MIT 8.01 Study Guide

Essentials of INTRODUCTORY CLASSICAL MECHANICS

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Sixth Edition

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A Study Guide to MIT Course 8.01

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Newton's Laws

First Law:	There exists inertial frames of reference in which any body left undisturbed maintains a constant velocity. (This is also known as the Law of Inertia.)
Second Law:	When a force is applied to a body, the body will experience an acceleration in the direction of the force; the magnitude of the acceleration is proportional to the magnitude of the force and inversely proportional to the mass of the body ($\vec{\mathbf{F}} = m\vec{\mathbf{a}} = \mathrm{d}\vec{\mathbf{p}}/\mathrm{d}t$).
Third Law:	The force exerted on a body A by a body B is equal in magnitude and opposite in direction to the force exerted by A on B.

Conserved Quantities

The following quantities are conserved in a closed (isolated) system:

- Total energy E_{tot}
- Total momentum $\vec{\mathbf{P}}_{\mathrm{tot}}$
- Total angular momentum $\vec{\mathbf{L}}_{\mathrm{tot}}$
- Total electric charge Q_{tot}

Some Macroscopic Forces

- Tension T
- Ideal spring force (Hooke's Law): $\vec{\mathbf{F}} = -k\vec{\mathbf{x}}$
- Force of kinetic friction: $|\vec{\mathbf{F}}_k| = \mu_k|\vec{\mathbf{N}}|$
- Force of static friction: $|\vec{\mathbf{F}}_s| \leq \mu_s|\vec{\mathbf{N}}|$
- Drag force (under certain conditions): $\vec{\mathbf{F}} = -k|\vec{\mathbf{v}}|\vec{\mathbf{v}}$

Some Definitions

Velocity $\vec{\mathbf{v}}$:	$\vec{\mathbf{v}} = \mathrm{d}\vec{\mathbf{r}}/\mathrm{d}t$	Angular velocity $\vec{\boldsymbol{\omega}}$:	$\lvert\vec{\boldsymbol{\omega}}\rvert = \mathrm{d}\theta/\mathrm{d}t$	Angular acceleration $\vec{\boldsymbol{\alpha}}$:	$\vec{\boldsymbol{\alpha}} = \mathrm{d}\vec{\boldsymbol{\omega}}/\mathrm{d}t$
Acceleration $\vec{\mathbf{a}}$:	$\vec{\mathbf{a}} = \mathrm{d}\vec{\mathbf{v}}/\mathrm{d}t$	Relation to $\vec{\mathbf{v}}$:	$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}$	Angular momentum $\vec{\mathbf{L}}$:	$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$
Momentum $\vec{\mathbf{p}}$:	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	Torque $\vec{\boldsymbol{\tau}}$:	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$	Pressure P:	$P = \lvert\vec{\mathbf{F}}\rvert/A$

Work W done by a force:	$\Delta W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$	Impulse $\vec{\mathbf{J}}$:	$\vec{\mathbf{J}} = \int_{t_1}^{t_2} \vec{\mathbf{F}}\mathrm{d}t = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1$
Rotational work about an axis:	$\Delta W = \lvert\vec{\boldsymbol{\tau}}\rvert\Delta\theta$	Coefficients of friction μ:	$\lvert\vec{\mathbf{F}}_s\rvert \leq \mu_s\lvert\vec{\mathbf{N}}\rvert$; $\lvert\vec{\mathbf{F}}_k\rvert = \mu_k\lvert\vec{\mathbf{N}}\rvert$
Moment of inertia I about an axis:	$I = \sum_i m_i r_{i,\perp}^2$	Kinetic temperature:	$\langle \frac{1}{2}mv^2 \rangle = \frac{3}{2}kT$
Power P:	$P = \mathrm{d}W/\mathrm{d}t$	Surface tension γ:	$\gamma = F/\ell = U/A$

Fundamental Forces

There are four fundamental forces in nature:

