

#### SAFE REINFORCEMENT LEARNING



Electrical and Computer Engineering

Jaime Fernández Fisac



#### A Little About Me

I'm an Assistant Professor at Princeton

Grew up and went to college in Spain

Got interested in robot safety while working with drones

PhD at UC Berkeley → Research Scientist at Waymo

same lab as Vicenç!



#### The Safe Robotics Lab



Goal: enable robots to operate safely around people

Theory + algorithms for active safety under uncertainty

Focus on changing & interactive environments

Control theory + Al + game theory + cognitive science





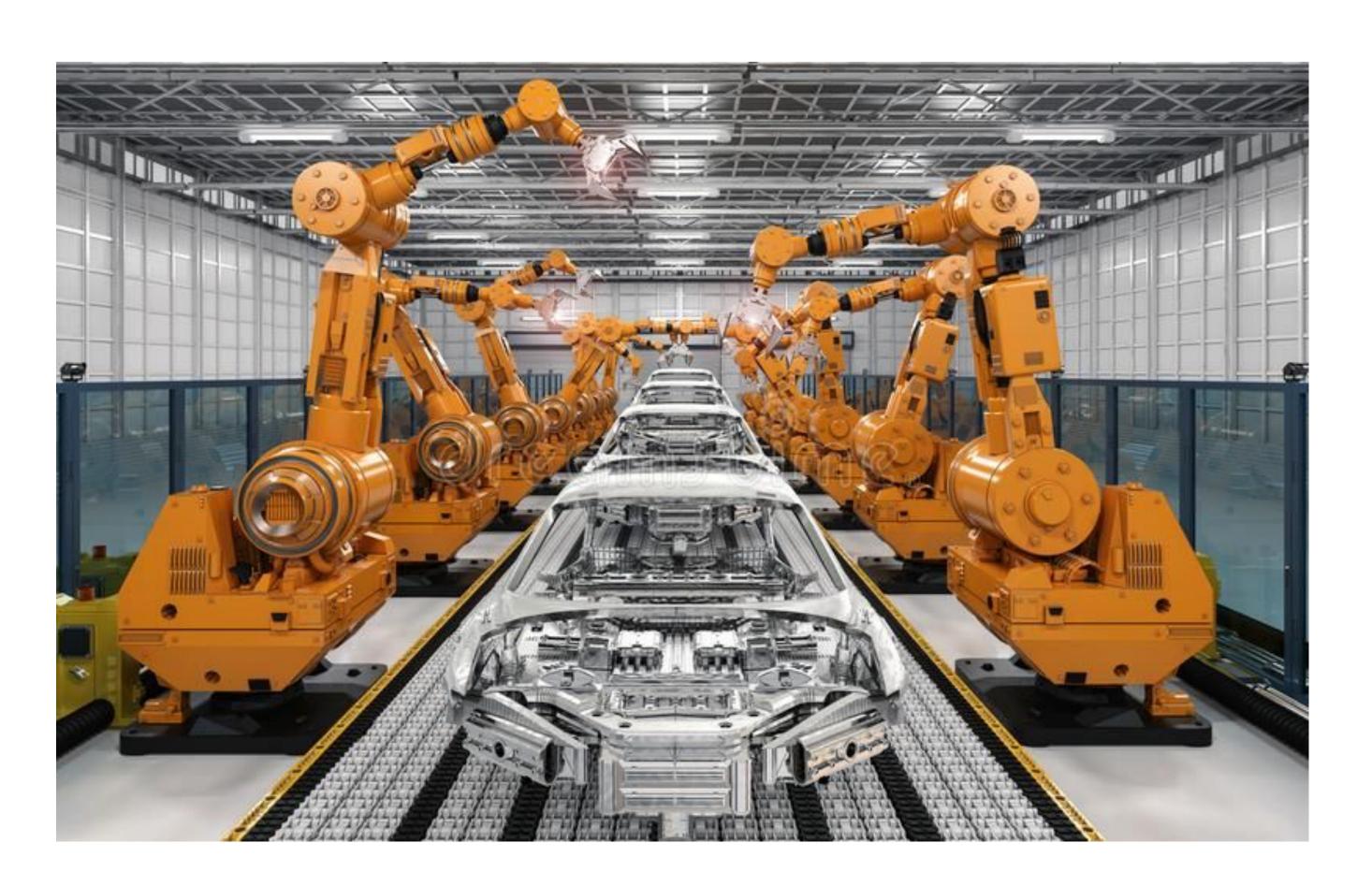






#### Why Have Robots Learn?

Robotics has existed for decades as a field primarily focused on industrial processes.



Repetitive (though complex) tasks

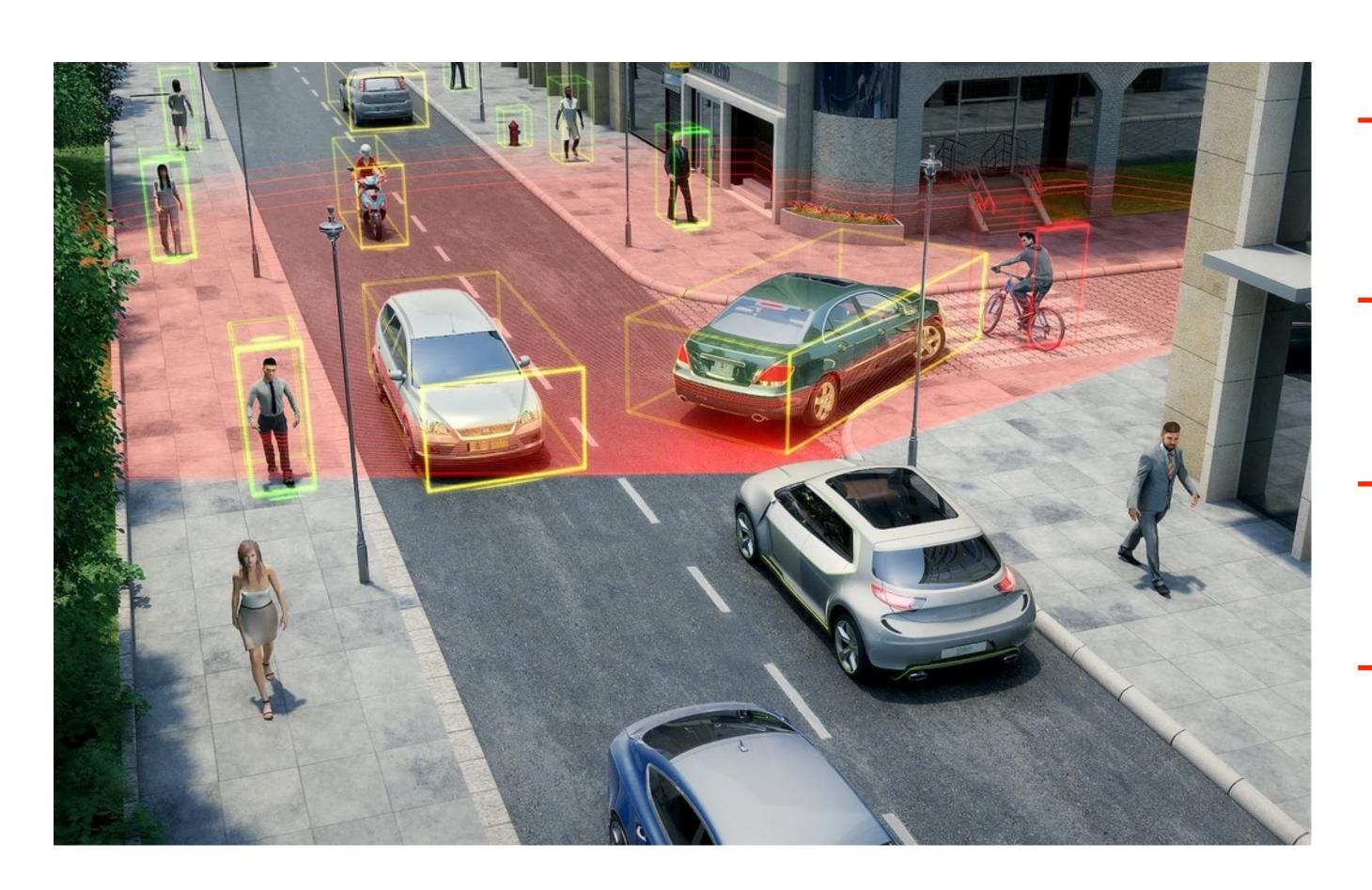
Structured environments

Well-understood dynamics

Isolated from humans

#### Why Have Robots Learn?

Advances in sensing, decision-making and control → new opportunities extend beyond factories



Repetitive (though complex) tasks

Structured environments

Well understood dynamics

Isolated from humans

#### Mitigating Uncertainty Through Adaptation

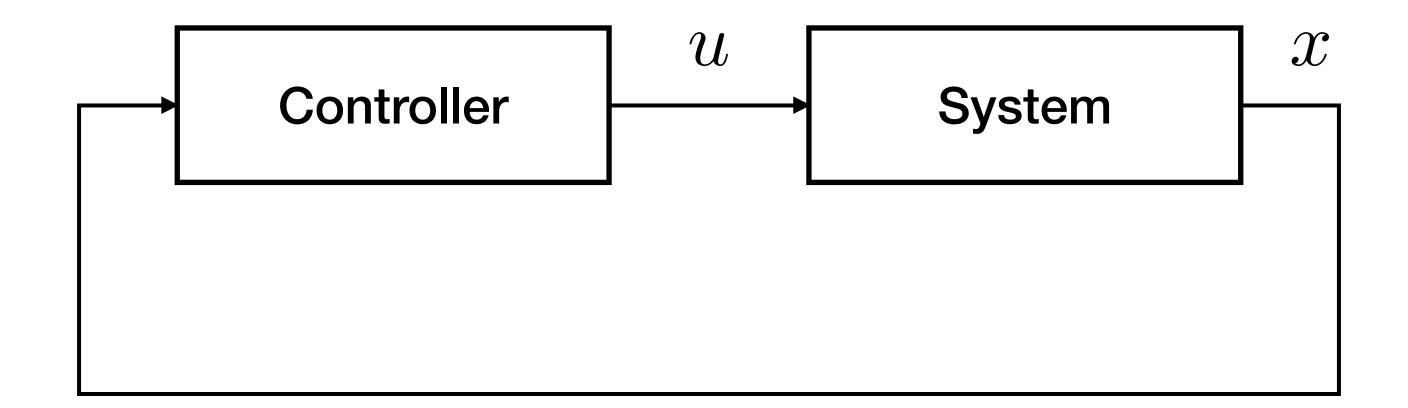
Design a fixed control policy that works in all conditions the system may find.

Adjust the variable control policy to the specific conditions encountered.

Exploit new information as it comes in.

Maintain consistent performance in the presence of changing conditions.

## Optimal Control (1950s–60s)

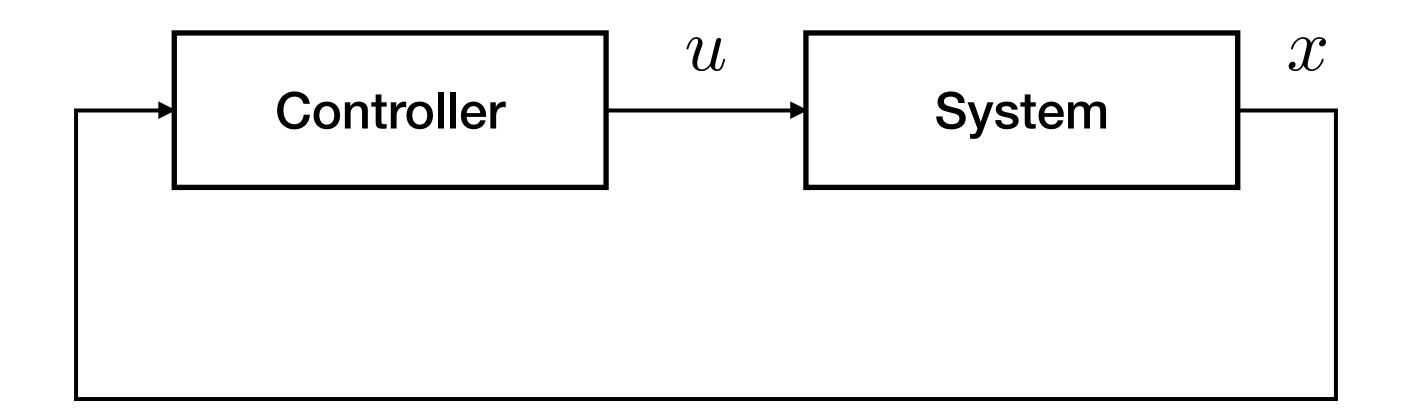


$$\max_{\boldsymbol{\theta}} \mathbb{E} J(\mathbf{x}_{x,t}^{\boldsymbol{\pi}_{\boldsymbol{\theta}},\mathbf{d}})$$

Performance Objective

Can we adjust the control policy  $\pi_{\theta}$  over time based on the robot's experience?

#### Adaptive Control (1960s-70s)

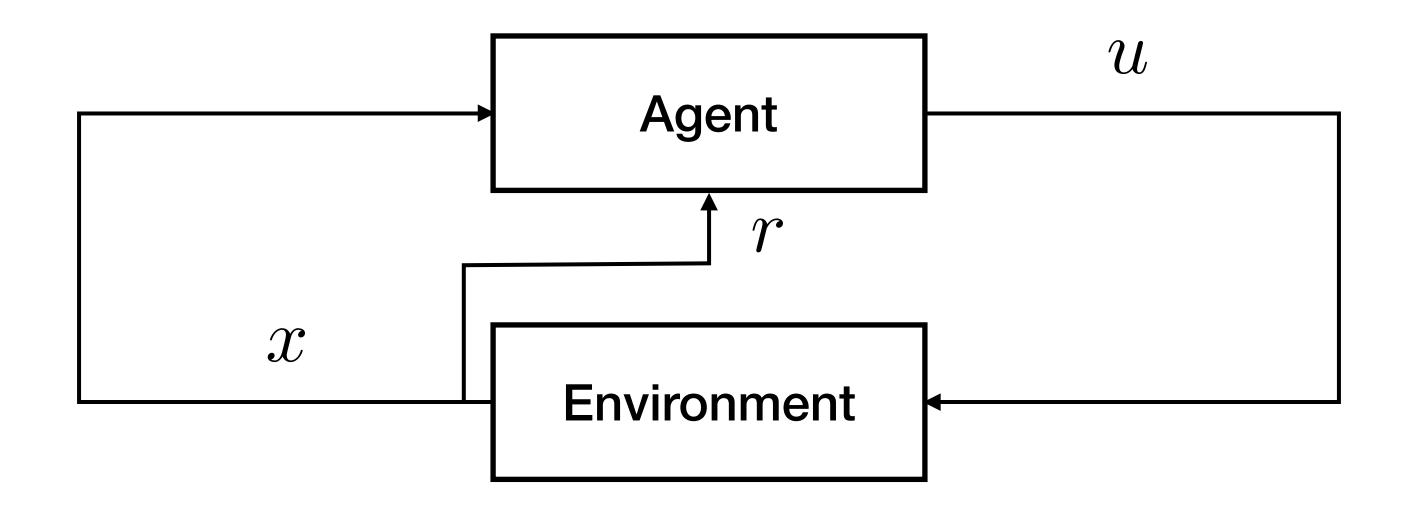


$$\max_{\boldsymbol{\theta}} \mathbb{E} J(\mathbf{x}_{x,t}^{\boldsymbol{\pi}_{\boldsymbol{\theta}},\mathbf{d}})$$

Performance Objective

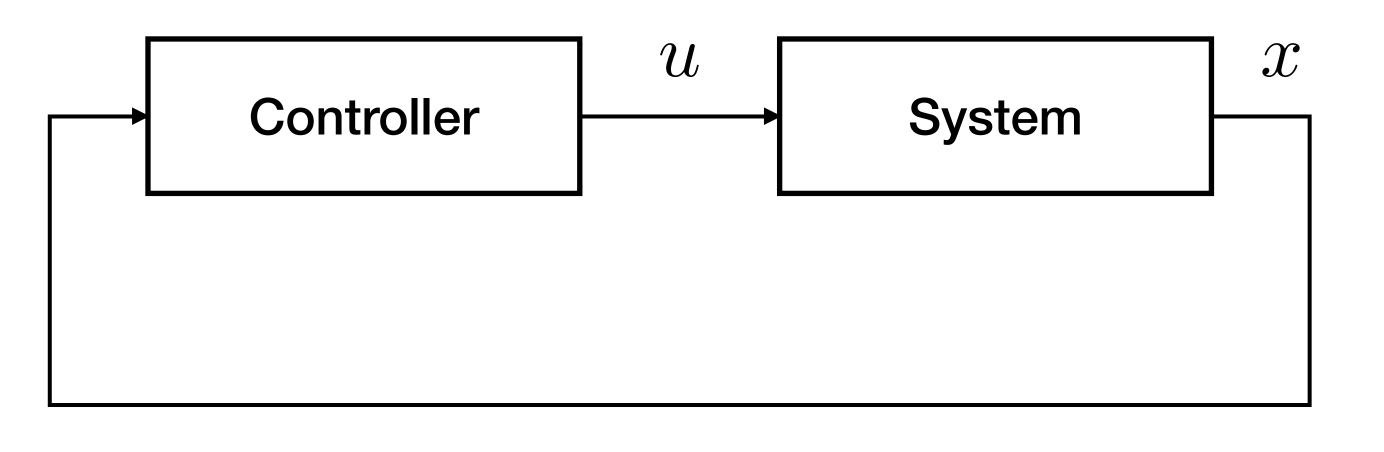
Can we adjust the control policy  $\pi_{\theta}$  over time based on the robot's experience?

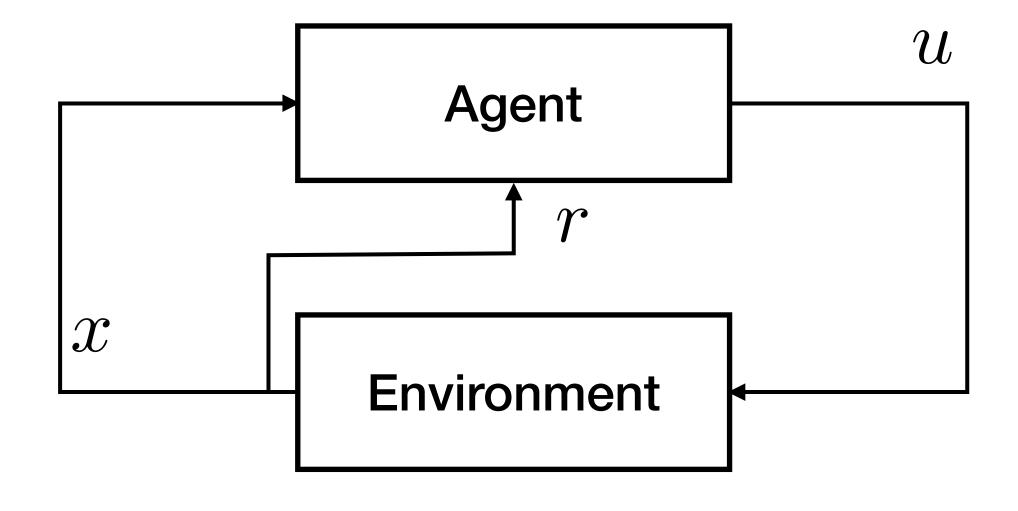
#### Reinforcement Learning





#### Learning-Based Control





#### Adaptive Control

Continual operation (single shot)

Typical objectives: stability, tracking error

World model: dynamical system

#### Reinforcement Learning

Episodic operation (many trials)

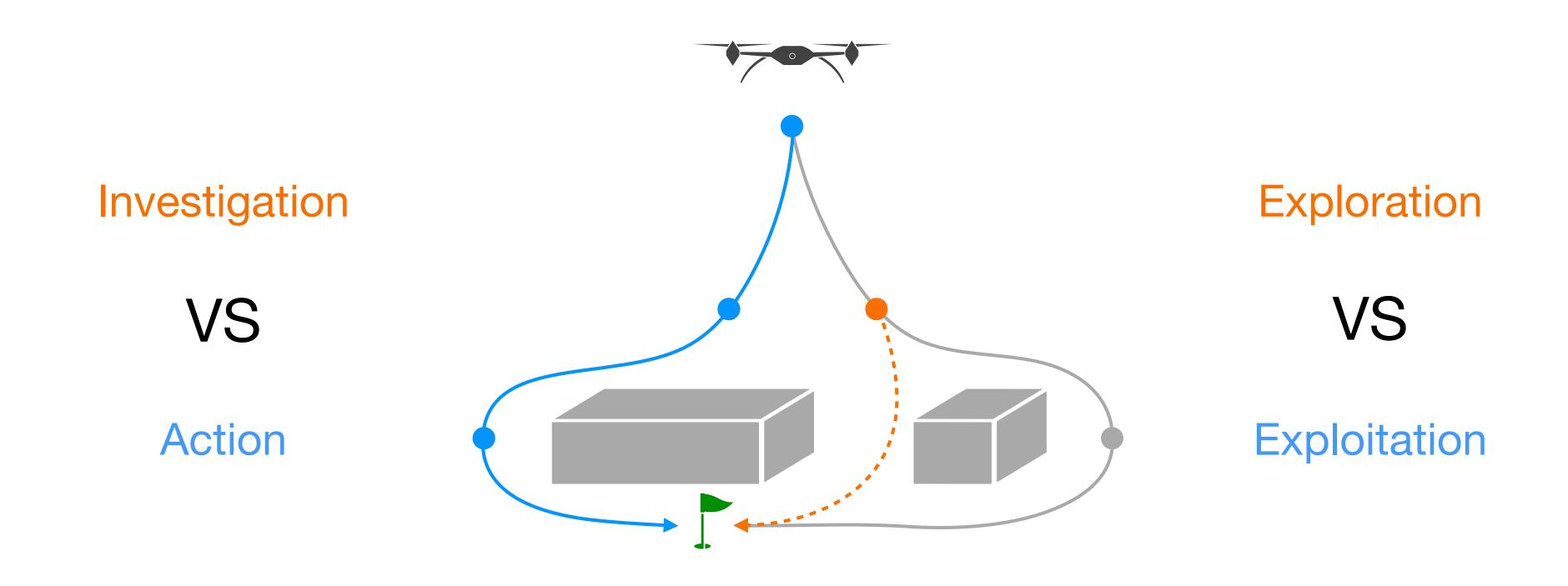
Typical objectives: accrued reward

World model: Markov chain (MDP)

#### The Dual Role of Control

#### Adaptive Control

#### Reinforcement Learning

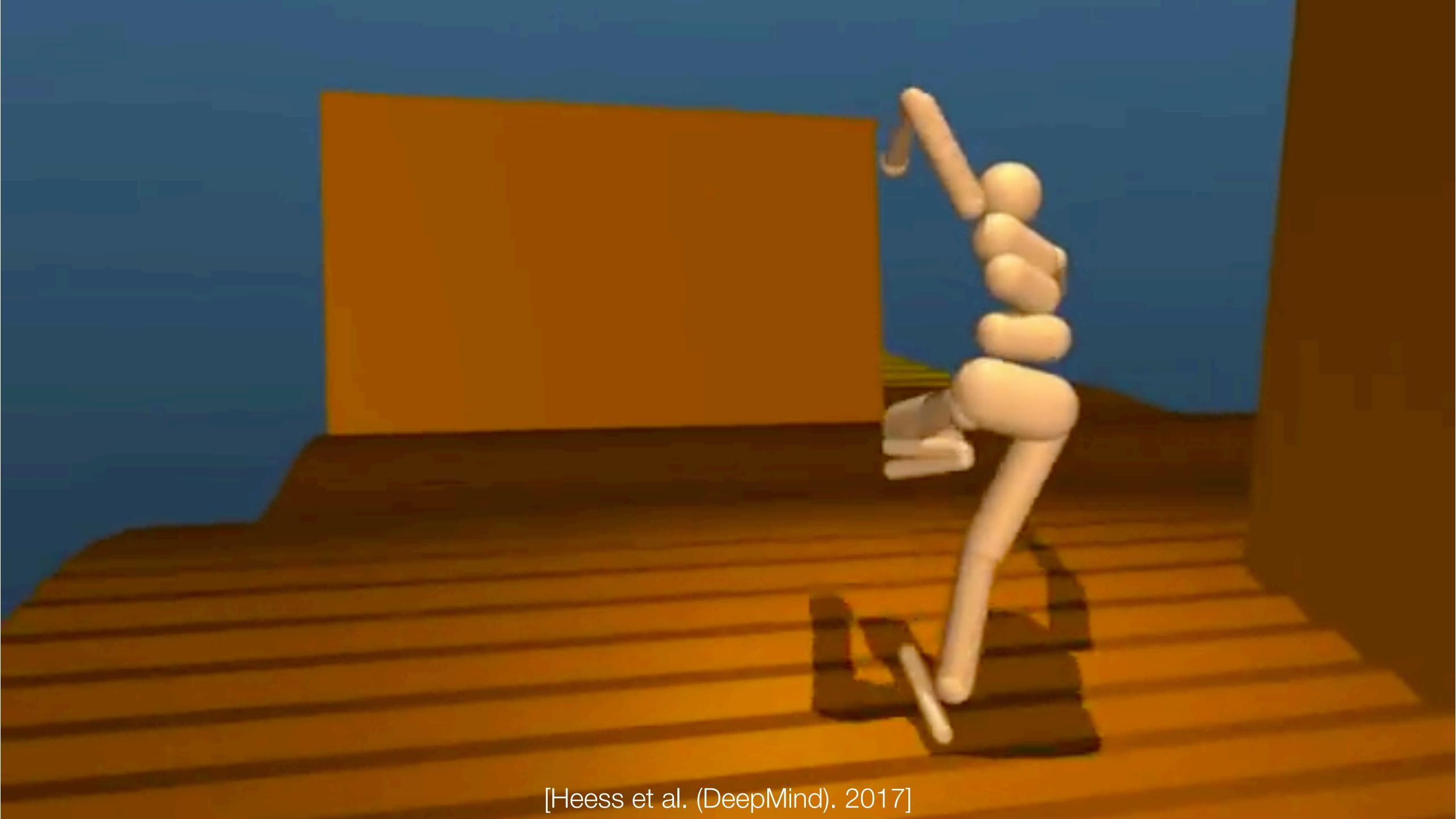


Note: neither of these functions can succeed if we fail to preserve safety.



