager interface with the title bar "PROJECT MANAGER - lab\_sol". The left sidebar contains sections for Flow Navigator, PROJECT MANAGER (Settings, Add Sources, Language Templates, IP Catalog), IP INTEGRATOR (Create Block Design, Open Block Design, Generate Block Design), SIMULATION (Run Simulation), RTL ANALYSIS (Open Elaborated Design), and SYNTHESIS (Run Synthesis). The main area is titled "IP Catalog" and shows a search bar with "Search: Q-". A table lists IP cores by Name, including "AXI4", "Status", "License", and "VLAN". The "Math Functions" category is expanded, showing sub-categories like Adders & Subtractors, Conversions, CORDIC, Dividers (with "Divider Generator" listed), Floating Point, Multipliers, Square Root, and Trig Functions. At the bottom, "Details" show the Path as "C:/Xilinx/Vivado/2019.1/data/" and the Number of IPs as "634". A callout arrow points from the text "Wide selection of math functions available" to the "Math Functions" category in the IP Catalog table.

# Divider Generator



[https://www.xilinx.com/support/documentation/ip\\_documentation/div\\_gen/v5\\_1/pg151-div-gen.pdf](https://www.xilinx.com/support/documentation/ip_documentation/div_gen/v5_1/pg151-div-gen.pdf)

# IP Catalog Divider

Divider Generator (5.1)

Documentation IP Location Switch to Defaults

Component Name: div\_gen\_0

Algorithm Type: Radix2

Operand Sign: Signed

Dividend Channel:

- Dividend Width: 16 [2 - 64]
- Has TLAST
- Has TUSER
- TUSER Width: 1

Divisor Channel:

- Divisor Width: 8 [2 - 64]
- Has TLAST
- Has TUSER
- TUSER Width: 1 [1 - 256]

IP Symbol Implementation Details

Show disabled ports

S\_AXIS\_DIVISOR

- s\_axis\_divisor\_tdata[7:0]
- s\_axis\_divisor\_tlast
- s\_axis\_divisor\_tready
- s\_axis\_divisor\_tuser[0:0]
- s\_axis\_divisor\_tvalid

M\_AXIS\_DOUT

- m\_axis\_dout\_tdata[23:0]
- m\_axis\_dout\_tlast
- m\_axis\_dout\_tready
- m\_axis\_dout\_tuser[0:0]
- m\_axis\_dout\_tvalid

S\_AXIS\_DIVIDEND

- aclk
- acklen
- aresetn

Data valid

Chose minimum number for application

# Coregen Divider

Divider Generator (5.1) 

Documentation IP Location Switch to Defaults

**IP Symbol** Implementation Details

Show disabled ports

S\_AXIS\_DIVISOR

- s\_axis\_divisor\_tdata[7:0]
- s\_axis\_divisor\_tlast
- s\_axis\_divisor\_tready
- s\_axis\_divisor\_tuser[0:0]
- s\_axis\_divisor\_tvalid

S\_AXIS\_DIVIDEND

- ackl
- acklen
- aresetn

M\_AXIS\_DOUT

- m\_axis\_dout\_tdata[23:0]
- m\_axis\_dout\_tlast
- m\_axis\_dout\_tready
- m\_axis\_dout\_tuser[0:0]
- m\_axis\_dout\_tvalid

Component Name: div\_gen\_0

Channel Settings Options

Clocks per Division: 1

Chose maximum for application

AXI4-Stream Options

- Flow Control: Non Blocking
- Optimize Goal: Performance
- Output has TREADY
- Output TLAST Behavior: Null

Latency Options

- Latency Configuration: Automatic
- Latency: 20 [20 - 20]

Control Signals

# Performance Metrics for Circuits

Circuit **Latency** ( $L$ ): time between arrival of new input and generation of corresponding output.  
For combinational circuits this is just  $t_{PD}$ .

Circuit **Throughput** ( $T$ ): Rate at which new outputs appear.  
For combinational circuits this is just  $1/t_{PD}$  or  $1/L$ .

# Coregen Divider Latency

Latency dependent on  
dividend width +  
fractioanl reminder width

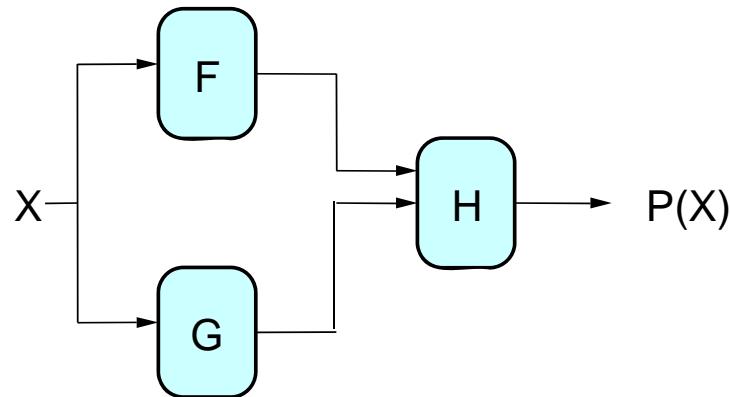
Table 2-1: Latency of Radix-2 Solution Based on Divider Parameters

Signed	Fractional	Clocks Per Division	Fully Pipelined Latency <sup>(1)</sup>
FALSE	FALSE	1	$M+A+2$
FALSE	FALSE	>1	$M+A+3$
FALSE	TRUE	1	$M+F+A+2$
FALSE	TRUE	>1	$M+F+A+3$
TRUE	FALSE	1	$M+A+4$
TRUE	FALSE	>1	$M+A+5$
TRUE	TRUE	1	$M+F+A+4$
TRUE	TRUE	>1	$M+F+A+5$

## Notes:

1. M = Dividend and Quotient Width, F = Fractional Width, A = total Latency of AXI interfaces.

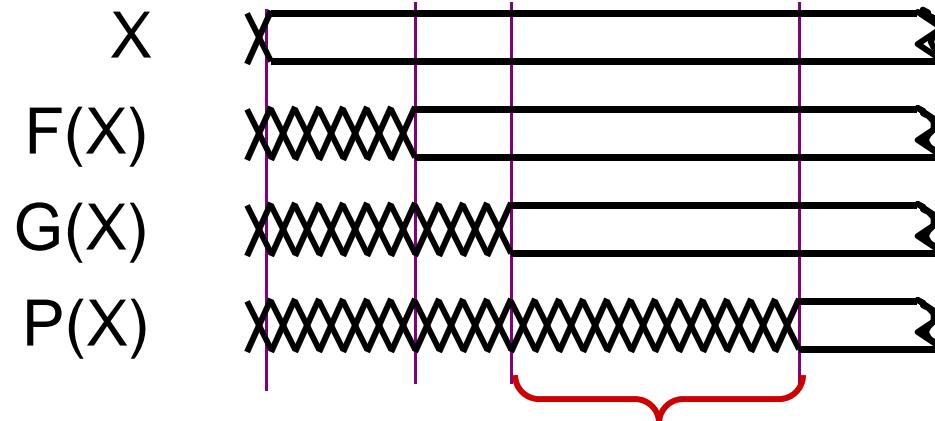
# Performance of Combinational Circuits



For combinational logic:

$$L = t_{PD},$$
$$T = 1/t_{PD}.$$

We can't get the answer faster, but  
are we making effective use of our  
hardware at all times?

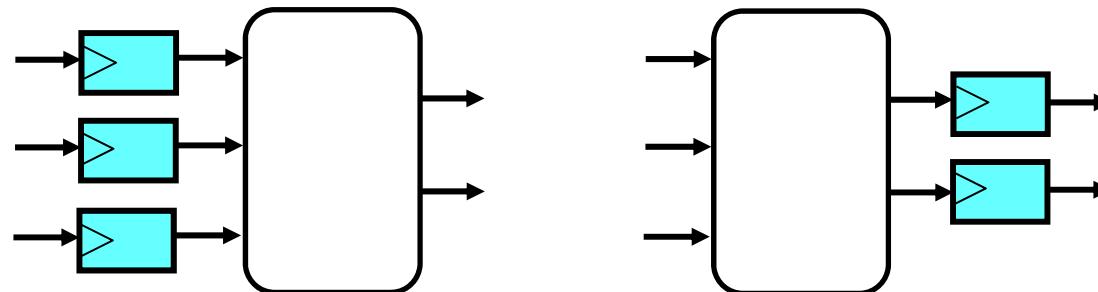


F & G are “idle”, just holding their outputs stable  
while H performs its computation

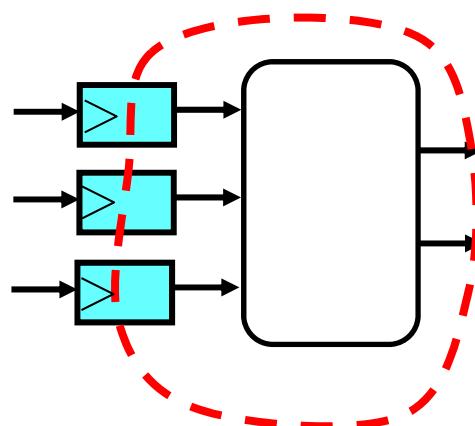
# Retiming: A very useful transform

Retiming is the action of moving registers around in the system

- Registers have to be moved from ALL inputs to ALL outputs or vice versa



**Cutset retiming:** A cutset intersects the edges, such that this would result in two disjoint partitions of the edges being cut. To retime, delays are moved from the ingoing to the outgoing edges or vice versa.

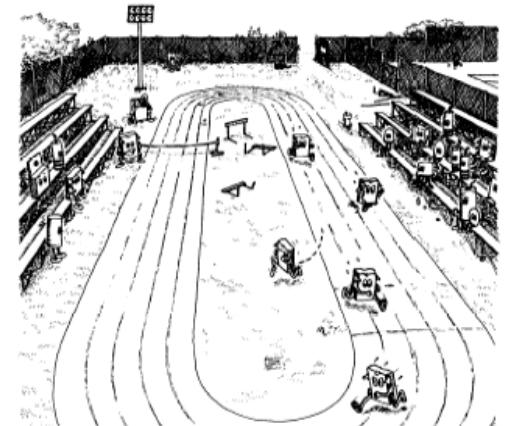


**Benefits of retiming:**

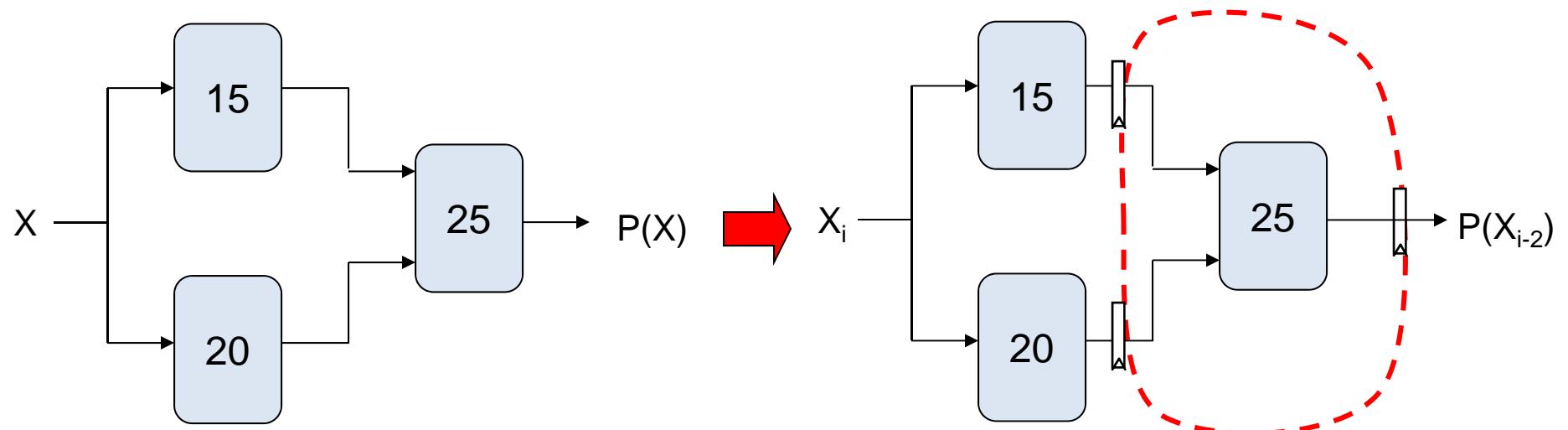
- Modify critical path delay
- Reduce total number of registers

Retiming Synchronous Circuitry

Charles E. Leiserson and James B. Saxe  
August 20, 1986.



