ESSENTIALS OF INTRODUCTORY CLASSICAL MECHANICS

Sixth Edition

CHAPTER 10

MOTION OF A RIGID BODY

OVERVIEW

This chapter contains no new ideas. Instead, we shall take the material covered in chapters 8 and 9 and use it to solve more complicated problems. Notice that in this chapter we shall not divide the problems up into sections dealing with specific topics; instead you will have to decide for yourself which of the physical principles you have learned are relevant to a given problem.

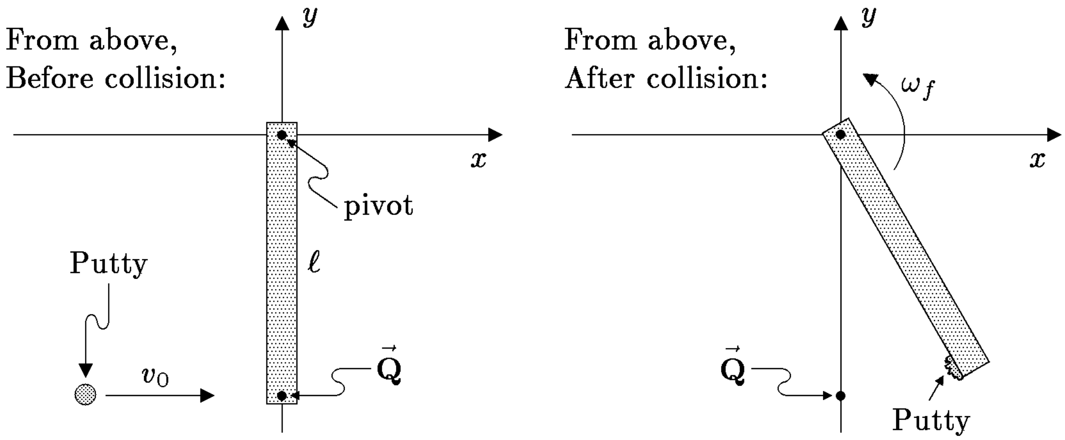
When you have completed this chapter you should:

- ✔ be able to extract the essential features of a problem and express them in mathematical equations;
- ✔ be capable of manipulating the equations relevant to a problem to obtain an expression for the required quantity, either symbolically or numerically;
- ✔ be able to analyze a hypothetical situation in terms of the physics presented in previous units, and explain that analysis clearly in non-mathematical terms.

PROBLEMS AND QUESTIONS

By the end of this chapter you should be able to answer or solve the types of questions or problems stated below.

At the end of the chapter there are answers to all the problems. In addition, for problems with an (H) or (S) after the number, there are respectively hints on how to solve the problems or completely worked-out solutions.



10.1 (S) On a frictionless horizontal table a slender rigid rod of mass M and length ℓ is attached at one end to a fixed pivot. A nonrotating disk of putty, with mass m, is moving with speed v_0 perpendicular to the line of the rod, and collides with it at point Q, at the opposite end from the pivot. The disk sticks to the end of the rod, so that after the collision they move off together. Assume that the disk is small enough to be treated as a point particle.

(a) Just before the collision, what is the angular momentum of the system of rod plus disk (i) about the the z-axis, (ii) about an axis perpendicular to the page, through the point Q?

(b) Just after the collision, what is the angular velocity of the rod/disk system about the pivot? What is the system's angular momentum about the axis perpendicular to the page and through the point Q? Explain.

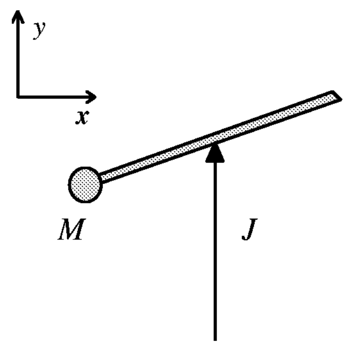
(c) Now suppose that the disk is uniform and has a small radius R. It is initially rotating counterclockwise (as seen from above) with an angular speed ω_0 so large that the resulting internal angular momentum cannot be neglected (although R is still negligible compared to ℓ). If the disk's translational speed when it hits the rod is the same as before, what is the final angular velocity about the pivot in this case?

10.2 (H) Classical mechanics indicates that it is theoretically possible to balance a pencil on its point, but in practice it can't be done. Discuss why this is so by considering the torque acting on the pencil when it is displaced by a small angle from the vertical. Further discuss why, if you attempt this experiment on a very smooth surface, the point of the pencil tends to move in the direction opposite to the direction in which the pencil is toppling.