- Gravity: by far the weakest, but long range and almost always attractive; the dominant force on astronomical distance scales; Newton's law: $\vec{\mathbf{F}} = -G\dfrac{m_1m_2}{r^2}\hat{\mathbf{r}}$.
- Weak force: very short range (of order the size of an atomic nucleus); responsible for some radioactive decays (e.g., free neutron $\longrightarrow$ proton + electron + antineutrino, half-life = 10.2 minutes) and all nongravitational interactions of neutrinos.
- Electromagnetism: encompasses electric and magnetic forces; holds atoms and molecules together, and is responsible for light waves, radio waves, X-rays, etc.; Coulomb's law: $\vec{\mathbf{F}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r^2}\hat{\mathbf{r}}$.
- Strong force: very short range (of order the size of an atomic nucleus); binds 3 quarks to make up each proton and neutron; also holds protons and neutrons together in the nucleus.

There is a well-established "electroweak" theory unifying weak and electromagnetic forces, and it is quite likely that all the forces are unified.

The Greek Alphabet

Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ϵ	Rho	P	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ,φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

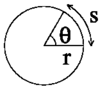
Some Useful Mathematics

Constants: $\pi = 3.1415927$
$e = 2.7182818$

Quadratic Formula:

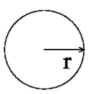
$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Circles



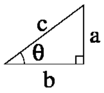
Circumference$=2\pi r$
Area$=\pi r^2$
$\theta_{rad} = s/r$

Spheres



Area$=4\pi r^2$
Volume$=\frac{4\pi}{3}r^3$

Triangles



$\sin\theta = a/c$
$\cos\theta = b/c$
$\tan\theta = a/b$
$a^2 + b^2 = c^2$

Derivatives

$${}^{d}/_{dx}\,[x^n] = nx^{n-1}$$
$${}^{d}/_{dx}\,[\sin(ax)] = a\cos(ax)$$
$${}^{d}/_{dx}\,[\cos(ax)] = -a\sin(ax)$$
$${}^{d}/_{dx}\,[\tan(ax)] = -a\sec^2(ax)$$
$${}^{d}/_{dx}\,[e^{ax}] = ae^{ax}$$
$${}^{d}/_{dx}\,[\ln(ax)] = 1/x$$

Integrals

$$\int x^n\,dx = x^{n+1}/(n+1) \text{ if } n \neq -1$$
$$\int (1/x)\,dx = \ln(x)$$
$$\int \sin(ax)\,dx = -\cos(ax)/a$$
$$\int \cos(ax)\,dx = \sin(ax)/a$$
$$\int \tan(ax)\,dx = -\ln[\cos(ax)]/a$$
$$\int e^{ax}\,dx = e^{ax}/a$$
$$\int \ln(ax)\,dx = x\ln(ax) - x$$

Diff Eqs

$$d^2x/dt^2 = a \Rightarrow x = x_0 + v_0\,t + \tfrac{1}{2}at^2$$
$$dx/dt = \alpha x \Rightarrow x = Ae^{\alpha t}$$
$$d^2x/dt^2 = -\omega^2 x \Rightarrow x = A\sin(\omega t + \phi)$$

Trigonometric Identities

$$\cos^2\theta + \sin^2\theta = 1$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin A \pm \sin B = 2\sin[(A \pm B)/2]\cos[(A \mp B)/2]$$
$$\cos A + \cos B = 2\cos[(A+B)/2]\cos[(A-B)/2]$$
$$\cos A - \cos B = 2\sin[(A+B)/2]\sin[(B-A)/2]$$