# Retiming Combinational Circuits aka “Pipelining”

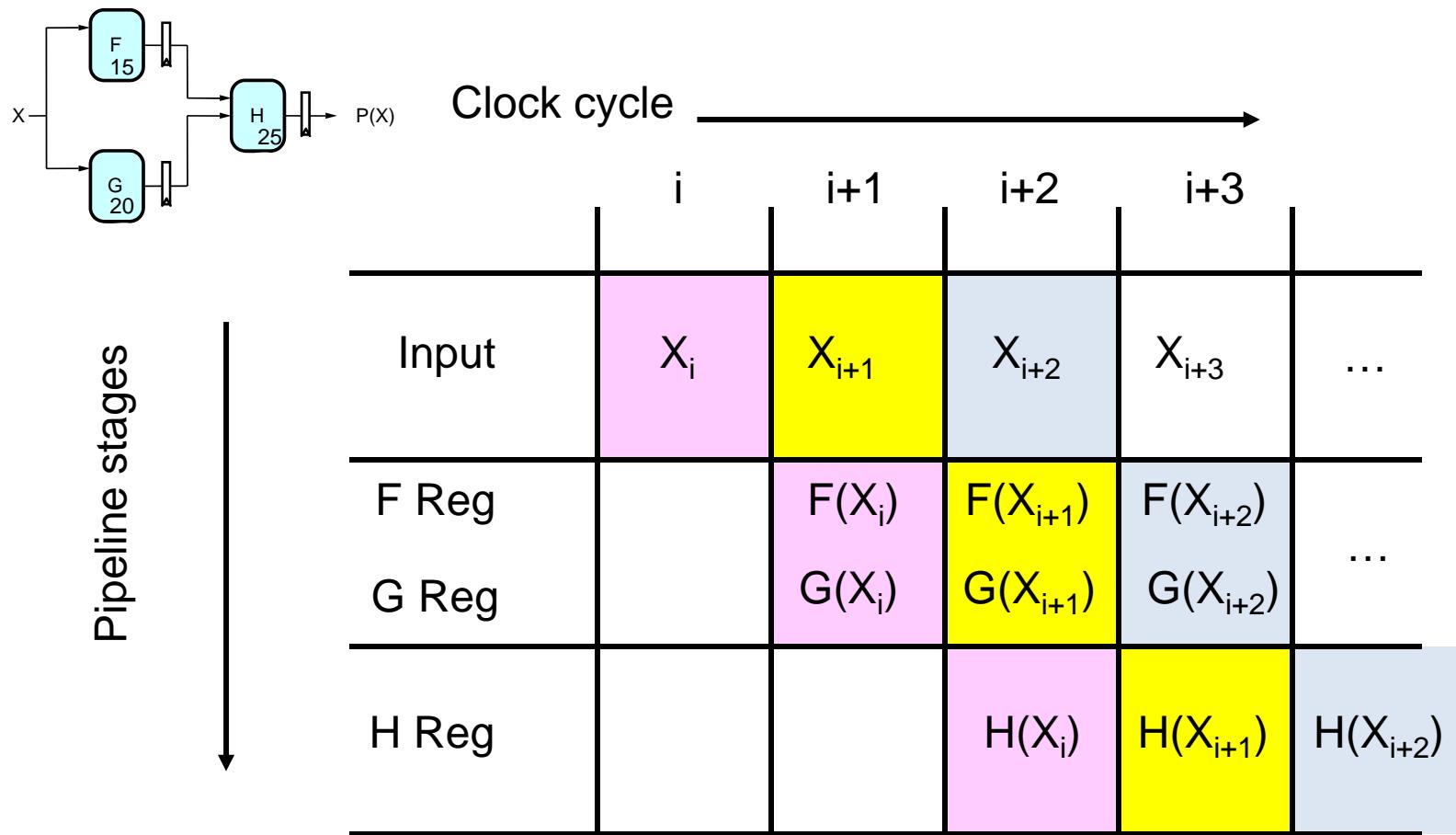


$$L = 45$$
$$T = 1/45$$

Assuming ideal registers:  
i.e.,  $t_{PD} = 0$ ,  $t_{SETUP} = 0$

$$t_{CLK} = 25$$
$$L = 2*t_{CLK} = 50$$
$$T = 1/t_{CLK} = 1/25$$

# Pipeline diagrams



The results associated with a particular set of input data moves diagonally through the diagram, progressing through one pipeline stage each clock cycle.

# Pipeline Conventions

## DEFINITION:

a **K-Stage Pipeline** (“K-pipeline”) is an acyclic circuit having exactly K registers on every path from an input to an output.

a COMBINATIONAL CIRCUIT is thus an 0-stage pipeline.

## CONVENTION:

Every pipeline stage, hence every K-Stage pipeline, has a register on its OUTPUT (not on its input).

## ALWAYS:

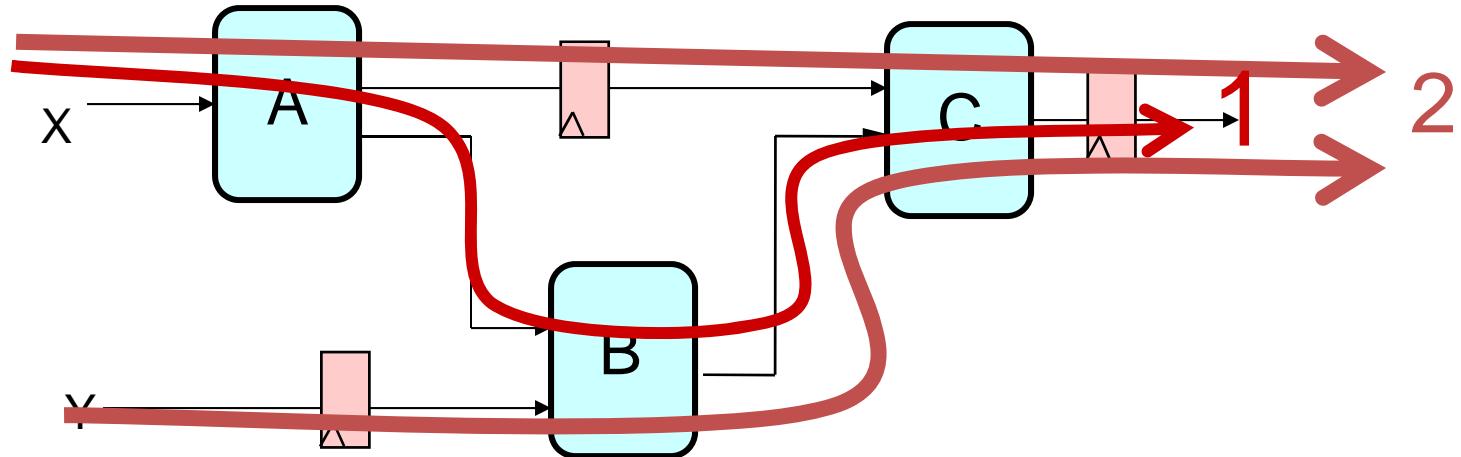
The CLOCK common to all registers must have a period sufficient to cover propagation over combinational paths PLUS (input) register  $t_{PD}$  PLUS (output) register  $t_{SETUP}$ .

The LATENCY of a K-pipeline is K times the period of the clock common to all registers.

The THROUGHPUT of a K-pipeline is the frequency of the clock.

# III-formed pipelines

Consider a BAD job of pipelining:



For what value of K is the following circuit a K-Pipeline? \_\_\_\_\_ none

Problem:

Successive inputs get mixed: e.g.,  $B(A(X_{i+1}), Y_i)$ . This happened because some paths from inputs to outputs have 2 registers, and some have only 1!

This CAN'T HAPPEN on a well-formed K pipeline!

# A pipelining methodology

Step 1:

Add a register on each output.

Step 2:

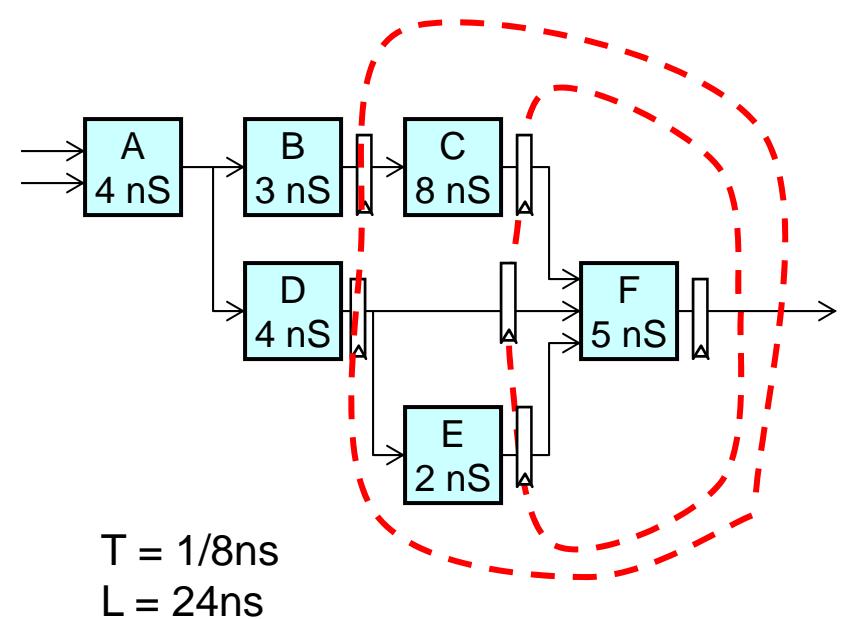
Add another register on each output. Draw a cut-set contour that includes all the new registers and some part of the circuit.