10.3 (H) A figure skater executing a spin starts with her arms extended and then brings them in close to her body. What happens to (a) her angular momentum, (b) her angular velocity, (c) her rotational kinetic energy during this procedure? How would the process be affected if she held a weight in each hand? Reconcile these results with the principle of energy conservation.

10.4 (H) At time $t = 0$, a particle of mass 5 kg is located at position [3, -2, 7] m and moving with velocity [1, 1, -2] m/s. Find its angular momentum about the origin. If it is acted on by a constant force of [0, 3, 0] N, what is the torque on it at $t = 0$, and what will its angular momentum be 10 s later?

10.5 A small disk of mass M is attached to one end of a slender rigid rod of mass M and length ℓ. (The radius of the disk is negligible compared to ℓ.) The assembly is placed on a frictionless horizontal table such that the rod is oriented at an angle θ to one edge of the table, which we define as the x-axis of coordinates, as shown. An impulse $\vec{\mathbf{J}} = [0, J, 0]$ is applied to the midpoint of the rod. What is the subsequent motion of the system, i.e. what is its velocity vector $\vec{\mathbf{v}}$ and its angular velocity $\vec{\omega}$?



10.6 A cylinder rotates about its horizontal axis of symmetry. It is dropped from a small distance onto a carpet. It does not bounce. Describe, as well as you can, what does happen and why.

10.7 (H) A car accelerates from rest to 100 km/h in 7.5 s. Its wheels have a radius of 25 cm. What is the average angular acceleration of the wheels relative to their axles? If we can approximate each wheel by a solid cylinder of mass 15 kg, and the mass of the car (without the four wheels) is 1,200 kg, (a) what average net torque must have been applied to each wheel; (b) what average torque must have been applied to each wheel by the axle?

10.8 A car of ***total*** mass M has four identical wheels, each with a radius R and a moment of inertia about its axle of I. The car is moving with a constant speed v_1 in a straight line on a level road, with the wheels rolling (not skidding). The coefficients of static and kinetic friction between the wheels and the road are μ_s and μ_k respectively. Assume that air drag is negligible and that there is no internal friction within the car (i.e. all wheel bearings etc. are frictionless). The acceleration due to gravity is g.

(a) What is the magnitude of the angular velocity ω of each wheel?

(b) In terms of ω and the given quantities, what is the angular momentum of each wheel about its axle?

(c) What is the net force on the car due to friction?

A deer runs on to the road ahead. The driver brakes suddenly, locking the wheels, so that they stop rotating. The car skids and decelerates until its speed is v_2. During the period when the wheels are locked,

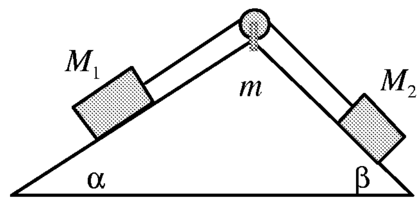
(d) What is the magnitude, a, of the acceleration of the car?

(e) In terms of v_1, v_2 and a, what distance does the car travel?

Meanwhile the deer has seen the car and retreated back into the bushes. The driver therefore releases the brakes when the speed of the car has reached v_2.

(f) In no more than 50 words, describe (without formulae or equations) what happens when the brakes are released, and the final motion of the car.

10.9 (H) Two blocks of mass M_1 and M_2 are connected by a light rope running over a pulley of mass m. The rope turns the pulley without slipping. If the coefficient of kinetic friction between the blocks and the slope is μ_k, find the acceleration of the blocks and the tension in each part of the rope. Assume the pulley is a solid uniform disk of radius R, and that M_2 is substantially greater than M_1.