[Mnih et al (DeepMind). Nature 2015]















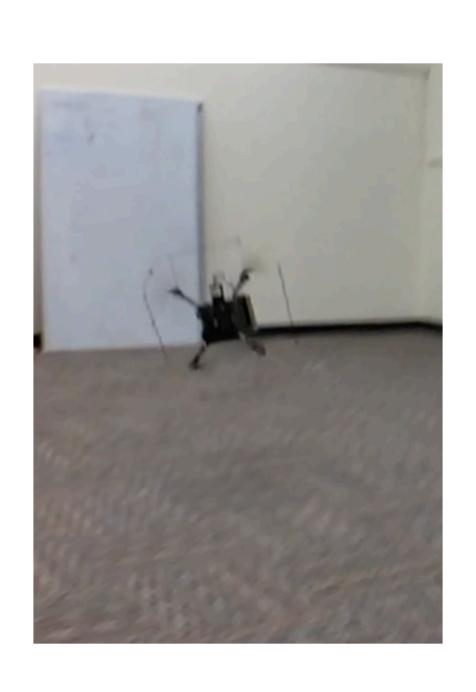
#### Safety-Critical System

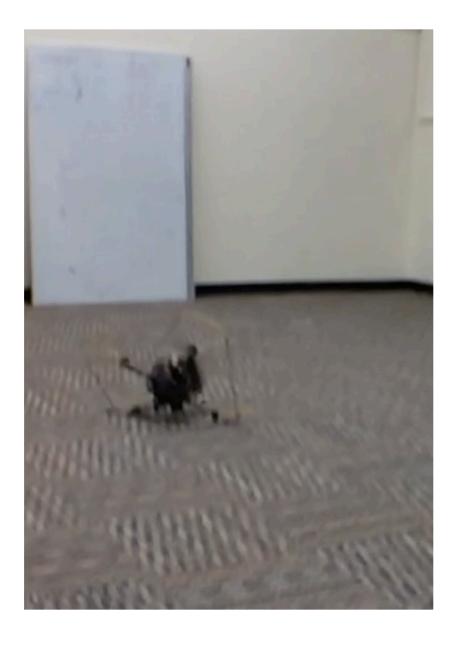
Any system in which there exist potential outcomes or failure modes that are deemed unacceptable, typically due to injury, loss of life, or severe material damage.

Failure Set: all system states that are unacceptable (we never want to reach them).

Unsafe Set: states from which it is not possible to avoid entering a failure state in the future.







Safe Unsafe Failure

## Is This a Safety-Critical System?

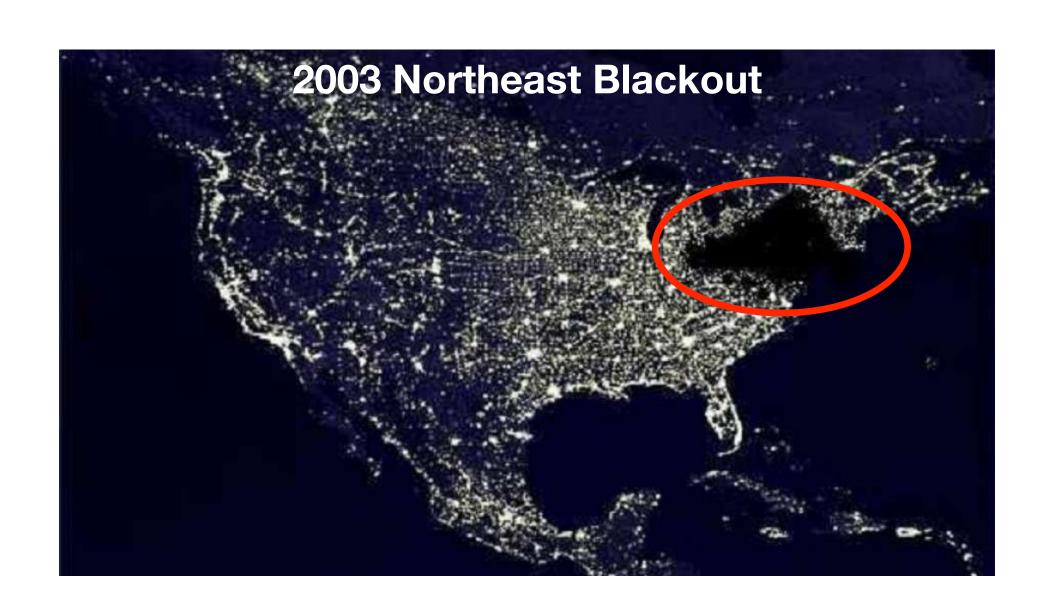


## Is This a Safety-Critical System?





## Critical Failures of Cyber-Physical Systems

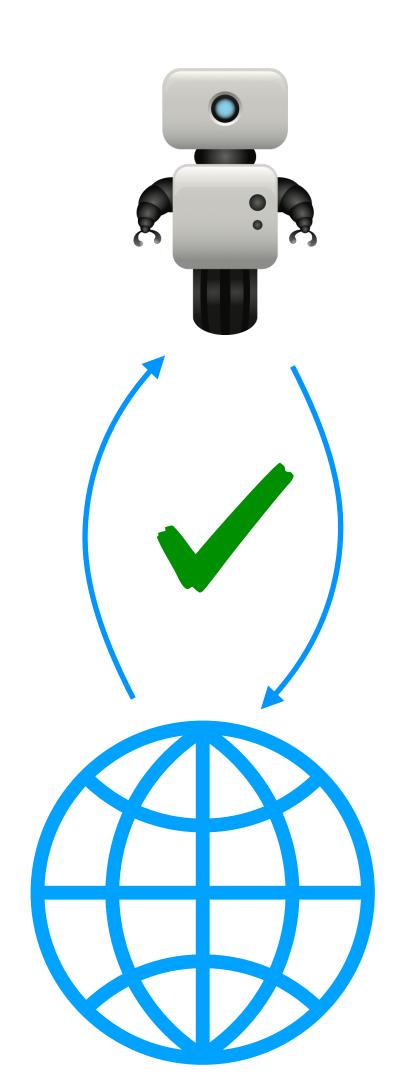


+50M people without power for 2 days



\$1T loss in market value

#### Autonomous System Safety Analysis

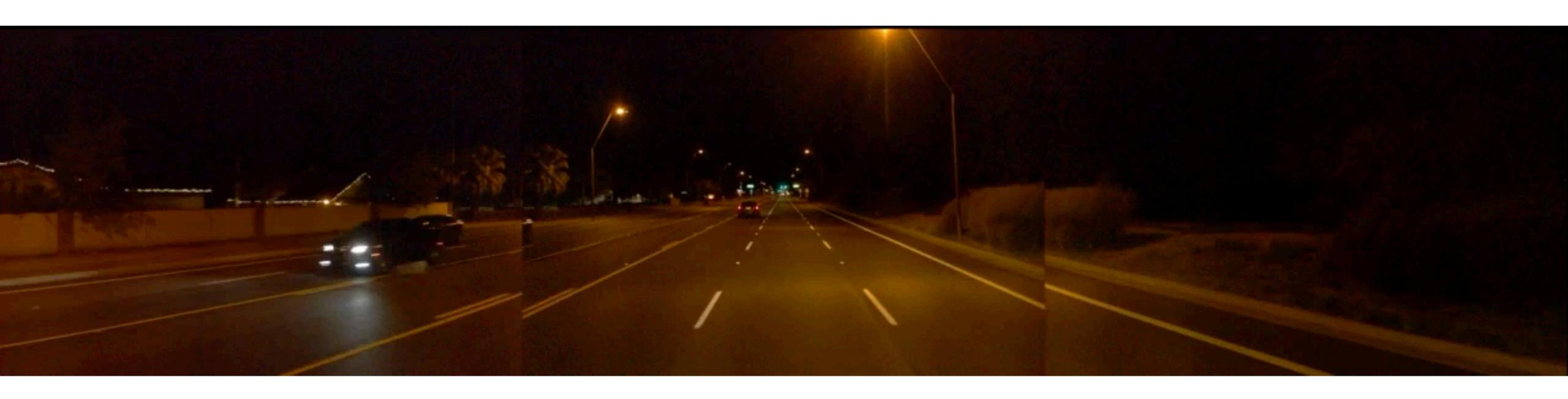


#### The Long Tail

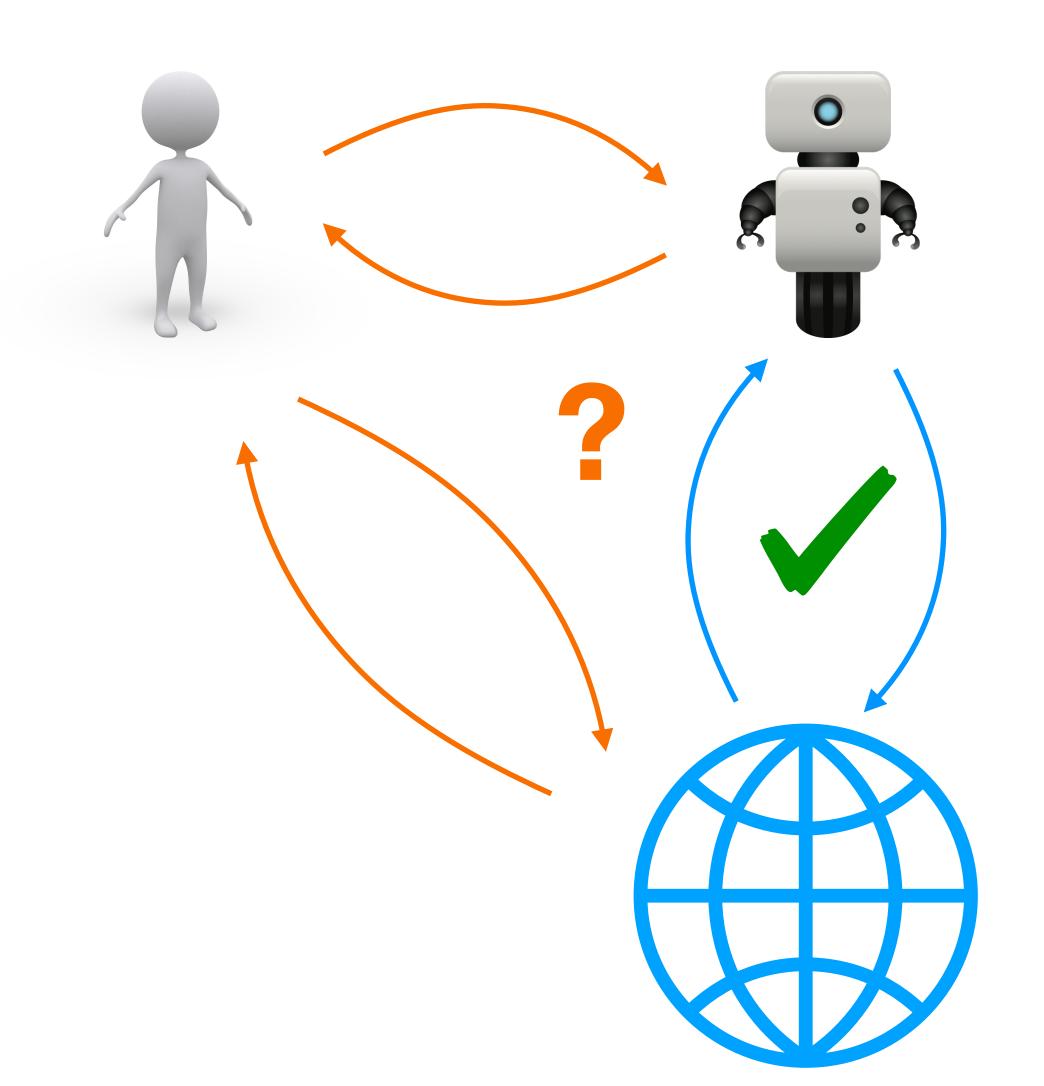
# 'Disgruntled' former Waymo self-driving car operator arrested for causing car crash

The 31-year-old swerved his car in front of the autonomous vehicle and then slammed on his brakes

By Andrew J. Hawkins | @andyjayhawk | Feb 13, 2020, 5:27pm EST

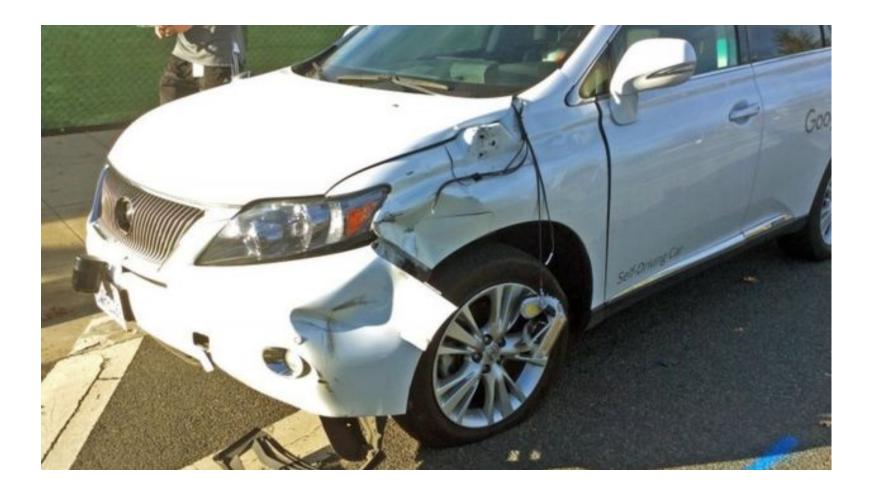


## Safety is not just up to the Autonomous System



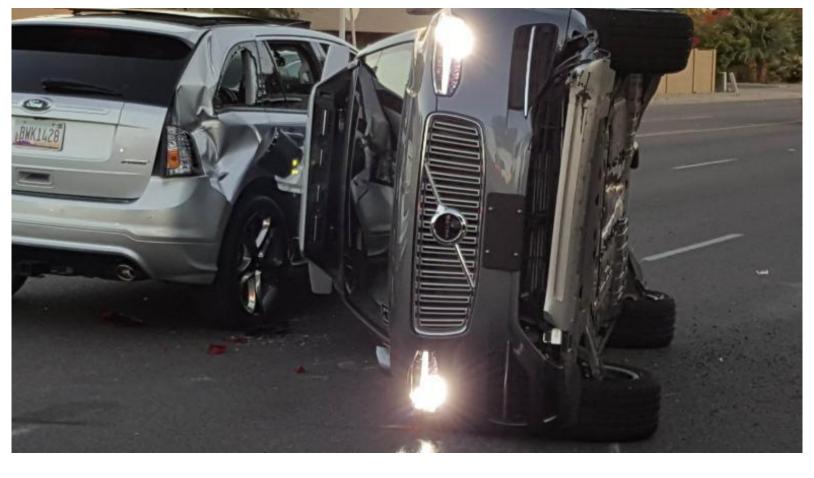
# Safety is compromised when humans don't behave the way Al systems assume, or vice versa.

Google, 2016



Turned into bus that did not stop

Uber, 2017



Hit by driver who did not yield

Tesla, 2018



Driver did not take over "fast enough"



#### Towards Intelligible if-then Safety Guarantees

Reliable "hard" guarantees on *under what conditions* the system can/cannot fail. Trust

Transparent to both system designers and the public. Social contract

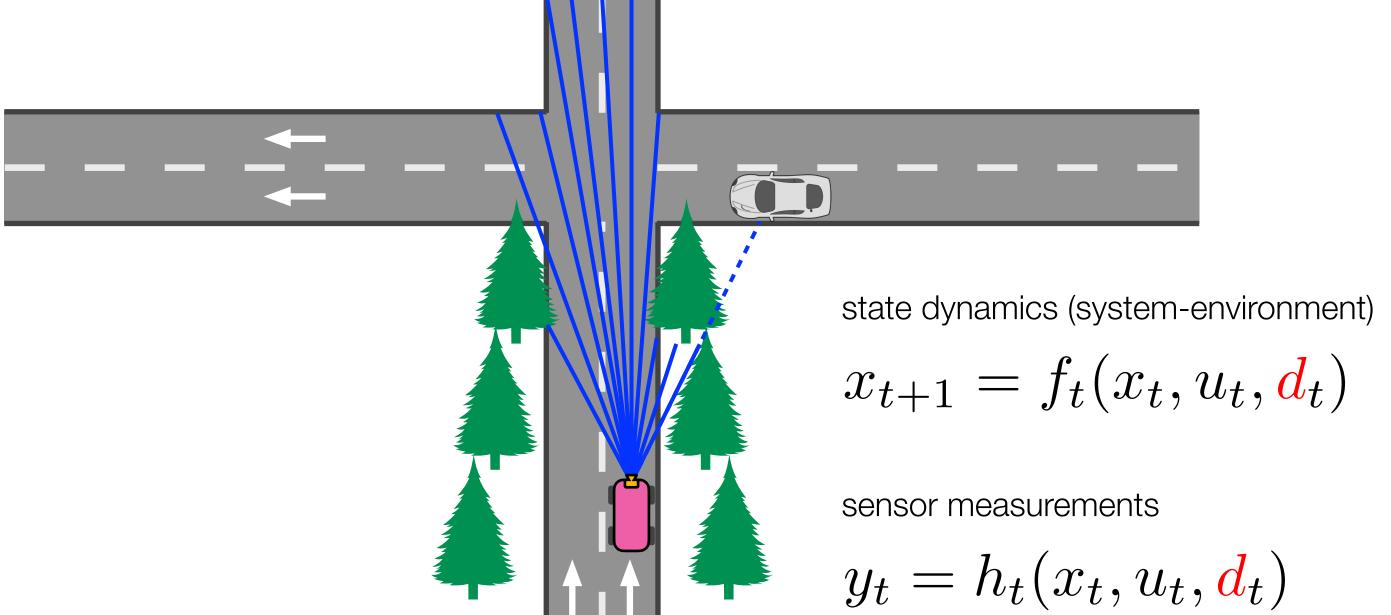
Checkable and enforceable at runtime by autonomy stack. Active safety

Traceable retrospectively/counterfactually in the event of a failure/near miss.

Accountability

## Operational Design Domain (ODD) Safety Theory

ODD: set of conditions under which the system must operate correctly and safely.

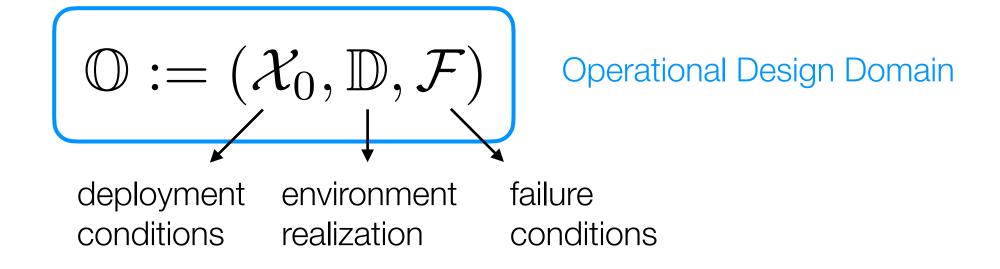


state dynamics (system-environment)

$$x_{t+1} = f_t(x_t, u_t, \mathbf{d}_t)$$

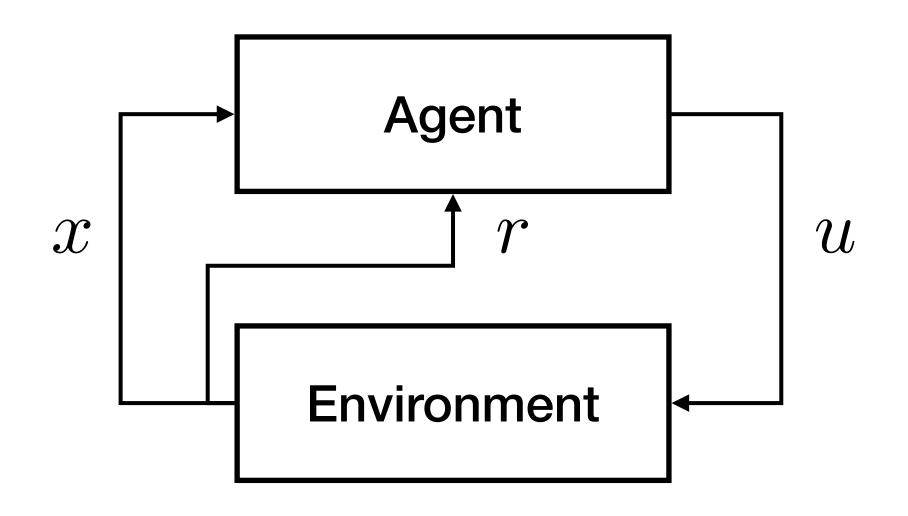
uncertainty (environment) realization (incl. behavior of other agents)

$$\mathbf{d} := (d_0, d_1, \dots)$$



ODD Theorems: let the environment realization satisfy  $d \in \mathbb{D}$ , then the system's operation from any deployment condition  $x_0 \in \mathcal{X}_0$  satisfies  $x_t \notin \mathcal{F}$  for all time  $t \geq 0$ .