Retime by moving regs from all outputs to all inputs of cut-set.

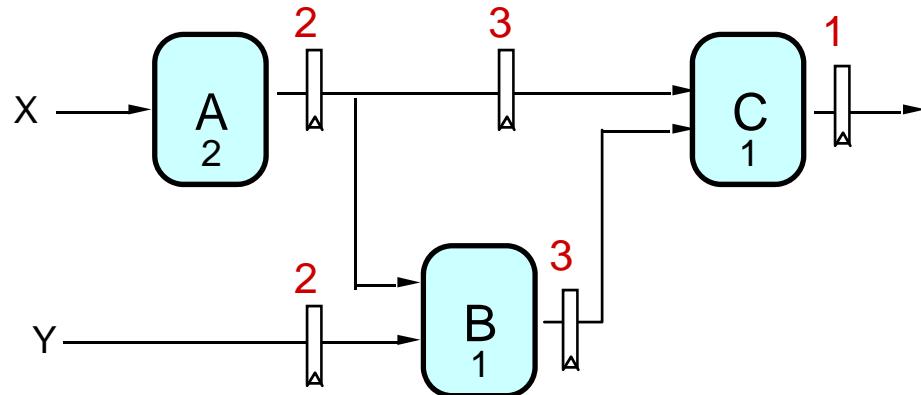
Repeat until satisfied with T.

STRATEGY:

Focus your attention on placing pipelining registers around the slowest circuit elements (BOTTLENECKS).



# Pipeline Example

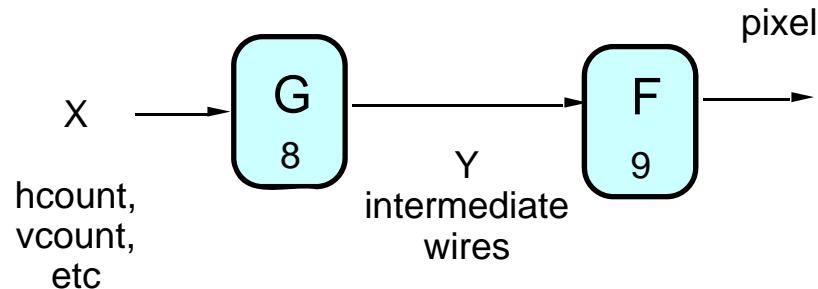


	LATENCY	THROUGHPUT
0-pipe:	4	1/4
1-pipe:	4	1/4
2-pipe:	4	1/2
3-pipe:	6	1/2

## OBSERVATIONS:

- 1-pipeline improves neither L or T.
- T improved by breaking long combinational paths, allowing faster clock.
- Too many stages cost L, don't improve T.
- Back-to-back registers are often required to keep pipeline well-formed.

# Pipeline Example - Verilog



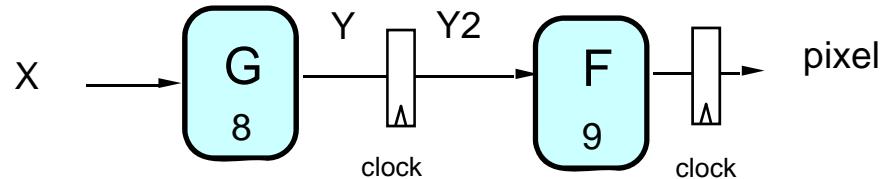
No pipeline

```
assign y = G(x);      // logic for y
assign pixel = C(y)  // logic for pixel
```

Lab 3 Pong

- $G$  = game logic 8ns tpd
- $C$  = draw fancy object puck, lots of multiplies with 9ns tpd
- System clock 65mhz = 15ns period – opps

```
reg [N:0] x,y;
reg [23:0] pixel
always @ * begin
    y=G(x);
    pixel = C(y);
end
```



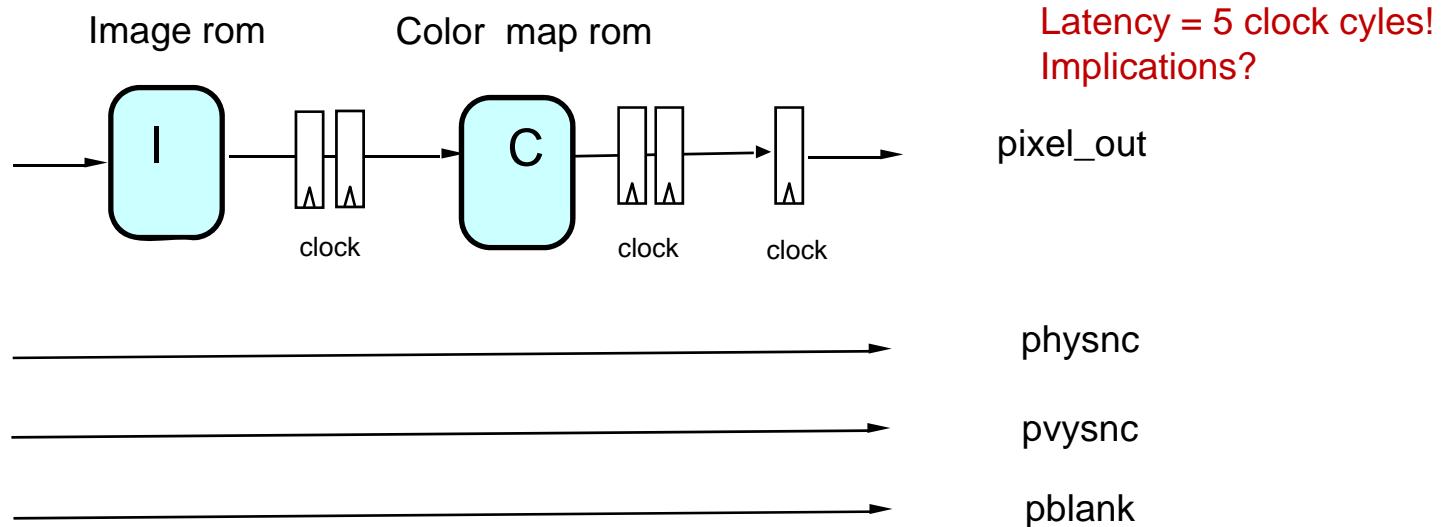
Pipeline

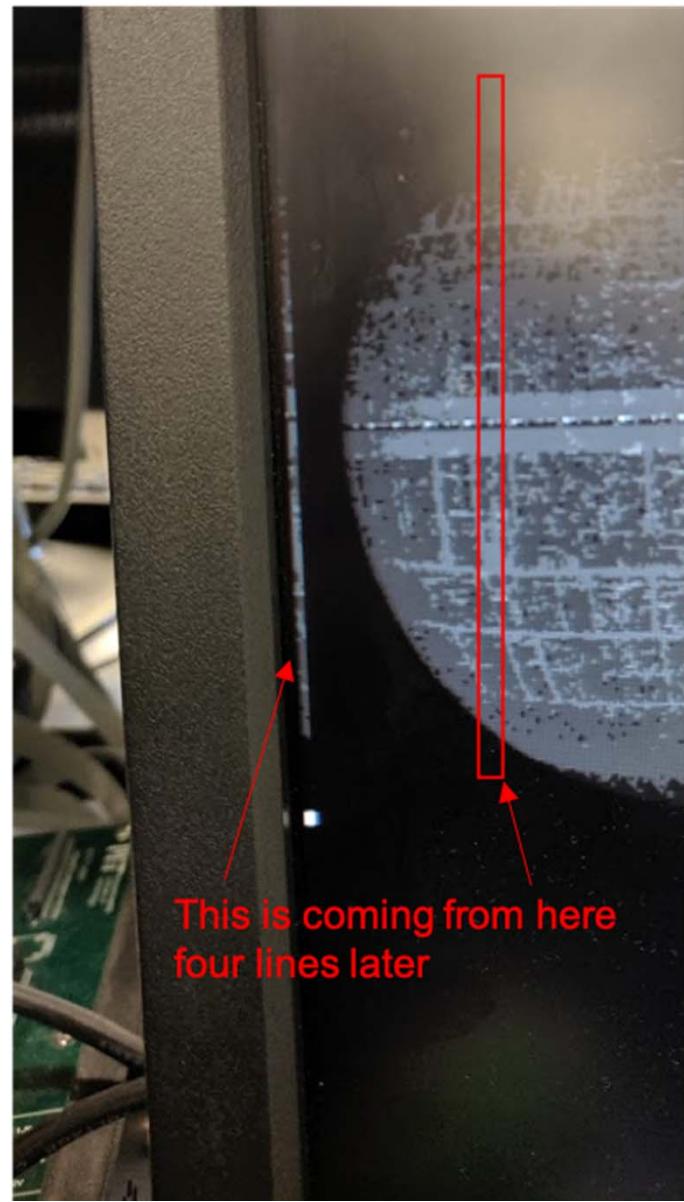
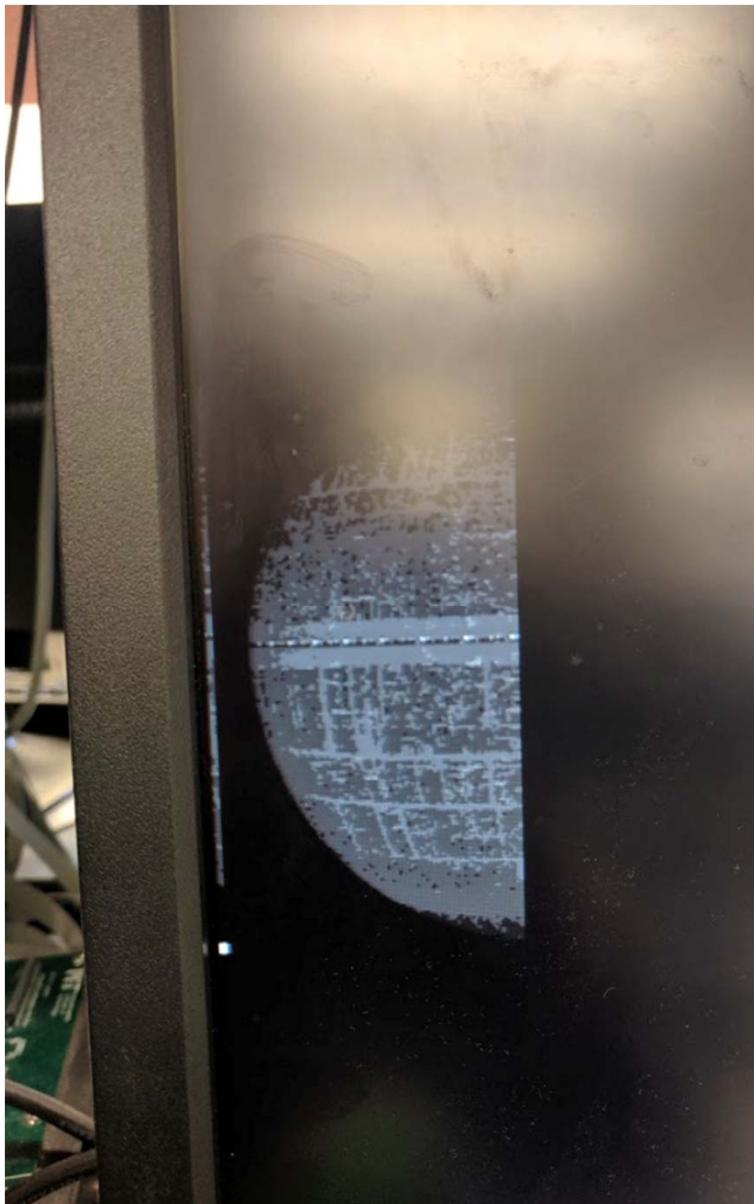
```
always @(posedge clock) begin
    ...
    y2 <= G(x);          // pipeline y
    pixel <= C(y2);     // pipeline pixel
end
```

Latency = 2 clock cycles!  
Implications?