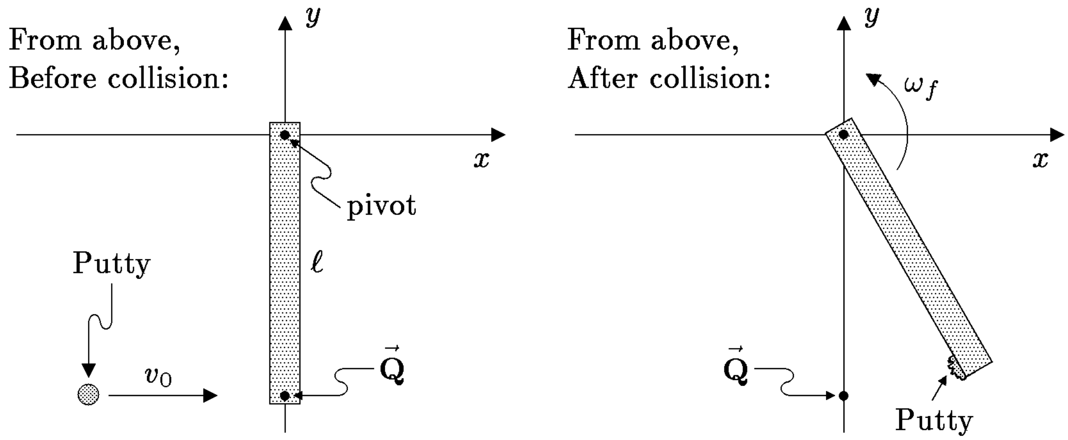
10.10 A yo-yo consists of two solid disks connected by a central spindle. Each disk has mass $M = 20$ g and radius $R = 2.5$ cm; the central spindle has radius $r = 0.5$ cm and negligible mass. The yo-yo has 1 m of string attached; you may assume the string is of negligible mass and thin enough that it does not change the effective diameter of the spindle when it is wound up. If the yo-yo is released from rest, use energy conservation methods to deduce its angular velocity when it reaches the end of the string (assume the string was fully wound at the start and unwinds without slipping). If the string is firmly attached to the yo-yo, what do you expect it to do after reaching the end? What if the last piece of string is tied in a loop around the spindle rather than being firmly attached? Take $g = 10$ m/s^2 and quote your results to two significant figures.

Draw a force diagram for the descending yo-yo, and hence calculate its acceleration. What effect would (a) decreasing the diameter of the central spindle; (b) decreasing the mass of the yo-yo; (c) decreasing the outer radius of the yo-yo have on its acceleration?

10.11 (S) A yo-yo consists of two solid disks, each of mass M and radius R, connected by a central spindle of radius r and negligible mass. A light string is coiled around the central spindle; we shall assume that its thickness is sufficiently small that winding or unwinding it has no effect on the effective radius of the spindle. The yo-yo is placed upright on a flat surface and the string is pulled gently at an angle θ to the horizontal. What happens? Assume that the surface is rough enough that the yo-yo rolls without slipping.

10.12 An anthropologist has discovered a cache of seven pre-Columbian statuettes in the forests of central America. The ceramic statues are approximately cylindrical in shape and all apparently identical, but our hero has reason to believe that one of them is a hollow fake constructed by ingenious drug smugglers. The fake has been ballasted to weigh exactly the same as the genuine articles, and of course our man is unwilling to do anything which might damage the precious artifacts. X-ray machines and similar sophisticated technology not being available in the jungle, use the physics of the last few chapters to suggest non-destructive ways of identifying the not-so-genuine article.

COMPLETE SOLUTIONS TO PROBLEMS WITH AN (S)



10.1 *On a frictionless horizontal table a slender rigid rod of mass M and length ℓ is attached at one end to a fixed pivot. A nonrotating disk of putty, with mass m, is moving with speed v_0 perpendicular to the line of the rod, and collides with it at point Q, at the opposite end from the pivot. The disk sticks to the end of the rod, so that after the collision they move off together. Assume that the disk is small enough to be treated as a point particle.*

(a) *Just before the collision, what is the angular momentum of the system of rod plus disk (i) about the the z-axis, (ii) about an axis perpendicular to the page, through the point Q?*

(b) *Just after the collision, what is the angular velocity of the rod/disk system about the pivot? What is the system's angular momentum about the axis perpendicular to the page and through the point Q? Explain.*

Conceptualize

All the motion in this problem takes place in the xy plane, and the question asks only for angular momenta about various axes that are perpendicular to the xy-plane. So we are only concerned with L_z, and can effectively treat angular momentum as a scalar quantity, using the techniques of Chapter 8. The external forces acting are gravity and the normal force from the table (which cancel, since there is no vertical acceleration), and possibly a contact force from the pivot.

Part (a) is straightforward. Initially the rod is stationary, and therefore has no angular momentum about any point, so we need only consider the angular momentum of the putty about the two axes (parallel to the z-axis), through the pivot and through Q.

Part (b) is tricky. During the impact of the putty with the rod, the rod exerts a force on the pivot. By Newton's third law, the pivot exerts an equal and opposite force on the rod. Since we assume the pivot is frictionless, it does not exert a torque on the rod about the pivot. Thus this is a collision problem with an external impulsive force acting on the rod at the pivot during the collision.

As usual in collision problems, we first ask what quantities are conserved.

10.1, continued:

The collision is inelastic (the putty sticks to the rod), so energy is not conserved. Because of the impulsive force, angular momentum is also not conserved except about the pivot.

Since angular momentum is conserved about the pivot, we can use this fact to solve for the motion of the rod and putty after the collision. Once we have found the motion of the rod and putty we can calculate their angular momentum about any other point, including Q, using the fact that for any rigid body, the total angular momentum about any point is equal to the angular momentum of the center of mass about that point plus the angular momentum of the body about the center of mass (see p. 320). This will avoid us having to deal with the impulsive force at the pivot.

