ESSENTIALS OF INTRODUCTORY CLASSICAL MECHANICS

Sixth Edition

CHAPTER 12

FLUID MECHANICS

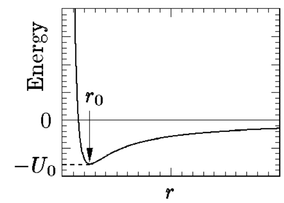
OVERVIEW

In real substances, the ideal-gas assumption of non-interacting molecules is never completely accurate. There is always some force acting between molecules. Under some conditions of pressure and temperature the intermolecular forces are strong enough to keep the molecules packed closely together, so that the substance no longer expands to fill its container completely: the gas has become a ***liquid***. In this chapter we shall investigate the behavior of liquids, using a combination of Newton's laws and some of the concepts we developed in our study of gases.

When you have completed this chapter you should:

- ✔ recognize the similarities and differences between liquids and gases;
- ✔ be able to calculate the pressure in a liquid under different conditions;
- ✔ understand the phenomenon of buoyancy and Archimedes' principle;
- ✔ know how to use Bernoulli's equation to solve problems involving fluid flow, and understand its derivation in terms of energy conservation for the moving fluid;
- ✔ know what is meant by surface tension, and have a qualitative understanding of its origin in terms of intermolecular forces;
- ✔ have a qualitative understanding of the properties of real liquids.

ESSENTIALS



We saw in the last chapter that in an ideal gas intermolecular forces are assumed to be negligible. However, if we imagine cooling a real gas (so that its constituent molecules are moving more slowly) or compressing it (so that they are closer together) we will reach a state where the intermolecular forces are ***not*** negligible. In these circumstances the gas will diverge from the behavior expected of an ideal gas; if we continue the process it will eventually condense into a ***liquid***. (We call the gas, liquid and solid states of a substance ***phases***, and the sudden change from one to another is a ***phase transition***.) The molecular interpretation of the gas-liquid phase transition is that the magnitude of the potential energy of the molecules due to the forces between them is comparable to their kinetic energy, so that they have a strong tendency to position themselves in the minimum of potential energy, at separations of r_0 (see diagram at right).

What are the characteristics of a liquid? Because the intermolecular separation is essentially fixed, the density must be nearly constant regardless of external conditions, and thus the volume occupied by a given mass of liquid is fixed. Unlike a gas, a liquid does not expand to fill its container. On the other hand, the molecules of the liquid are still free to move randomly, and so the ***shape*** of the liquid is not fixed (this is the basic difference between liquids and solids). Since the rise in intermolecular potential energy with decreasing separation below r_0 is not quite vertical, it is in fact possible to compress real liquids slightly; they also (like solids) expand or contract slightly with changing temperature. However, these changes are so small compared to those we found in the case of an ideal gas that it is reasonable to idealize liquids as having absolutely fixed density (such an ideal liquid is called ***incompressible***) and no sensitivity to temperature. Because the intermolecular forces are important, the motion of molecules in a liquid is much more complicated to calculate than the ideal gas: we cannot construct a simple kinetic theory of liquids. Instead we use Newton's laws to analyse behavior of liquids ***in bulk***. Much of this analysis is also applicable to gases, and it is therefore given the name ***fluid mechanics*** (a fluid is either a liquid or a gas — literally something that flows).

Our analysis will obviously use the standard concepts of mass, volume and density. In addition, the concept of pressure that we developed in the ideal-gas model is still applicable (the molecules in a liquid, though subject to intermolecular forces, are still moving and can collide with walls and other molecules). We do not have an equivalent of the ideal-gas law to calculate the pressure or temperature directly, but we can use Newton's laws to determine how the pressure within a liquid varies with position.

Consider a small volume element ΔV of liquid within a larger sample. If the liquid is in stable equilibrium, no net force must act on this small volume element. We know there is a downward force on it: its weight $\Delta mg = \rho g \Delta V$, where Δm is the mass of our small element and ρ is its density (so $\Delta m = \rho \Delta V$). This must be balanced by a difference between the pressure from below (acting upwards) and the pressure from above (acting down) to give zero net force, i.e.

$$\rho g \, \Delta V - PA + (P + \Delta P)A = 0 \ ,$$

where A is the horizontal cross-section of our volume element, i.e.

$$\Delta V = A \, \Delta y \ ,$$

taking the y-axis to be vertical. Dividing the equation by ΔV yields, in the limit of small Δy,

$$\frac{\mathrm{d}P}{\mathrm{d}y} = -\rho g \ .$$

This is valid for any fluid, even if the density depends on the pressure as in a gas. For a liquid ρ is a constant, so we can integrate this equation very easily to get

$$\boxed{P_2 - P_1 = -\rho g (y_2 - y_1)}$$

for the difference in pressure between two different vertical coordinates within the liquid. The obvious choice of reference point is the surface of the liquid, which we can define to be $y = 0$; the pressure at any depth h below the surface is then

$$P = P_0 + \rho g h$$

Problems 12A.

(note that a positive h corresponds to a negative y, and thus to a higher pressure). We see that the pressure depends only on the pressure at the surface and the depth below the surface — as in the case of a gas, it does not depend on the shape of the container. Our analysis deals directly only with the vertical pressures, but since we expect the motion of molecules in liquids to be randomly oriented our molecular understanding of pressure tells us that the pressure in other directions will also be given by this equation.

This equation is simpler than the corresponding relation for gases, because the density in this case is a constant; in a gas, the density increases with increasing depth.

The same argument can be applied to understanding the phenomenon of ***buoyancy***. If we have a mass m of liquid (of arbitrary shape), its weight is $m\vec{g}$ acting through its center of mass. If it is contained within a larger sample of the same liquid, the net force on it due to pressure from the surrounding liquid must be $-m\vec{g}$ through the center of mass, assuming the whole system is in equilibrium. If we then replace this mass of liquid by another body of the same shape, the net force from pressure remains the same (since the surrounding liquid is unaffected). The object therefore feels an upward ***buoyant force*** of magnitude mg and a ***net*** force $(M - m)\vec{g}$, where M is its own mass.

This result is known as Archimedes' principle. It is valid for gases as well as liquids, since it does not require constant density.

As we saw in Chapter 9, the weight $M\vec{g}$ acts through the center of mass of the body. Since the buoyant force $-m\vec{g}$ balances the weight of the original mass of liquid, it acts as if through ***the liquid's*** center of mass, which is referred to as the ***center of buoyancy*** of the immersed object. Since the center of mass and the center of buoyancy do not in general coincide, the buoyant force may exert a net torque about the object's center of mass.

Problem 13.3

If $M > m$ obviously the net force is downward, and the object sinks. If $M < m$ there is a net upward force and the object will rise towards the surface of the liquid. Once it breaks the surface the upward force on it changes, because only part of its volume is surrounded by liquid: the net force is thus $(M - \rho V)\vec{g}$, where ρ is the density of the liquid and V is the volume of the submerged part of the object. Equilibrium is reached when

Problems 12B.

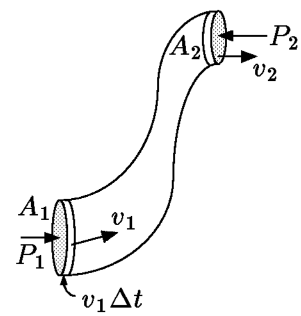
$$M = \rho V \ ,$$

i.e. the mass of the object is equal to the mass of liquid corresponding to the submerged part of its volume. We call this the amount of liquid ***displaced*** by the floating object. [The mass of a ship is often quoted in terms of its ***displacement***, i.e. the mass of water it displaces when afloat.]