# Pipeline Example – Lab 3

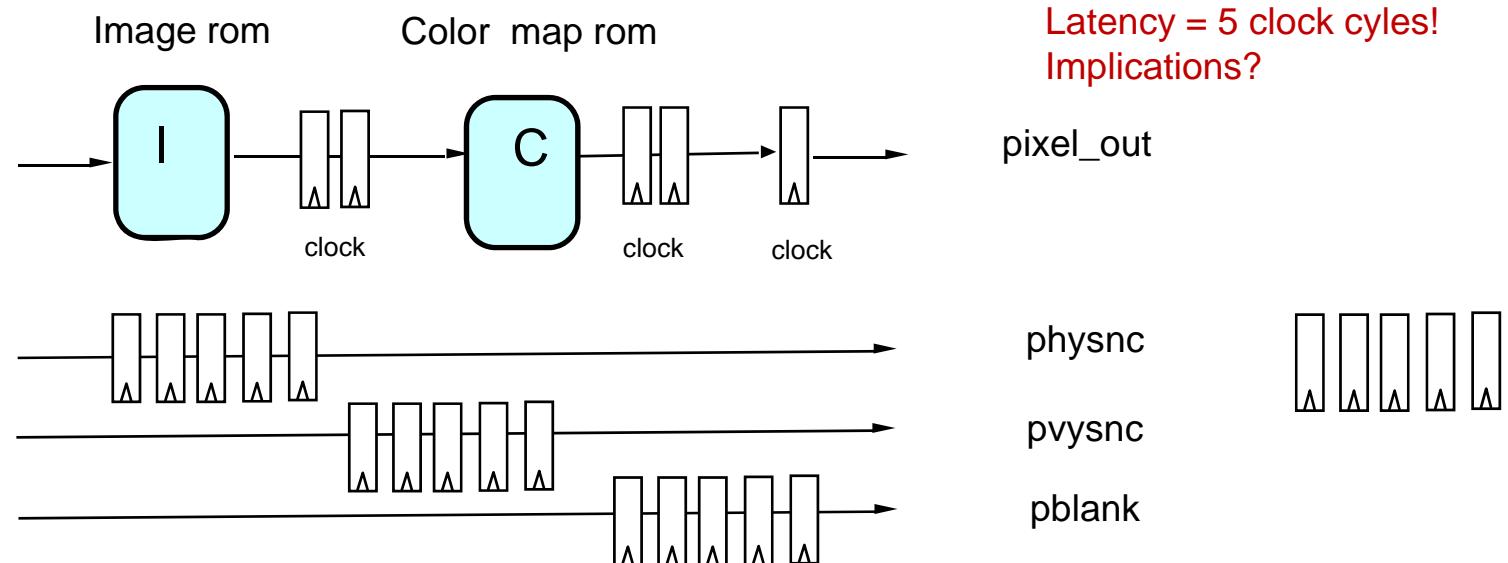
```
// calculate rom address and read the location  
assign image_addr = (hcount_in-x_in) + (vcount_in-y_in) * WIDTH;  
image_rom rom1(.clka(pixel_clk_in), .addra(image_addr), .douta(image_bits));  
  
red_coe rcm (.clka(pixel_clk_in), .addra(image_bits), .douta(red_mapped));  
  
always @ (posedge pixel_clk) begin  
    if ((hcount_in >= x && hcount_in < (x_in+WIDTH)) && (vcount_in >= y_in && vcount_in < (y_in+HEIGHT)))  
  
        pixel_out <= {red_mapped[7:4], red_mapped[7:4], red_mapped[7:4]}; // greyscale  
  
    else pixel_out <= 0;  
end
```





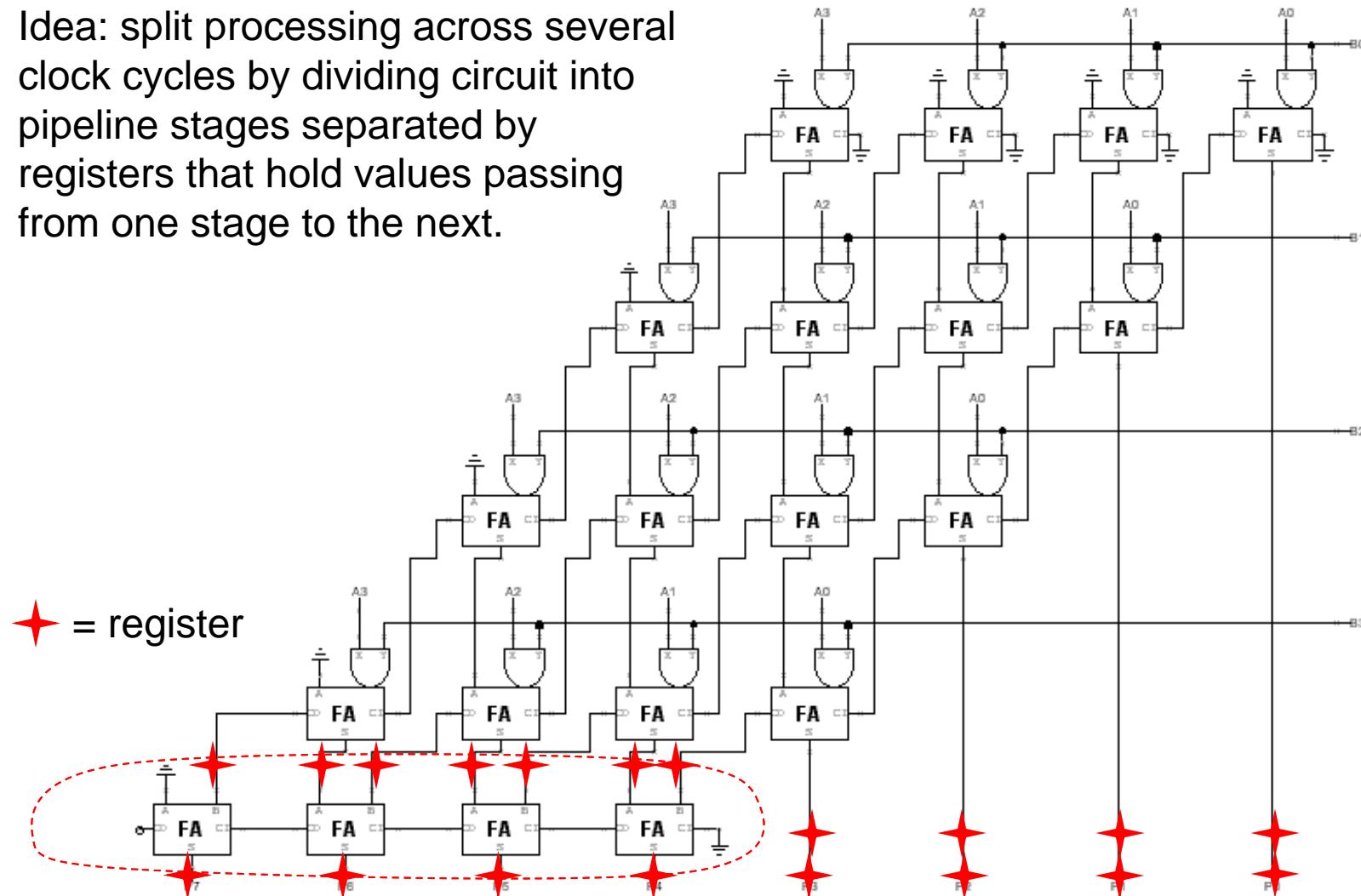
# Pipeline Example – Lab 3

```
// calculate rom address and read the location  
assign image_addr = (hcount_in-x_in) + (vcount_in-y_in) * WIDTH;  
image_rom rom1(.clka(pixel_clk_in), .addra(image_addr), .douta(image_bits));  
  
red_coe rcm (.clka(pixel_clk_in), .addra(image_bits), .douta(red_mapped));  
  
always @ (posedge pixel_clk) begin  
    if ((hcount_in >= x && hcount_in < (x_in+WIDTH)) && (vcount_in >= y_in && vcount_in < (y_in+HEIGHT)))  
  
        pixel_out <= {red_mapped[7:4], red_mapped[7:4], red_mapped[7:4]}; // greyscale  
  
    else pixel_out <= 0;  
end
```



# Increasing Throughput: Pipelining

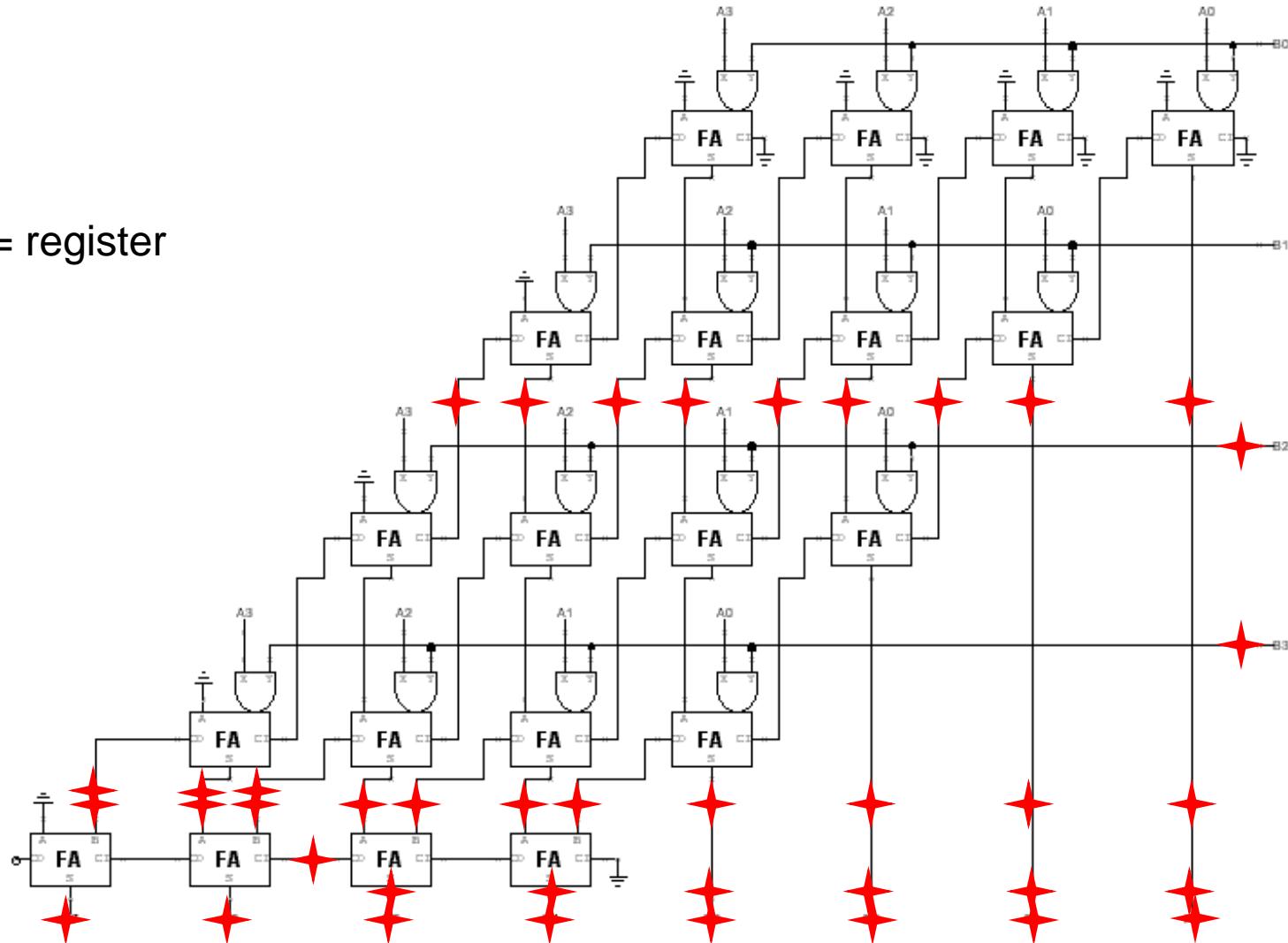
Idea: split processing across several clock cycles by dividing circuit into pipeline stages separated by registers that hold values passing from one stage to the next.