Formulate and Solve (a)
Just before the collision the position and velocity of the putty are given by $\vec{\mathbf{r}}_{\text{putty}} = [0, -\ell, 0]$ and $\vec{\mathbf{v}}_0 = [v_0, 0, 0]$ so $\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\text{putty}} \times m\vec{\mathbf{v}}_0 = [0, 0, m\ell v_0]$. So the angular momentum about the z-axis is $L_z = m\ell v_0$. With respect to Q, immediately before the collision $\vec{\mathbf{r}}_{\text{putty}} = [0, 0, 0]$, so clearly $\vec{\mathbf{L}} = [0, 0, 0]$. The angular momentum about the axis through the point Q is therefore zero.

Formulate (b)
About the pivot the total angular momentum of the system after the collision is

$$L_f = (I_{\text{rod}} + I_{\text{putty}})\,\omega_f,$$

where ω_f is the final angular velocity. Since we concluded that angular momentum about the pivot is conserved, this must be equal to $m\ell v_0$.

We saw in Chapter 8 that the moment of inertia of a slender rod about one end is $\frac{1}{3}M\ell^2$ (see problem 8C.4(d)). Therefore

$$m\ell v_0 = \omega_f\left(\tfrac{1}{3}M\ell^2 + m\ell^2\right)$$
$$\omega_f = \frac{m v_0}{\left(\frac{1}{3}M + m\right)\ell},$$

counterclockwise around the z-axis.

Immediately after the collision, the rod's center of mass has velocity $v = \omega_f \ell/2$, and the angular velocity of the rod about its center of mass is ω_f.

Thus the angular momentum of the center of mass of the rod about an axis through Q and parallel to the z-axis is

$$-Mv\frac{\ell}{2} = -M\omega_f\frac{\ell}{2}\frac{\ell}{2} = -M\omega_f\frac{\ell^2}{4}\ ,$$

and the angular momentum of the rod about its center of mass is

$$\frac{1}{12}M\ell^2\omega_f\ ,$$

10.1, continued:

where we have obtained the moment of inertia of a rod about the axis through its center from Chapter 8, p. 277.

Thus the total angular momentum of the rod about Q is

$$-\frac{1}{4}M\ell^2\omega_f + \frac{1}{12}M\ell^2\omega_f = -\frac{1}{6}M\ell^2\omega_f \ ,$$

where we have used

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\rm cm} \times \vec{\mathbf{p}}_{\rm tot} + \sum_i \vec{\mathbf{r}}_{{\rm rel},i} \times m_i \vec{\mathbf{v}}_{{\rm rel},i}$$

from Chapter 9. The putty still has zero angular momentum about Q, so this is the angular momentum of the system.

Angular momentum about Q clearly is ***not*** conserved. We noted in the conceptualization that there is a net external force on the system, namely the contact force from the pivot. In the instant after the collision, this force must have a component in the negative x-direction, since this end of the rod remains stationary instead of moving off in the positive x-direction as it would if it were unrestrained. Such a force contributes no torque ***about the pivot***, but it clearly ***does*** contribute a torque about Q. Therefore it is entirely reasonable that angular momentum about Q is not conserved in the collision.

Scrutinize

In (b), our expression for ω_f has dimensions of 1/[time], which is correct for an angular velocity. If m becomes very small, ω_f tends to zero, which is reasonable: a zero-mass lump of putty would obviously not set the rod in motion. If M becomes negligible, $\omega_f = v_0/\ell$: the angular velocity of the putty does not change as a result of the collision. This is also reasonable—the putty is moving in a circle, but its speed has not changed.

Learn

This problem demonstrates that conservation of angular momentum can only be used to analyze the motion of a system ***about an appropriate point or axis***. This is because angular momentum depends on $\vec{\mathbf{r}}$, and $\vec{\mathbf{r}}$ depends on the choice of origin. In particular, angular momentum is conserved when a body moves under the influence of a central force, but only if the angular momentum is calculated with respect to the source of the force. The effects of this can be seen in considering, for example, the effect of Jupiter on the orbit of a comet: if the comet passes close to Jupiter, the ensuing deflection will conserve the comet's angular momentum ***with respect to Jupiter***, but it will not conserve the comet's angular momentum with respect to the Sun, and the comet's orbit can be very significantly changed as a result.