Series

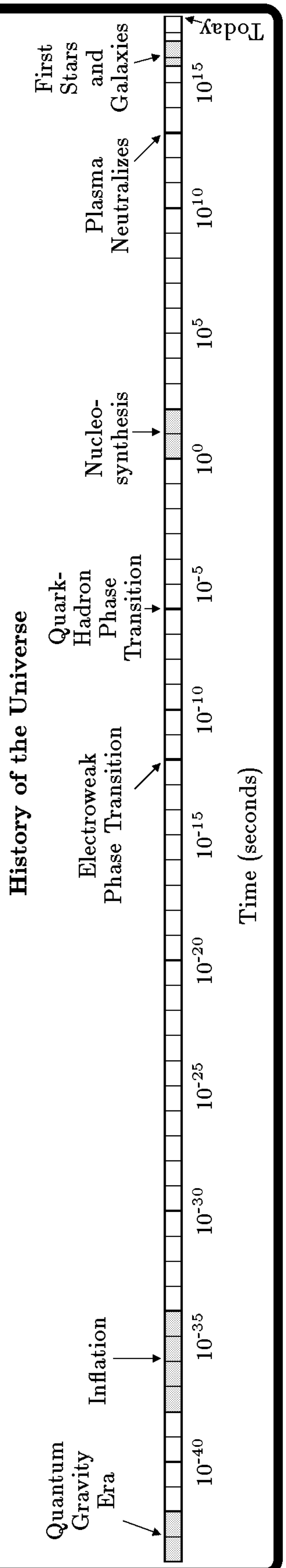
$$\sin\theta = \theta - \theta^3/3! + \ldots \quad \therefore \text{ for } |\theta| \ll 1 \quad \sin\theta \approx \theta$$
$$\cos\theta = 1 - \theta^2/2! + \ldots \quad \therefore \text{ for } |\theta| \ll 1 \quad \cos\theta \approx 1$$
$$e^x = 1 + x + x^2/2! + \ldots$$
$$f(x+h) \approx f(x) + hf'(x) + (h^2/2!)f''(x) + \ldots$$
$$(1+x)^n = 1 + nx + [n(n-1)/2!]\,x^2 + \ldots \quad \text{for } |x| < 1$$

Vector Identities

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = |\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}|\,\cos\theta = A_xB_x + A_yB_y + A_zB_z$$
$$\vec{\mathbf{A}} \times \vec{\mathbf{B}} = -\vec{\mathbf{B}} \times \vec{\mathbf{A}} = (A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x)$$
$$|\vec{\mathbf{A}} \times \vec{\mathbf{B}}| = |\vec{\mathbf{A}}|\,|\vec{\mathbf{B}}|\,\sin\theta$$

Fundamental Constants

Speed of light	c	2.99792458	$\times 10^{8}$ m/s (exact)
Permeability of vacuum	μ_0	4π	$\times 10^{-7}$ N/A^2 (exact)
Permittivity of vacuum	ϵ_0	8.85418782	$\times 10^{-12}$ F/m
Gravitational constant	G	6.6726	$\times 10^{-11}$ m^3/(kg $\cdot$ s^2)
Elementary charge	e	1.6021773	$\times 10^{-19}$ C
Planck's constant	h	6.626076	$\times 10^{-34}$ J $\cdot$ s
h-bar $\equiv h/2\pi$	$\hbar$	1.054573	$\times 10^{-34}$ J $\cdot$ s
Avogadro's number	N_A	6.022137	$\times 10^{23}$/mol
Electron mass	m_e	9.109390	$\times 10^{-31}$ kg
Proton mass	m_p	1.672623	$\times 10^{-27}$ kg
Rydberg constant	R_∞	1.09737315	$\times 10^{7}$/m
Fine structure const $e^2/(4\pi\epsilon_0\hbar c)$	α	7.2973531	$\times 10^{-3}$
$1/\alpha$	$1/\alpha$	137.03599	
Classical electron radius	r_e	2.8179409	$\times 10^{-15}$ m
Electron Compton wavelength	λ_C	2.4263106	$\times 10^{-12}$ m
Bohr radius	a_0	5.2917725	$\times 10^{-11}$ m
Boltzmann's constant	k	1.38066	$\times 10^{-23}$ J/K
Universal gas constant	R	8.31452	J/mol $\cdot$ K
Volume of ideal gas, STP		22414	cm^3/mol