#### Reinforcement Learning and Constraints



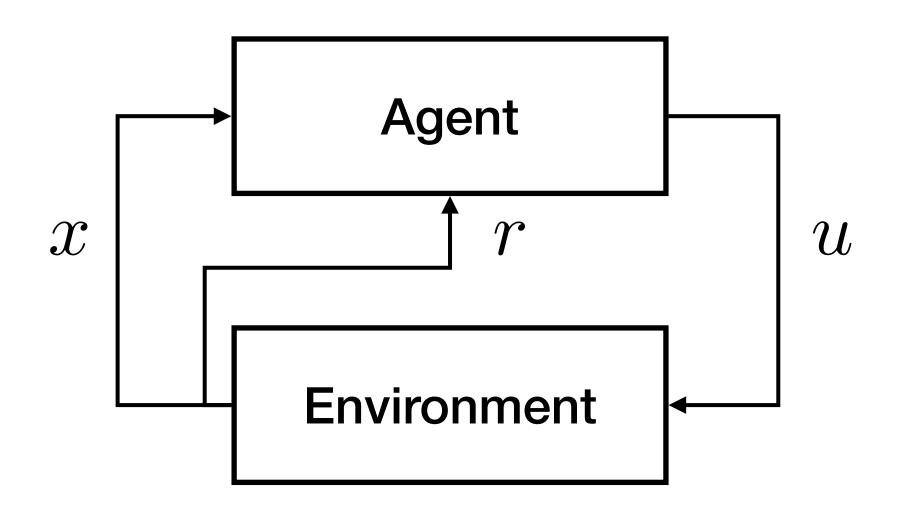
$$\max_{\pi} \mathbb{E} \sum_{t=0}^{\infty} \gamma^{t} r(\mathbf{x}(t))$$

s.t. 
$$l(\mathbf{x}(t)) \ge 0 \quad \forall t \ge 0$$

Additive/Average Performance

**Property Satisfaction** 

#### Reinforcement Learning and Constraints



$$\max_{\pi} \mathbb{E} \sum_{t=0}^{\infty} \gamma^{t} r(\mathbf{x}(t))$$

s.t. 
$$\inf_{t \ge 0} l(\mathbf{x}(t)) \ge 0$$

Additive/Average Performance

**Property Satisfaction** 

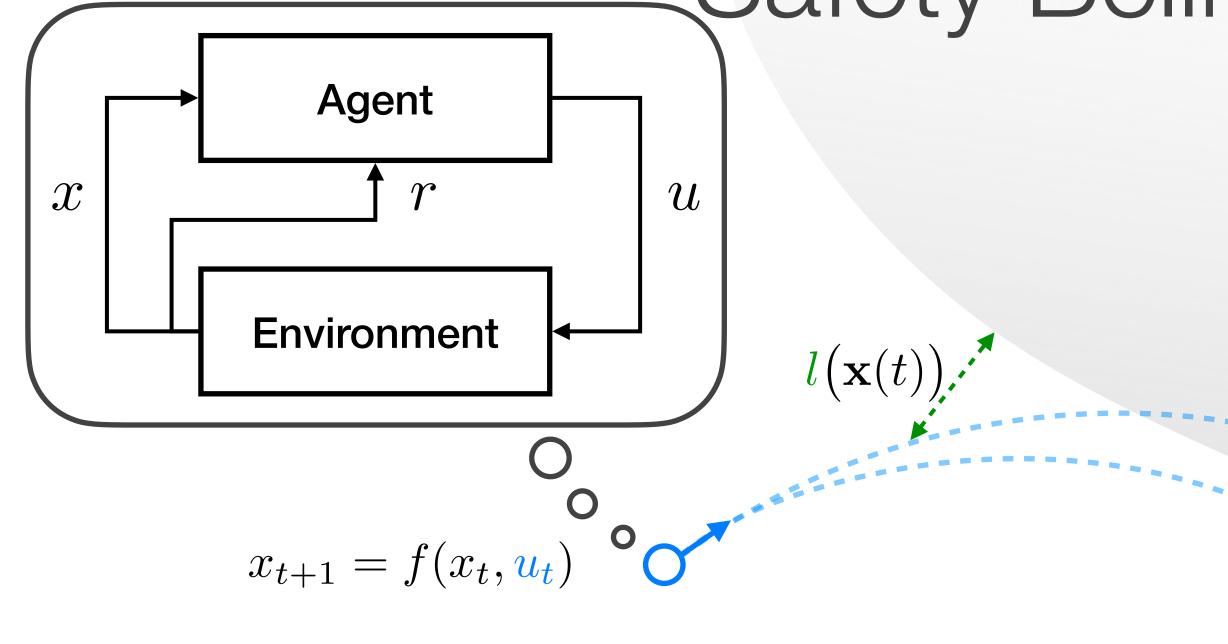
#### The Safe Learning Problem

# CAN ROBOTIC SYSTEMS **LEARN** BY EXPERIENCE WHILE ALWAYS SATISFYING **SAFETY** CONSTRAINTS?

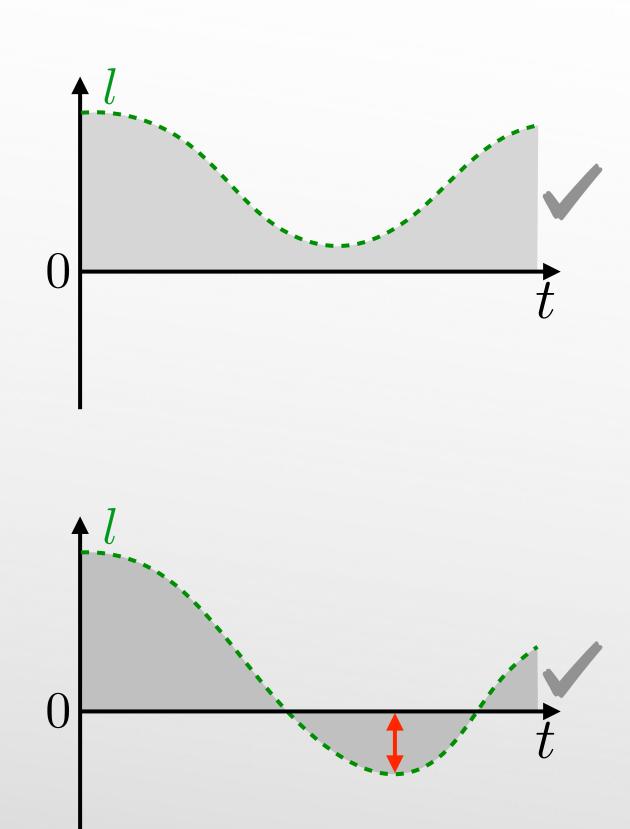
Can the learned control policy maintain safety once it is deployed on the robot?

Can the learning process maintain safety while the robot is learning its control policy?

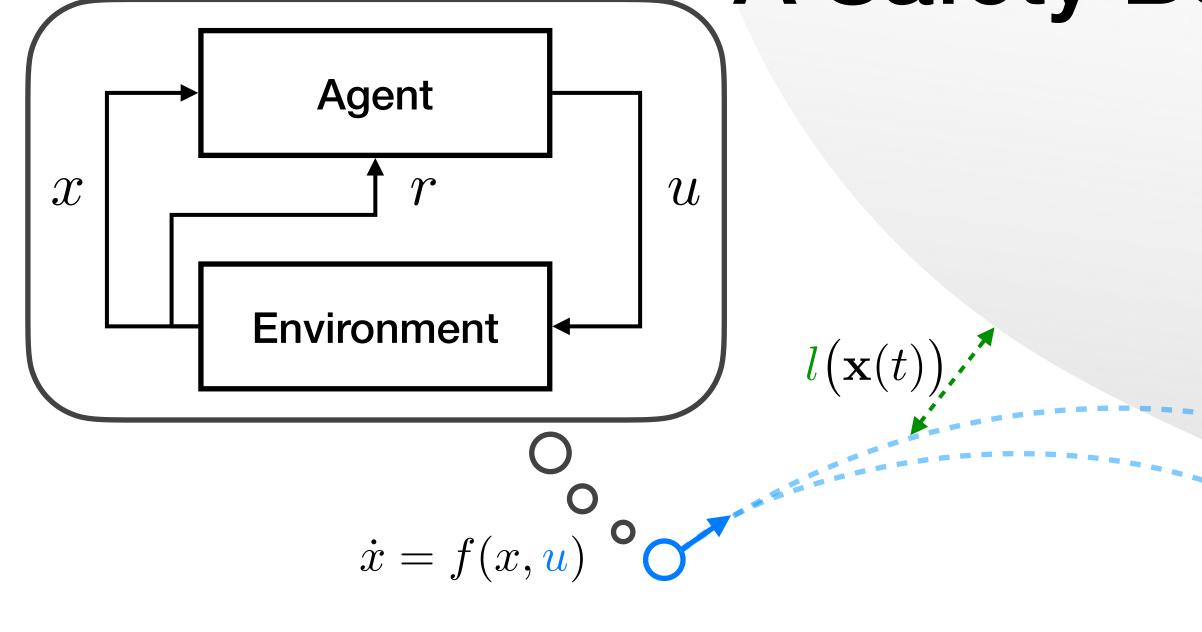
Safety Bellman Equation



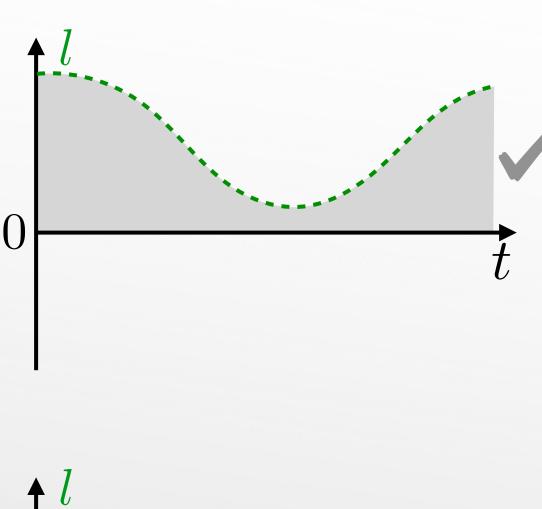
 $\exists \mathbf{u} : \forall t, \mathbf{x}(t) \notin \mathcal{F}$ 

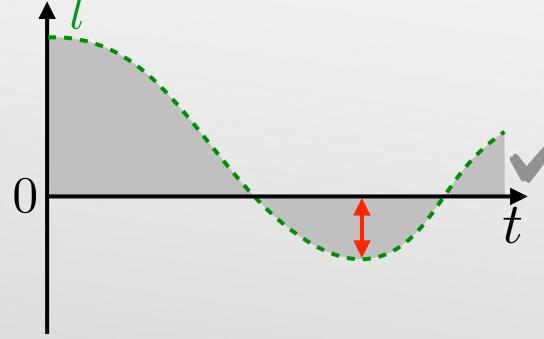


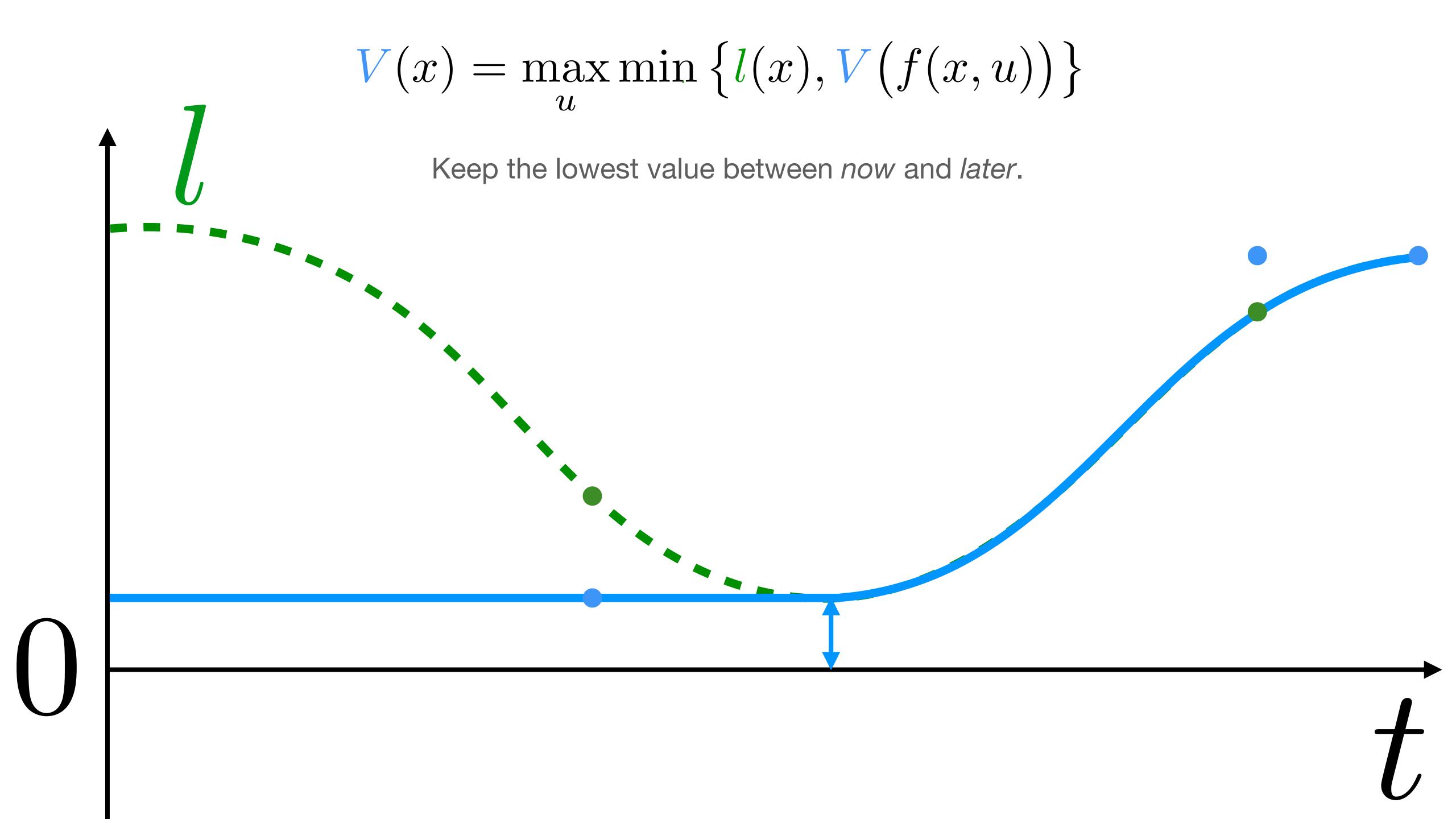
## A Safety Bellman Backup

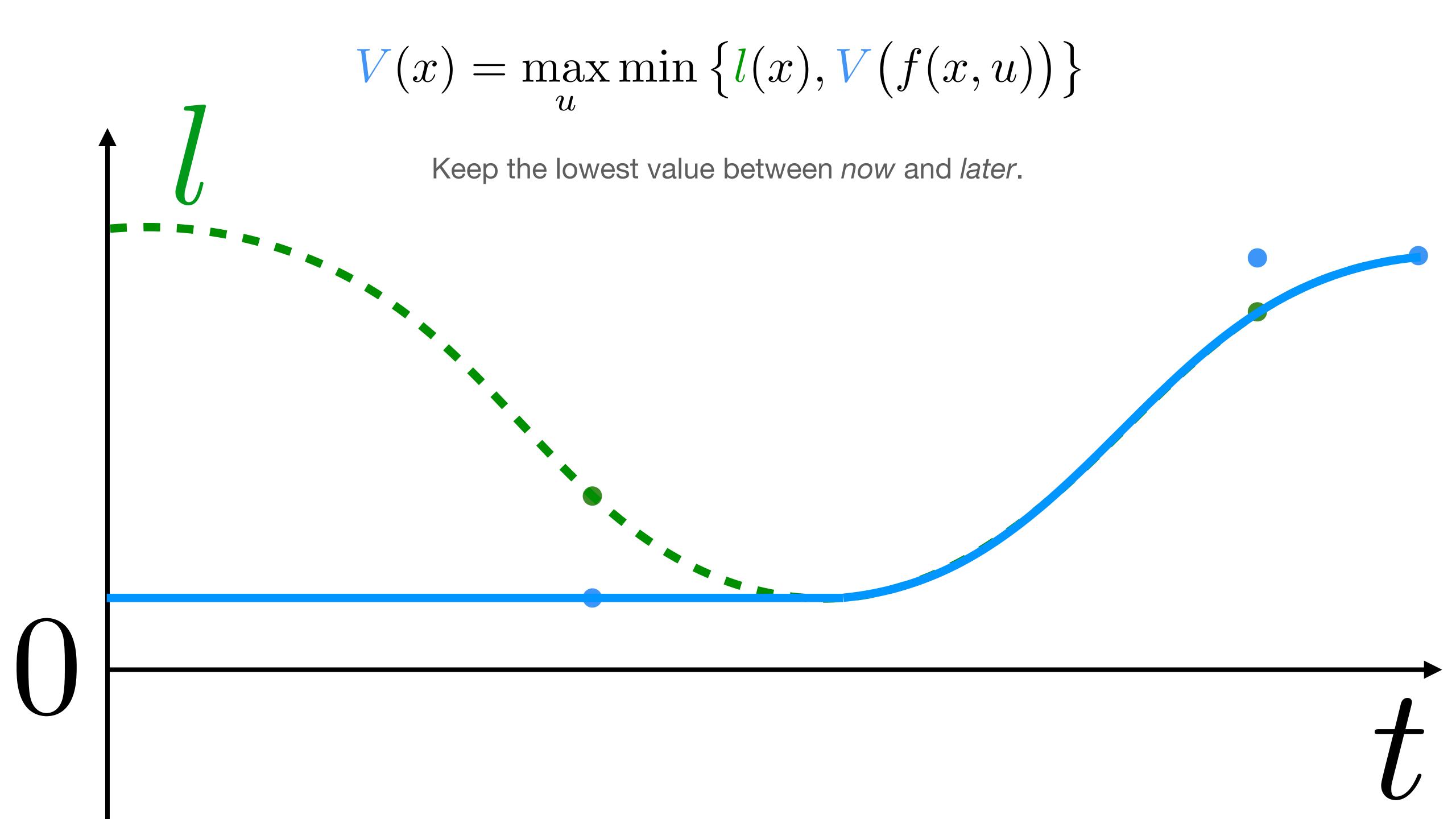


 $\exists \mathbf{u} : \forall t, \mathbf{x}(t) \not\in \mathcal{F}$ 

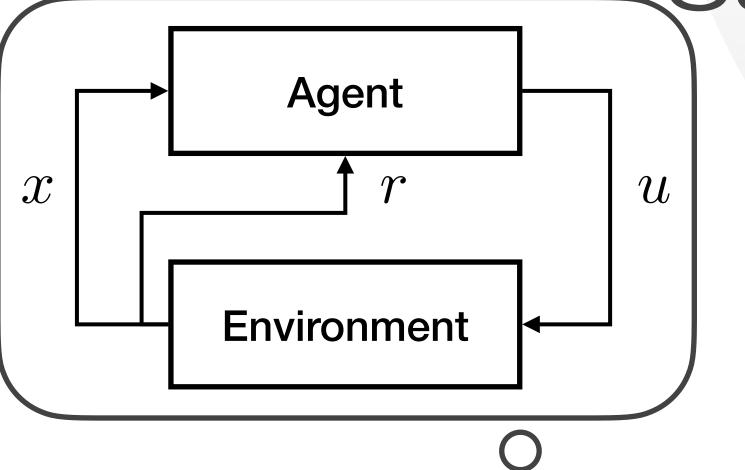








# Safety Bellman Equation



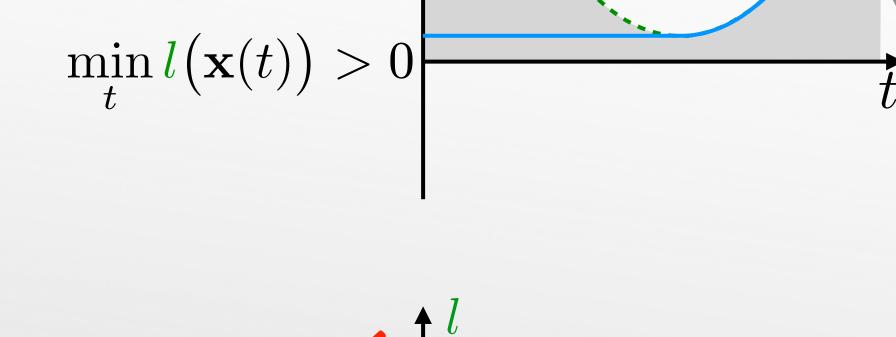
$$l(\mathbf{x}(t))$$

$$x_{t+1} = f(x_t, \mathbf{u_t})$$

$$\exists \mathbf{u} : \forall t, \mathbf{x}(t) \not\in \mathcal{F}$$

$$J(\mathbf{x}) = \sum_{t=0}^{\infty} \gamma^t r(\mathbf{x}(t))$$

$$J(\mathbf{x}) = \inf_{t \ge 0} l(\mathbf{x}(t))$$



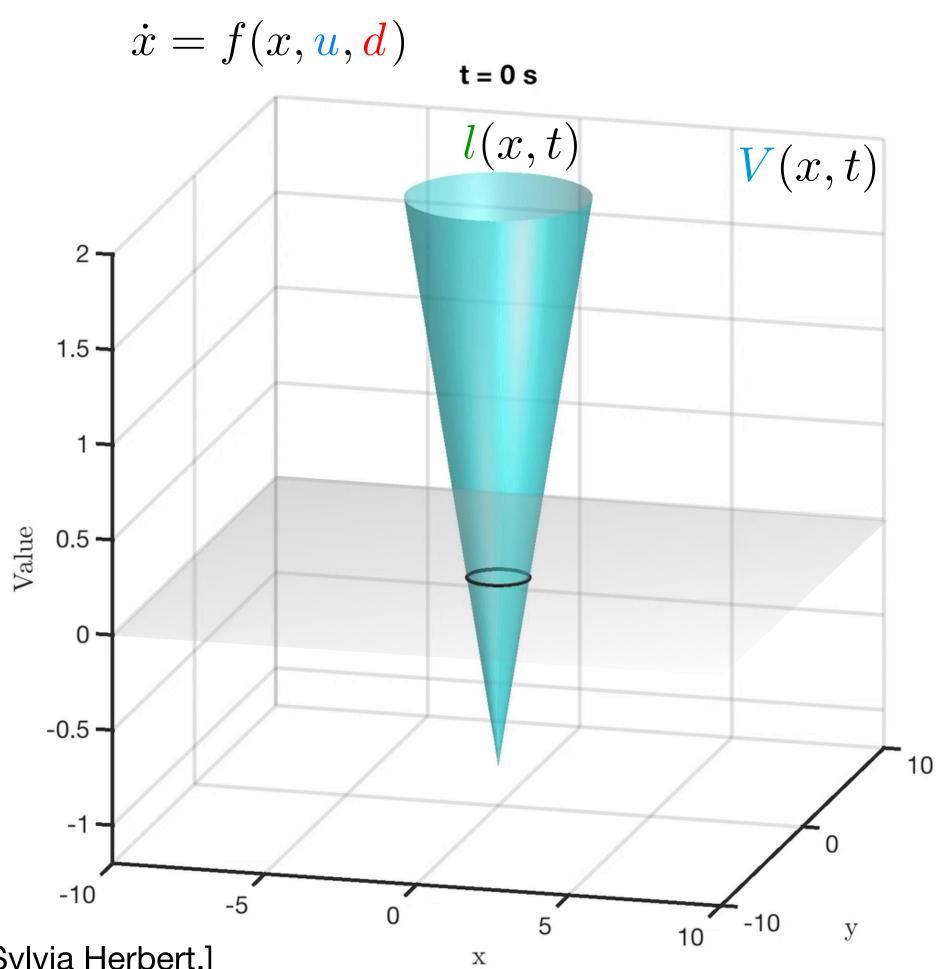
$$\min_{t} l(\mathbf{x}(t)) < 0$$

$$V(x) = \max_{u} \left[ r(x, u) + \frac{\gamma V}{\gamma V} (f(x, u)) \right]$$

$$V(x) = \max_{u} \left[ \min \left\{ l(x), V(f(x, u)) \right\} \right]$$

## Hamilton-Jacobi Safety Analysis

Safe set: states from which the controller can keep the system from entering any failure state in the future  $\iff V(x) \ge 0$ 



Continuous-Time Dynamic Programming

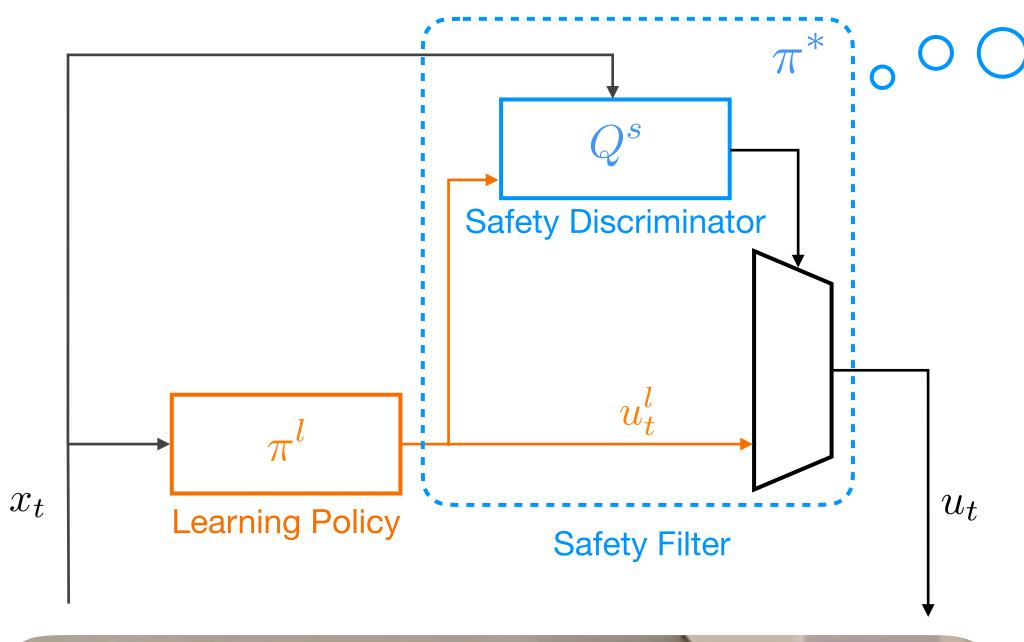
$$0 = \min \left\{ l(x, t) - V(x, t), \frac{\partial V(x, t)}{\partial t} + \max_{u \in \mathcal{U}} \min_{\mathbf{d} \in \mathcal{D}} \nabla_x V(x, t)^{\top} f(x, u, \mathbf{d}) \right\}$$

Discrete-Time Dynamic Programming

$$V(x,t) = \max_{u \in \mathcal{U}} \min_{\mathbf{d} \in \mathcal{D}} \min \left\{ l(x,t), V(x+f(x,u,\mathbf{d})\Delta t, t+\Delta t) \right\}$$

[Fisac, Chen, Tomlin, and Sastry. HSCC 2015]

## Learning with Safety Filters



HJI safety analysis

Control barrier function

Modelpredictive shielding

Safety critic

#### **Safety Discriminator**

 $Q^s(x,u)>0 \implies$  known  $\pi^s$  maintains safety after taking action u from x .

#### Safety Filter

 $\pi^*(x,u)$  modifies the proposed action to avoid future safety violations.