$$\text{Throughput} = 1/4t_{PD,FA} \text{ instead of } 1/8t_{PD,FA})$$

# How about $t_{PD} = 1/2t_{PD,FA}$ ?

★ = register

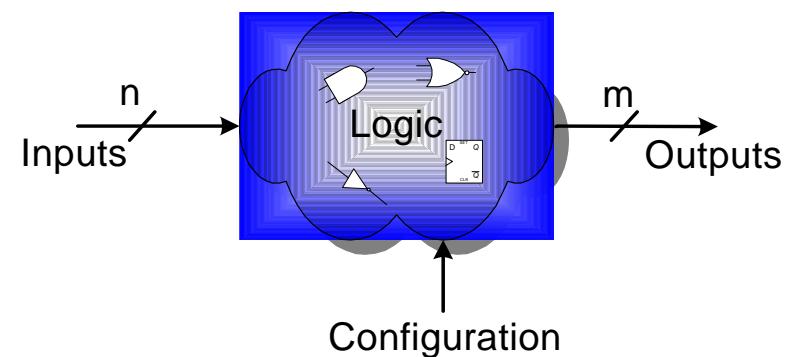
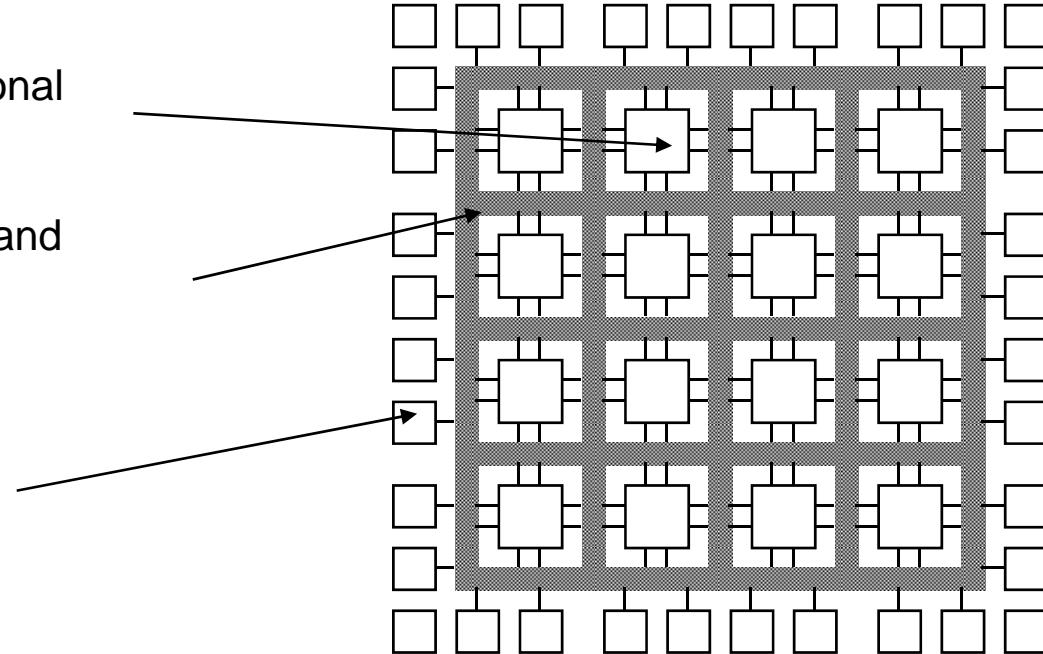


# History of Computational Fabrics

- Discrete devices: relays, transistors (1940s-50s)
- Discrete logic gates (1950s-60s)
- Integrated circuits (1960s-70s)
  - e.g. TTL packages: Data Book for 100's of different parts
- Gate Arrays (IBM 1970s)
  - Transistors are pre-placed on the chip & Place and Route software puts the chip together automatically – only program the interconnect (mask programming)
- Software Based Schemes (1970's- present)
  - Run instructions on a general purpose core
- Programmable Logic (1980's to present)
  - A chip that can be reprogrammed after it has been fabricated
  - Examples: **PALs**, EPROM, EEPROM, PLDs, **FPGAs**
  - Excellent support for mapping from Verilog
- ASIC Design (1980's to present)
  - Turn Verilog directly into layout using a library of standard cells
  - Effective for high-volume and efficient use of silicon area

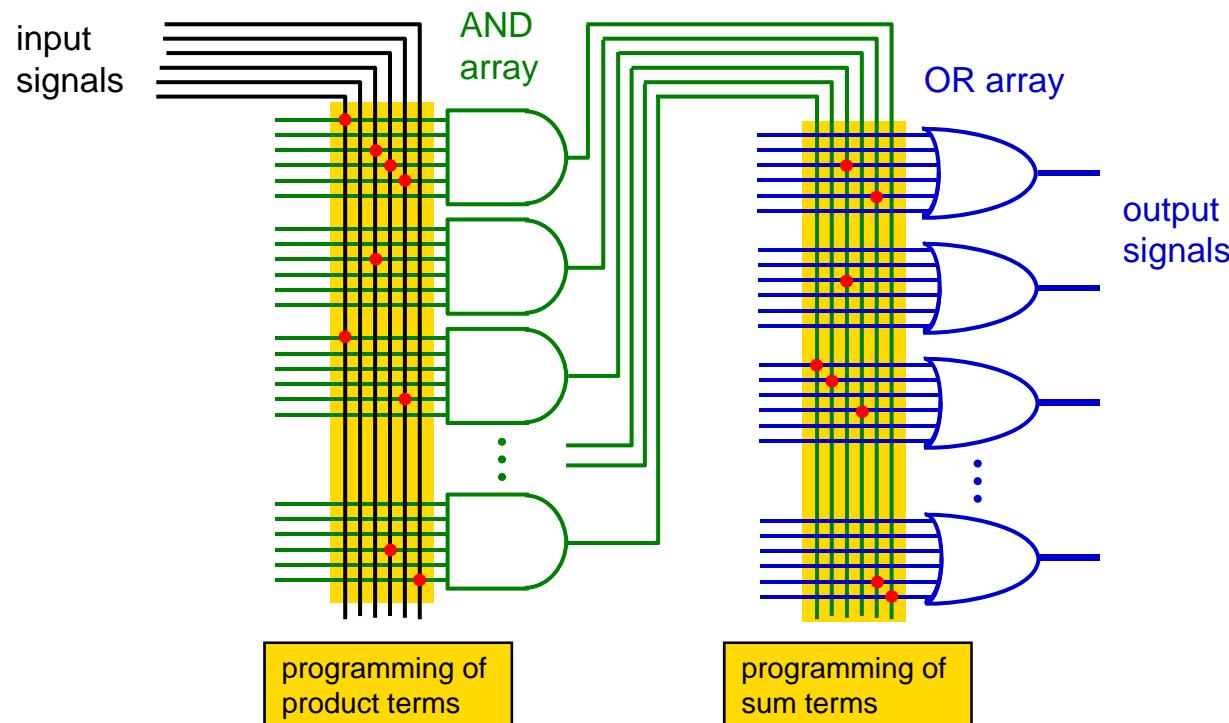
# Reconfigurable Logic

- **Logic blocks**
  - To implement combinational and sequential logic
- **Interconnect**
  - Wires to connect inputs and outputs to logic blocks
- **I/O blocks**
  - Special logic blocks at periphery of device for external connections
- **Key questions:**
  - How to make logic blocks programmable? (after chip has been fabbed!)
  - What should the logic granularity be?
  - How to make the wires programmable? (after chip has been fabbed!)
  - Specialized wiring structures for local vs. long distance routes?
  - How many wires per logic block?