So far we have considered static liquids. This approach ignores an obvious feature of fluids: as the name implies, they flow. Fluid flow is actually a very complicated phenomenon and extremely difficult to calculate for real situations (hence the use of wind tunnels to ***measure*** fluid flow around complex shapes such as auto bodies and airplanes), but we can use basic conservation principles to deduce results which are useful in simple situations.

The simple situations we shall consider involve steady flow of an ***ideal liquid***. Steady flow means that the flow pattern is constant over time: a small element of liquid that is initially at some point (x, y, z) will always follow the same subsequent path. (The paths followed by such small elements of liquid are called ***flow lines***; the curves defined by the direction of the liquid's velocity at any point are called ***streamlines***. For steady flow these concepts are interchangeable, since the path of an element of liquid is clearly determined by the direction of its velocity.) An ideal liquid is incompressible and has no internal friction, so the motion of a small element is not affected by that of neighboring elements.

In steady flow, flow lines do not cross, because an element of liquid at a given point has a unique, well-defined velocity which in turn defines a unique flow line: we cannot have a point with two possible flow lines. Therefore we can define a ***flow tube*** as a bundle of neighboring flow lines, and any liquid element which is inside this flow tube at any given time will stay inside it. Given that we have a steady flow, it follows that the mass of liquid ***entering*** any section of the tube in a time interval Δt must be equal to the amount ***leaving*** the section in the same interval.



Problems 12C.1 and 12C.2.

Consider a narrow flow tube, so that the velocity and pressure of the fluid do not vary across it. If the cross-sectional area of the tube where the fluid enters is A_1 and its speed at that point is v_1, the mass of fluid entering the tube in a time interval Δt is $\rho A_1 v_1 \,\Delta t$, and the mass leaving is similarly $\rho A_2 v_2 \,\Delta t$. Equating these gives us the ***equation of continuity***

$$\boxed{A_2 v_2 = A_1 v_1 \ .}$$

Since $Av\,\Delta t$ is just the volume of the small cylinder of fluid which moves past a specific point on the flow tube in time Δt, we can also express this as the ***volume flow rate***

$$\frac{\mathrm{d}V}{\mathrm{d}t} = Av = \text{constant} \ .$$

The ***mass flow rate***, $\mathrm{d}m/\mathrm{d}t$, is given by $\rho\,\mathrm{d}V/\mathrm{d}t$.

If we now consider the work done on the fluid in this section of the tube during time Δt, we observe that the work done by the external pressure P_1 is

$$P_1 A_1 v_1 \,\Delta t \ ,$$

since $P_1 A_1$ is the force and $v_1 \Delta t$ is the distance through which it acts. Similarly, the work done by external pressure P_2 is

$$-P_2 A_2 v_2 \,\Delta t \ .$$

(the minus sign enters because this pressure is directed opposite to the direction of motion). The net work done by pressure inside the section of tube is zero, by Newton's third law—only external forces contribute. Hence the total work done is

$$P_1 A_1 v_1 \,\Delta t - P_2 A_2 v_2 \,\Delta t = (P_1 - P_2)\,\Delta V \ .$$

The kinetic energy of the water in the volume $A_1 v_1 \,\Delta t$ is

$$\frac{1}{2}(\rho A_1 v_1 \,\Delta t) v_1^2 = \frac{1}{2}\rho v_1^2 \,\Delta V \ ,$$

and likewise for volume $A_2 v_2 \Delta t$, so the change in kinetic energy of the fluid originally inside our section of flow tube is

$$\Delta K = \frac{1}{2}\rho \,\Delta V \,(v_2^2 - v_1^2) \ ,$$

(The middle of the section is full of fluid in both cases, so clearly we need only consider the difference between the two small volumes at the ends.)

The change in ***potential*** energy is clearly

$$\Delta U = \rho g \,\Delta V \,(y_2 - y_1) \ ,$$

where y_1, y_2 are the heights of each end of the tube. Assuming the internal energy of an ideal fluid is constant (part of the same assumption that fluid is incompressible and at constant temperature) we can apply conservation of energy $\Delta W = \Delta K + \Delta U$, and obtain

$$P_1 - P_2 = \frac{1}{2}\rho \left(v_2^2 - v_1^2\right) + \rho g (y_2 - y_1) \ .$$

This is ***Bernoulli's equation***. It is often convenient to rearrange it into the form

Problems 12C.3 through 12C.8.

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2}$$

i.e. the sum of the pressure, the kinetic energy per unit volume, and the potential energy per unit volume is the same at any point in the flow. If the liquid is ***stationary***, this equation reduces to the equation for the variation of pressure with depth for a static liquid, as it must for consistency.

We derived this equation for an incompressible fluid in steady flow, but it can also be used for gases provided the pressure differences involved are not too large. It has many important practical applications, some obvious (calculating and measuring the flow speed of fluid in a pipe, for instance) and some less so (one can explain, albeit in a somewhat oversimplified fashion, why an airplane wing provides lift, and why the direction of spin affects the trajectory of the ball in games such as golf, tennis and baseball). Bernoulli's equation does not work if the flow is ***turbulent*** rather than steady—white-water rapids on a river, for example. In turbulent flow the flow patterns are constantly changing and our assumptions about well-defined flow lines break down. Fluid flow in a given situation is usually steady, or ***laminar***, at low speeds but becomes turbulent (often very suddenly) above a certain critical speed. The study of turbulent flow is an application of chaos theory and far beyond the scope of this book.

Problem 12C.6.

Our derivation also ignored frictional effects. We know that real fluids—particularly liquids—are composed of molecules which exert forces on one another, so it is not surprising that frictional effects are actually important in many cases. In particular, because of friction, an object moving through a fluid is generally surrounded by a thin ***boundary layer*** of fluid which is almost at rest relative to the object; in laminar flow there is a smooth transition from this boundary layer to the steady flow of the fluid as a whole. This effect is significant in many applications of Bernoulli's equation: for example, the fact that the trajectory of a spinning ball curves is due to the difference in air speed on the two sides of the ball resulting from the formation of a boundary layer.

The property of a liquid which measures its internal friction is its ***viscosity***, a quantity related to the force necessary to maintain a given flow rate relative to a stationary wall. A fluid with a high viscosity, molasses for example, has a high resistance to flow; gases, which flow much more readily than liquids, have low viscosity.

So far we have concentrated on the behavior of a volume element ***within*** the liquid. What happens when we consider a volume element at the surface?

We can make some predictions about this from the molecular picture. A molecule at the surface of the liquid is in an asymmetric situation—it has molecules below it and on either side, but not above it (of course the air or other gas above the liquid consists of molecules, but their number density is much less than within the liquid).

It is therefore subject to a net force tending to pull it back into the body of the liquid; alternatively, it has a positive potential energy relative to a typical molecule within the liquid. Our model therefore predicts that the equilibrium state for a sample of liquid is the one which minimizes its surface area. (Zero potential energy would actually require zero surface, but this is clearly impractical!) Any change in the surface area produces a change in the associated potential energy, and therefore results in a net force (recall from Chapter 4 that a conservative force can be expressed as $-\mathrm{d}U/\mathrm{d}x$). We conclude that there should be a force associated with the surface of a liquid which acts to reduce the surface area. This is indeed the case: the effect is known as ***surface tension***. Surface tension causes liquids to behave rather as though their surfaces were covered by a thin stretched membrane: to increase the surface area one has to stretch the membrane, and this requires energy, as for example in blowing up a balloon.