The International System of Units (SI)

Quantity	**Unit**	**Symbol**	**Definition**
SI Base Units:			
Length	meter	m	"The meter is the length of path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."
Time	second	s	"The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom."
Mass	kilogram	kg	"The kilogram is the unit of mass, it is equal to the mass of the international prototype of the kilogram." (The international prototype is a platinum-iridium cylinder kept at the BIPM in Sèvres (Paris), France).
Electric current	ampere	A	"The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length."
Temperature	kelvin	K	"The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water."
Amount of substance	mole	mol	"The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon-12."
Luminous intensity	candela	cd	"The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of (1/683) watt per steradian."
SI Supplementary Units:			
Plane angle	radian	rad	"The radian is the plane angle between two radii of a circle that cut off on the circumference an arc equal in length to the radius."
Solid angle	steradian	sr	"The steradian is the solid angle that, having its vertex in the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere."

Some Derived Units

Quantity	Unit	Symbol	Base Units
Force	newton	N	$\mathrm{kg \cdot m/s^2}$
Energy	joule	J	$\mathrm{kg \cdot m^2/s^2}$
Power	watt	W	$\mathrm{kg \cdot m^2/s^3}$
Pressure	pascal	Pa	$\mathrm{kg/(m \cdot s^2)}$
Electric Charge	coulomb	C	$\mathrm{A \cdot s}$
Frequency	hertz	Hz	$\mathrm{s^{-1}}$

Metric (SI) Multipliers

Prefix	Symb	Value	Prefix	Symb	Value
exa	E	10^{18}	deci	d	10^{-1}
peta	P	10^{15}	centi	c	10^{-2}
tera	T	10^{12}	milli	m	10^{-3}
giga	G	10^{9}	micro	μ	10^{-6}
mega	M	10^{6}	nano	n	10^{-9}
kilo	k	10^{3}	pico	p	10^{-12}
hecto	h	10^{2}	femto	f	10^{-15}
deka	da	10^{1}	atto	a	10^{-18}

Conversion Factors

Length	1 fermi = 1 fm = 10^{-15} m
	1 angstrom (Å) = 10^{-10} m
	1 inch = 2.54 cm = 0.0254 m
	1 mile = 1609 m
	1 light-year (ly) = 9.46×10^{15} m
	1 parsec = 3.26 ly = 3.09×10^{16} m
Mass	1 atomic mass unit (u) = 1.6605×10^{-27} kg
	1 pound (lb avoirdupois) = 0.4536 kg
Time	1 year = 3.156×10^{7} s
Volume	1 liter (L) = 10^{-3} m^3
Angle	1 degree (°) = $\pi/180$ rad = 0.01745 rad
Pressure	1 atmosphere (atm) = 1.013×10^{5} Pa
Temperature	Zero degree Celsius (0° C) = 273.15 K
	$T\,(^\circ\mathrm{F}) = (9/5)T\,(^\circ\mathrm{C}) + 32^\circ$
Energy	1 electron volt (eV) = 1.602×10^{-19} J