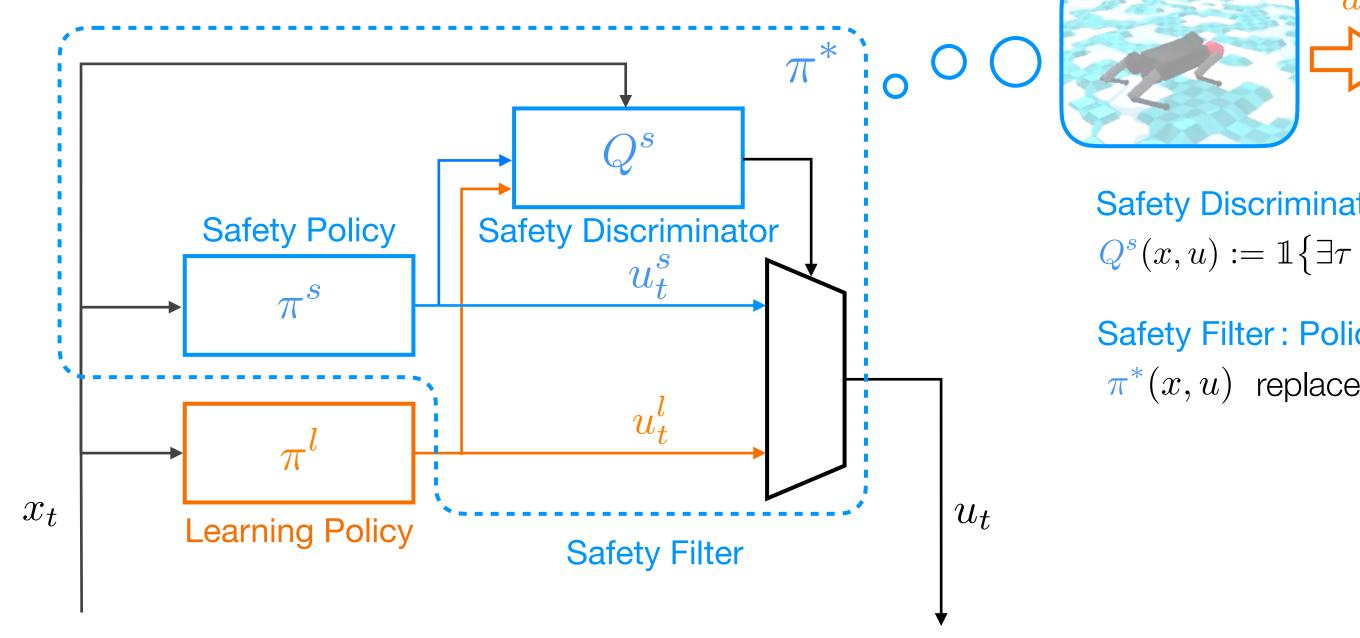
**Theorem:** any safety filter  $\pi^*$  that enforces

$$Q^{s}\left(x, \pi^{*}\left(x, u^{l}\right)\right) > 0 \tag{1}$$

maintains safety for all time from any initial state  $x_0$  that is safe under  $\pi^s$ , i.e.  $Q^s(x_0, \pi^s(x_0)) > 0$ .

Moreover, (1) is recursively enforceable, since  $\pi^*$  can always choose  $\pi^s(x)$ .

## Model-Predictive Shielding





Safety Discriminator: Policy Rollout

$$Q^{s}(x,u) := \mathbb{1}\left\{\exists \tau \in [t_{1},T], \mathbf{x}_{x_{1},t_{1}}^{\pi^{s}}(\tau) \in \Omega^{\pi} \land \forall s \in [t_{1},\tau], \mathbf{x}_{x_{1},t_{1}}^{\pi^{s}}(s) \not\in \mathcal{F}\right\}$$

$$x_{1} := \mathbf{x}_{x,t_{0}}^{u}(t_{1})$$

$$t_{1} := t_{0} + \Delta t$$

Safety Filter: Policy Switch

 $\pi^*(x,u)$  replaces  $u^l$  by  $u^s$  whenever  $Q^s(x,u^l)=0$ .

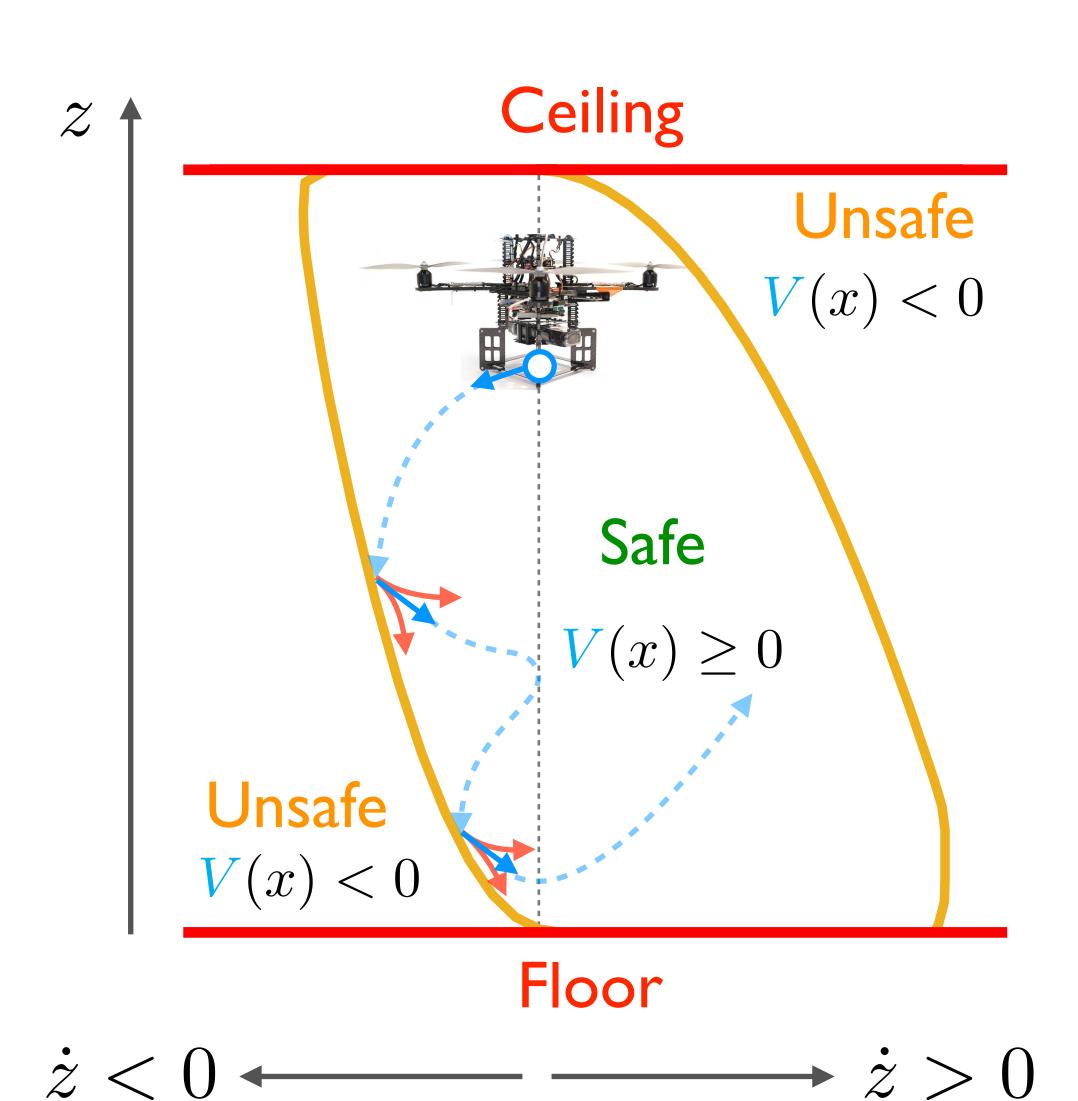


**Theorem:** the safety filter  $\pi^*$  given by

$$\pi^*(x, \mathbf{u}^l) := \begin{cases} \mathbf{u}^l, & Q^s(x, \mathbf{u}^l) = 1\\ \pi^s(x), & Q^s(x, \mathbf{u}^l) = 0 \end{cases}$$

maintains safety for all time from any initial state  $x_0$ that is safe under  $\pi^s$ , i.e.  $Q^s(x_0, \pi^s(x_0)) > 0$ .

## Hamilton-Jacobi Safety Analysis



$$\ddot{z} = k_T u - g + d$$

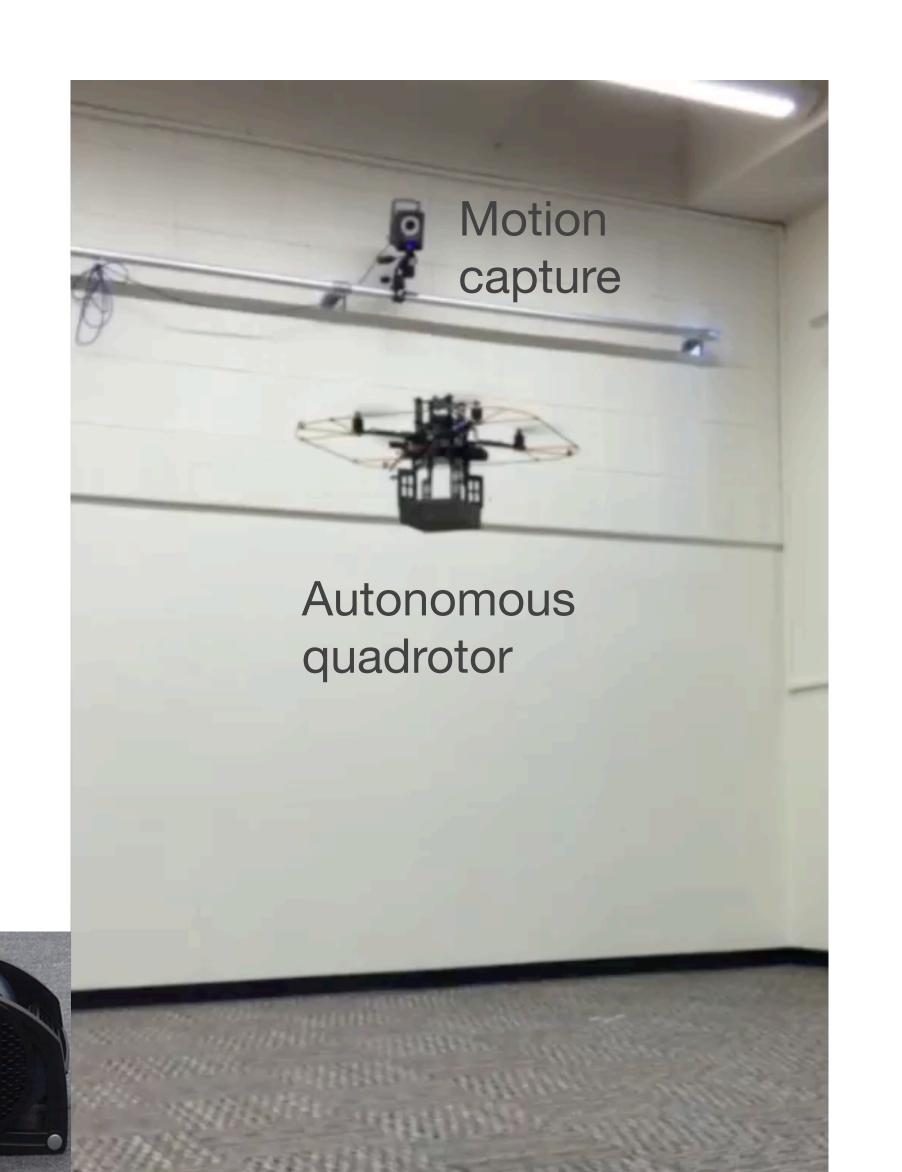
$$u \in \mathcal{U} \quad d \in \hat{\mathcal{D}}(x)$$

Theorem: the least-restrictive control law

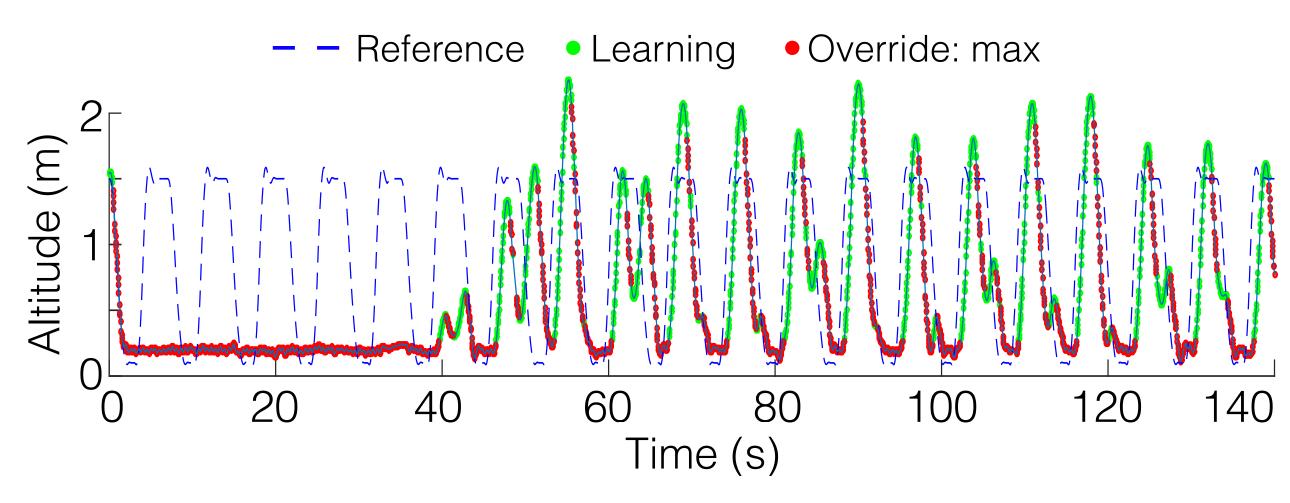
$$\mathbf{u}(x) \in \begin{cases} \mathcal{U} & V(x) \ge \epsilon > 0 \\ \{\mathbf{u}^*(x)\} & V(x) < \epsilon \end{cases}$$

renders the *safe set* controlled-invariant if  $d \in \operatorname{int} \hat{\mathcal{D}}(x)$  on the boundary  $\{V(x) = 0\}$ .

## Learning With Safety Envelope Protection



Policy gradient reinforcement learning (feature weights initialized to 0)



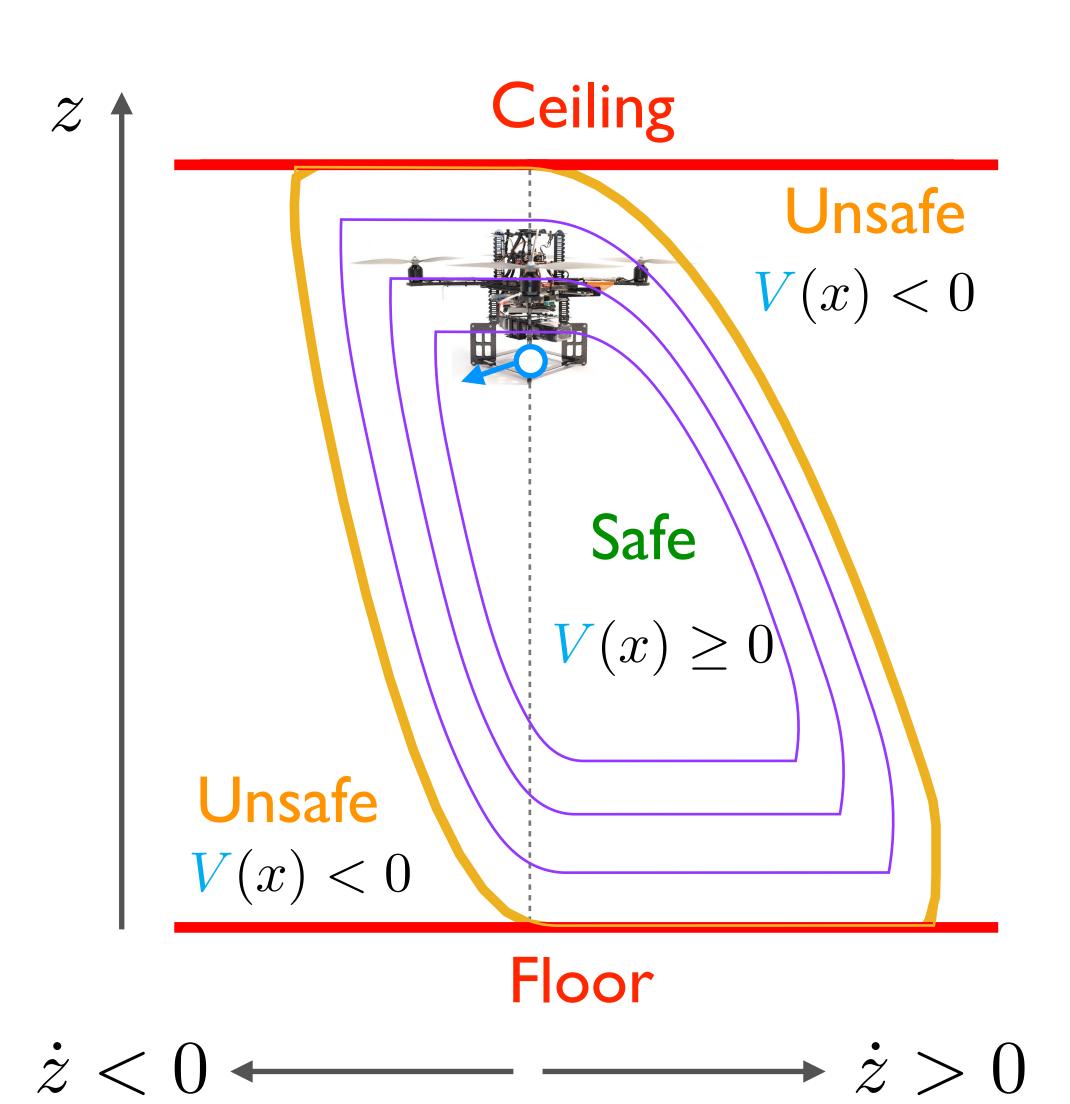
From Fall to Flight

A guarantee is as good as the model it is based on.

Model error is inevitable in real-world environments.



## The Safety Onion



$$\ddot{z} = k_T u - g + d$$

$$u \in \mathcal{U} \quad d \in \hat{\mathcal{D}}(x)$$

Theorem: the least-restrictive control law

$$\frac{\mathbf{u}(x)}{\mathbf{u}} \in \begin{cases} \mathcal{U} & V(x) \ge \alpha + \epsilon & \epsilon > 0 \\ \{ \mathbf{u}^*(x) \} & V(x) < \alpha + \epsilon & \alpha \ge 0 \end{cases}$$

renders the  $\alpha$ -level set controlled-invariant if  $d \in \operatorname{int} \hat{\mathcal{D}}(x)$  on the boundary  $\{V(x) = \alpha\}$ .

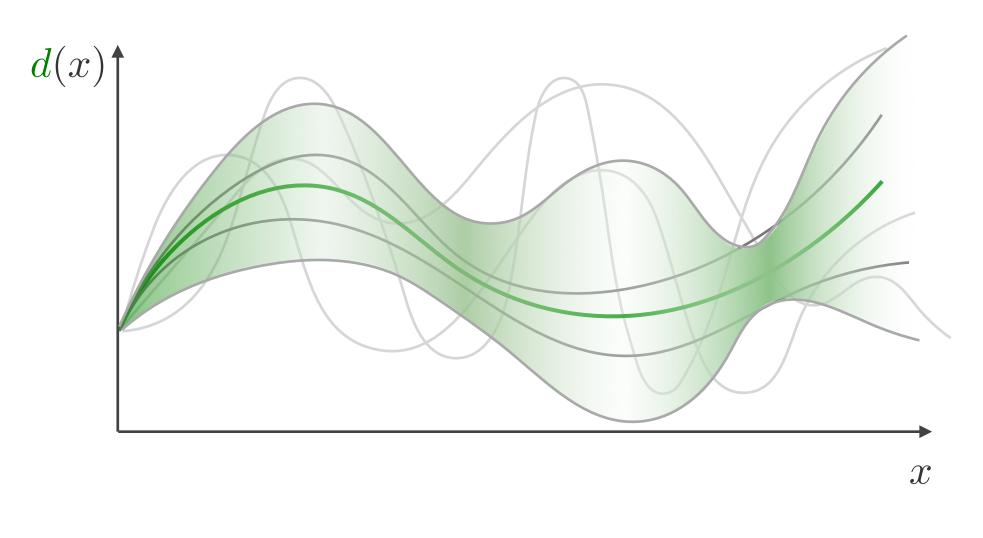
Use data-driven analysis to update the probability that each theoretical safety level curve is usable for the real system.

## Bayesian Quantification of Model Error

$$\dot{x} = f(x, \mathbf{u}, \mathbf{d})$$

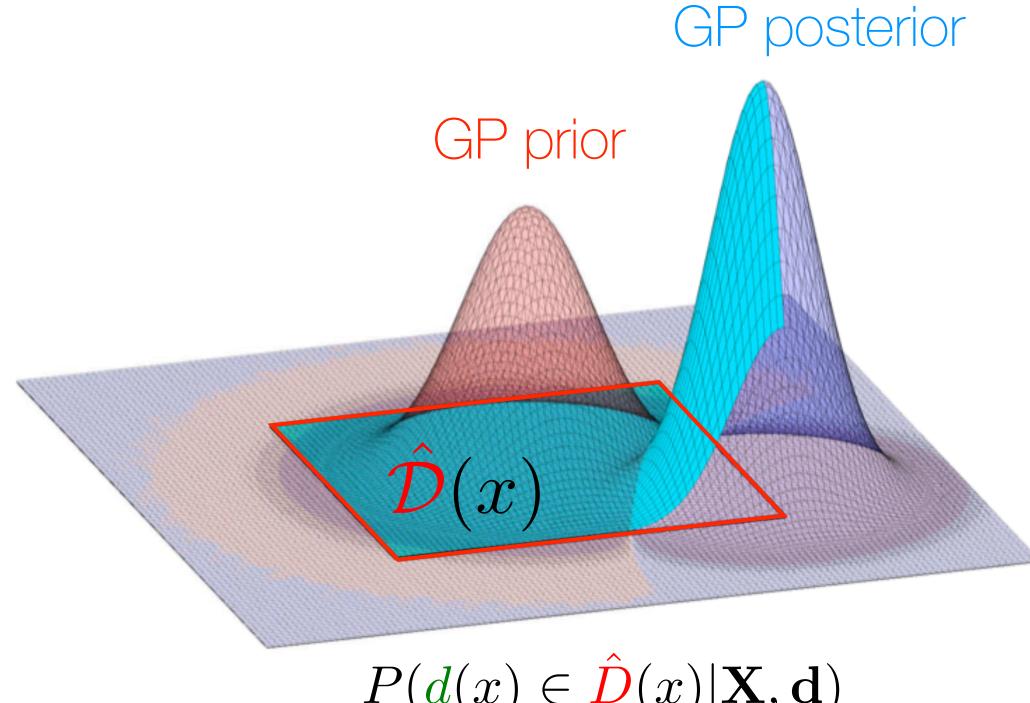
$$\mathbf{d} \in \hat{\mathcal{D}}(x)$$

Gaussian process (GP)



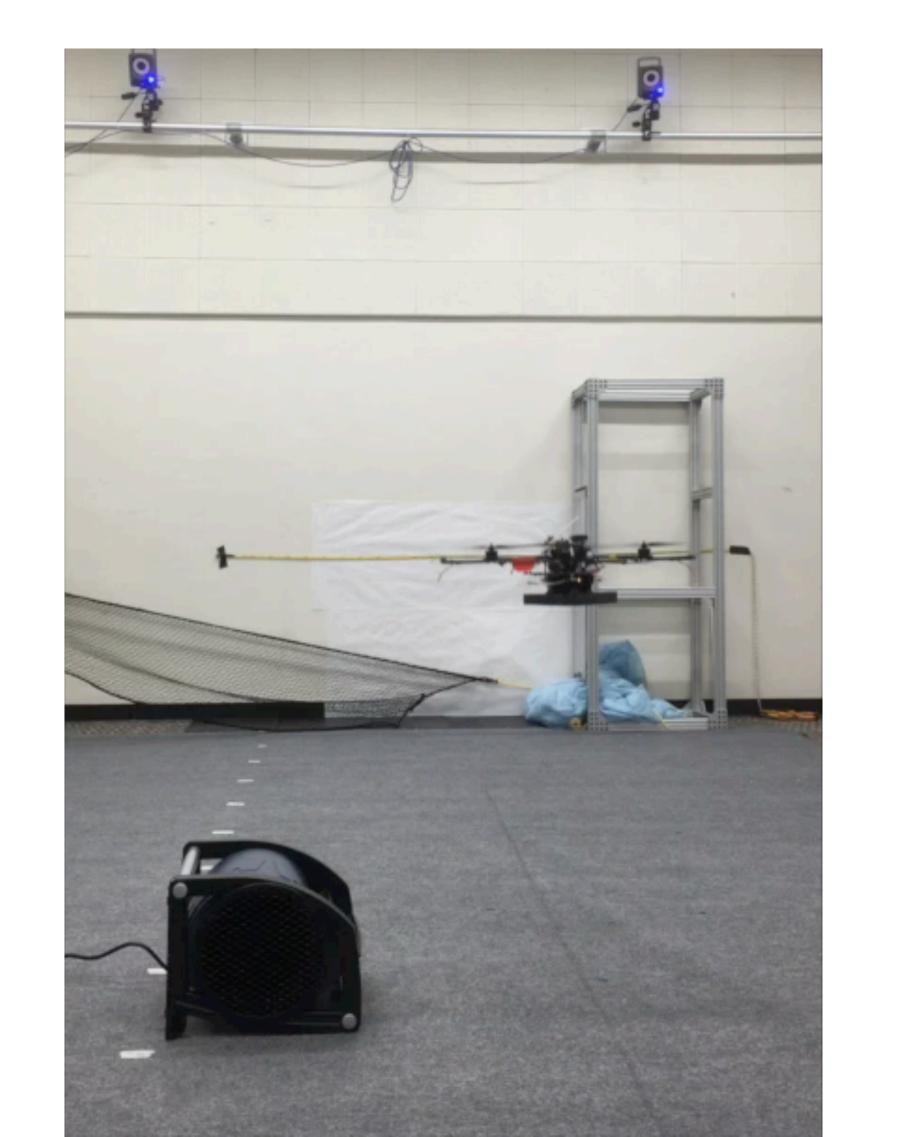
$$d(\cdot) \sim \mathcal{GP}(\mu(\cdot), k(\cdot, \cdot))$$