More precisely, the surface tension γ of a liquid is defined as the ratio of the net surface force to the length along which the force acts:

$$\boxed{\gamma = \frac{F}{\ell}\,.}$$

(Note that this definition of γ has nothing whatsoever to do with the ratio of specific heats—the fact that they have the same standard symbol is an unfortunate coincidence.)

Surface tension is thus a force per unit length, with SI unit N/m. The molecular picture tells us that it is actually more helpful to picture it as an energy per unit area (you can easily check that this has the same units)

Problems 12D.

$$\boxed{\gamma = \frac{U}{A}\,.}$$

To check that the two pictures are self-consistent, consider a thin film of liquid (e.g. a soap bubble) held in a wire frame with one movable side. We expand the frame by an area ΔA by moving our wire a distance Δx. The work done by the wire is $F\Delta x$, where F is the force we exerted to move it; if the wire has length ℓ, the area of the soap bubble has increased by $2\Delta A = 2\ell\Delta x$ (the factor of 2 allows for the two surfaces of the film). The increase in energy associated with this increase in surface area must, by energy conservation, equal the work done by the wire, so

$$\gamma = \frac{\Delta U}{\Delta A} = \frac{F}{2\ell}.$$

The force we exerted on each of the two surfaces is, by symmetry, $\frac{1}{2}F$, so the first definition of γ also gives $F/2\ell$: the two definitions are consistent.

The phase transition between the gas and liquid states also involves the intermolecular potential energy: we noted that in the liquid state the separation between molecules corresponds to the minimum potential energy, whereas in a gas under the same external conditions of pressure and temperature the separation is much larger. Therefore in going from liquid to gas the molecules gain potential energy, which has to be supplied from outside as heat, whereas in condensing from gas to liquid the molecules lose potential energy, which is released to the surroundings as heat. The amount of heat lost or gained per kilogram is called the ***latent heat of vaporization*** of the substance. (The equivalent quantity for the liquid-solid phase transition is the ***latent heat of fusion***.)

Both surface tension and latent heat therefore depend on the intermolecular potential energy. This interpretation of two experimentally measurable quantities was used in the mid-19th century to provide one of the earliest estimates of the actual size of molecules. The results obtained are accurate to about a factor of 5, which is astonishing given that we are using macroscopic quantities to measure sizes of the order of 10^{-10} m. This illustrates how even very simple and idealized models can provide important information about real physical quantities.

Problem 13.11.

SUMMARY

* The molecules of a substance in the liquid state are strongly constrained to intermolecular separations corresponding to the minimum potential energy of the intermolecular forces. Within this limitation, the molecules can still move randomly.
* An ideal liquid therefore has a fixed density (since molecules are at fixed separation), but is free to assume any shape (since molecules can move randomly). Changes of volume with pressure or temperature do occur in real liquids, but are very small compared to the equivalent changes in gases, and can be neglected in an idealized model.
* The pressure in a liquid is the same in all directions, as with gases, and at rest depends only on the surface pressure and the depth below the surface. For an ideal incompressible liquid, the pressure increases linearly with depth.
* A fluid exerts a ***buoyant force*** on a body immersed in it which is equal in magnitude and opposite in direction to the weight of the fluid displaced by the body (***Archimedes' Principle***). For purposes of calculating the torque, the buoyant force can be treated as if it acts directly on the center of mass of the displaced liquid, which is called the ***center of buoyancy***.
* In steady flow of an incompressible fluid the volume flow rate is constant; the speed of flow is therefore inversely proportional to the cross-sectional area of the flow tube.
* Conservation of energy implies that the sum of the pressure, the kinetic energy per unit volume and the potential energy per unit volume is the same at any point in the steady flow of an incompressible fluid (***Bernoulli's theorem***).
* The molecules at the surface of a liquid have greater potential energy than the molecules within the liquid volume. A force known as ***surface tension*** therefore acts to resist any attempt to increase the surface area of a given mass of liquid.
* Condensation of a gas to the liquid state releases energy (known as ***latent heat***) since the potential energy of the molecules decreases. The transition from one state to another is called a phase transition.
* Physical concepts introduced in this chapter: liquid; buoyancy; phase transition; latent heat; surface tension.
* Mathematical concepts introduced in this chapter: none.
* Equations introduced in this chapter:

$P_2 - P_1 = -\rho g(y_2 - y_1)$	(Pressure in a liquid as a function of height, for a stationary liquid);
$A_2 v_2 = A_1 v_1$	(equation of continuity for steady flow);
$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$	(Bernoulli's equation);
$\gamma = \frac{F}{\ell} = \frac{U}{A}$	(surface tension).

PROBLEMS AND QUESTIONS

By the end of this chapter you should be able to answer or solve the types of questions or problems stated below.

Note: throughout the book, in multiple-choice problems, the answers have been rounded off to 2 significant figures, unless otherwise stated.

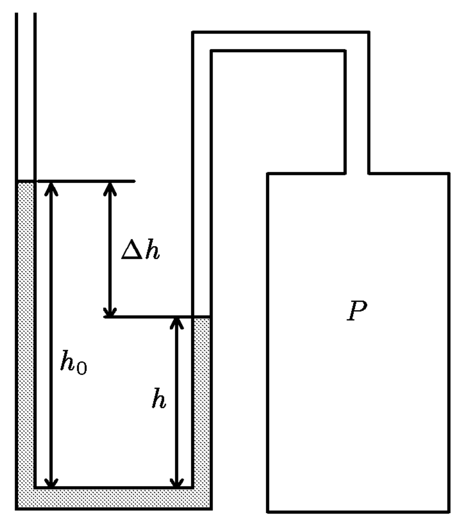
At the end of the chapter there are answers to all the problems. In addition, for problems with an (H) or (S) after the number, there are respectively hints on how to solve the problems or completely worked-out solutions.

12A PRESSURE AND ITS VARIATION WITH HEIGHT

12A.1 A large glass tank in an aquarium is filled with seawater (density 1030 kg/m^3) to a depth of 5 m. The top of the tank is open to the air. How much pressure should the glass used to construct the tank be able to withstand if the tank is not to crack at the base? (Take g to be 9.8 m/s^2 and atmospheric pressure to be 1.01×10^5 Pa.)

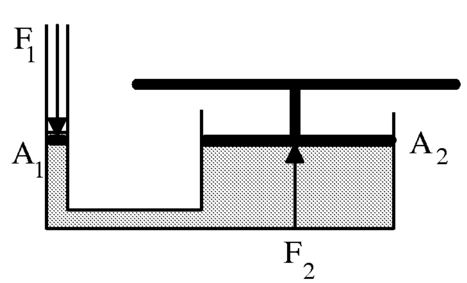
(a) 1.5×10^5 Pa; (b) 5.0×10^4 Pa; (c) 2.5×10^5 Pa; (d) 1.0×10^5 Pa.

12A.2 (S) A U-shaped tube of constant cross-sectional area A is filled with a liquid of density ρ. One end of the tube is open to the atmosphere, while the other side is connected to a vessel containing gas at some unknown pressure P. Calculate the unknown pressure in terms of the difference in level between the liquid on the two sides of the U.