The Chemical Elements

Element	Sym	No	Weight
Actinium	Ac	89	227.028
Aluminum	Al	13	26.9815
Americium	Am	95	(243)
Antimony	Sb	51	121.75
Argon	Ar	18	39.948
Arsenic	As	33	74.9216
Astatine	At	85	(210)
Barium	Ba	56	137.33
Berkelium	Bk	97	(247)
Beryllium	Be	4	9.01218
Bismuth	Bi	83	208.9804
Boron	B	5	10.81
Bromine	Br	35	79.904
Cadmium	Cd	48	112.41
Calcium	Ca	20	40.08
Californium	Cf	98	(251)
Carbon	C	6	12.011
Cerium	Ce	58	140.12
Cesium	Cs	55	132.9054
Chlorine	Cl	17	35.453
Chromium	Cr	24	51.996
Cobalt	Co	27	58.9332
Copper	Cu	29	63.546
Curium	Cm	96	(247)
Dysprosium	Dy	66	162.50
Einsteinium	Es	99	(252)
Erbium	Er	68	167.26
Europium	Eu	63	151.96
Fermium	Fm	100	(257)
Fluorine	F	9	18.9984
Francium	Fr	87	(223)
Gadolinium	Gd	64	157.25
Gallium	Ga	31	69.72
Germanium	Ge	32	72.59
Gold	Au	79	196.9665
Hafnium	Hf	72	178.49
Helium	He	2	4.00260
Holmium	Ho	67	164.930
Hydrogen	H	1	1.0079
Indium	In	49	114.82
Iodine	I	53	126.9045
Iridium	Ir	77	192.22
Iron	Fe	26	55.847
Krypton	Kr	36	83.80
Lanthanum	La	57	138.9055
Lawrencium	Lr	103	(260)
Lead	Pb	82	207.2
Lithium	Li	3	6.941
Lutetium	Lu	71	174.967
Magnesium	Mg	12	24.305
Manganese	Mn	25	54.9380
Mendelevium	Md	101	(258)
Mercury	Hg	80	200.59
Molybdenum	Mo	42	95.94
Neodymium	Nd	60	144.24
Neon	Ne	10	20.179
Neptunium	Np	93	237.0482
Nickel	Ni	28	58.69
Niobium	Nb	41	92.9064
Nitrogen	N	7	14.0067
Nobelium	No	102	(259)
Osmium	Os	76	190.2
Oxygen	O	8	15.9994
Palladium	Pd	46	106.42
Phosphorus	P	15	30.97376
Platinum	Pt	78	195.08
Plutonium	Pu	94	(244)
Polonium	Po	84	(209)
Potassium	K	19	39.0983
Praseodymium	Pr	59	140.9077
Promethium	Pm	61	(145)
Protactinium	Pa	91	231.0359
Radium	Ra	88	226.0254
Radon	Rn	86	(222)
Rhenium	Re	75	186.207
Rhodium	Rh	45	102.9055
Rubidium	Rb	37	85.4678
Ruthenium	Ru	44	101.07
Samarium	Sm	62	150.36
Scandium	Sc	21	44.9559
Selenium	Se	34	78.96
Silicon	Si	14	28.0855
Silver	Ag	47	107.868
Sodium	Na	11	22.98977
Strontium	Sr	38	87.62
Sulfur	S	16	32.06
Tantalum	Ta	73	180.9479
Technetium	Tc	43	(98)
Tellurium	Te	52	127.60
Terbium	Tb	65	158.9254
Thallium	Tl	81	204.383
Thorium	Th	90	232.0381
Thulium	Tm	69	168.9342
Tin	Sn	50	118.69
Titanium	Ti	22	47.88
Tungsten	W	74	183.85
Unnilennium	Une	109	?
Unnilhexium	Unh	106	(263)
Unniloctium	Uno	108	?
Unnilpentium	Unp	105	(262)
Unnilquadium	Unq	104	(261)
Unnilseptium	Uns	107	(264)
Ununnilium	Uun	110	?
Uranium	U	92	238.029
Vanadium	V	23	50.9415
Xenon	Xe	54	131.29
Ytterbium	Yb	70	173.04
Yttrium	Y	39	88.9059
Zinc	Zn	30	65.39
Zirconium	Zr	40	91.224

Parentheses indicate most stable isotope of radioactive elements. "?" indicates value is in dispute.