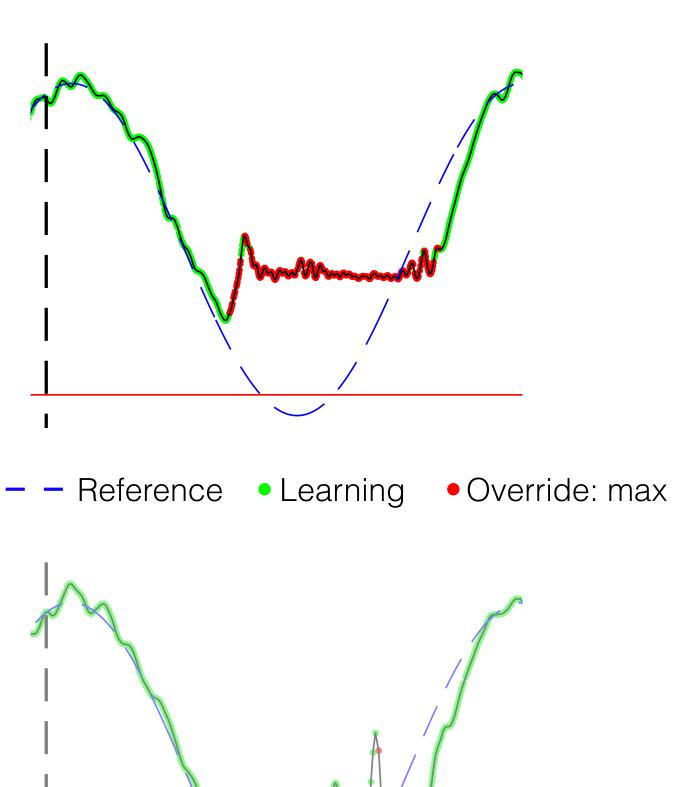
Observations: X, d



$$P(d(x) \in \hat{D}(x)|\mathbf{X}, \mathbf{d})$$

# Preserving Safety in Unforeseen Conditions





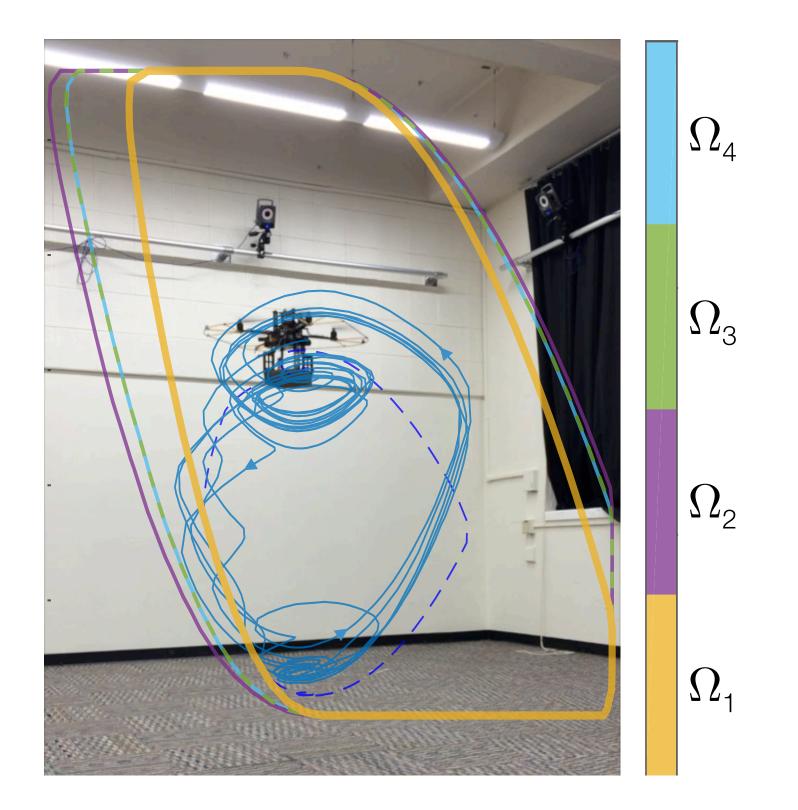
Bayesian safety validation

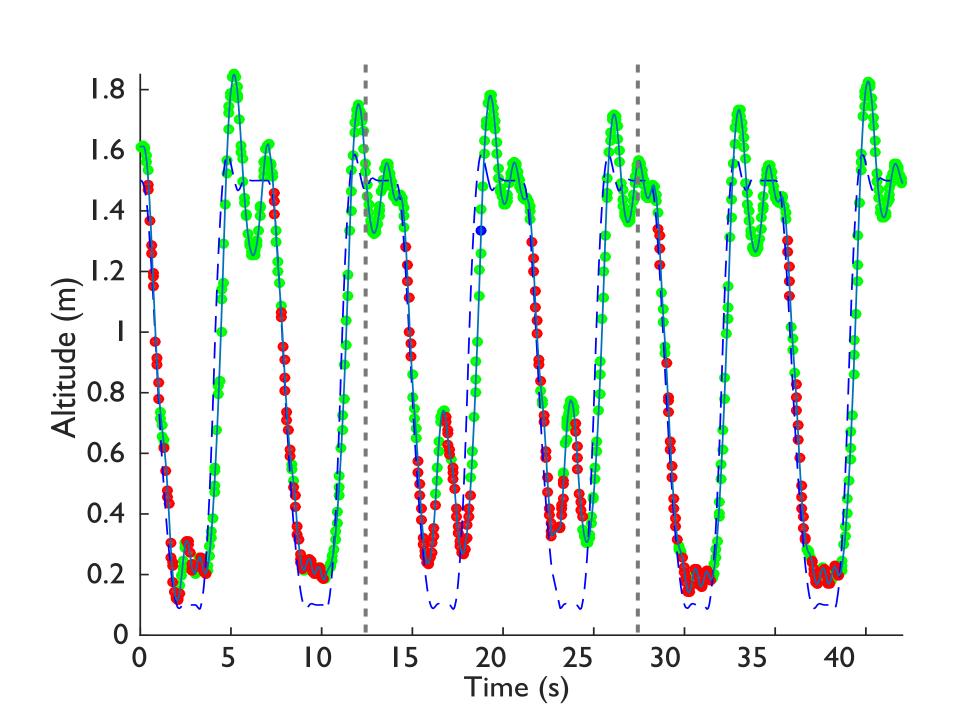
Assume safety analysis holds

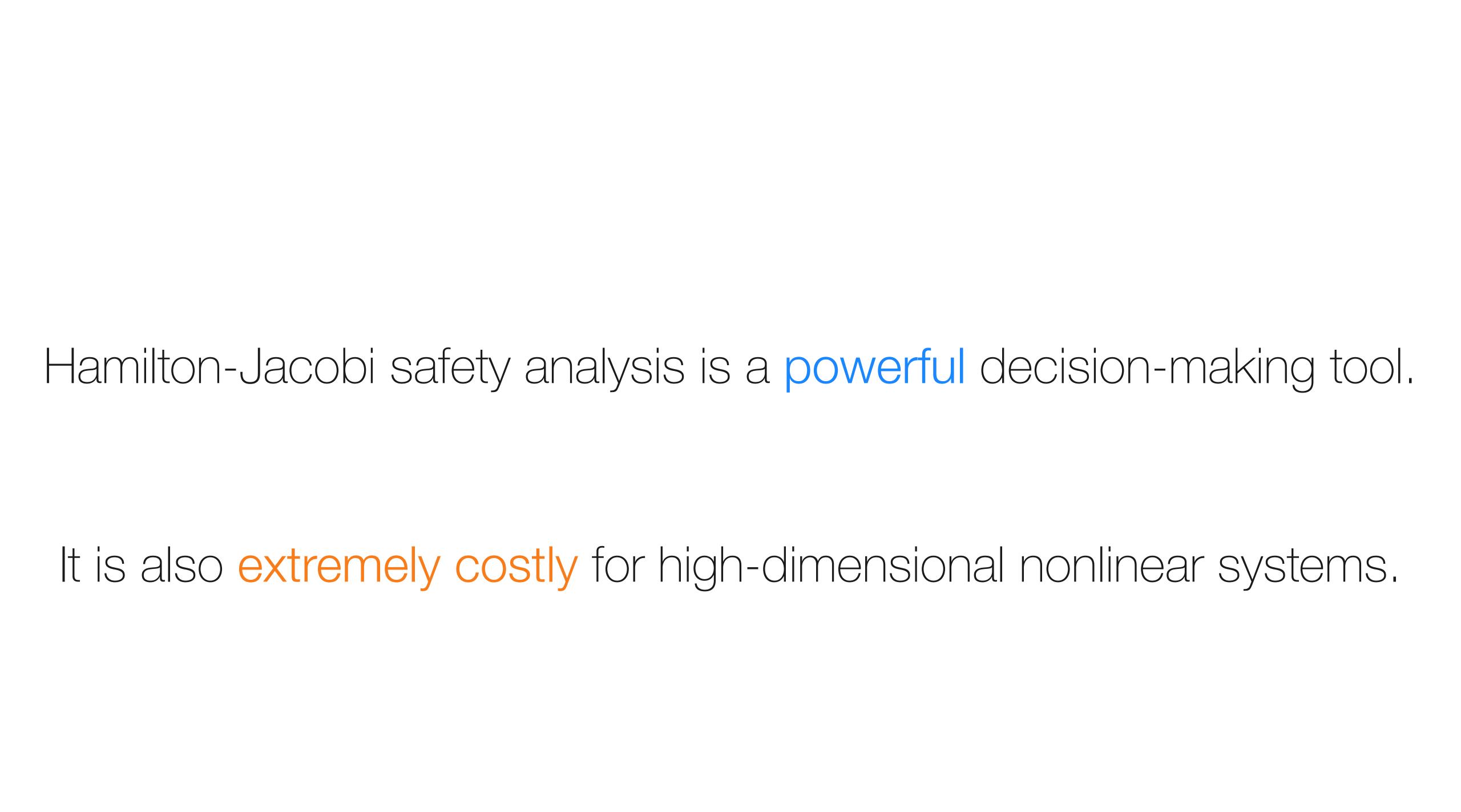
## Iteratively Recomputing Safety

As we acquire new information about the environment (in this case the unknown part d(x) of the dynamics) we can recompute the safe set and safety policy.

At the same time, online (local) safety validation helps prevent reliance on any overly optimistic analysis resulting from incomplete or misleading exploration.

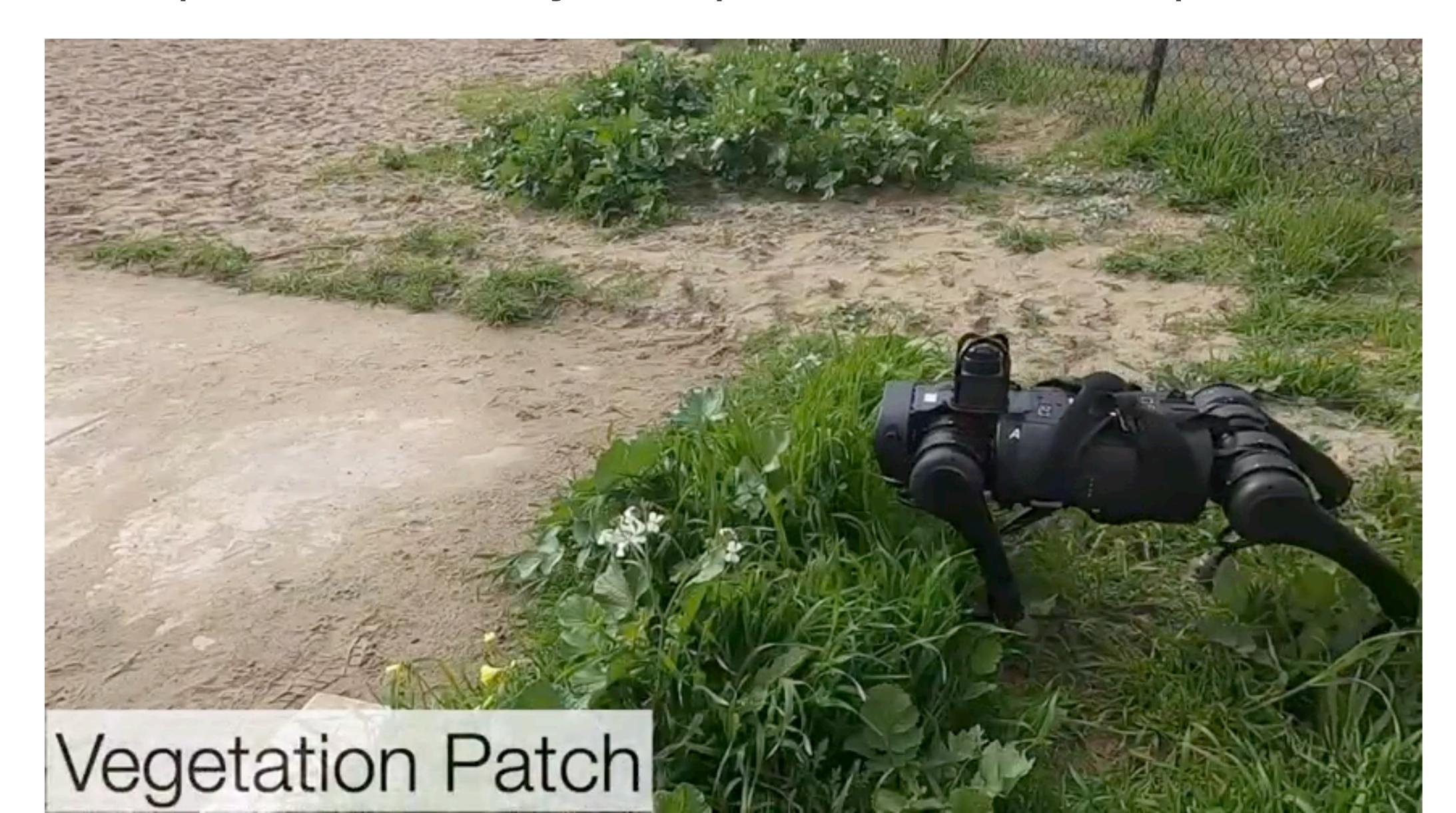






What if we don't have a Hamilton-Jacobi safety value function?
Is there a next best thing?

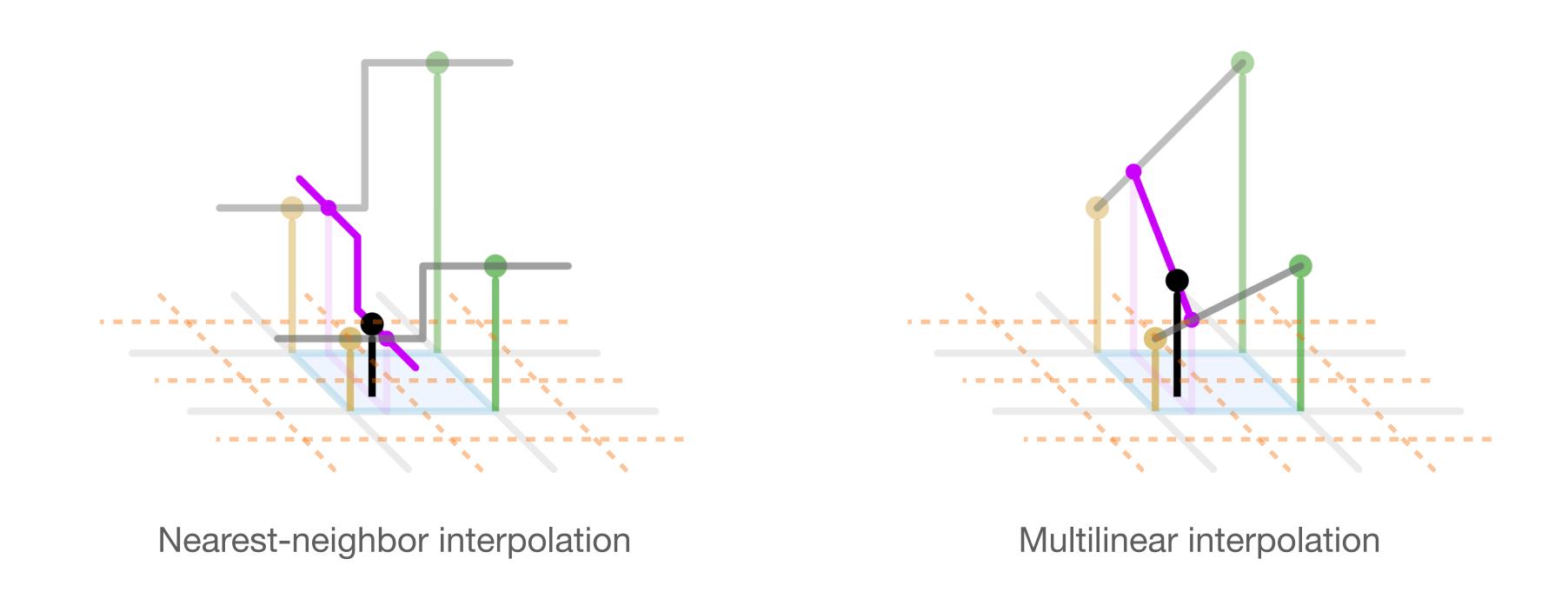
## Empirical Safety: Rapid Motor Adaptation



Can we use **learning** to compute best-effort safety controllers with sound **guarantees**?

## From Grids to Neural Networks

A grid discretization is essentially a (rather inefficient) function approximator.

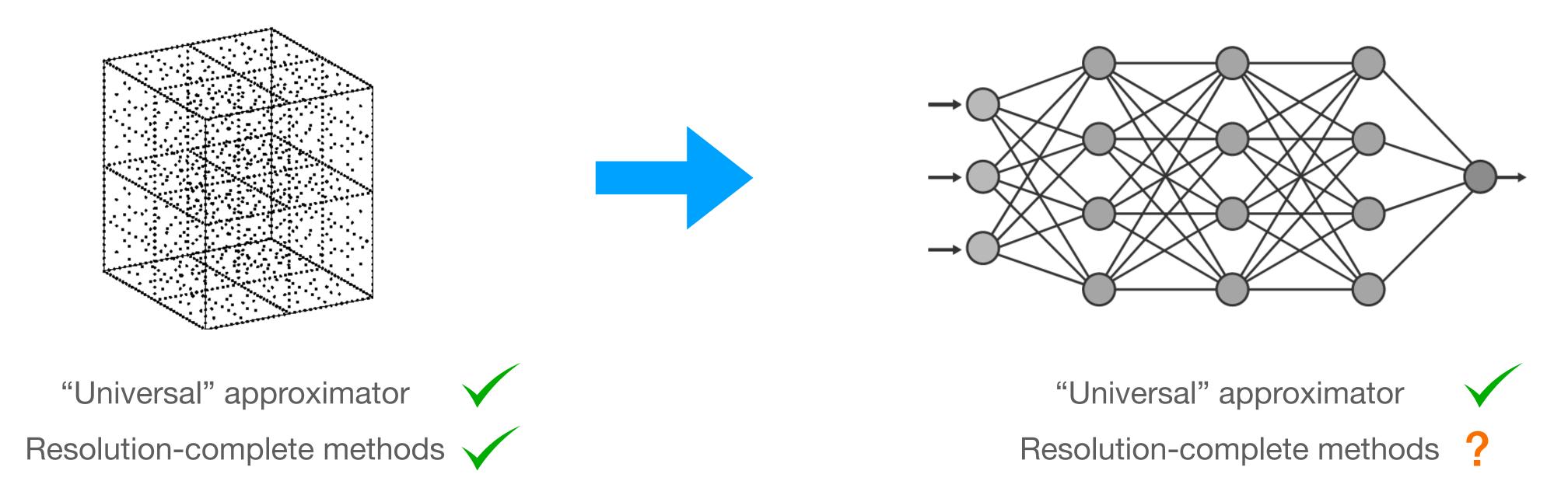


Can we represent the value function/optimal policy in a more scalable form?

## From Grids to Neural Networks

A neural network is a "universal" gradient-trainable function approximator.

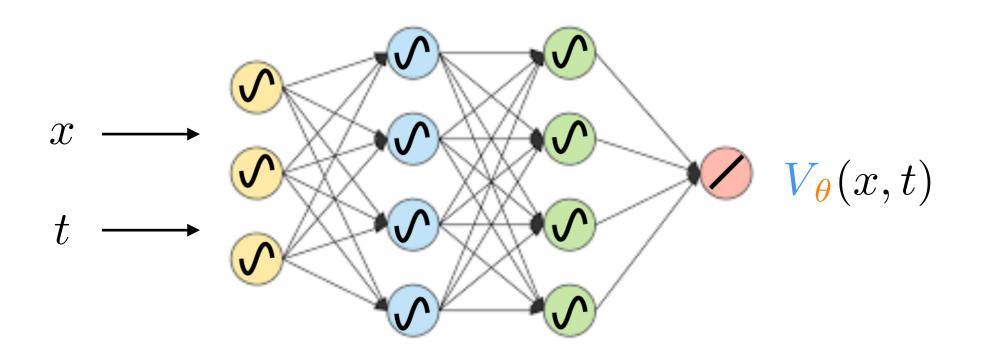
can represent any continuous function to arbitrary accuracy with enough units (Universal Representation Theorem)



Similar to deep RL, we can approximate the safe control problem using neural networks.

# Self-Supervised Value Learning: DeepReach

**Deep representation:** the value function is represented through a neural network  $V_{\theta}$ .



$$\min \left\{ \partial_t V + \max_u \min_{\mathbf{d}} \nabla_x V^\top f(x, u, \mathbf{d}), l(x, t) - V(x, t) \right\} = 0$$
 
$$V(\cdot, T) \equiv l(\cdot, T) \qquad \text{HJI safety equation (variational inequality)}$$

Loss function: penalizes HJI error incurred by the learned value function  $V_{\theta}$ .

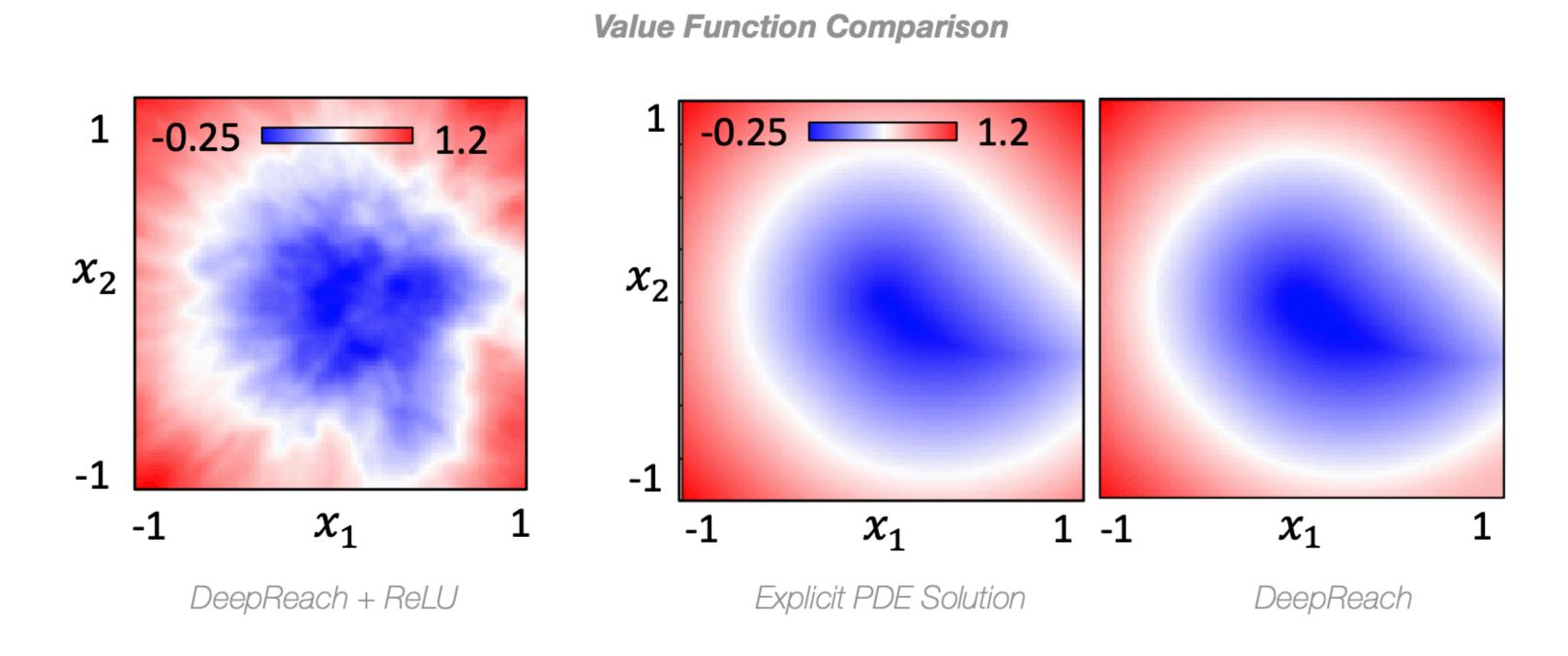
$$\left[ L({\color{red}\theta}) := \sum_i \left| \min \left\{ \partial_t V_{\color{red}\theta} + \max_u \min_{\color{red} \textbf{d}} \nabla_x V_{\color{red}\theta}^{\top} f(x_i, \underline{u}, \underline{d}), l(x_i, t_i) - V_{\color{red}\theta}(x_i, t_i) \right\} \right| \\ + \lambda \left| l(x_i, T) - V_{\color{red}\theta}(x_i, T) \right| \right]$$
 Training loss

Self-supervision: repeatedly sample a batch of space-time points  $\{(x_i, t_i)\}$  and update  $V_{\theta}$ .

$${\color{red} \theta} \leftarrow {\color{red} \theta} - \alpha \, \nabla L({\color{red} \theta})$$
 Update rule

# Self-Supervised Value Learning: DeepReach

Sinusoidal networks: sinusoidal activation function works well in practice.