12A.3 (H) Some types of pump ('suction' pumps) operate by producing a reduced pressure in the area towards which you want the fluid to flow. A landscape gardener wishing to create an artificial waterfall plans to use such a device to pump water from a lake to the head of her waterfall, from where it will cascade decoratively back into the lake. What is the absolute maximum possible height of fall she can achieve using a pump of this type (neglecting viscosity and similar effects)? How would you go about pumping water up to greater heights (e.g. to service the restrooms on the observation deck of the John Hancock tower)? (Take atmospheric pressure to be 1.01×10^5 Pa; the density of fresh water is 1000 kg/m^3.)

12A.4 (S) An auto mechanic is using a hydraulic jack to lift a car off the ground in order to fit new tires. If the car has a mass of 1500 kg and the mechanic applies a force of 300 N to the jack, what ratio of piston areas will be required? If the car is to be lifted 20 cm, how far must the piston at the mechanic's end move, and how much work is done, assuming that both ends of the



12A.4, continued:

jack—the car's and the mechanic's—are at the same height? (Take $g = 10$ m/s^2; the diagram shows a schematic drawing of the jack.)

12A.5 (S) Three flasks each contain (when full) one liter of water. One has straight sides and a circular base of area A; one has inward-sloping sides and a base of area $2A$, and the last has outward-sloping sides and a base of area $A/2$. The flasks are shaped so that when they are full the level of the water above the base is the same in each case, namely h. Calculate the force on the base of each container due to the water pressure. Explain qualitatively why the shape of the container does not affect the reading on a weigh-scale.

12A.6 (H) You are designing a diving vessel to serve as an underwater laboratory at a depth of 500 m. Assuming that seawater is completely incompressible, what pressure do the walls of your vessel have to withstand if it is (a) completely sealed, with its internal air maintained at atmospheric pressure; (b) provided with an exit underneath which is open to the ocean, with the inside air maintained at the pressure required to prevent flooding of the vessel? Given that human beings cannot tolerate rapid decreases in pressure, under what circumstances would you prefer a design of type (a) rather than type (b) and vice versa? (The density of sea-water is 1030 kg/m^3; take $g = 9.8$ m/s^2.)

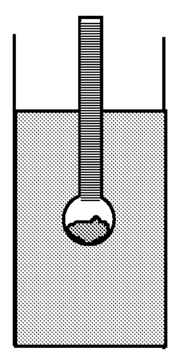
12B BUOYANCY AND ARCHIMEDES' PRINCIPLE

12B.1 A cylindrical glass vase is 15 cm in diameter and 15 cm high. You notice while washing it that when empty it floats so that its rim is 4 cm above the surface of the water. What is its mass?

(a) 1.9 g; (b) 1.9 N; (c) 19 N; (d) none of these.

12B.2 (S) Water is unusual in that its solid phase—ice—is less dense than its liquid phase. In fact ice has a density of 920 kg/m^3. To analyze the old proverb, how much of an iceberg really is under water? (Assume seawater, which has a density of 1030 kg/m^3. By how much does your answer change if you assume fresh water?)

12B.3 (S) (a) A hydrometer is basically a calibrated float, resembling a standard liquid-in-glass thermometer in shape, weighted at the bottom so it always floats in the same orientation. If the stem of such a hydrometer has a cross-sectional area of 0.5cm^2, the total volume of the float is 15 cm^3, and in fresh water with a density of 1000 kg/m^3 4.0 cm of the stem is above water level, how much of the stem will be exposed if the hydrometer floats in seawater with a density of 1030 kg/m^3? The hydrometer is then placed in a sample of unknown liquid where it floats with 2.0 cm of stem exposed: what is the density of this liquid?



(b) A beaker containing one liter of water is placed on a scale and found to weigh 12 N (this of course includes the weight of the beaker). A cubical block of wood 8 cm on a side is suspended from a spring balance and lowered into the water. The density of the wood is 700 kg/m^3. How far is the base of the cube below the surface of the water, and what is the reading on the scale, when the reading on the spring balance is 2.5 N? Take $g = 9.8$ m/s^2.

12B.4, continued:

12B.4 (H) A beaker contains a thick layer of oil, of density 650 kg/m^3, floating on water (density 1000 kg/m^3). A cubical block of wood of density 750 kg/m^3 and with dimensions $10 \times 10 \times 10$ cm^3 is lowered very gently into the beaker, taking care not to disturb the layers of liquid, until it is completely submerged. At what position relative to the interface between oil and water does the block come to rest? If the beaker has a circular cross-section of diameter 20 cm and the oil layer was 10 cm deep before the block was inserted, what is the pressure on the upper and lower surfaces of the block when it is in equilibrium?

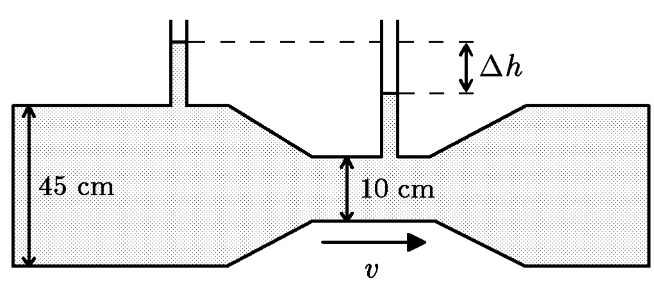
12C FLUIDS IN STEADY FLOW: BERNOULLI'S LAW

12C.1 A pipe in a factory has a diameter of 10 cm and carries water flowing at a speed of 5.0 m/s. At one point on its route it has to pass behind a large piece of equipment, and here the pipe has been squashed from its original circular cross-section to a rectangle of dimensions 14 cm by 1.7 cm. What is the speed of the water in this region? Assume that the pipe runs at the same height above sea-level throughout its route, and that the flow of the water is laminar.

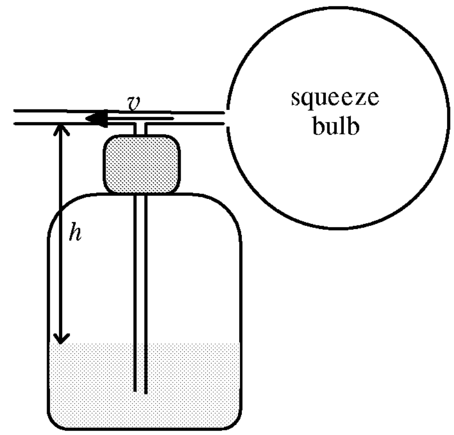
(a) 5.0 m/s; (b) 16.5 m/s; (c) 1.5 m/s; (d) none of these.

12C.2 (S) A water faucet turned on at a very low rate will produce a smooth laminar stream of water whose initial diameter is equal to the diameter d of the faucet. What is the diameter of the water stream when it has fallen through a height h? Assume that the water leaves the faucet with speed v, and that surface tension is sufficient to maintain the water in a single steady stream.

12C.3 (H) The diagram shows a Venturi meter installed in a water main. If the water in the pipe is flowing at 5×10^{-3} m^3/s (volume flow rate), what is (a) the speed of flow in each section of pipe; (b) the difference in the water level in the two tubes? The pipe has a circular cross-section at all points.



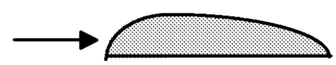
12C.4 Many spray devices, e.g. perfume atomizers, some insecticide sprays, etc., are designed along the lines of the diagram at right. Treating air as an incompressible fluid, explain how these devices work. If the height of the spray tube above the level of the fluid is h, at what speed v must the spray be expelled if it is to contain liquid? Assume that the bottle is vented, so that the air inside it is at atmospheric pressure.



Most practical versions of this design have a constriction in the spray tube where the vertical tube joins it, so that the diameter of this part of the spray is smaller. What is the advantage of this arrangement?