Some Physical Quantities

Densities	Air at STP	1.29 kg/m^3
	Water at 20° C, 1 atm	1.00×10^3 kg/m^3
	Ice at STP	0.917×10^3 kg/m^3
	Aluminum at 20° C, 1 atm	2.702×10^3 kg/m^3
	Iron at 20° C, 1 atm	7.860×10^3 kg/m^3
	Lead at 20° C, 1 atm	11.33×10^3 kg/m^3
Specific Heats	Water at 20° C, 1 atm	4190 J/(kg·K)
	Aluminum at 20° C, 1 atm	900 J/(kg·K)
	Iron at 20° C, 1 atm	447 J/(kg·K)
	Lead at 20° C, 1 atm	159 J/(kg·K)
Melting/ Boiling Points	Water	273/373 K
	Aluminum	934/2740 K
	Iron	1808/3023 K
	Lead	601/2013 K

Latent heat:	
of fusion of water	3.33×10^5 J/kg
of vaporization of water	2.26×10^6 J/kg
Speed of sound in air	343 m/s
Typical range of audible frequencies	20 Hz-16,000 Hz
Free-fall acceleration	9.80 m/s^2

Astronomical Quantities

Solar Temp	Surface	5.8×10^3 K
	Center	1.6×10^7 K
Masses	Earth	5.97×10^{24} kg
	Moon	7.35×10^{22} kg
	Sun	1.99×10^{30} kg
Radii	Earth	6.38×10^6 m
	Moon	1.74×10^6 m
	Sun	6.96×10^8 m
Distances	Earth-sun	1.496×10^{11} m
	Earth-moon	3.84×10^8 m
	Nearest star	4.04×10^{16} m
	Galactic Center	2.2×10^{20} m
	Andromeda Galaxy	2.1×10^{22} m

Energy Scales

Supernova Explosion	10^{46} J
Solar power incident on Earth	2×10^{17} W
350-kiloton nuclear warhead	10^{15} J
1 ton of coal	2.6×10^{10} J
Electric output, large power plant	10^9 W
1 gallon of gasoline	1.3×10^8 J
Food energy used by a human	10^2 W

INTRODUCTION

WHAT IS CLASSICAL MECHANICS?

This is an introductory book on Classical Mechanics. Mechanics is the branch of science that deals with the motion of objects, how that motion changes with time, the conditions required to induce certain types of motion, etc. *Classical* Mechanics restricts us to circumstances where the speeds we encounter are small compared to the speed of light and the objects we deal with are generally of macroscopic size. Fortunately, almost any situation we are likely to meet in everyday life satisfies these restrictions, so the results of classical mechanics have a wide variety of applications in science and engineering. Furthermore, some of the most important principles of mechanics—such as the conservation laws for energy and momentum—can be fully explored within classical mechanics.

Why is it important to study classical mechanics? We can think of four reasons:

- The modern scientific view of the world, to a large extent, begins with classical mechanics. Newer developments, such as quantum theory and relativity, have all grown from roots in classical mechanics.
- The contents of the subject—the physical laws and principles you will learn, and the methods of applying them to practical problems—are important and relevant in many other fields. A civil engineer designing a bridge, an automobile designer laying out the specifications for the engine or the safety air-bag of a new model, a geologist estimating the likely severity of the next California earthquake: all are using, directly or indirectly, the principles of classical mechanics.
- The structure and development of classical mechanics is a good example of the aims and methods of scientific study. We will see how experimental results and mathematical representations are combined to create testable scientific theories, and how the impossible complexities of most real-life physical situations can be reduced to soluble problems by identifying the essential physical features of the system. This way of working is what distinguishes the scientific approach to situations from the many other ways of looking at them (e.g. artistic, political, business.).
- The study of classical mechanics is an excellent introduction to the art of problem solving. When you finish this book you should be able to extract the essential features of a problem, use them to set up and solve the appropriate mathematical equations, and make quick and easy checks on your answer to catch simple mistakes.