10.1 (c) *Now suppose that the disk is uniform and has a small radius R. It is initially rotating counterclockwise (as seen from above) with an angular speed ω_0 so large that the resulting internal angular momentum cannot be neglected (although R is still negligible compared to ℓ). If the disk's translational speed when it hits the rod is the same as before, what is the final angular velocity about the pivot in this case?*

10.1, continued:

Conceptualize

The only difference between this problem and part (b) is that the putty's angular momentum now includes a contribution from its own rotation (its ***spin angular momentum***, as opposed to its ***orbital angular momentum*** about the pivot). This is not a serious difficulty, because as we saw earlier the total angular momentum of a rigid body can be expressed as the sum of two parts: its angular momentum about its center of mass, and the angular momentum of its center of mass about the origin. Thus we can simply add the spin angular momentum to the orbital angular momentum we calculated in part (b). Angular momentum is still conserved in the collision: the forces that act to squash the putty against the rod are internal forces in the rod/disk system, and any torques they create must cancel out.

Formulate

As before the angular momentum of a rigid body is given by

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}_{\mathrm{cm}} \times \vec{\mathbf{p}}_{\mathrm{tot}} + \sum_i \vec{\mathbf{r}}_i \times m_i \vec{\mathbf{v}}_i ,$$

where $\vec{\mathbf{r}}_i$ and $\vec{\mathbf{v}}_i$ are measured relative to the body's center of mass, and $\vec{\mathbf{r}}_{\mathrm{cm}}$ is measured relative to the reference point for the angular momentum. For our spinning disk this gives

$$L_z = mv_0\ell + I_{\mathrm{disk}}\omega_0 .$$

The moment of inertia of a uniform disk about a perpendicular axis through its center is $\frac{1}{2}mR^2$, so

$$L_z = m\left(v_0\ell + \frac{1}{2}\omega_0 R^2\right) .$$

Solve

We simply replace $mv_0\ell$ in our solution for part (b) by this new value of L_z to obtain

$$\omega_f = \frac{m\left(v_0\ell + \frac{1}{2}\omega_0 R^2\right)}{\left(\frac{1}{3}M + m\right)\ell^2} .$$

Scrutinize

This obviously reduces to our previous answer if $\omega_0 R^2$ is negligible, which is reassuring. The effect of the disk's spin is to increase the overall angular momentum, since the spin is in the same direction as the post-collision rotation of the rod. We can see how this comes about: as the disk collides with the rod, its lower edge is moving faster than its upper edge ($v_0 + \omega_0 R$ and $v_0 - \omega_0 R$, respectively), and therefore the lower edge imparts a greater impulse to the rod than does the upper edge. The result is that the disk exerts a net torque on the rod, causing the rod to rotate more rapidly than it would have done if the disk were not rotating. We would have seen the opposite effect if the disk had been rotating clockwise rather than counterclockwise.

10.11 *A yo-yo consists of two solid disks, each of mass M and radius R, connected by a central spindle of radius r and negligible mass. A light string is coiled around the central spindle; you may assume that its thickness is sufficiently small that winding or unwinding it has no effect on the effective radius of the spindle. The yo-yo is placed upright on a flat surface and the string is pulled gently at an angle θ to the horizontal. What happens? Assume that the surface is rough enough that the yo-yo rolls without slipping.*

Conceptualize

In principle this set-up contains two distinct problems: what is the net force on the yo-yo (what will its linear acceleration be?) and what is the net ***torque*** on it about some suitable point (what will its angular acceleration about that point be?)? However, if the yo-yo rolls without slipping, the linear acceleration determines the angular acceleration and vice versa. We can therefore combine the force and torque equations to solve the problem.

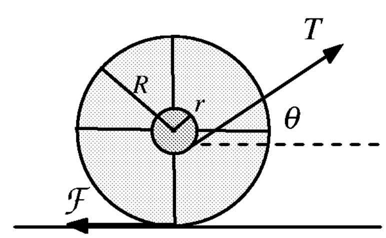
Before formulating the torque equations, we must decide on a suitable reference point. There is more than one possible choice, but the most natural approach is probably to consider rotation about the yo-yo's center of mass. If we do this we have effectively split the problem into rotation ***about*** the center of mass and translational motion ***of*** the center of mass, with the condition of rolling without slipping to link the two.

Since the question states that the string is pulled ***gently***, we can assume that the force applied is not sufficient to lift the yo-yo off the floor, so the vertical component of the net force remains zero throughout. All the forces act, and the yo-yo moves, in the plane of the page, and therefore if we call this the xy-plane only the z-component of torque is non-zero: the vector nature of torque is not going to be important in solving this problem.