12C.5 (S) Two flat sheets of metal are suspended so that they hang parallel, separated by a short distance d. We then arrange, e.g. by using a blow dryer with a suitably shaped nozzle, to blow a stream of air between the plates. What happens?

12C.6 (H) If an incompressible fluid flows past the object shown in the diagram in the direction indicated by the arrow, what happens? Do any pressure differences develop, and if so what is the direction of the net force which results? Assume that the fluid flow is laminar. (Take the diagram to be a side view of a long object oriented perpendicular to the plane of the page.)



Discuss how your conclusions relate to the shape of aircraft wings.

12C.7 Due to frictional effects, the velocity of air in an air jet from a blower is highest in the center of the jet. What will happen to a ping-pong ball placed in the center of such a jet if the airstream is directed vertically upwards, and why?

12D SURFACE TENSION

12D.1 (H) The various types of small insects which can 'walk on water' tend to have feet which are covered with an oily or waxy substance to which water does not readily adhere. How does this help them to avoid sinking into the water? What would happen if they instead had feet covered with a substance with a strong affinity for water?

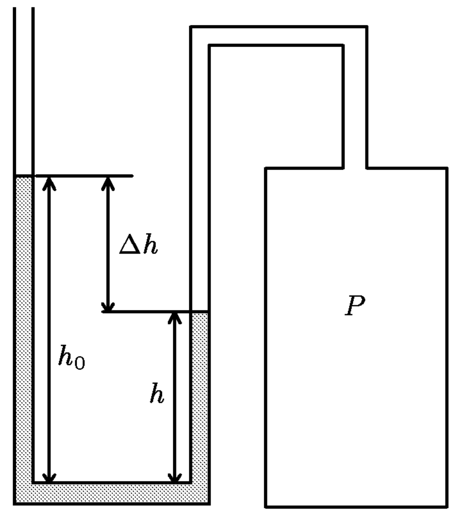
12D.2 (S) In terms of the surface tension γ of the liquid, what is the pressure difference between the liquid inside a liquid drop and the surrounding gas? What is the corresponding result for the difference in air pressure inside and outside a soap bubble?

12D.3 (H) What is the difference between the pressure inside and outside a soap bubble of diameter 5 cm in air at atmospheric pressure? What will happen if the bubble drifts into an area where, due to local wind conditions, the pressure is 5 Pa less than it was where the bubble was originally formed, although the temperature is the same? (Atmospheric pressure: 101.3 kPa; surface tension of soap solution: 25 mN/m.)

12D.4 In 100 words or less, explain the origin of surface tension.

COMPLETE SOLUTIONS TO PROBLEMS WITH AN (S)

12A.2 *A U-shaped tube of constant cross-sectional area A is filled with a liquid of density ρ. One end of the tube is open to the atmosphere, while the other side is connected to a vessel containing gas at some unknown pressure P. Calculate the unknown pressure in terms of the difference in level between the liquid on the two sides of the U.*



Conceptualize

We assume that the liquid is incompressible and that the system is in a steady state. If this is so, there must be no net force on any part of the liquid (it is not moving). In particular, the downward pressure of the liquid at the base of the left-hand vertical tube must equal the upward pressure of the liquid in the horizontal section, and likewise for the right-hand vertical tube. Since the pressure at both ends of the horizontal section must be the same, it follows that the pressures at the bases of the two vertical tubes must also be equal. Because the liquid is not flowing, we can use the static pressure-height relationship to calculate these pressures.

Formulate and Solve

Thus,

$$P_0 + \rho g h_0 = P + \rho g h,$$

$$\text{i.e.} \qquad P - P_0 = -\rho g (h - h_0).$$

Scrutinize

If both ends of the tube were open, we would expect $h = h_0$, and indeed the equation gives this result for $P = P_0$. Everyday experience (e.g., sucking on a straw) says that the height should be greater on the side with lower pressure, and the minus sign in the equation ensures that this is so. The special case where $P = 0$ is discussed below.

Learn

The tube can be thought of as a pressure gauge measuring the ***difference*** between the unknown pressure P and atmospheric pressure. In general pressure gauges do indeed measure pressure differences, usually with respect to atmospheric pressure, and this has resulted in the definition of the ***gauge pressure*** as $P - P_{\text{amb}}$, where P is the pressure in question and P_{amb} is the reference pressure of the gauge (the ambient pressure; usually local atmospheric pressure), in contrast to the ***absolute pressure*** P. Many pressures we encounter in everyday life (e.g. tire pressures) are gauge pressures rather than absolute pressures.

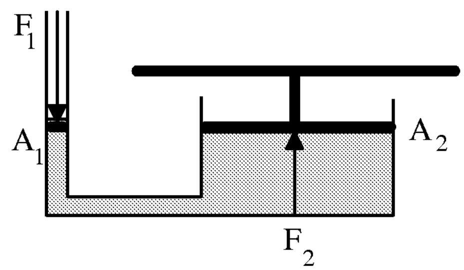
A special case is where $P = 0$, i.e. the right-hand side of the tube is evacuated. In this case our gauge has become a ***barometer***, measuring atmospheric pressure:

$$P_0 = \rho g \Delta h.$$

12A.2, continued:

For mercury, with a density of 13.6×10^3 kg/m^3, the value of Δh corresponding to standard atmospheric pressure is 760 mm, or 29.9 inches. Atmospheric pressures are often quoted in terms of the equivalent Δh for mercury, in mm Hg (or inches of mercury, as in TV weather forecasts). Mercury is conventionally used in such barometers and pressure gauges (***manometers***) because it is by far the densest room-temperature liquid: using a less dense liquid would require an impractically large tube (a water barometer, for example, would have a height of over thirty feet).

12A.4 *An auto mechanic is using a hydraulic jack to lift a car off the ground in order to fit new tires. If the car has a mass of 1500 kg and the mechanic applies a force of 300 N to the jack, what ratio of piston areas will be required? If the car is to be lifted 20 cm, how far must the piston at the mechanic's end move, and how much work is done, assuming that both ends of the jack—the car's and the mechanic's—are at the same height. (Take* $g = 10$ m/s^2*; the diagram shows a schematic drawing of the jack.)*



Conceptualize

We assume that the liquid is incompressible and that it is not flowing (or is flowing at negligible speed). We can therefore apply the static pressure-height relationship, which states that if the height difference between the two sides is negligible, the pressures must be equal. Since pressure is force per unit area, it follows that the ratio of forces, F_1/F_2, will give the ratio of areas, A_1/A_2.

To calculate the work done we need the dot product of force and displacement. The displacement on the right-hand side is known; we can calculate the displacement on the left-hand side from the ratio of areas, since the total volume of liquid in the system is assumed to be constant.

Formulate

The surface pressure at the left-hand end is $P_1 = F_1/A_1$, (by Newton's Third Law, the pressure of the piston on the water is equal to the pressure of the water on the piston) and at the right-hand end it is $P_2 = F_2/A_2$. Since $P_1 = P_2$, the ratio of forces is given by the ratio of areas:

$$\frac{F_2}{F_1} = \frac{A_2}{A_1}.$$

For the work done, we note that if the piston A_2 moves a distance Δh_2, the right-hand end of the jack needs an additional volume of liquid $A_2 \Delta h_2$. This must come from depressing the left-hand piston a distance Δh_1, which decreases the volume of the left-hand end by $A_1 \Delta h_1$. The total volume of liquid is constant, since this is a sealed system and we assume an ideal incompressible liquid, so $A_2 \Delta h_2 = A_1 \Delta h_1$, giving

$$\frac{\Delta h_1}{\Delta h_2} = \frac{A_2}{A_1}.$$

12A.4, continued:

Solve

To solve the problem we simply put in the numbers. We have $F_1 = 300$ N and want $F_2 = 15000$ N (mg, where m is the mass of the car), so we need $A_2/A_1 = 50$. It follows that $\Delta h_1 = 50 \times \Delta h_2$: to move the large piston by 20 cm we must move the small piston 10 m! The work done is $F_1 \Delta h_1 = F_2 \Delta h_2 = 3.0$ kJ.