The book will have succeeded in its aims if you come away from it with a grasp of the basic principles governing the motion of objects, a feel for the scientific method, and a strengthened ability to wrestle with difficult problems until they are solved.

HOW IS THE BOOK ORGANIZED?

The book is organized with a fairly rigid structure, to make it as easy as possible for you to locate the information that you want. There are 13 chapters, each of which consists of about one week's work for a student in a freshman physics course. A typical chapter contains:

- a brief ***Overview*** setting out the main themes of the chapter;
- ***the Essentials***, a concise but complete discussion of the topic, explaining what you need to know and giving cross-references to related problems;
- a ***Summary*** of the material covered to help you review the topic and to provide a handy reference guide for problem solving;
- a set of ***Problems and Questions*** designed for self-testing and for sharpening problem-solving skills.

Answers are given to all numerical problems. In addition, some problems come with hints to help you get started, while others have fully worked-out solutions to show you how to apply the ideas and equations in the *Essentials* to problem solving. Some of the worked solutions include comments on general problem-solving techniques or on the relevance of the particular problem to other areas of physics.

Furthermore, many chapters include ***Supplementary Notes*** which discuss some aspects of the material in a wider context, such as how particular points relate to the real world or how they may be developed into more advanced concepts. You don't need to know this material to progress to the next chapter, but it should provide a starting point if you are curious to see how the artificial-seeming problems you may be doing fit into the rest of physics.

About every third chapter (Chapters 3, 7, 10, and 13) consists of review problems rather than new material. The problems in these chapters tend to be slightly more challenging and may use physics from more than one of the preceding chapters. In this sense they are a better representation of "real" applications of classical mechanics than the more specialized problems in normal chapters.

HOW TO USE THIS BOOK

Each chapter (except for the review problem chapters) consists of two different types of material. One type ***defines*** what you ought to know: this includes the *Overview,* the *Essentials,* and the checklist of new ideas in the *Summary.* The second type ***applies*** this knowledge to problem solving: in this category are the *Problems and Questions, Solutions,* and *Hints.*

The *Essentials,* as the name suggests, are the heart of the book, and your main tool for acquiring the information you will need. They are intended to include everything that you will need to solve the problems and master the material of the chapter. Our goal in writing the *Essentials* was to be as concise as possible, but not more so. We hope that in most cases you will appreciate and benefit from this conciseness, but we recognize that you may sometimes want a more detailed discussion. For such cases we recommend that you consult one of the more standard introductory physics textbooks.

If the *Essentials* are the main tool for ***accessing*** the necessary information, the main ***learning activity*** should center on the *Problems and Questions.* You haven't really understood a given topic until you can apply it in solving problems; conversely, the step-by-step process of setting up and solving a problem will often be of more help in grasping a complicated idea than reading an abstract theoretical explanation. For that reason, problems come in three varieties:

- S-type problems, which come with completely worked out solutions;
- H-type problems, which come with hints in the form of questions, and answers to these questions;
- problems with just the answer given.

You will probably find that in many cases the worked solutions will be very useful, but you need not study them in detail if you already know how to solve them. You should nonetheless check them for comments (marked ☺ Learn), which may be of more general relevance.

SOLVING PROBLEMS

Solving problems is a key part of classical mechanics, or indeed any field of science. The theoretical and mathematical frameworks we construct are only of value if they can be applied to understand the behavior of the physical world. Therefore, one of the objectives of this course is to help you to develop your problem-solving abilities. One way to do this is to adopt a general ***problem-solving strategy***. This section outlines such a strategy, and the worked solutions you find in this book will normally follow the steps shown here. We believe this is a useful framework for attacking any new problem—but feel free to use any method that works for you! The guidelines are not rigid: for some problems, one or two of the steps shown may be unnecessary, while for more complicated situations you may have to apply some of them more than once.