$\Sigma\int$

Formulate

There are four forces acting on the yo-yo: the gravitational force $2M\vec{\mathbf{g}}$, a normal force $\vec{\mathbf{N}}$ from the floor, friction $\vec{\mathcal{F}}$ and the tension T in the string. However, we are only interested in the torque and the horizontal component of the net force. To avoid complicating the force diagram too much we will therefore omit $\vec{\mathbf{N}}$ and $2M\vec{\mathbf{g}}$, both of which act vertically through the center of mass and therefore contribute neither a torque nor a horizontal component.



If the yo-yo rolls without slipping, there must be no relative motion at the point of contact with the floor. Therefore the speed of the outer edge of the yo-yo must be $-v$, where v is the speed of the yo-yo's center of mass. This implies $v = \pm\omega R$ and $a = \pm\alpha R$, where the relative sign depends on the directions of translation and rotation we define to be positive. Choosing counterclockwise rotation and rightward translation to be positive selects $v = -\omega R$ and $a = -\alpha R$. With this sign choice the force and torque equations are:

$$F = T\cos\theta - \mathcal{F}$$

$$\tau = Tr - \mathcal{F}R.$$

10.11, continued:

To calculate the acceleration of the yo-yo we use $F = ma$ and $\tau = I\alpha$. The total mass of the yo-yo is obviously $2M$; its moment of inertia about its central axis is MR^2 (since the two solid disks each contribute $\frac{1}{2}MR^2$).

Solve

Setting $F = 2Ma$ and $\tau = MR^2\alpha = -MRa$ gives

$$2Ma = T cos\theta - \mathcal{F}$$
$$MaR = -Tr + \mathcal{F}R,$$

and hence (dividing the second equation by R and adding it to the first)

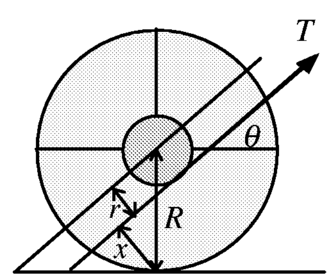
$$a = \frac{T}{3M}\left(\cos\theta - \frac{r}{R}\right).$$

This is positive for $\cos\theta > r/R$ (yo-yo will roll to the right) and negative for $\cos\theta < r/R$ (yo-yo will roll to the left). If $\cos\theta = r/R$, the torque and the horizontal force are both zero and the yo-yo won't roll at all. If you pull hard enough, either the vertical component of the tension will exceed the weight of the yo-yo (in which case the yo-yo will lift off the ground), or the horizontal component of tension will exceed the maximum static friction force (in which case it will skid, rather than roll, to the right). Analysis of the two limiting cases (zero normal force, i.e. $T\sin\theta = 2Mg$, and $\mathcal{F} = \mu_s N$) shows that, regardless of μ_s, the limit for skidding occurs at a lower tension than the limit for lifting: the yo-yo skids before it lifts.

Scrutinize and Learn

Note that these solutions are only valid if the yo-yo rolls without slipping, since we used the relation between a and α to solve the equations. If the yo-yo rolls and slips, we have nothing to tie the torque and force equations together, and thus cannot solve the problem without more information (e.g. the value of μ_k). We can determine the minimum value of μ_s needed to ensure that the yo-yo does roll without slipping by substituting our derived value for a back into the equations to determine $\mathcal{F}$, and then setting this equal to $\mu_s N$.

What is the significance of r/R as a value of $\cos\theta$? We can see this most easily by looking at the yo-yo's motion in an unconventional way. Because the yo-yo does not slip, the point of contact between the yo-yo and the ground has zero velocity when it starts to move. We can therefore look at this initial instant of motion as a rotation of the yo-yo about this point of contact. (The point of contact forms an instantaneous axis of rotation for the yo-yo.) If we use this view-point, then the moment of inertia of the yo-yo is (by the parallel-axis theorem) $3MR^2$, and only the tension in the string contributes a torque (friction, the normal force and gravity all act through the point of contact and thus produce no torque). The torque produced is Tx, where x is the perpendicular distance between the line of action of the tension and the point of contact. From the diagram, $\cos\theta = (r + x)/R$, and thus the net torque



10.11, continued:

is $T(R\cos\theta - r)$. Dividing by the appropriate moment of inertia gives us the same angular acceleration as we had using the more obvious choice of axis through the center of the yo-yo, but this time the significance of r/R is clear—this is the value of $\cos\theta$ for which $x = 0$, i.e. the line of action of the tension passes through the point of contact, and therefore contributes no torque. There is then zero net torque around this axis, and the yo-yo cannot roll. Looking a little more carefully, you can also see that the sign of the torque around this axis changes for $\theta > r/R$, because the line of action of the tension will pass to the right rather than the left of the point of contact.