Scrutinize

Note that the same amount of work is done on both sides, as we expect from energy conservation. The hydraulic jack is very like the block and tackle of Problem 7.7: it is a device to enable you to do the same amount of work by applying a smaller force over a longer distance.

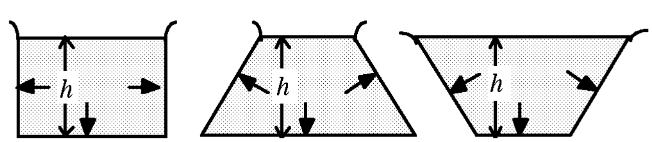
Learn

Obviously a hydraulic jack 10 m long is not very practical! This difficulty arises because we assumed the working fluid was a fixed amount of incompressible liquid. In fact we could instead use a compressible gas—air, for instance. This is less efficient, since some of the work goes into compressing and heating the gas instead of lifting the car, but it has the great advantage that with an appropriate system of valves it is possible to do the lift by repeated short strokes of the small piston, drawing in more air with each return, rather than one long stroke.

12A.5 *Three flasks each contain (when full) one liter of water. One has straight sides and a circular base of area A; one has inward-sloping sides and a base of area 2A, and the last has outward-sloping sides and a base of area A/2. The flasks are shaped so that when they are full the level of the water above the base is the same in each case, namely h. Calculate the force on the base of each container due to the water pressure. Explain qualitatively why the shape of the container does not affect the reading on a weigh-scale.*

Conceptualize

Since the water is not flowing, we are again entitled to use the static pressure-height relationship. The surface pressure and the water depth are the same for all three flasks, so it follows that the pressure on the base is also the same in all cases. Since pressure is force per unit area, the downward ***force*** exerted by the water on the base of the container is proportional to the base area, so it will be different for each container.



The readings on the scale will nonetheless be the same in each case because this is not the ***net*** force acting. There is also a pressure on the ***sides*** of the container: the resulting force is purely horizontal for the flask with straight sides, but it has a net upward component for the flask with inward-sloping sides and a net downward component for the third flask.

12A.5, continued:

Formulate and Solve
The pressure at depth h is $P = P_0 + \rho g h$ for all three containers, where ρ is the density of water. If the flasks were suspended in air, the net pressure would be $\rho g h$, since there would be an inward pressure P_0 from the surrounding air. This gives the net pressure force on the base of each container as $\rho g h A$, $2\rho g h A$ and $\rho g h A/2$, respectively. In each case, however, the ***net*** downward force, taking into account the pressure forces on the sides of the container, is equal to the weight of the water, and so the scale readings will be the same.

Scrutinize
Although we have presented a qualitative argument that the forces on the sides of the container act to counterbalance the effect of the different base areas, we have not in fact shown quantitatively that the cancellation is exact, nor can we do so without more information about the slopes of the sides of the containers. However, we know that the quantitative calculation will work, because this principle of balancing pressure forces against the weight of the liquid is exactly what we used to derive the pressure-height relationship in the *Essentials.*

12B.2 *Water is unusual in that its solid phase—ice—is less dense than its liquid phase. In fact ice has a density of* $920\ \mathrm{kg/m^3}$. *To analyze the old proverb, how much of an iceberg really is under water? (Assume seawater, which has a density of* $1030\ \mathrm{kg/m^3}$. *By how much does your answer change if you assume fresh water?)*

Conceptualize
Archimedes' principle tells us that the iceberg is in equilibrium when it is floating at a level such that it displaces an amount of water equal to its own mass. The mass of water displaced is $\rho_{\mathrm{water}} V_{\mathrm{sub}}$, where V_{sub} is the volume of berg submerged, and the total mass of the iceberg is $\rho_{\mathrm{ice}} V_{\mathrm{tot}}$, where V_{tot} is the total volume. Equating these will give us the ratio $V_{\mathrm{sub}}/V_{\mathrm{tot}}$.

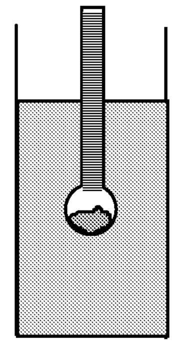
Formulate and Solve
If $\rho_{\mathrm{water}} V_{\mathrm{sub}} = \rho_{\mathrm{ice}} V_{\mathrm{tot}}$, it follows that

$$\frac{V_{\mathrm{sub}}}{V_{tot}} = \frac{\rho_{\mathrm{ice}}}{\rho_{\mathrm{water}}} = \frac{920\ \mathrm{kg/m^3}}{1030\ \mathrm{kg/m^3}} = 0.89.$$

(Almost) nine-tenths of an iceberg really is under water! If we consider fresh water, 92% of the berg is submerged. (Note that icebergs are frozen ***fresh*** water, so we don't have to worry about whether the density of frozen seawater is different).

Scrutinize and Learn
This calculation agrees well with our intuitive idea of how things float: the less dense the object, the higher it floats—e.g. balsa wood versus teak, or a heavily laden cargo ship versus an empty one. Note that we can calculate the mass of a floating object by measuring the amount of water it displaces: this is how ships' masses are measured, and indeed they are often quoted as "so many tons ***displacement***".

12B.3 (a) *A hydrometer is basically a calibrated float, resembling a standard liquid-in-glass thermometer in shape, weighted at the bottom so it always floats in the same orientation. If the stem of such a hydrometer has a cross-sectional area of* $0.5\ \text{cm}^2$*, the total volume of the float is* $15\ \text{cm}^3$*, and in fresh water with a density of* $1000\ \text{kg/m}^3$ $4.0\ \text{cm}$ *of the stem is above water level, how much of the stem will be exposed if the hydrometer floats in seawater with a density of* $1030\ \text{kg/m}^3$*? The hydrometer is then placed in a sample of unknown liquid where it floats with* $2.0\ \text{cm}$ *of stem exposed: what is the density of this liquid?*



Conceptualize

The situation is essentially the same as Problem 12B.2, except that here we are looking for the volume exposed rather than the volume submerged. Since we know the cross-sectional area of the hydrometer tube, calculating the exposed volume will determine the length of tube exposed.