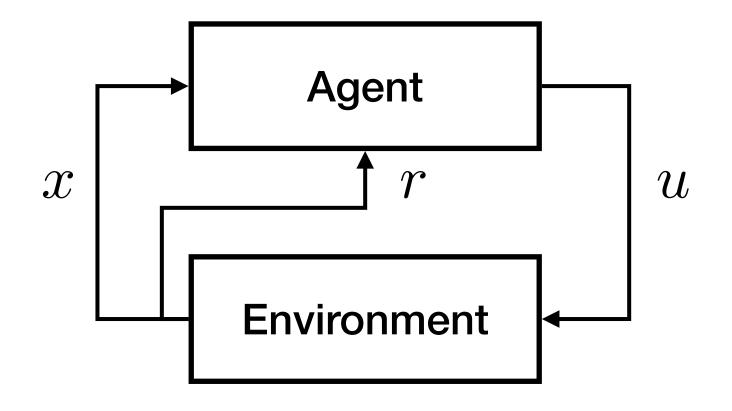


Comparison graphic by Somil Bansal (USC).

Other activation functions struggle to approximate the value function well

[Bansal and Tomlin. DeepReach: A Deep Learning Approach to High-Dimensional Reachability. ICRA 2021]

## Reinforcement Learning: Beyond Rewards



$$J(\mathbf{x}) = \sum_{t=0}^{\infty} \gamma^t r(\mathbf{x}(t))$$

$$J(\mathbf{x}) = \sum_{t=0}^{\infty} \gamma^t r(\mathbf{x}(t)) \qquad V(x) = \max_{u \in \mathcal{U}} r(x, u) + \gamma V(x')$$

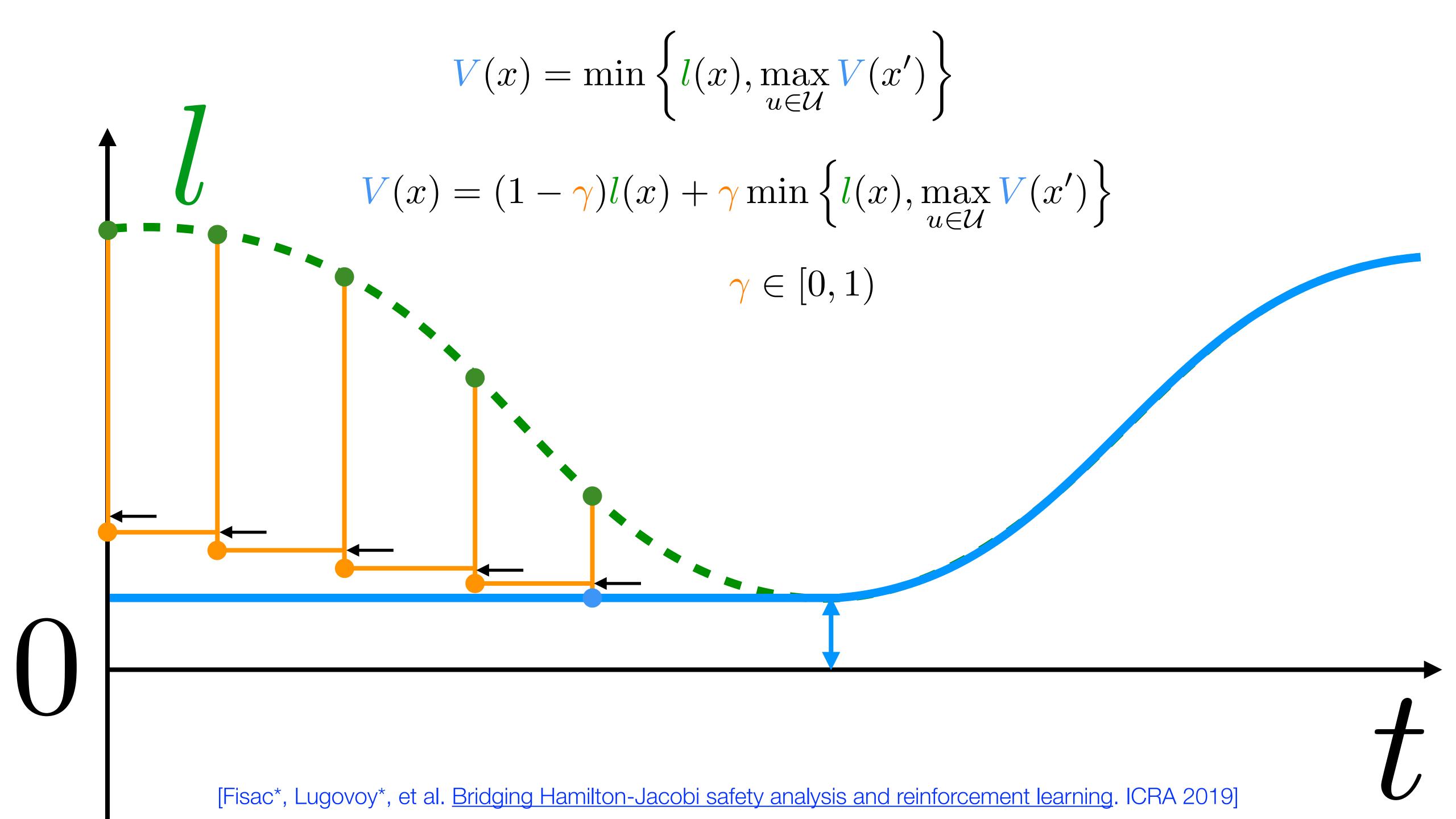
$$V(x) = \max_{u \in \mathcal{U}} (1 - \gamma)r(x, u) + \gamma \left(r(x, u) + V(x')\right)$$

$$J(\mathbf{x}) = \inf_{t > 0} l(\mathbf{x}(t))$$

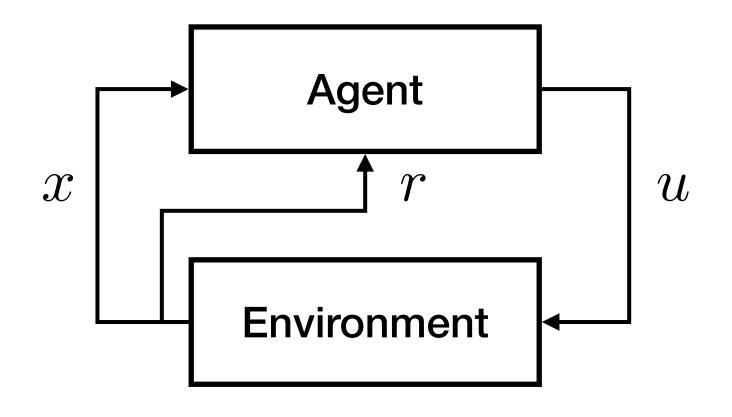
$$J(\mathbf{x}) = \inf_{t>0} l(\mathbf{x}(t)) \qquad V(x) = \min \left\{ l(x), \max_{u \in \mathcal{U}} V(x') \right\}$$

Not a contraction mapping

How do we discount a minimum?



## Reinforcement Learning: Beyond Rewards



$$J(\mathbf{x}) = \sum_{t=0}^{\infty} \gamma^t r(\mathbf{x}(t))$$

$$J(\mathbf{x}) = \sum_{t=0}^{\infty} \gamma^t r(\mathbf{x}(t)) \qquad V(x) = \max_{u \in \mathcal{U}} r(x, u) + \gamma V(x')$$

Contraction mapping

$$V(x) = \max_{u \in \mathcal{U}} (1 - \gamma) r(x, u) + \gamma \left( r(x, u) + V(x') \right)$$

$$J(\mathbf{x}) = \inf_{t>0} l(\mathbf{x}(t))$$

$$J(\mathbf{x}) = \inf_{t \ge 0} l(\mathbf{x}(t)) \qquad V(x) = \min \left\{ l(x), \max_{u \in \mathcal{U}} V(x') \right\}$$

Contraction mapping

$$V(x) = (1 - \gamma)l(x) + \gamma \min \left\{ l(x), \max_{u \in \mathcal{U}} V(x') \right\}$$

## Contraction Mapping Result

Theorem: The time-discounted safety Bellman equation

$$V(x) = (1 - \gamma)l(x) + \gamma \min \left\{ l(x), \max_{u \in \mathcal{U}} V(x') \right\}$$

induces a contraction in the space of value functions under the supremum norm.

Let  $V, V: \mathcal{X} \to \mathbb{R}$  then there exists a constant  $\kappa \in [0,1)$  such that

$$||B[V] - B[\tilde{V}]||_{\infty} \leq \kappa |V - \tilde{V}||_{\infty}.$$

Proof: For all states  $x \in \mathcal{X}$ , the following bounds hold:

$$|B[V](x) - B[\tilde{V}](x)| = \gamma |\min\{l(x), \max_{u \in \mathcal{U}} V(x + f(x, u)\Delta t)\} - \min\{l(x), \max_{\tilde{u} \in \mathcal{U}} \tilde{V}(x + f(x, \tilde{u})\Delta t)\}|$$

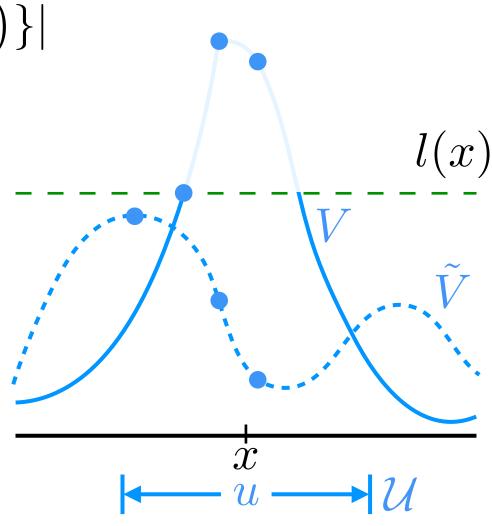
Let the first max be greater, and achieved by  $u^* \in \mathcal{U}$ .

$$\leq \gamma |\max_{u \in \mathcal{U}} V(x + f(x, u)\Delta t) - \max_{\tilde{u} \in \mathcal{U}} \tilde{V}(x + f(x, \tilde{u})\Delta t)|$$

$$\leq \gamma |V(x+f(x,u^*)\Delta t) - \tilde{V}(x+f(x,u^*)\Delta t)|$$

$$\leq \gamma \max_{u \in \mathcal{U}} |V(x + f(x, u)\Delta t) - \tilde{V}(x + f(x, u)\Delta t)|$$

$$\leq \gamma \sup_{\tilde{x}} |V(\tilde{x}) - \tilde{V}(\tilde{x})| = \gamma |V - \tilde{V}|_{\infty} \quad \Box$$

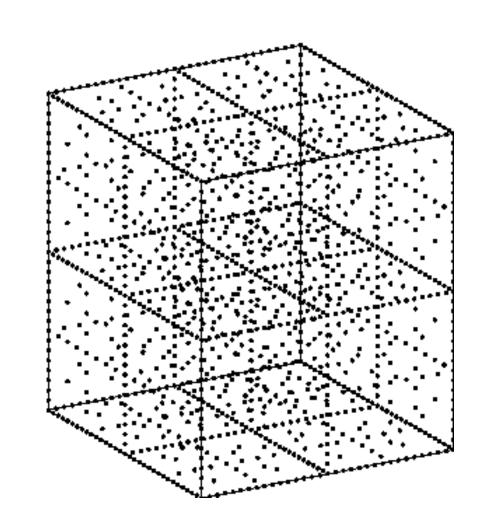


## Safety Q-Learning

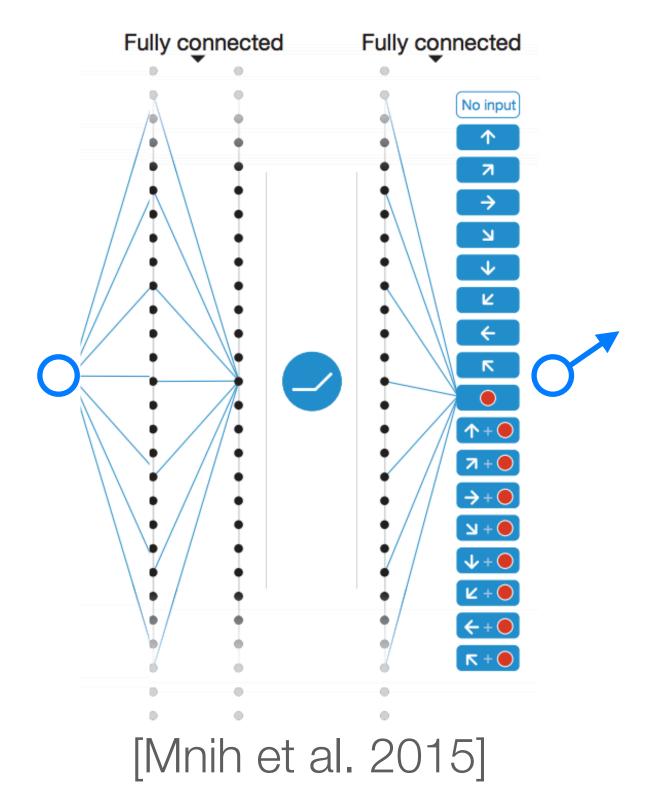
Time-discounted state-action Safety Bellman Equation

$$Q(x,u) \leftarrow (1-\alpha)Q(x,u) + \alpha \left[ (1-\gamma)l(x) + \gamma \min\left\{l(x), \max_{u' \in \mathcal{U}} Q(x',u')\right\} \right]$$

#### **Tabular Q-Learning**



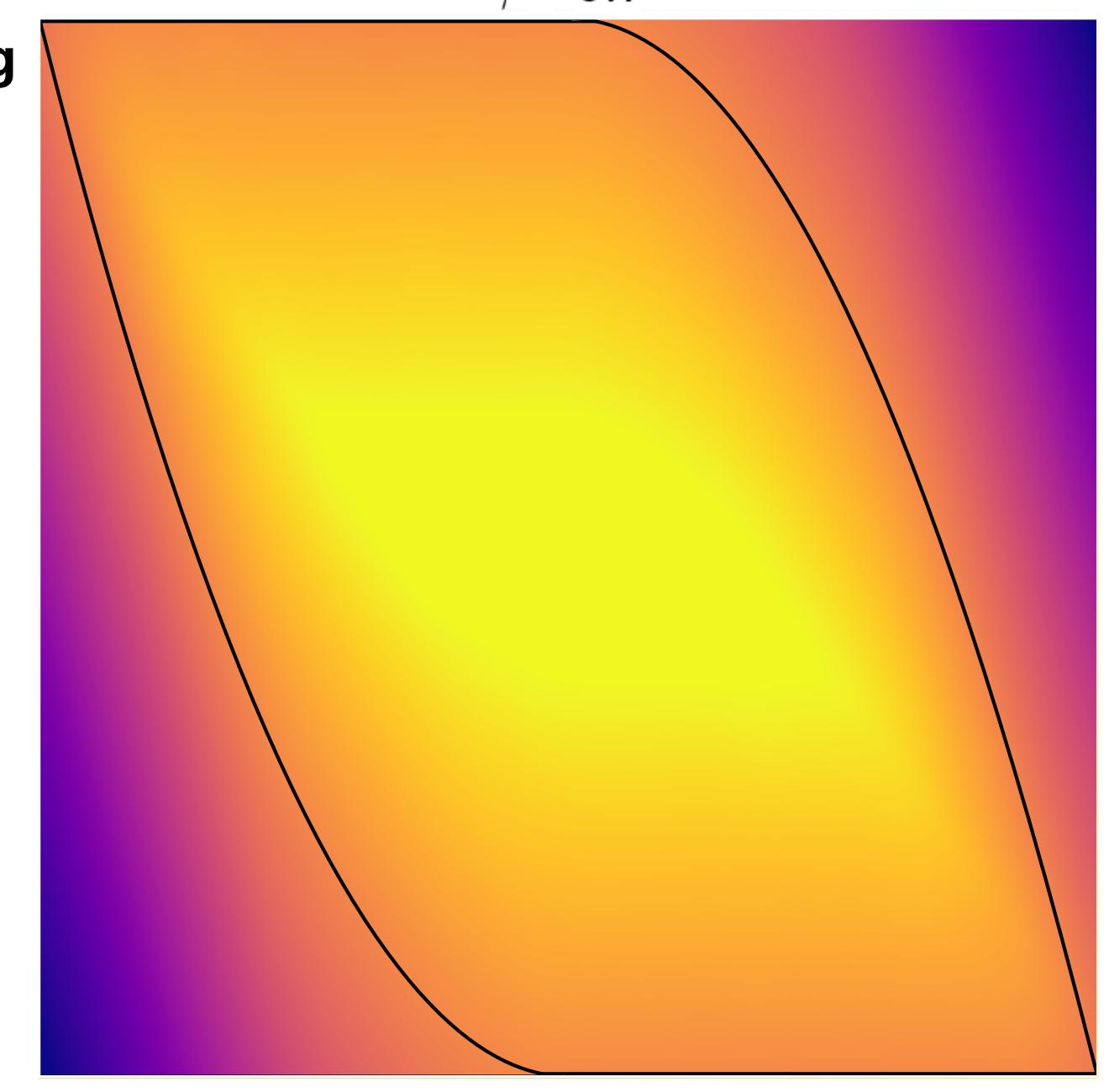
### **Deep Q-Learning**



### $\gamma$ = 0.7

### Deep Q-Learning

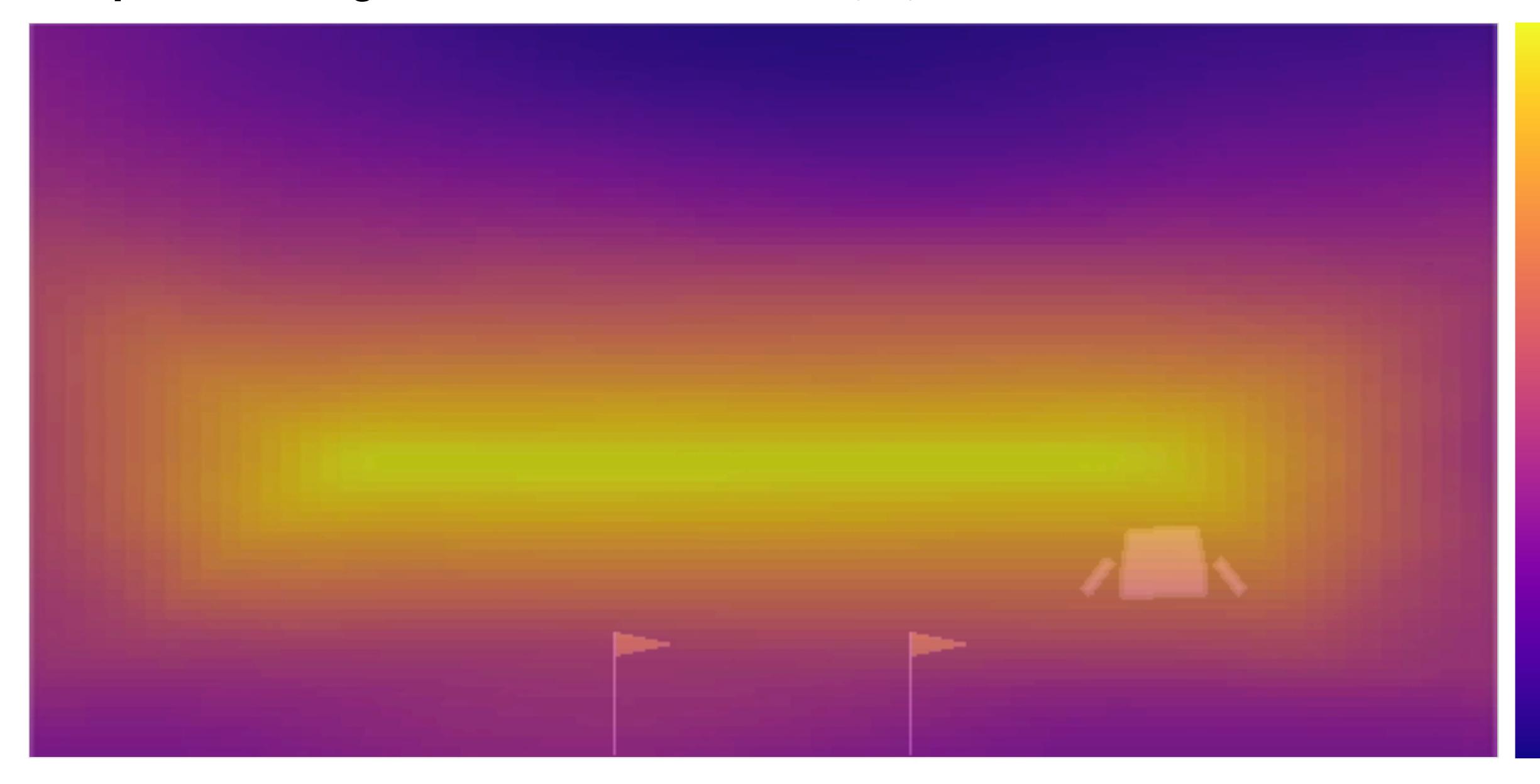
Double Integrator (2D)



Gradient Updates = 0

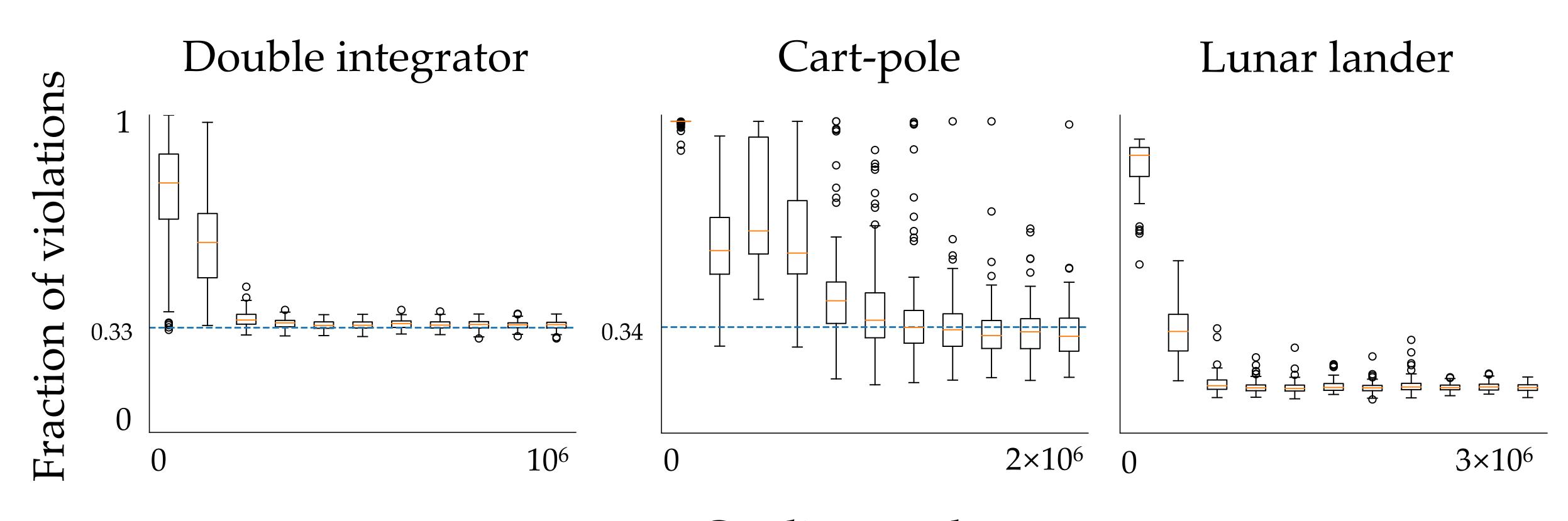
### **Deep Q-Learning**

### Lunar Lander (6D)



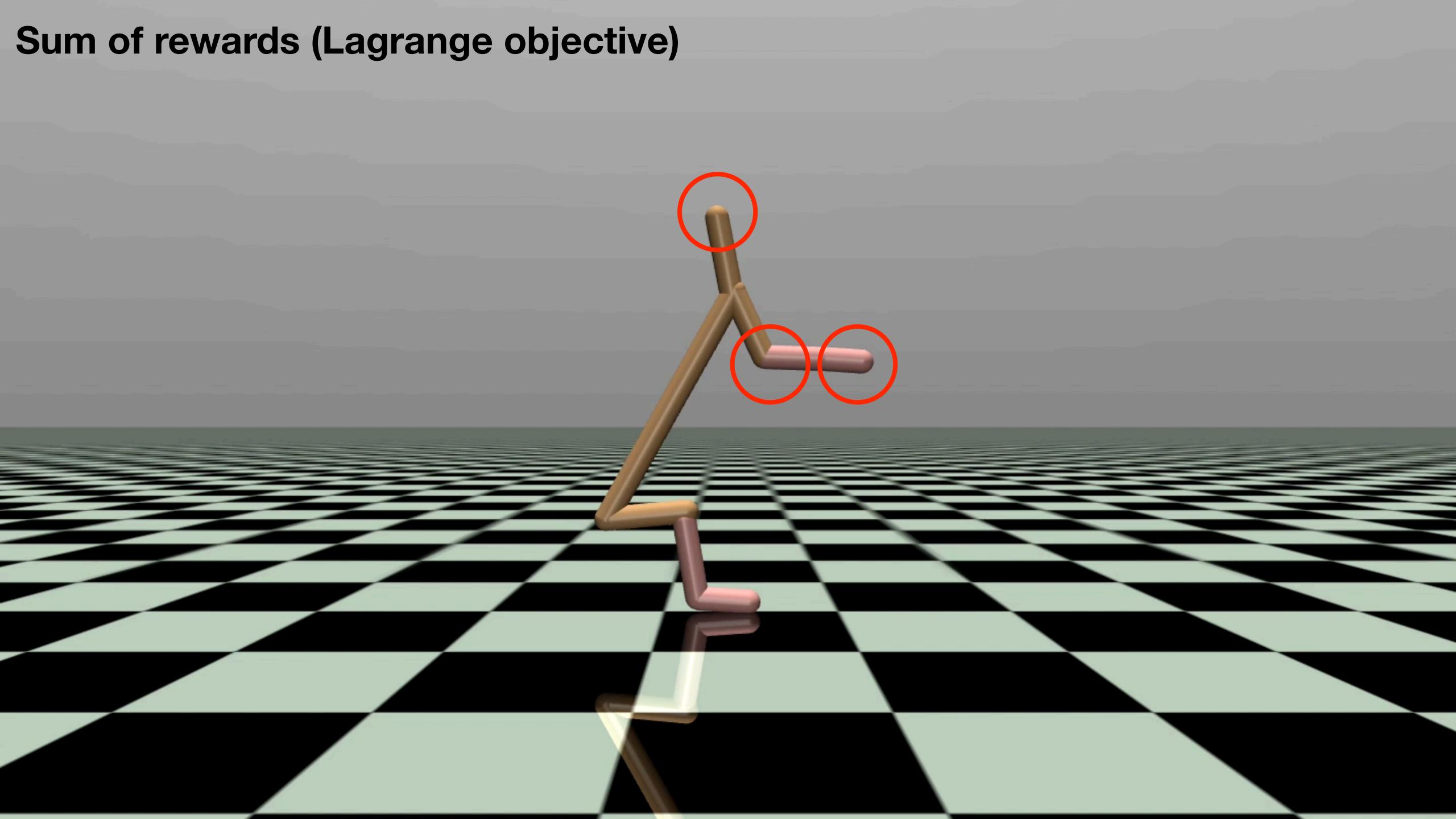
## Learning to stay safe if staying safe is possible