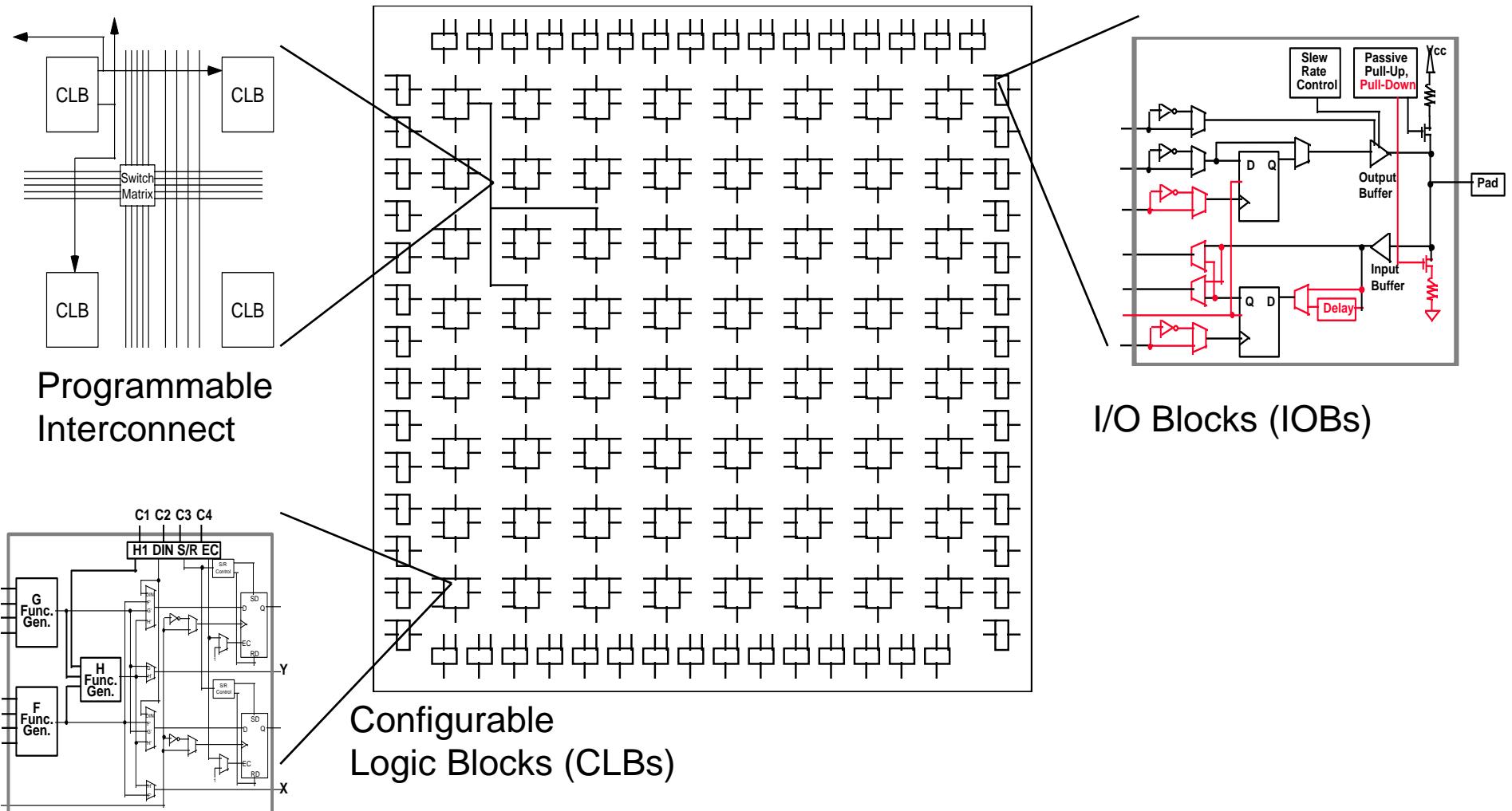


# Programmable Array Logic (PAL)

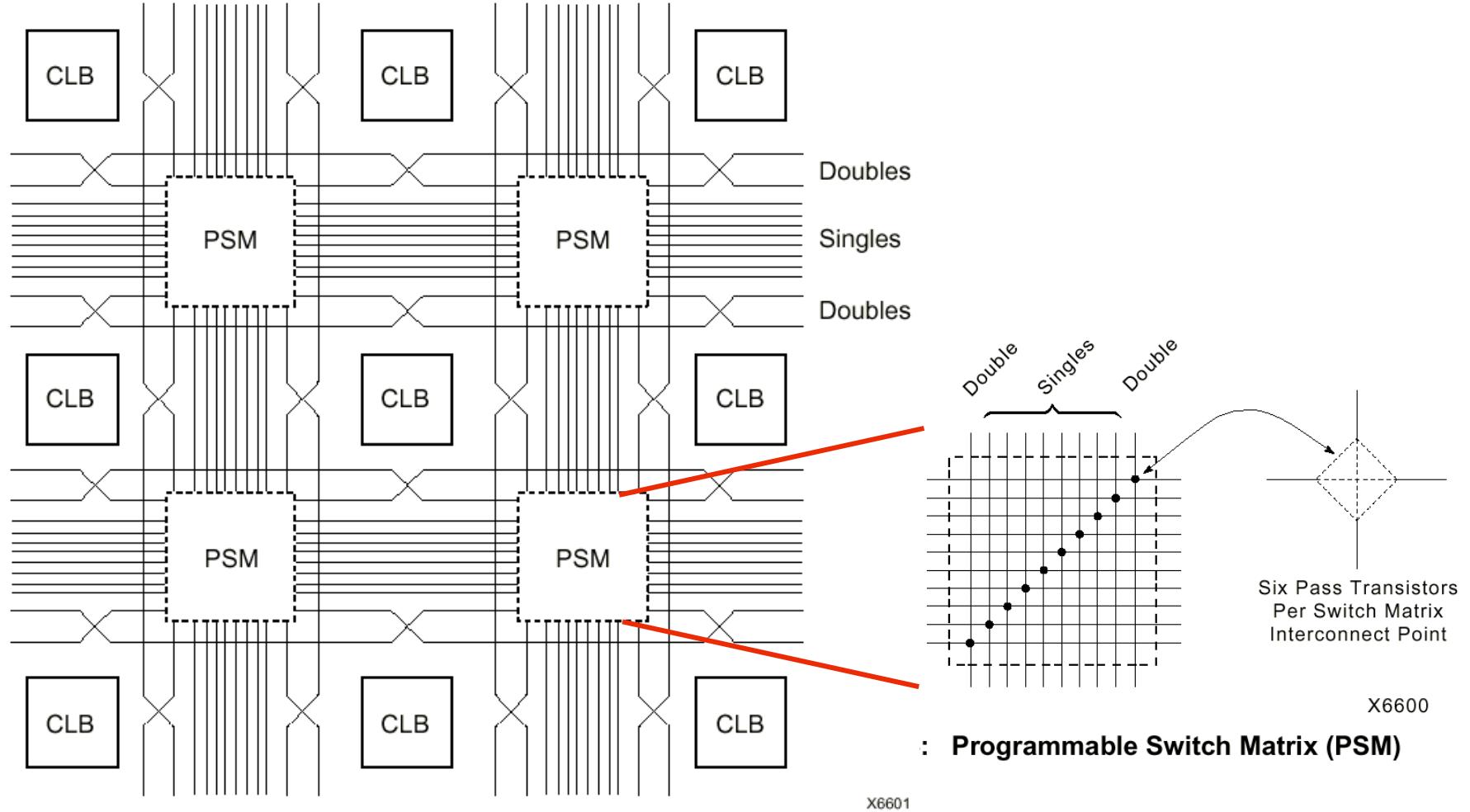
- Based on the fact that any combinational logic can be realized as a sum-of-products
- PALs feature an array of AND-OR gates with programmable interconnect



# RAM Based Field Programmable Logic - FPGA

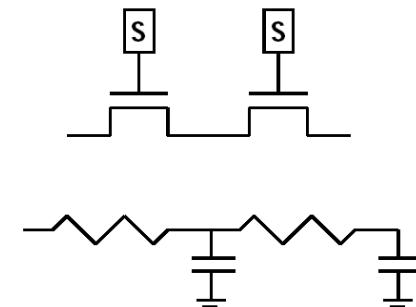
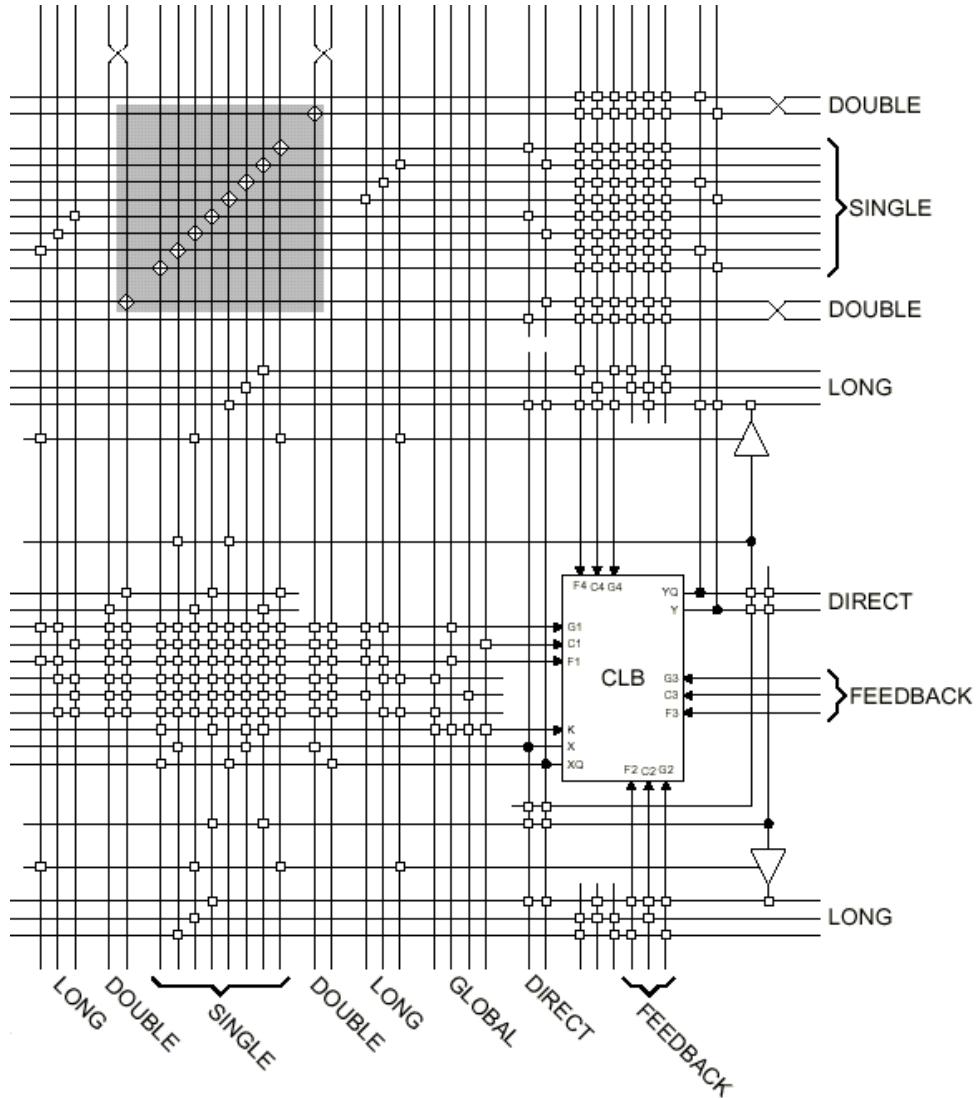


# FPGA RAM based Interconnect



**Figure 28: Single- and Double-Length Lines, with Programmable Switch Matrices (PSMs)**

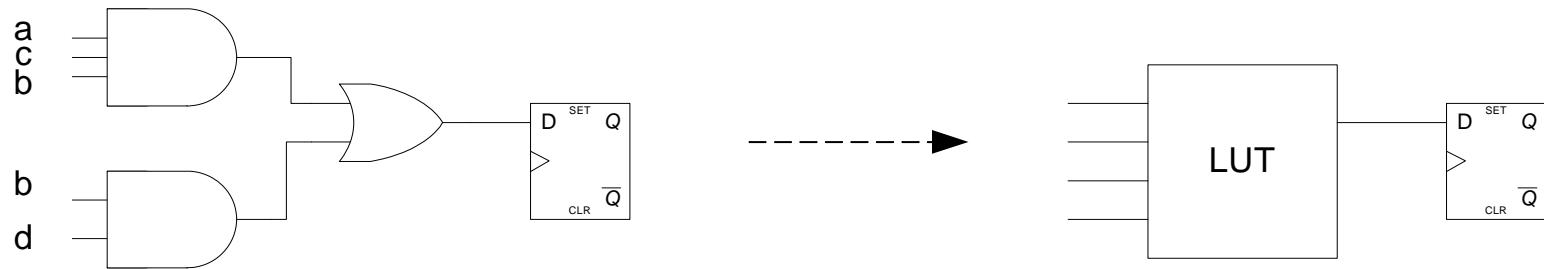
# Xilinx Interconnect Details



Wires are not ideal!