Formulate

The basic equation is the same one we derived in the solution to problem 12B.2:

$$\frac{V_{\text{sub}}}{V_{\text{tot}}} = \frac{\rho_{\text{h}}}{\rho_{\text{liquid}}},$$

where ρ_{h} is the overall density of the hydrometer. We could calculate this from the information we have, but we don't need to: we can simply take this equation for water and divide it by the same equation for the liquid we want to measure (the seawater, or our unknown sample):

$$\frac{V_{\text{sub}}^{\text{water}}}{V_{\text{sub}}^{\text{liquid}}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}}.$$

Solve

The submerged volume for water is

$$(15.0\ \text{cm}^3) - (0.5\ \text{cm}^2 \times 4.0\ \text{cm}) = 13.0\ \text{cm}^3.$$

We conclude that the submerged volume for seawater must be

$$(13.0\ \text{cm}^3)/1.03 = 12.6\ \text{cm}^3,$$

leaving $(2.4\ \text{cm}^3)/(0.5\ \text{cm}^2) = 4.8\ \text{cm}$ of stem exposed. The submerged volume for the unknown liquid is

$$(15.0\ \text{cm}^3) - (0.5\ \text{cm}^2 \times 2.0\ \text{cm}) = 14.0\ \text{cm}^3,$$

so its density is $(1000\ \text{kg/m}^3) \times (13\ \text{cm}^3)/(14\ \text{cm}^3) = 930\ \text{kg/m}^3$.

Scrutinize

As we expect, the denser the liquid, the higher the hydrometer floats, and the more of its stem is exposed. Note, however, that a density change of only 3% produced a change in hydrometer reading of 20%. This is because we measure the exposed volume,

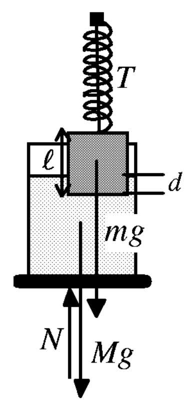
12B.3, continued:

which for a suitable ballasting of the hydrometer can be made a rather small fraction of the total volume. As a result hydrometers are a very useful tool for making quick measurements of fairly small differences in liquid density. This has a variety of practical applications, e.g. strength of car battery acid, alcohol content of beer, etc.

(b) *A beaker containing one liter of water is placed on a scale and found to weigh* 12 N *(this of course includes the weight of the beaker). A cubical block of wood* 8 cm *on a side is suspended from a spring balance and lowered into the water. The density of the wood is* 700 kg/m^3. *How far is the base of the cube below the surface of the water, and what is the reading on the scale, when the reading on the spring balance is* 2.5 N*? Take* $g = 9.8$ m/s^2.

Conceptualize

The force diagram for the system is shown below. The spring balance reading is the tension T in the spring, while the scale reading is the normal force N exerted by the pan of the scale on the beaker and its contents. Gravity exerts a downward force Mg on the beaker-plus-water system, where M is the combined mass of beaker and water, and mg on the block of wood, where m is its mass. Neglecting ambient pressure, which is the same in all directions and will therefore cancel, these are the only external forces acting on the whole beaker-plus-water-plus-wood system, and so if the block and beaker are both stationary it follows that $N + T = (M + m)g$. This will solve the second part of the problem.



For the first part, we recall that the net pressure force on a partially or wholly submerged body is $-m_{\mathrm{W}}g$, where m_{W} is the mass of the displaced water. The downward force on the block of wood is mg, so for zero net force the tension in the spring balance must be the difference between these, $T = (m - m_{\mathrm{W}})g$.

Formulate

The mass of water displaced is $m_{\mathrm{W}} = \rho V_{\mathrm{sub}}$, where ρ is the density of water (1000 kg/m^3) and $V_{\mathrm{sub}} = \ell^2 d$ is the submerged volume of the cube (writing ℓ for the side of the cube and d for the depth to which it is submerged). The cube's mass is $m = \rho' V_{\mathrm{tot}}$, where ρ' is 700 kg/m^3 and V_{tot} is the total volume of the cube. The tension in the spring balance is therefore

$$T = (m - m_{\mathrm{W}})g = \rho'\ell^3 g - \rho\ell^2 dg = (\rho'\ell - \rho d)\ell^2 g.$$

Solve

If we put in the numbers we find that

$$\begin{aligned} d &= \frac{1}{\rho}\left(\rho'\ell - \frac{T}{g\ell^2}\right) \\ &= \frac{1}{1000\ \mathrm{kg/m^3}}\left((700\ \mathrm{kg/m^3}) \times (0.08\ \mathrm{m}) - \frac{2.5\ \mathrm{N}}{(9.8\ \mathrm{m/s^2}) \times (0.08\ \mathrm{m})^2}\right) \\ &= 0.016\ \mathrm{m},\ \text{or}\ 1.6\ \mathrm{cm}. \end{aligned}$$

12B.3, continued:

The mass of the whole block of wood is $\rho' V_{\text{tot}} = 0.36$ kg, so

$$N = (M+m)g - T = (12\ \text{N}) + (0.36\ \text{kg} \times 9.8\ \text{m/s}^2) - (2.5\ \text{N}) = 13\ \text{N}.$$

Scrutinize

The spring balance tension should be zero when the block is floating on the water. Our equation says that this will happen when $\rho'\ell = \rho d$. This corresponds exactly to the relation $\rho'/\rho = V_{\text{sub}}/V_{\text{tot}}$ that we used in part (a).

Learn

If we had used a metal block, with $\rho' > \rho$, the logic of this problem would be unaltered. If we then completely submerge the block, such that $d = \ell$, the tension in the spring balance is $(\rho' - \rho)V_{\text{tot}}g$, and we can determine V_{tot} by measuring the rise in the level of the water. Thus this set-up can serve to determine the density of an object of unknown composition, provided that said object is denser than water. This, legend has it, was Archimedes' own application of Archimedes' Principle: he had been asked by the king of Sicily to determine whether a particular crown was made of pure gold or of a less dense gold/silver alloy.

12C.2 *A water faucet turned on at a very low rate will produce a smooth laminar stream of water whose initial diameter is equal to the diameter d of the faucet. What is the diameter of the water stream when it has fallen through a height h? Assume that the water leaves the faucet with speed v, and that surface tension is sufficient to maintain the water in a single steady stream.*

Conceptualize

If the water falls in a steady stream, the volume flow rate must be constant, i.e. the volume of water leaving the faucet in one second must equal the volume disappearing down the drain in the same time interval. Once the water has left the faucet it is in free fall, so (assuming air resistance can be neglected) its speed v_h after descending a distance h is given by energy conservation:

$$\tfrac{1}{2}v_h^2 = \tfrac{1}{2}v^2 + gh$$

(i.e. the gain in kinetic energy is equal to the loss of potential energy; we have canceled a common factor of m, the mass of our volume element of water). To maintain the same volume flow rate, this increase in linear speed will have to be balanced by a decrease in the cross-sectional area of the stream.

Formulate and Solve

Assuming the stream of water has a circular cross-section, its cross-sectional area will be $\pi d^2/4$, where d is its diameter. Then the equation of continuity gives for the diameter d_h of the stream after falling through distance h

$$\tfrac{1}{4}\pi d_h^2 v_h = \tfrac{1}{4}\pi d^2 v,$$

12C.2, continued:

i.e., using the above equation for v_h,

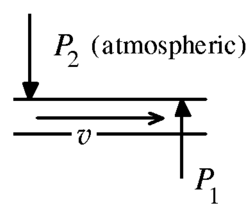
$$d_h = d\left(1 + \frac{2gh}{v^2}\right)^{-\frac{1}{4}}.$$

Scrutinize
This equation appears to make sense, in that the change in diameter is less for smaller heights or higher initial velocities. It is, at first sight, somewhat worrying that it does not work for initial $v = 0$ (surely we are entitled to have our water start from rest?), but on second thought we can see that the volume flow rate is meaningless if the liquid is not moving—the stream of water never actually emerges from the faucet.

Learn
It is possible to observe this effect with a real faucet, but the experiment only works for low flow rates. At higher flow rates the stream becomes turbulent, and the analysis of this chapter no longer applies. Turbulent flow is very common in real situations, but extremely difficult to analyze. The study of turbulence is one of the areas of application of chaos theory.

