Problem Set 2 ⟶ Issued September 12, 2005
Due September 21, 2005

Problem 1: A two-coil rotary system has self- and mutual inductances of this form:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_1 & M\cos\theta \\ M\cos\theta & L_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Assume values of the inductances to be $L_1 = L_2 = 100$mHy and $M = 80$mHy.

Note that this system has coupling coefficient less than one ($M^2 < L_1 L_2$). It is rotated at a constant speed of 3,600 RPM (60 Hz). Current in Coil 1 is held constant $i_1 = I_1 = 5A$. It has been on long enough to be in steady state operation.

Find and plot the answers to the following questions. Your plots should show two complete rotations of the system. Time origin is not important.

- What torque does this system produce?
- What is the voltage (ignore resistance) at the terminals of Coil 1?

under these three cases:

- Current in Coil 2 is held constant ($i_2 = I_2 = 5A$)
- *Flux* linked by coil 2 is constrained to be zero ($\lambda_2 = 0$). Note: this is what would be approximately true were the terminals of Coil 2 to be directly shorted and the resistance of Coil 2 is small).
- Coil 2 is shorted through a resistor so that the total resistance (including the coil itself) is 20Ω.

Problem 2: This problem relates to magnetic lifting systems such as are used in so-called "attractive levitation' schemes for high speed ground transport. See Figure 1. An example of a transportation system that uses such a scheme (Not quite this embodiment of it!) is the German built Transrapid system. The stationary element is the fixed 'rail'. Mounted on and supporting the car is a ferromagnetic yoke and a current carrying coil. When the coil is excited it produces magnetic fields that attract the yoke to the rail. A section of the system is shown in a little more detail in Figure 2.

This would be only one of several magnets supporting a 'car'. Assume that this magnet is supporting about 5 metric tons, which implies about 50 kN of required lift force. Dimensions and othe data are as follows:

Support Force Required	F	50,000 N
Magnet Length	L	0.5 m
Pole width	w	0.1 m
Relative Motion Gap	g	0.01 m

Note that the pole width is w and we assume the coil dimensions are $w \times w$. The rail width is such that, when the yoke is centered, each side has an overlap of $\frac{1}{2}w$.
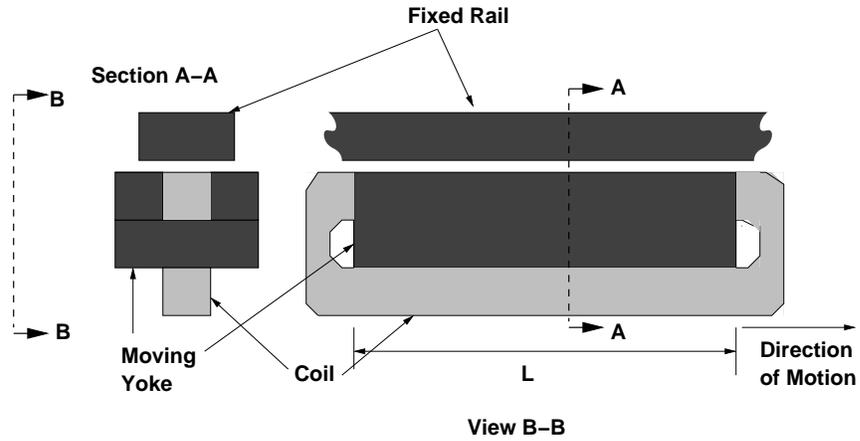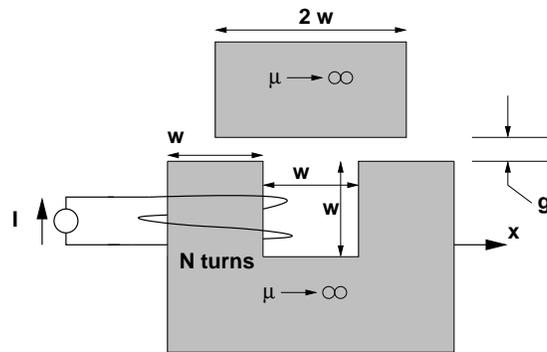
Figure 1: Suspension Magnet

Figure 2: Suspension Magnet Details

1. With the yoke centered and providing the nominal support force, what is the required number of ampere-turns in the exciting coil?

2. How does this required current change with yoke lateral position x? Calculate and plot the required current as a function of position, ranging from -w/4 to +w/4.

3. If the car moves laterally (direction x), this system provides a restoring force. Find that lateral force as a function of x and exciting current NI.

4. Now combine these two parts and, assuming that current is controlled to maintain a 1 cm relative motion gap, find and sketch (or have MATLAB plot) restoring force as a function of lateral position.

NOTE: You may find a reading of Chapter 2 of the notes, on magnetic circuits, will make some of these calculations easier.