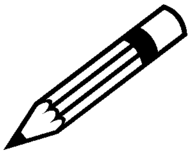
Some parts of the approach described here may not seem natural at first. Why think through the whole problem conceptually before starting on the math, instead of writing down the equations straightaway? Why calculate everything with symbols first, instead of putting in the numbers immediately? With practice, we think you will agree that working from general physical concepts down to specific numerical values is usually the most effective way to solve problems: it minimizes the risk of making simple numerical errors, and it usually does more to help develop your physical intuition.

Step 1: ***Conceptualize***

Read the problem through carefully, noting the information you are given and the information you are asked for. If appropriate, draw a diagram of the situation. Decide which physical concepts are involved and which areas of the theory you have learned will be relevant. Think through your approach to solving the problem.

Step 2: ***Formulate***

Express your verbal concepts in mathematical terms. This implies identifying the necessary equations, defining the proper symbols, choosing the appropriate reference frames, etc. We strongly recommend that you introduce symbols to represent any numerical values that you are given. Make sure that you know the physical significance of all the symbols you have introduced. Check that your formulation makes sense: do you have enough equations to calculate all your unknown quantities, for example? Work out a strategy for solving the equations.



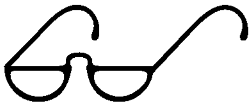
Step 3: ***Solve***

Solve your equations algebraically, i.e. rearrange them so that the quantity you want to evaluate is expressed in terms of other quantities whose values are known. It is usually best to do this symbolically, with algebra, rather than numerically, with arithmetic, for several reasons:

- the algebraic solution is more general: you can substitute in more than one set of numbers, which may be useful later in the problem;
- mistakes are easier to find;
- the physical behavior of the system should be easier to visualize.

Once you have the algebraic solution, substitute numerical values if you have been asked to do so.

(The only common exceptions to this advice are problems with multiple parts which are not closely connected; in such cases it may well be easier to evaluate each answer numerically before going on.)



Step 4: ***Scrutinize***

Always check to see if your answer makes sense.

- One of the most powerful tools for doing this is ***dimensional analysis***. To find the ***dimensions*** of a quantity, we express it in terms of more basic concepts: for example, a velocity, whether measured in m/s, miles per hour, or furlongs per fortnight, is always a length divided by a time: [velocity] = [length]/[time], where square brackets denote "dimensions of". Dimensional analysis involves determining the dimensions of each term in an equation and asking two questions: (i) are they the same (it is meaningless to add quantities with different dimensions—the sum "1 kg + 2 m" is nonsensical), and (ii) are they what we expect (if we are calculating a distance, we expect its dimensions to be [length], not, say, [length]/[mass])? Note, however, that dimensional analysis cannot uncover errors which involve pure numbers, such as a missing minus sign, or a factor of $\frac{1}{2}$ or 2π.
- Missing minus signs or numerical factors can often be caught by considering ***special cases*** which are easy to visualize. In a problem involving two masses, for example, we might ask if the solution behaves sensibly when one mass becomes vanishingly small, or extremely large.
- In problems which have numerical solutions, you should also ask yourself if the magnitude of the numerical value seems reasonable: for example, if you were asked to calculate the speed of a car engaged in a collision, an answer of 700 mph would seem unlikely to be correct!

If the authors of this book were given 10 cents every time a student submitted a test answer which he or she could have known was obviously wrong, we would be quite rich.

Step 5: ***Learn***

Once you are convinced that your solution is correct, take the time to look at how it fits into what you already know. Does it explain phenomena you have noticed in everyday life, but not understood? Is it unexpected or surprising? Does it lead you to make predictions about more complicated systems? Does it illustrate the use of some technique that might be useful for other problems? Have you understood the problem well enough so that now you will be able to quickly solve problems that are closely related? You should find that problem solving gives you much more insight into the physics you are learning than simply reading the theory.

NOTE: This approach to problem-solving was suggested by a similar strategy outlined in ***Physics: The Nature of Things***, by Susan M. Lea and John Robert Burke.