# Design Flow - Mapping

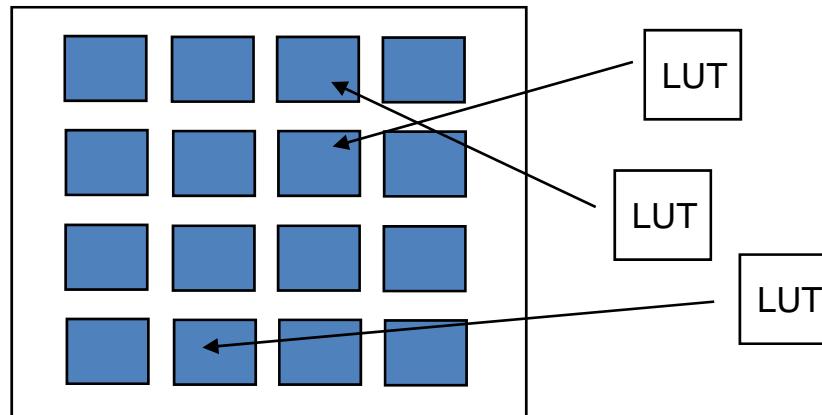
- Technology Mapping: Schematic/HDL to Physical Logic units
- Compile functions into basic LUT-based groups (function of target architecture)



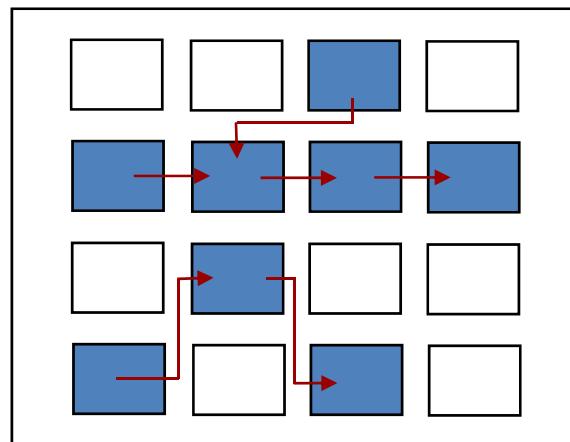
```
always @(posedge clock or negedge reset)
begin
    if (!reset)
        q <= 0;
    else
        q <= (a&b&c) || (b&d);
end
```

# Design Flow – Placement & Route

- **Placement** – assign logic location on a particular device



- **Routing** – iterative process to connect CLB inputs/outputs and IOBs. Optimizes critical path delay – *can take hours or days for large, dense designs*



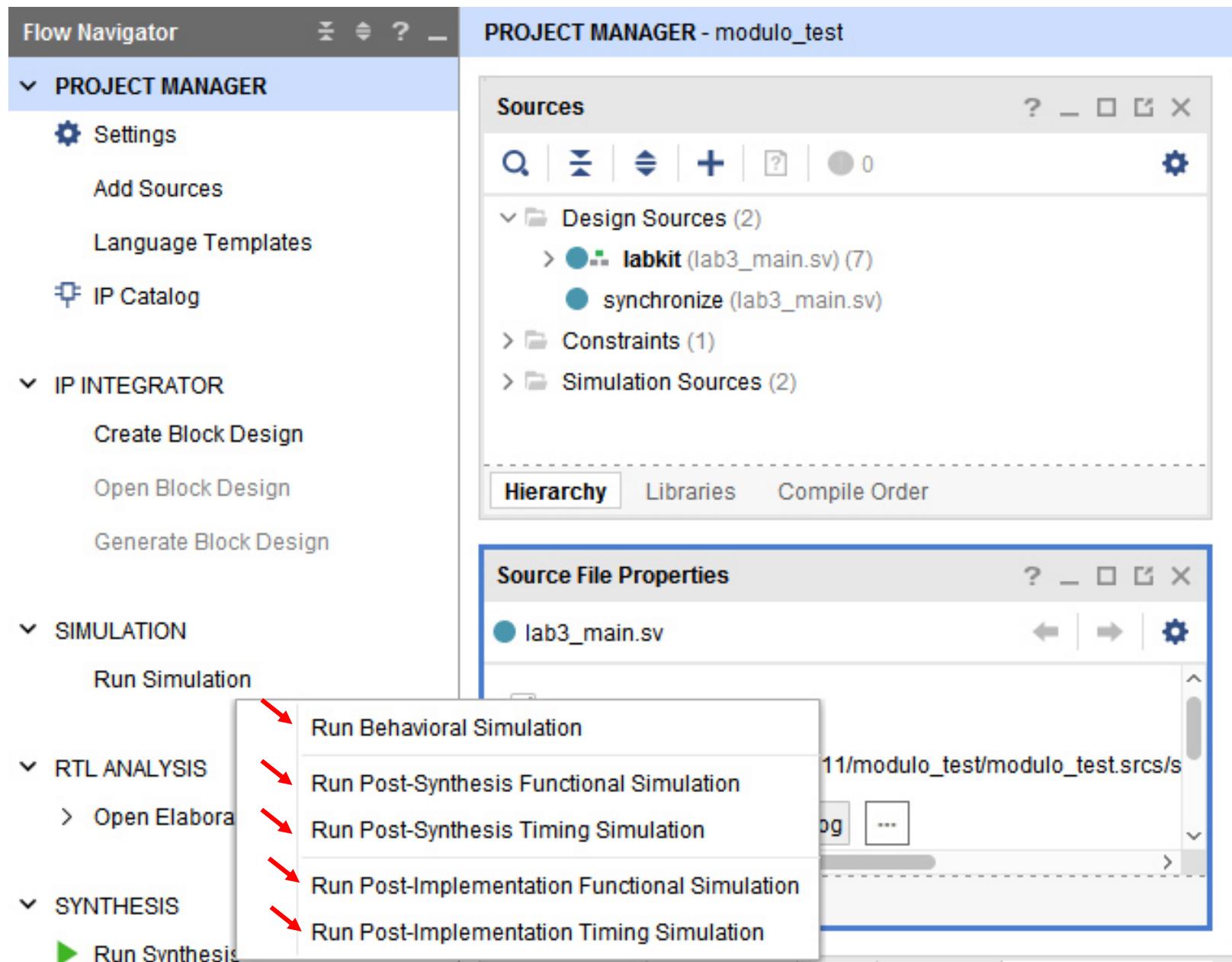
Iterate placement if timing not met

Satisfy timing? → Generate Bitstream to config device

Challenge! Cannot use full chip for reasonable speeds (wires are not ideal).

Typically no more than 50% utilization.

# Simulation – Five Options



# Simulations

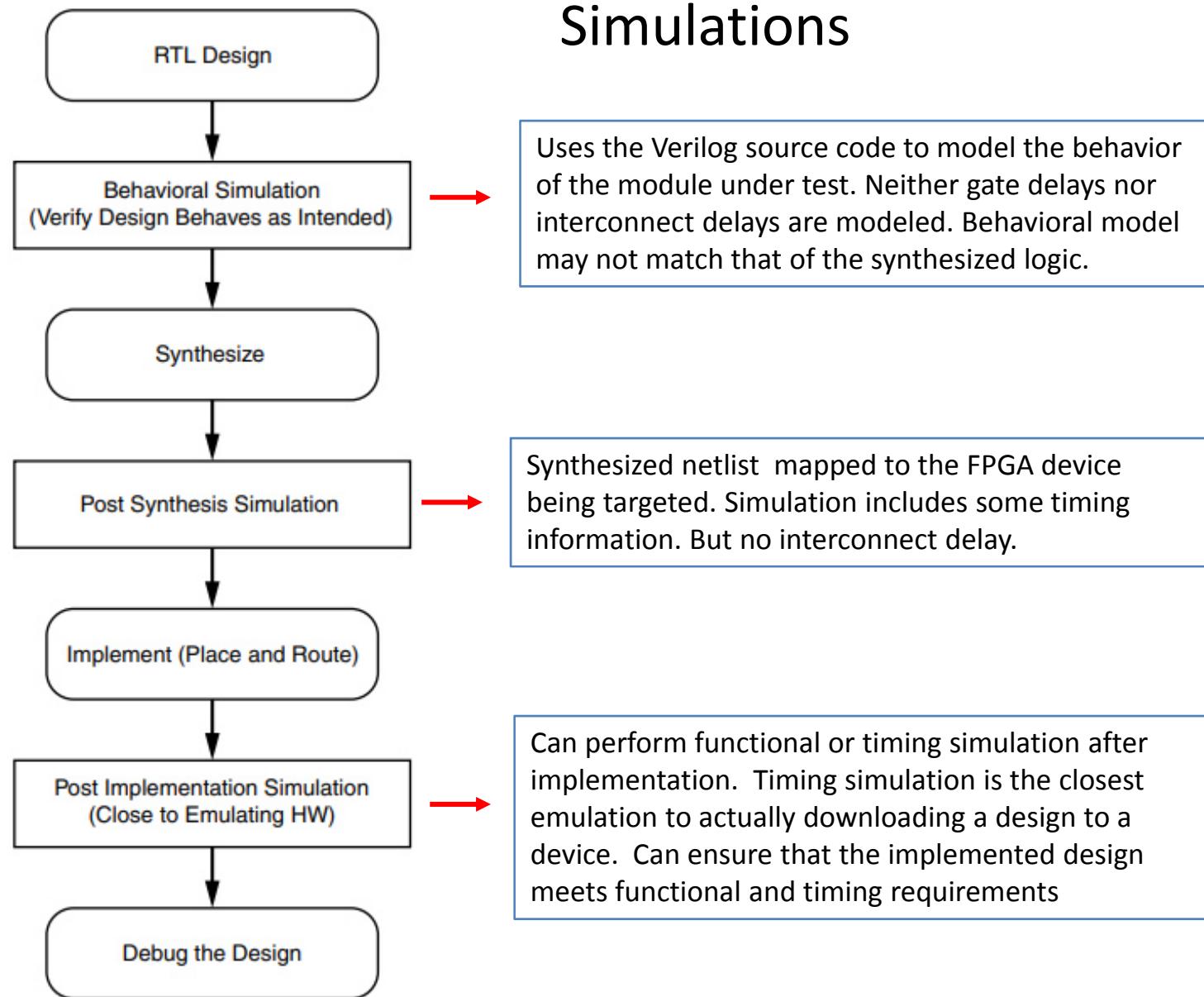
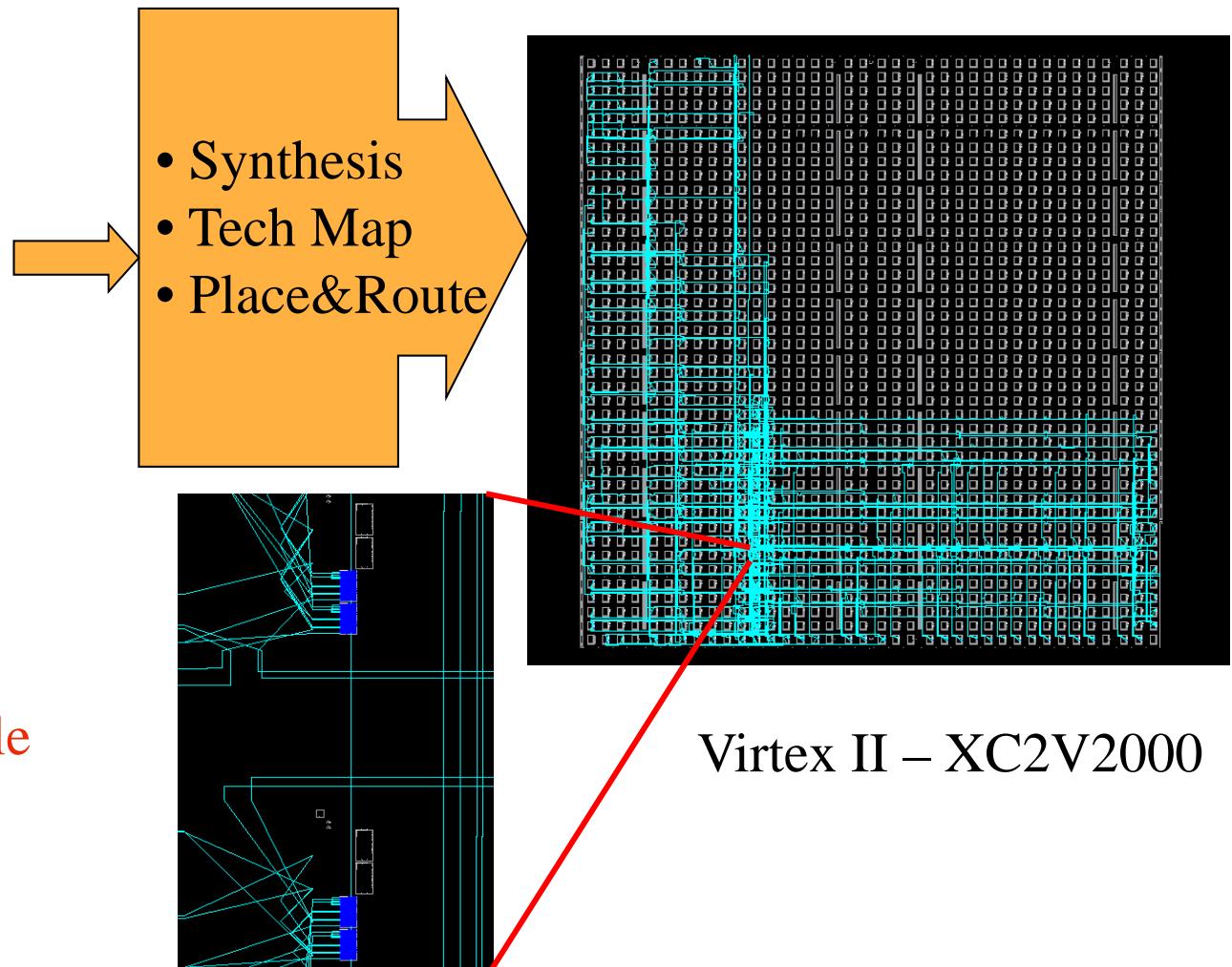