Working with an instantaneous axis of rotation is somewhat like choosing a different coordinate system or reference frame in a translational-motion context. It is never ***necessary*** to work this way in order to solve a problem, but it may ***simplify*** the solution or make the underlying physical principles more apparent.

HINTS FOR PROBLEMS WITH AN (H)

The number of the hint refers to the number of the problem

10.2 Draw a force diagram for the pencil at a small angle to the vertical. What are the forces acting? Which contributes a torque about the pencil point, and in what direction does that torque act?

What is happening to the ***linear*** momentum of the pencil as it topples? What force must act to cause this?

10.3 Is the skater an isolated system? What quantities are conserved for isolated systems?

Does the skater exert any net force or torque on herself? Does she do any work?

If you are still puzzled, look back at problem 4D.1.

10.4 This problem is clearly an exercise in the use of vectors in three dimensions. What do we need to know to calculate the torque exerted on and angular momentum of a particle about a specified point?

What is the momentum of the particle at $t = 10$ s? What is its position then?

(Use the coordinate form of the vector product to calculate $\vec{\mathbf{r}} \times \vec{\mathbf{p}}$ and $\vec{\mathbf{r}} \times \vec{\mathbf{F}}$ in this problem.)

10.7 (a) How is the angular acceleration of the wheels related to the linear acceleration of the car?

(b) Draw a force diagram for one wheel.

(c) What average force does the road surface apply to the car if the magnitude of its average acceleration is a? What torque about the wheel axis does this force apply to the wheels?

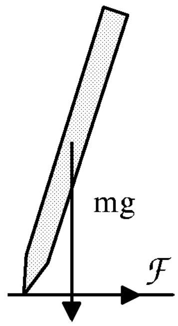
10.9 If you are having trouble with this problem, first look back at problem 8E.6, a very similar setup without the complication of friction. The same approach applies:

Draw free-body diagrams for the pulley and each mass. Does the tension in the two parts of the rope (on either side of the pulley) have to be the same? What quantity is the same for the two masses and the rim of the pulley?

How is the angular acceleration of the pulley related to the linear acceleration of the blocks?

ANSWERS TO HINTS

10.2 Gravity and friction; gravity; away from the equilibrium position.



The pencil gains both vertical and horizontal linear momentum; gravity produces vertical component; horizontal component must come from friction.

10.3 Yes, there is no net external force or torque on her (her center of mass is on her axis of rotation). Linear and angular momentum.

No, (by the Third Law, she cannot exert a net force on herself; in pulling her arms in she exerts a force parallel to the line joining her arms to the axis, so there is no torque).

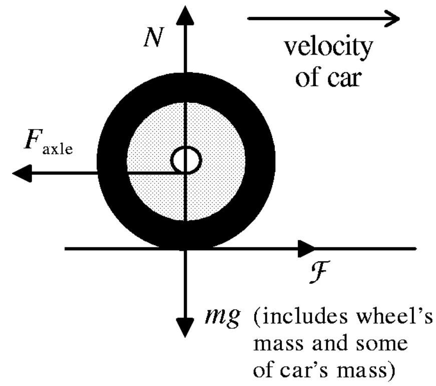
Yes, (in pulling in her arms, she exerts a force on her hands and forearms, which undergo a non-zero displacement in the direction of the force).

10.4 The vectors representing (a) the position of the particle relative to the point in question; (b) the force on the particle and (c) the particle's linear momentum.

$[5, 35, -10]$ kg · m/s;
$[13, 38, -13]$ m.

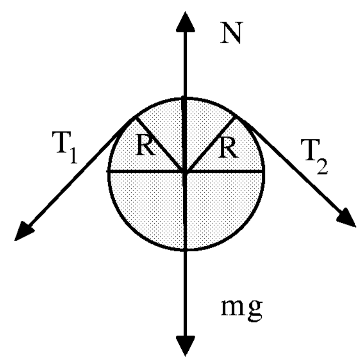
10.7 (a) $a = r\alpha$.

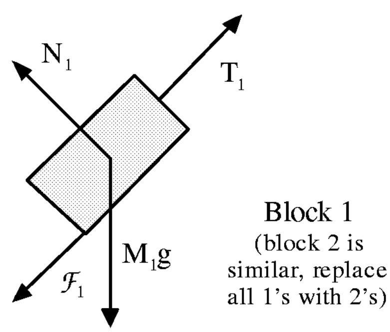
(b)



(c) Ma; a total of Mar, where M is the mass of the car and r is the radius of the wheels.

10.9





No: T_1 for block 1 $= T_1$ on pulley, and likewise for T_2, but $T_1 \neq T_2$.

$\alpha = a/R$, where a is the linear acceleration of either block (they are equal), and α is the angular acceleration of the pulley.