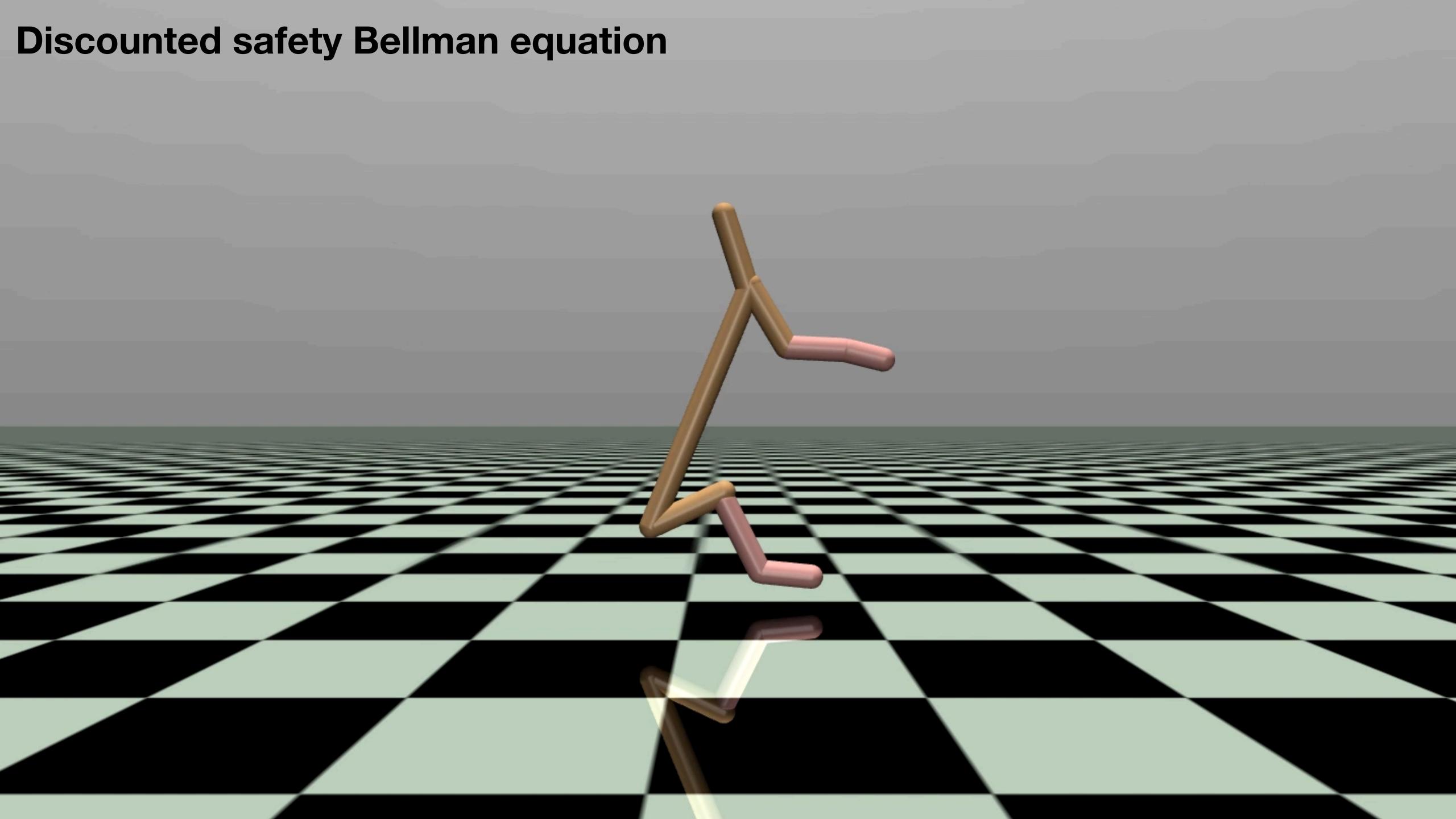
Sample initial states uniformly throughout the state space and apply the learned safety policy.

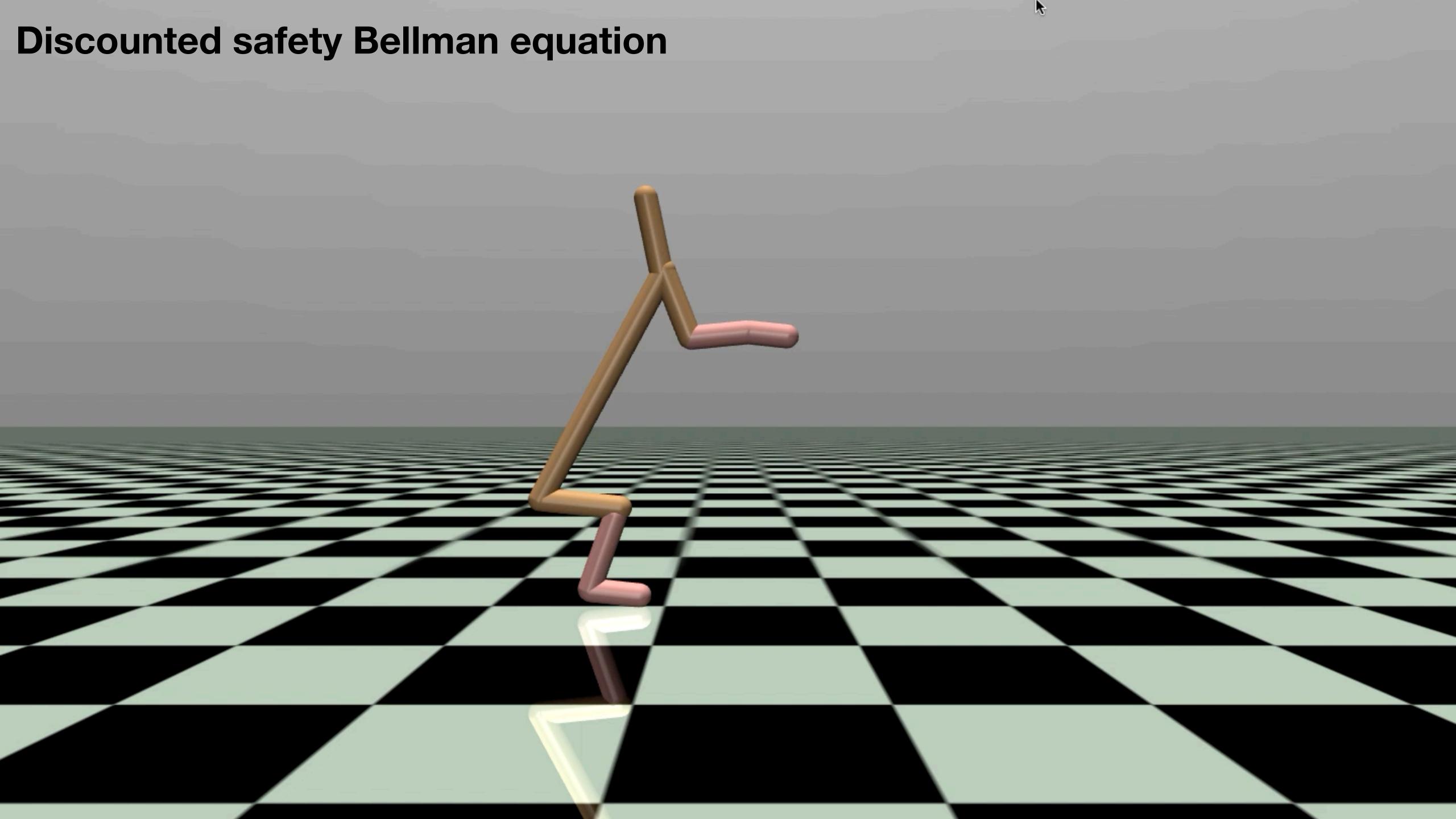


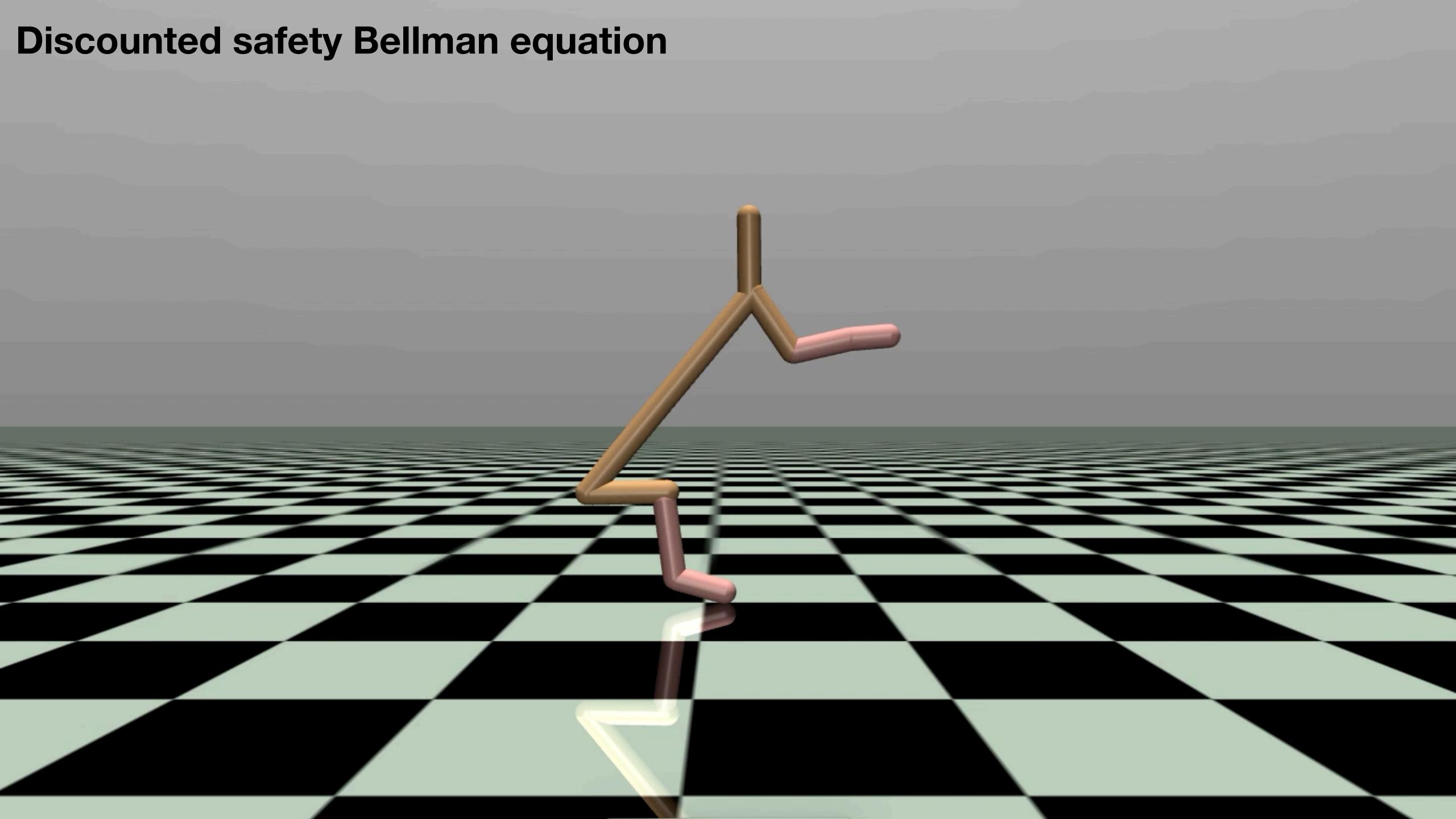
Gradient updates

Fraction of violations under the learned policy decreases towards volume of the true unsafe set.





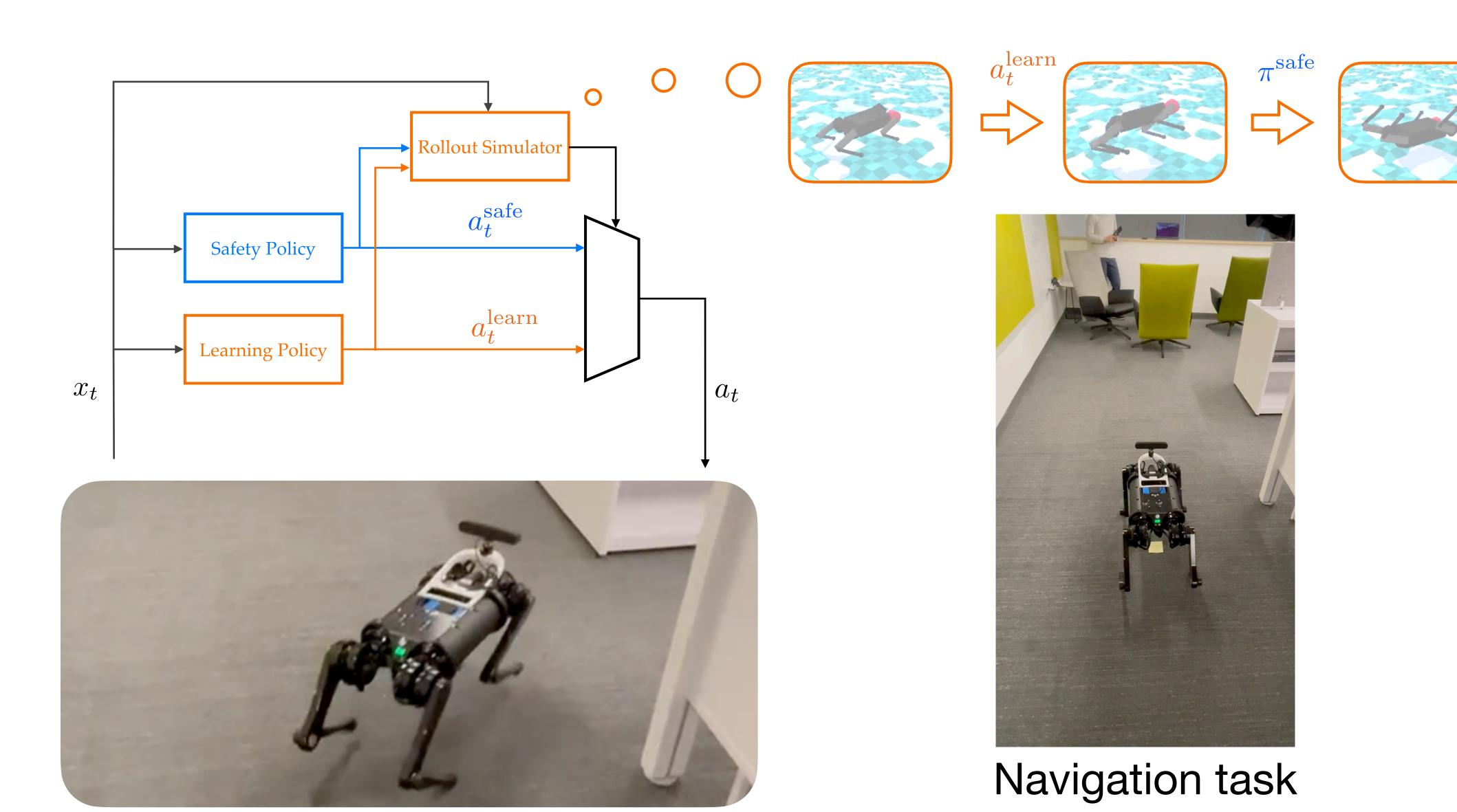


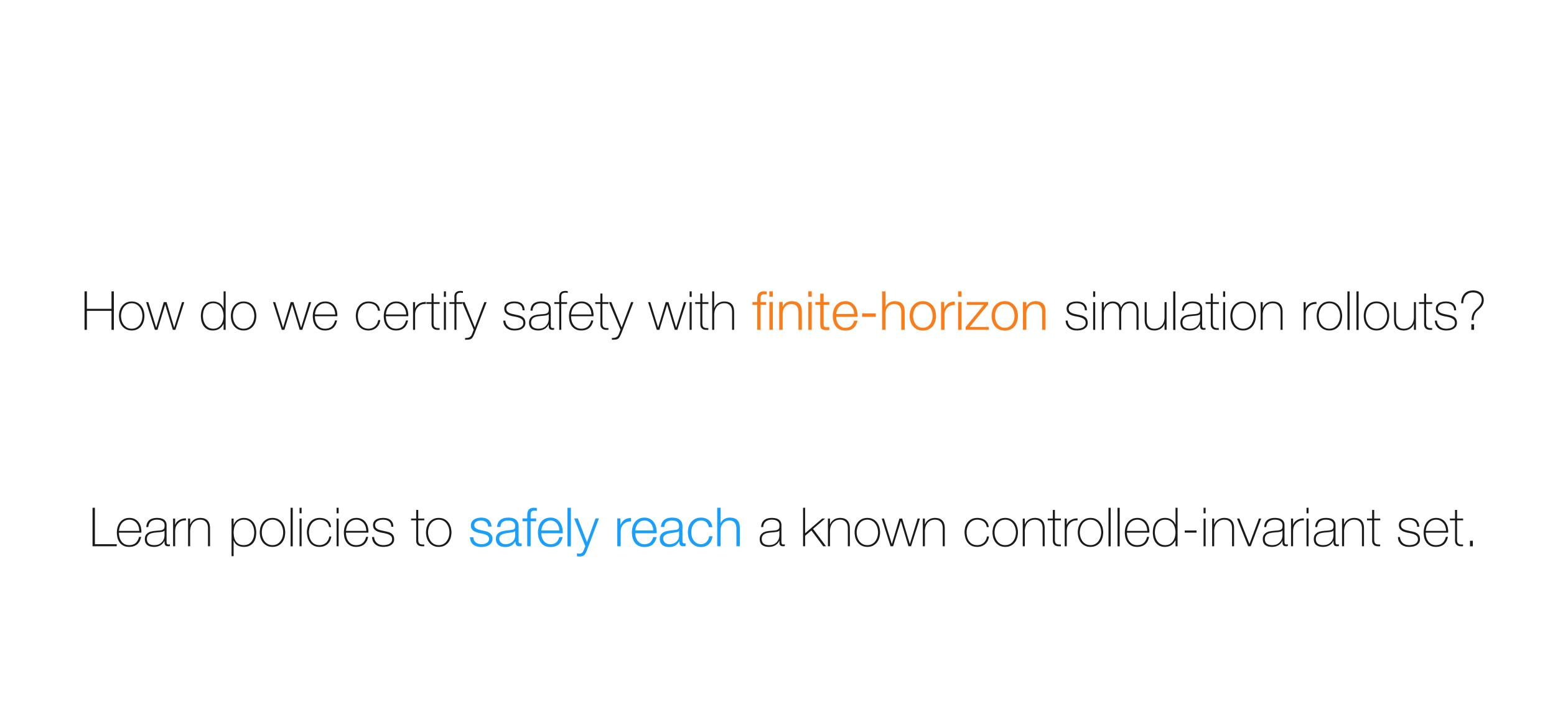


Works well in practice, but what about guarantees?

Do we have safety filters that work with an arbitrary safety policy?

# Safety RL Meets Shielding!

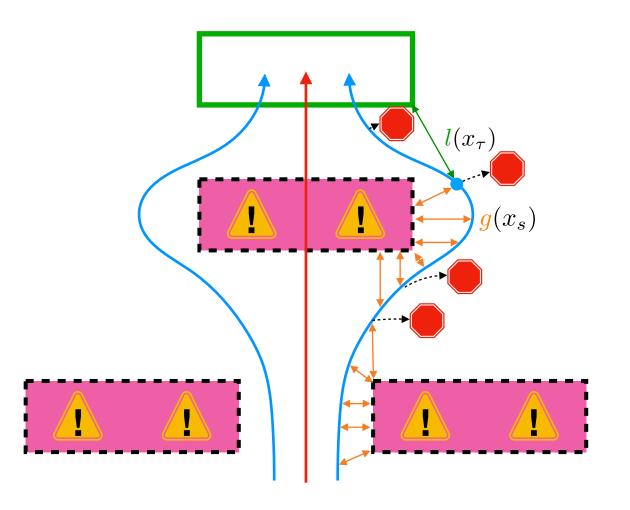




## Reach-Avoid Reinforcement Learning

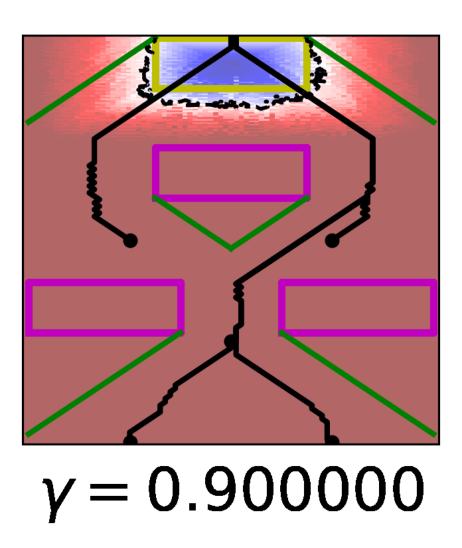
Time-discounted reach-avoid Bellman equation

$$V(s) = \gamma \max \left\{ \min \left\{ l(x), \min_{u \in \mathcal{U}} V(x') \right\}, g(x) \right\}$$
$$+ (1 - \gamma) \max \left\{ l(x), g(x) \right\}$$
$$\gamma \in [0, 1)$$

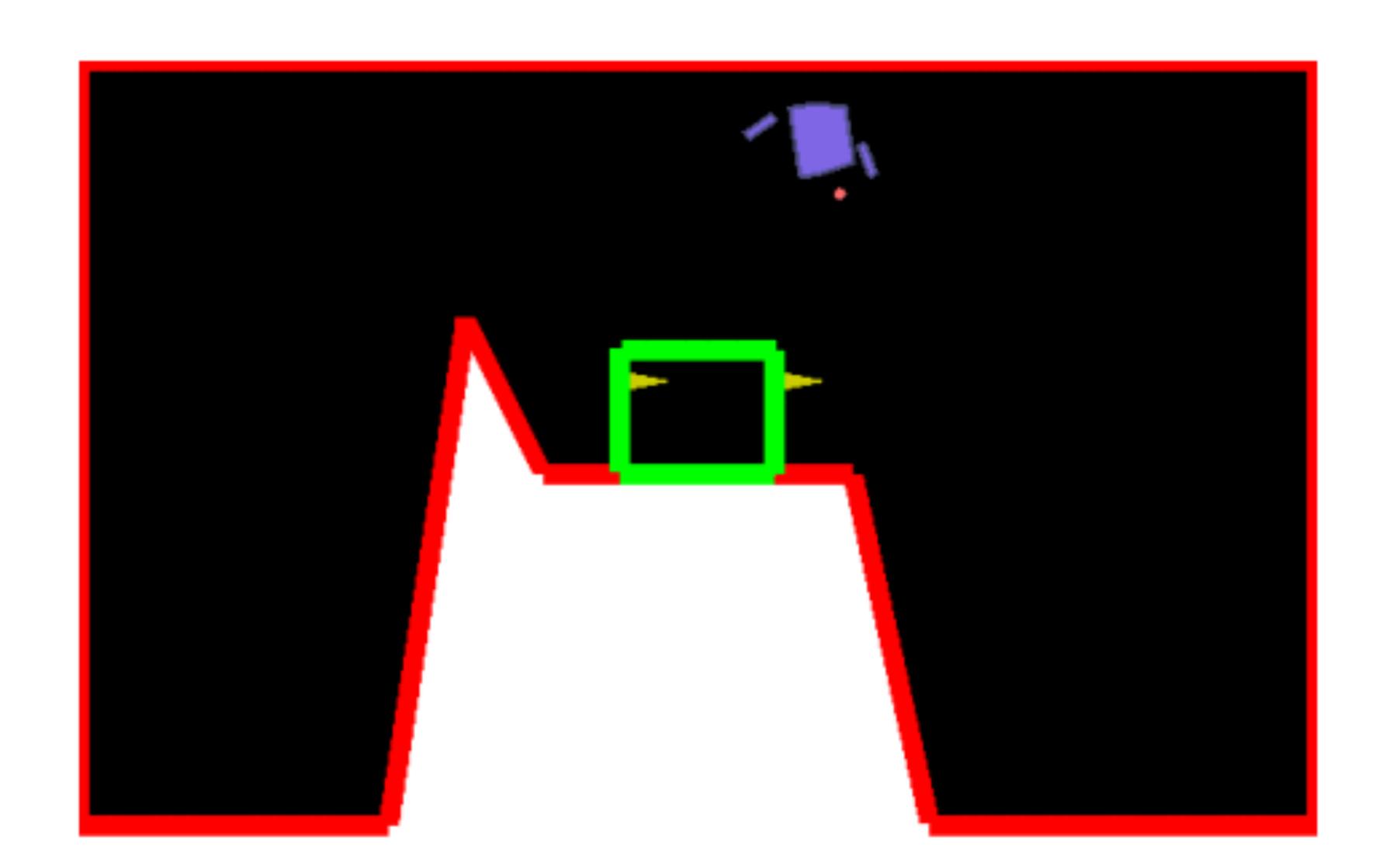


#### Convergent under-approximation:

The sublevel set  $\{x:V(x)<0\}$  is a subset of  $\overline{\mathcal{RA}}(\mathcal{T},\mathcal{F})$  and converges to  $\overline{\mathcal{RA}}(\mathcal{T},\mathcal{F})$  as  $\gamma\to 1$ .