Figure 1-1: Simulation Flow

# Example: Verilog to FPGA

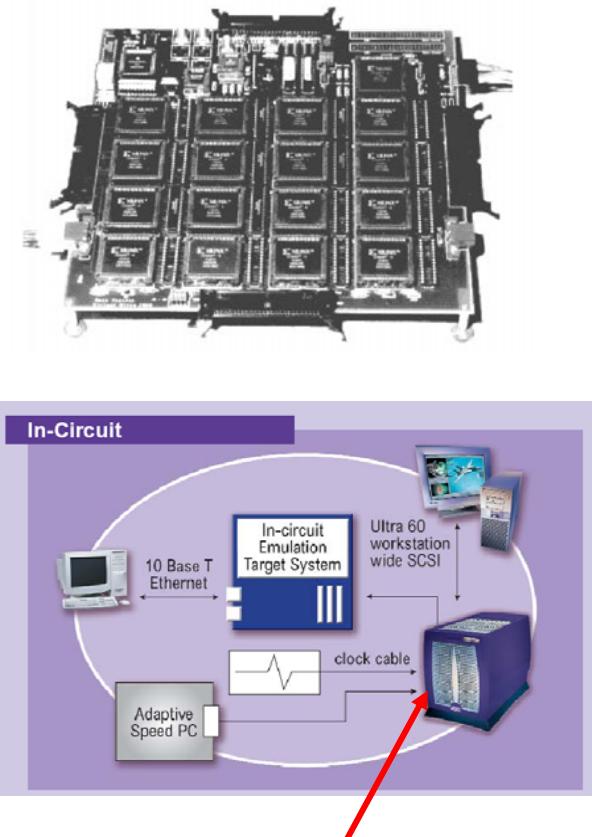
```
module adder64 (
    input [63:0] a, b;
    output [63:0] sum);

    assign sum = a + b;
endmodule
```



# How are FPGAs Used?

## Logic Emulation



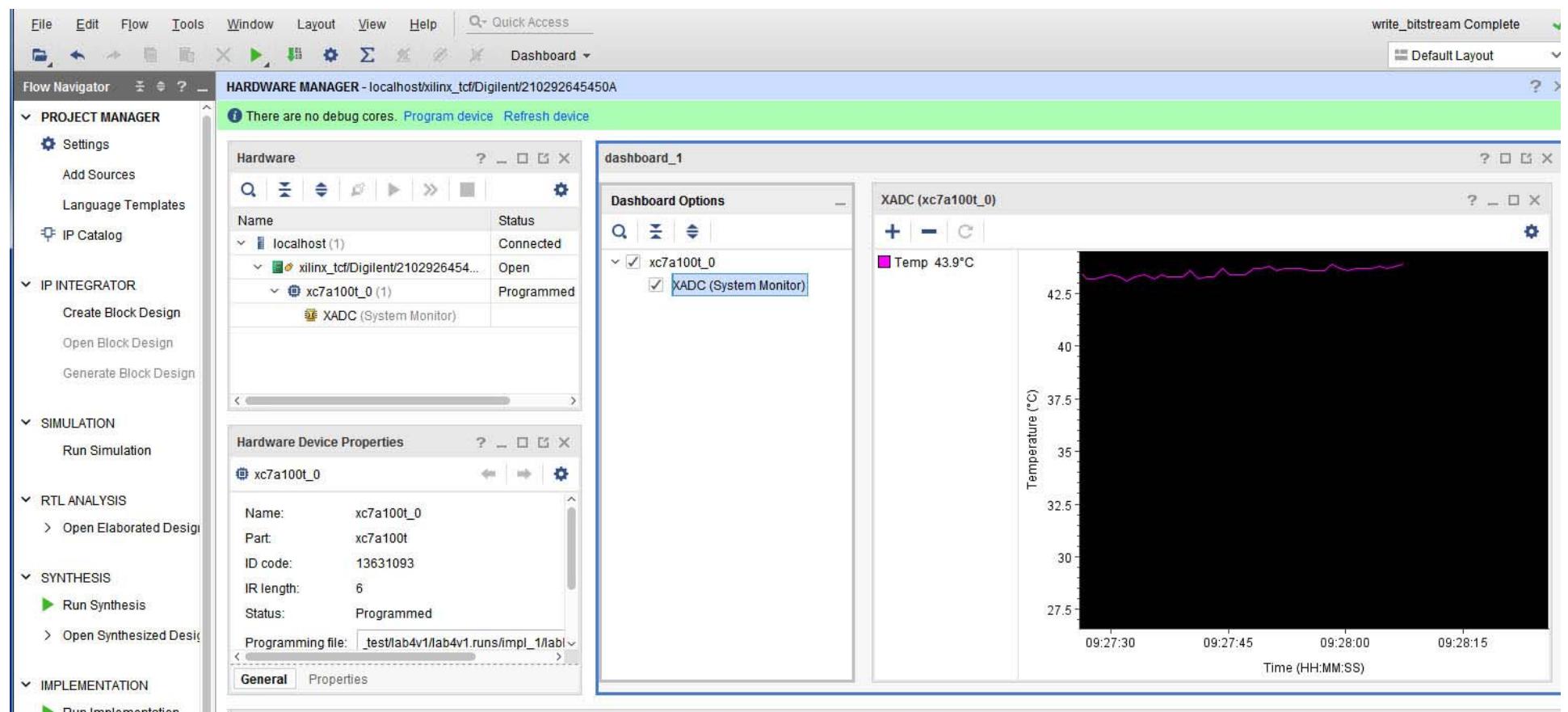
FPGA-based Emulator  
(courtesy of IKOS)

- Prototyping
  - Ensemble of gate arrays used to emulate a circuit to be manufactured
  - Get more/better/faster debugging done than with simulation
- Reconfigurable hardware
  - One hardware block used to implement more than one function
- Special-purpose computation engines
  - Hardware dedicated to solving one problem (or class of problems)
  - Accelerators attached to general-purpose computers (e.g., in a cell phone!)

# Summary

- FPGA provide a flexible platform for implementing digital computing
- A rich set of macros and I/Os supported (multipliers, block RAMS, ROMS, high-speed I/O)
- A wide range of applications from prototyping (to validate a design before ASIC mapping) to high-performance spatial computing
- Interconnects are a major bottleneck (physical design and locality are important considerations)

# Dashboard



# Loading Nexys4 Flash

1. Format a flash drive to have 1 fat32 partition
2. In vivado, click generate bitstream and afterwards do file->Export->Export\_Bitstream\_File to flash top-level directory
3. On the nexys 4, switch jumper JP1 to be on the USB/SD mode
4. Plug the usb stick into the nexys 4 while it's off and then power on. A yellow LED will flash while the bitstream is being loaded. When it's done, the green DONE led will turn on
5. You can remove the usb drive after your code is running

# Test Bench

```
module sample_tb;  
  
    // Inputs  
    logic clk;  
    logic data_in;  
  
    // Outputs  
    wire [7:0] data_out;  
  
    // Instantiate the Unit Under Test (UUT)  
    sample uut (  
        .clk(clk),  
        .data_in(data_in),  
        .data_out(data_out)  
    );  
  
    always #5 clk = !clk; // create a clock  
  
    initial begin  
        // Initialize Inputs  
        clk = 0;  
        data_in = 0;  
  
        // Wait 100 ns for global reset to finish  
        #100;  
  
        // Add stimulus here  
    end
```

```
module sample(  
    input clk,  
    input data_in,  
    output [7:0] data_out  
);  
  
    // Verilog  
  
endmodule
```

inputs must be initialized