ANSWERS TO ALL PROBLEMS

10.1 See complete solution.

10.2 Any small displacement from vertical produces a torque which tends to increase the displacement; if point does not move, center of mass of pencil gains net horizontal momentum, which implies a net horizontal force—the only such force acting is friction.

10.3 (a) unchanged; (b) increased; (c) increased; increase of angular velocity and K.E. would be greater; work done by skater's arms in moving weights.

10.4 $[-15, 65, 25]$ kg · m²/s; $[-21, 0, 9]$ N · m; $[75, 65, 265]$ kg · m²/s.

10.5 $\vec{\mathbf{v}} = \left[0, \dfrac{J}{2M}, 0\right]$; $\omega = \dfrac{6J}{5M\ell}\cos\theta$ ($\vec{\omega}$ in positive z direction).

As soon as the cylinder touches the carpet, a horizontal frictional force will act to oppose the relative motion between the carpet and the cylinder. This will have two effects: first, it will obviously tend to slow down the rotation (it produces a net torque acting in the direction opposite to the direction of rotation); secondly, as it is the only horizontal force acting, it will produce a horizontal acceleration of the center of mass of the cylinder. Initially, the frictional force will be kinetic friction, which has a constant magnitude of $\mu_k N$, so the cylinder will have a uniform horizontal acceleration and a constant (negative) angular acceleration: it will skid sideways with steadily increasing velocity, while continuing to rotate at a steadily decreasing rate. Eventually, however, the condition for rolling without slipping must be met (we started with $v = 0$ and $\omega > 0$, so $v < \omega R$: v is increasing and ω is decreasing, so at some point we must have $v = \omega R$). At that moment there is no relative motion between the carpet and the part of the cylinder in contact with it, so the frictional force goes to zero and there is no net force on the cylinder. It should then continue to roll at constant speed indefinitely. In practice, there are additional drag forces we have not considered (air resistance, deformation of the carpet pile, etc.), so the cylinder will slow down and finally stop.

10.7 15 rad/s²; (a) 6.9 N·m; (b) 300 N·m

10.8 (a) $\omega = v_1/R$

(b) $L = I\omega$

(c) Zero

(d) $g\mu_k$

(e) $\left(v_1^2 - v_2^2\right)/2a$

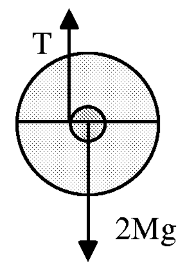
(f) Kinetic friction continues to decelerate the car while simultaneously increasing the angular velocity of the wheels. Eventually the condition for rolling without slipping is met, friction drops to zero, and the car continues at constant speed (if the driver does not accelerate).

10.9 $a = g\dfrac{M_2\sin\beta - M_1\sin\alpha - \mu_k(M_2\cos\beta + M_1\cos\alpha)}{M_1 + M_2 + \frac{1}{2}m}$;

$T_1 = M_1(a + g(\sin\alpha + \mu_k\cos\alpha))$;

$T_2 = M_2(g(\sin\beta - \mu_k \cos\beta) - a)$, where a is given above.

10.10 $\sqrt{2gh/\left(r^2 + \frac{1}{2}R^2\right)} = 240$ rad/s, where $h = 1$ m is the length of the string;



climb back up string to previous height; continue to spin, slowing due to friction between loop of string and surface of spindle (or climb back up, if friction sufficient to prevent slipping).

$$a = \frac{2r^2 g}{2r^2 + R^2} = 0.74 \text{ m/s}^2.$$

(a) decrease; (b) none; (c) increase.

10.11 See complete solution.

10.12 Although the fake statue has been ballasted to give the same total mass as the genuine objects, it is most unlikely that the ***distribution*** of mass within the statue is the same. Our hero can check this in a number of ways:

- find the position of the center of mass of each statue (by balancing it on something, or hanging it from a rope and adjusting the rope position until it hangs horizontally);
- measure the moment of inertia of the statue about one end, by suspending the statue by that end from a short rope and measuring the period of small oscillations, as in Problem 8E.4;
- measure the moment of inertia about a short axis through the center of mass, by suspending the statue horizontally by a rope (or a pair of ropes) around its middle and measuring the period of small oscillations produced by twisting the ropes (this is a torsion pendulum, as in problem 8D.3: the torque produced by the twisted ropes is proportional to $-\Delta\theta$, where $\Delta\theta$ is the angle of twist);
- measure the moment of inertia about the long axis of the statue, by rolling it down an inclined plane and measuring its velocity, as in Problem 8E.8. (He would, of course, package the statue in a light but well-padded cylindrical barrel before attempting this!)