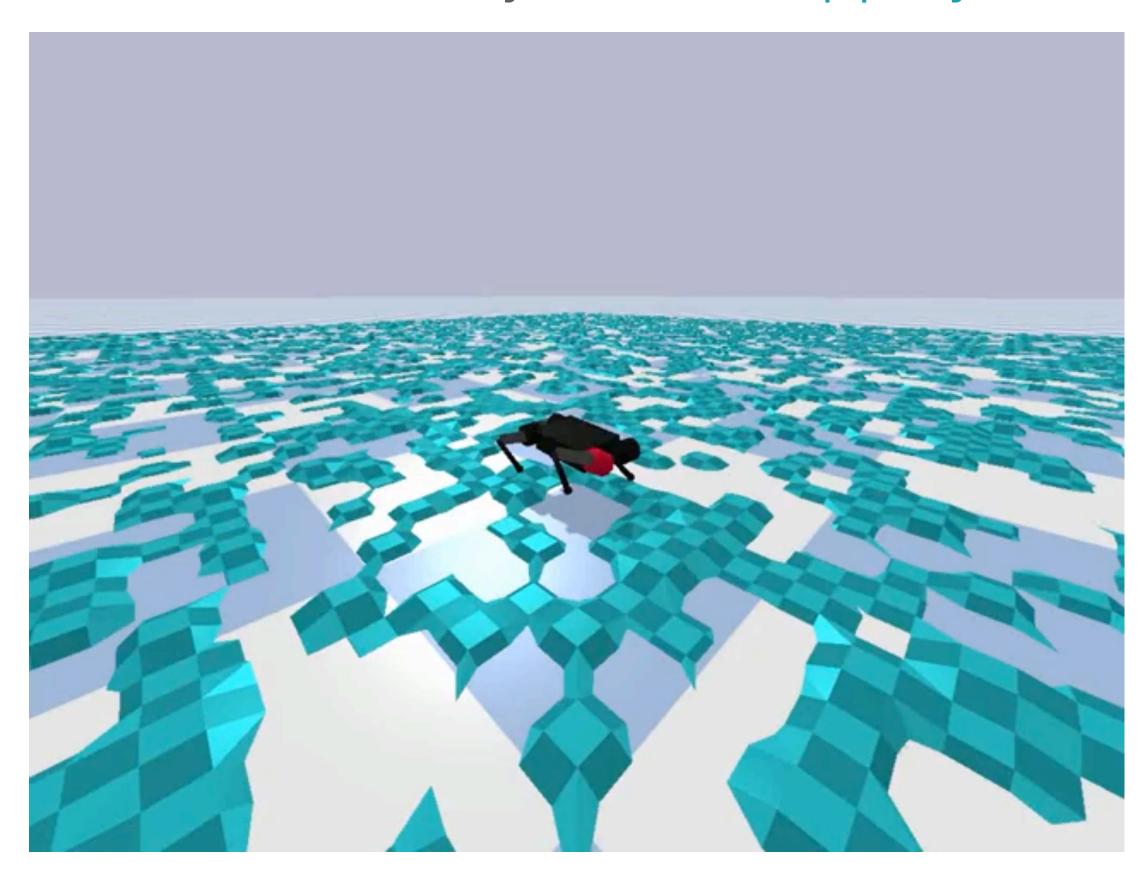
## Reach-Avoid Reinforcement Learning



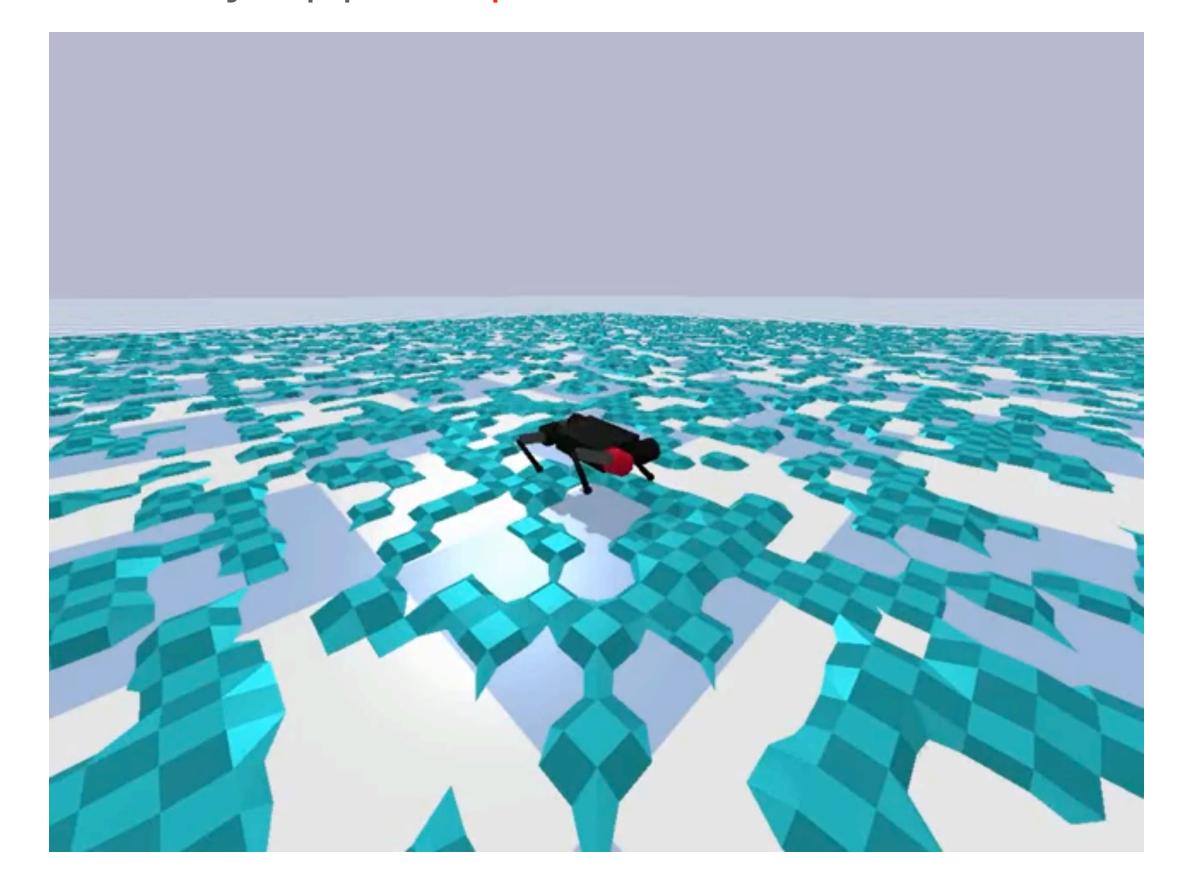
## Simulation Stress Test

#### Gait control task

Previously unseen slippery terrain + randomly applied perturbation forces







Safety policy

### Generalization Guarantees

PAC-Bayes theory: probabilistic bounds on performance and safety as long as deployment environments have the same distribution as training environments.

**Theorem 2** (PAC-Bayes Bound for Control Policies). For any  $\delta \in (0,1)$ , with probability at least  $1 - \delta$  over sampled environments  $S \sim \mathcal{D}^N$ , the following inequality holds:

$$\underbrace{C_{\mathcal{D}}(P)}_{True\ expected\ cost} \leq C_{PAC}(P) := \underbrace{C_{S}(P)}_{Training\ cost} + \underbrace{\sqrt{\frac{\mathbb{D}(P\|P_{0}) + \log(\frac{2\sqrt{N}}{\delta})}{2N}}}_{"Regularizer"}.$$

#### Algorithm 1 PAC-Bayes Policy Learning

- 1: Fix prior distribution  $P_0 \in \mathcal{P}$  over policies
- 2: Inputs:  $S = \{E_1, \ldots, E_N\}$ : Training environments,  $\delta$ : Probability threshold
- 3: Outputs:

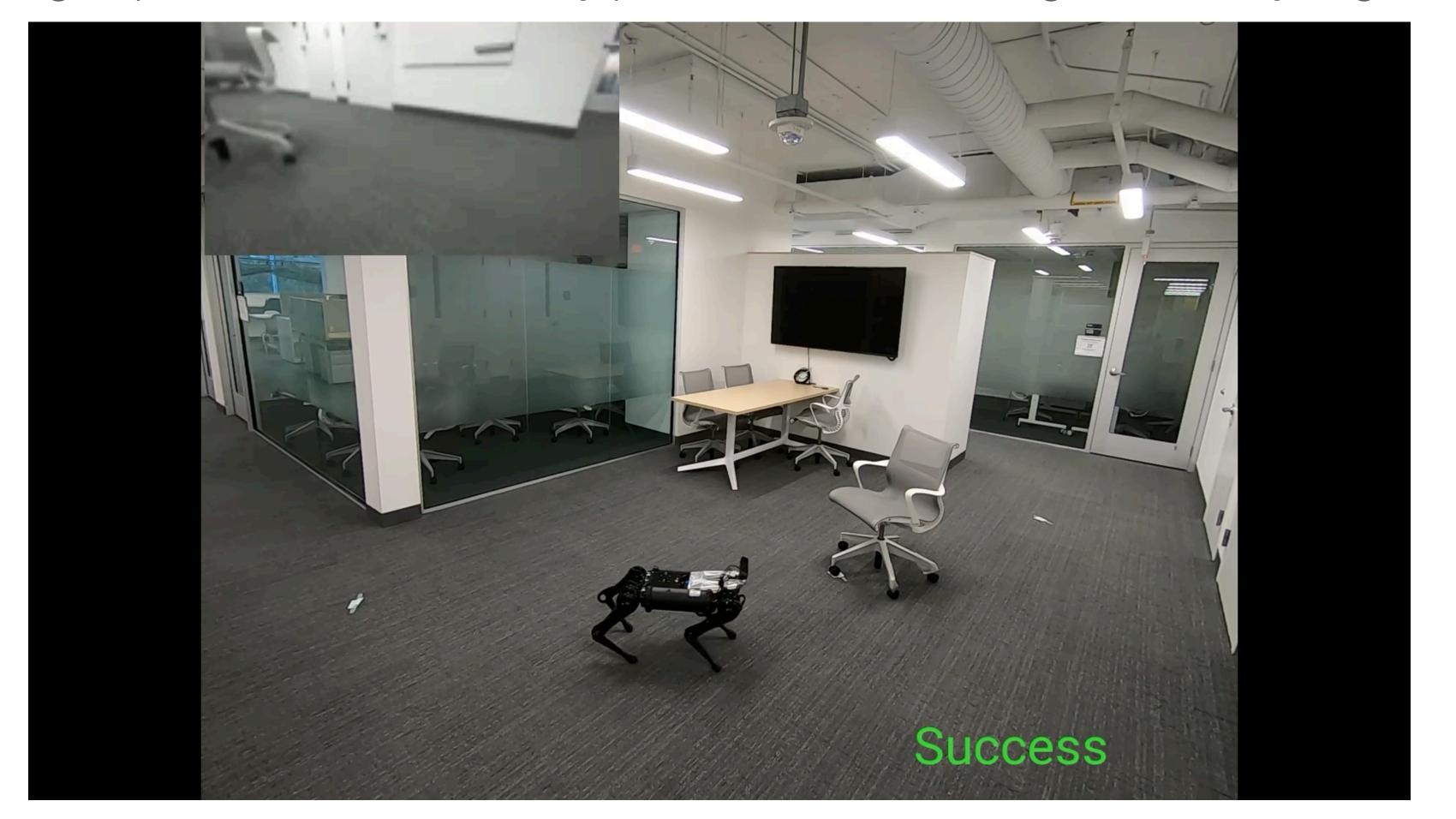
4: 
$$P_{\text{PAC}}^{\star} = \underset{P \in P}{\operatorname{argmin}} \ C_{\text{PAC}}(P) := \frac{1}{N} \sum_{E \in S} \underset{w \sim P}{\mathbb{E}} [C(r_w; E)] + \sqrt{\frac{\mathbb{D}(P \parallel P_0) + \log(\frac{2\sqrt{N}}{\delta})}{2N}}$$

5: 
$$C_{\text{bound}}^{\star} := \mathbb{D}^{-1} \left( C_S(P_{\text{PAC}}^{\star}) \| \frac{\mathbb{D}(P_{\text{PAC}}^{\star} \| P_0) + \log(\frac{2\sqrt{N}}{\delta})}{N} \right)$$

## Sim-to-Lab-to-Real

Fine-tune control policies in controlled environments before deployment.

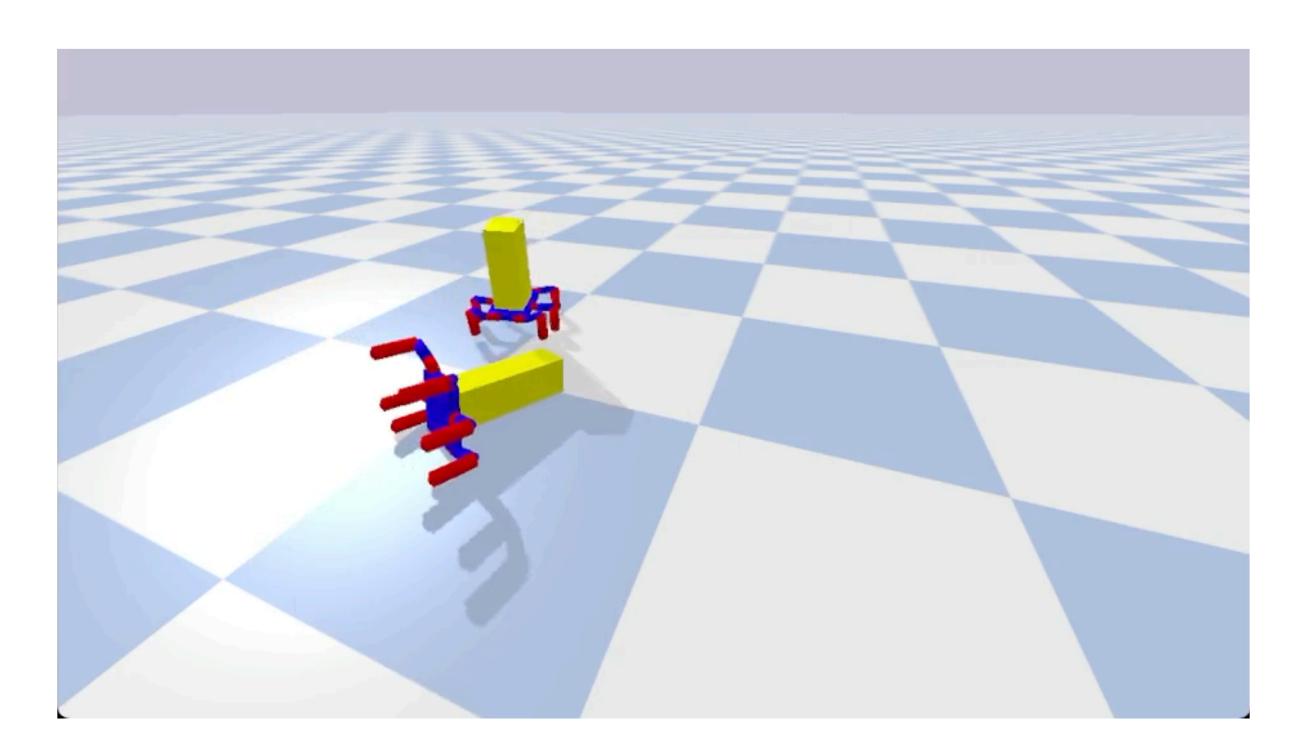
Co-training of performance and safety policies leads to stronger PAC-Bayes guarantees.

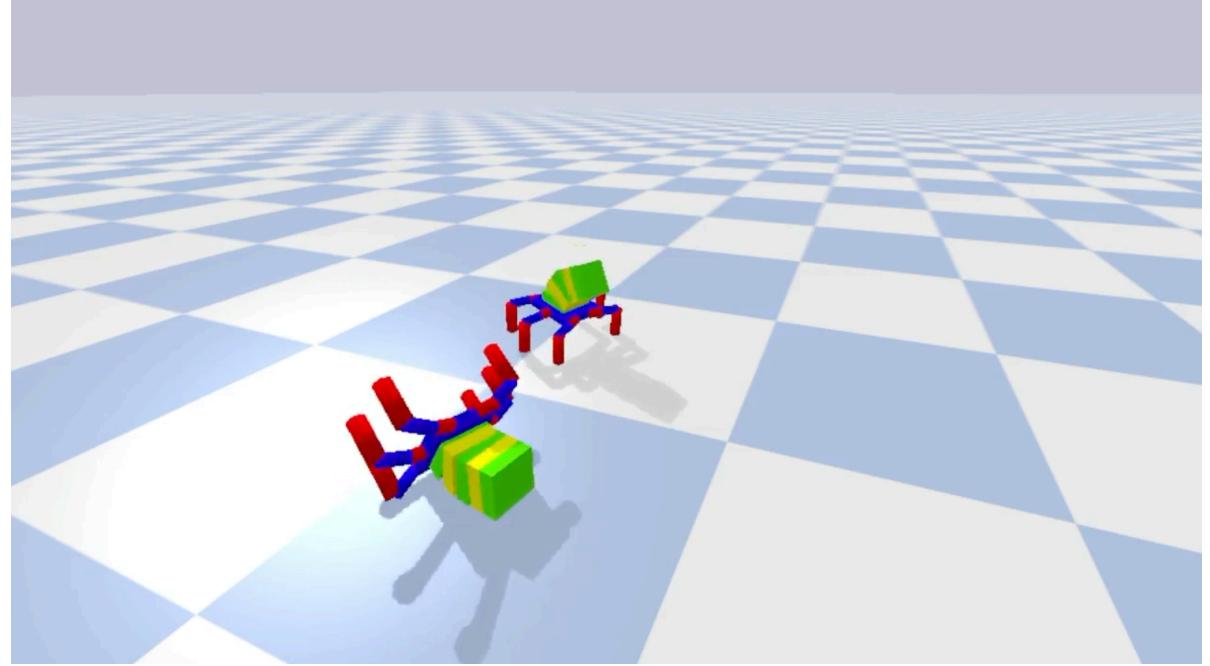


[Hsu, Ren, et al. Safe Reinforcement Learning with Shielding and Generalization Guarantees. Artificial Intelligence, 2022]

## Robustness beyond Training Conditions

Soft Actor-Critic trained with random disturbance (domain randomization) and no payload





Adversarial Safety Reinforcement Learning?













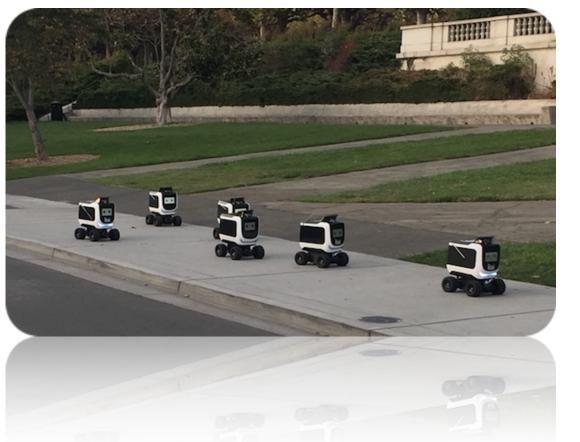












## Some Takeaways

Safe Learning is critical for many real-world RL applications (can't learn if you break).

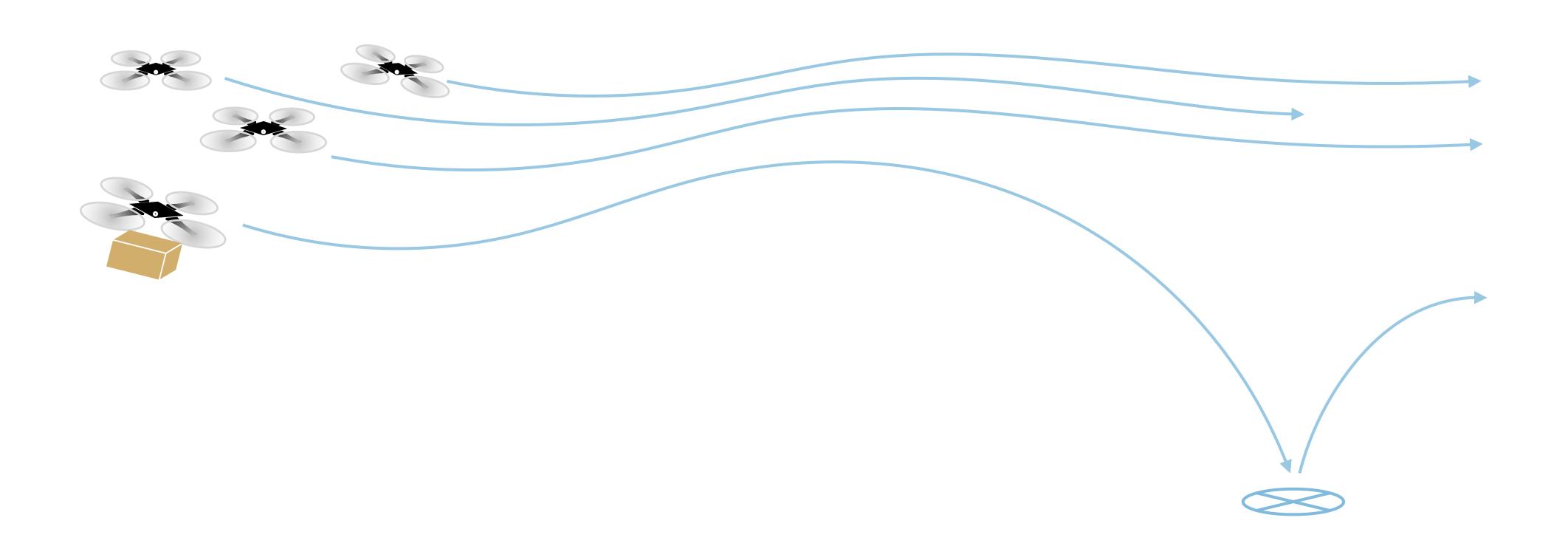
Safety Filters allow RL systems to explore and learn within a safety envelope.

Learning-based Safety Analysis improves scalability, flexibility and robustness of Safe RL.









jfisac@princeton.edu