12C.5 *Two flat sheets of metal are suspended so that they hang parallel, separated by a short distance d. We then arrange, e.g. by using a blow dryer with a suitably shaped nozzle, to blow a stream of air between the plates. What happens?*

Conceptualize
Although we are considering a gas here, the velocities involved are small enough that we can still apply Bernoulli's equation. If we consider a flow tube which goes between the two plates and then out to the atmosphere, it's clear that at a sufficiently large distance from the plates we will have atmospheric pressure and zero velocity (meaning here zero net velocity of a volume element, of course, not zero molecular velocity), whereas in the region between the plates we have a non-zero velocity and therefore, according to Bernoulli's equation, a lower pressure.



Formulate
Using a point between the plates as point 1, and for point 2 our distant point where the pressure is ambient and the velocity zero, Bernoulli's equation gives

$$P_1 + \tfrac{1}{2}\rho v^2 = P_2.$$

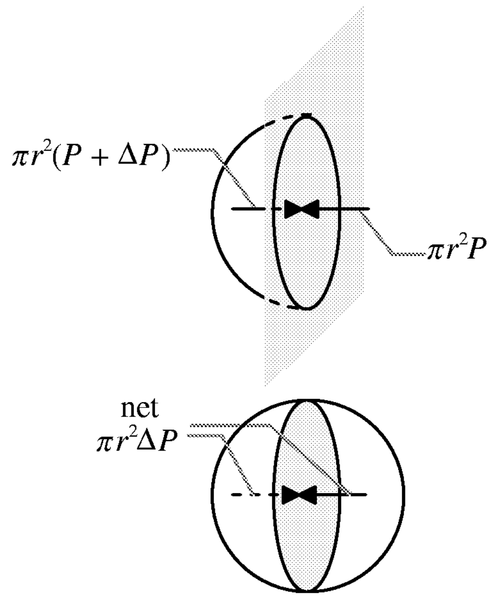
Solve
The pressure between the plates is less than atmospheric pressure, by an amount proportional to v^2. Since the plates still have atmospheric pressure acting on them on their outer sides, they will be subject to a net inward force and will move together.

Scrutinize
This rather counterintuitive result can be easily demonstrated by holding two sheets of paper a short distance apart and blowing between them. As in Problem 12C.2, it is best to blow slowly, so that the flow remains laminar (and so that the air is not significantly compressed).

12D.2 *In terms of the surface tension γ of the liquid, what is the pressure difference between the liquid inside a liquid drop and the surrounding gas? What is the corresponding result for the difference in air pressure inside and outside a soap bubble?*

Conceptualize

To avoid having to worry about curved surfaces, let's initially think about half a drop, say a hemispherical blob of liquid on a thin membrane. If the difference between the pressure in the liquid and atmospheric pressure is ΔP, then the net force on the membrane is $\Delta P \pi r^2$, where r is the radius of the drop. Putting two half-drops together to form a spherical drop, we conclude that each half of the drop exerts a net force $\Delta P \pi r^2$ on the other half. Why do the two halves not fly apart? The answer is that surface forces counteract this force. Each half of the drop will pull on the other half of the drop, at the surface along the junction between the two halves, with a force $2\pi r\gamma$ (surface tension force per unit length, γ, times the circumference of $2\pi r$).



Formulate and Solve

Since the drop is in equilibrium, outward pressure force and the inward surface tension force must balance, i.e.

$$\Delta P \pi r^2 = 2\pi r\gamma$$

or

$$\Delta P = \frac{2\gamma}{r}.$$

In the case of a bubble, there are two surfaces at approximately the same radius (since the thickness of the bubble is negligible compared to its diameter), so the force from surface tension is doubled. Hence there is twice as large a pressure difference.

Scrutinize and Learn

For typical surface tensions these pressure differences are very small: for example soap solution has a surface tension of about 25 mN/m, so a soap bubble 2.5 cm in radius would have an internal pressure 4 Pa higher than the ambient pressure—a difference of 0.004%! Notice that the larger the bubble, the smaller the pressure difference required to maintain it (though the total ***force*** being exerted is greater, since the pressure difference acts over a larger surface area).

The energy-per-unit-area picture of surface tension can help when considering the ***stability*** of bubbles and droplets. If a droplet is not spherical, its surface area is higher (for a given volume) than a spherical drop, and therefore the energy stored in its surface tension is larger: this is why liquid drops tend to be spherical (unless some external force is acting).

HINTS FOR PROBLEMS WITH AN (H)

The number of the hint refers to the number of the problem

12A.3 What is the maximum possible pressure difference that can be achieved between the surface of the lake and the other end of the suction pump?

If you're stuck, study the solution to problem 12A.2.

12A.6 What is the pressure outside the vessel? How does this compare with the internal pressure in the two cases?

12B.4 Draw a force diagram for the block. What are the buoyant forces from the oil and the water? Check that the total buoyant force is equal to the force due to the difference in the pressures on the top and bottom of the block—why should this be so?

12C.3 How does the speed of flow relate to the volume flow rate?

What is the relation between the pressures at the water surface in the two tubes?

12C.6 How does the path length of a streamline deflected by the object compare with one flowing just underneath it and hence undisturbed? What does this imply for the speed of the fluid in the two cases?

12D.1 Draw the surface in the two cases. In which direction is the net force from surface tension?

12D.3 Is the pressure inside a soap bubble equal to the pressure outside? If not, how is it that the bubble can be in equilibrium? (If you're not sure, review the solution to problem 12D.2.)

Taking the air in the bubble to be an ideal gas, can you find an equation for the outside pressure in terms of the radius of the bubble?

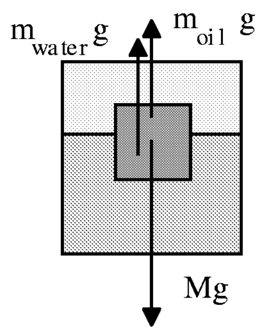
Now differentiate this equation to find the relation between a small change ΔP in this pressure and the resulting small change in the radius of the bubble.

ANSWERS TO HINTS

12A.3 101 kPa, which corresponds to atmospheric pressure at the bottom and zero at the top.

12A.6 5.05×10^6 Pa, compared to 10^5 Pa in case (a) and 5.05×10^6 Pa in case (b).

12B.4 $\rho_{\text{water}} V_1 g$, where V_1 is the volume of the cube under water, and $\rho_{\text{oil}}(V - V_1)g$, where V is the total volume of the cube.



(Water buoyancy shown displaced for clarity)

Buoyant force *is* simply total upward pressure force, and as sides of block are vertical the only vertical pressure forces are those on top and bottom.

12C.3 $\dfrac{\mathrm{d}V}{\mathrm{d}t} = Av$, i.e. volume flow rate = speed of flow times cross-sectional area of pipe.

Both are equal to the ambient pressure.

12C.6 Longer; faster.

12D.1 See answer to problem.

12D.3 No, inside is greater. Surface tension provides force opposing expansion of bubble.

$$P = \frac{3P_0V_0}{4\pi r^3} - \frac{4\gamma}{r},$$

where r is the radius and P_0, V_0 the starting inside pressure and volume of the bubble. (The first term is the pressure inside the bubble, using $PV = NkT$; the second is the difference between this and the outside pressure.) The second term turns out to have a negligible effect (i.e. we can really regard the pressures inside and outside the bubble as equal). To find the small change in radius resulting from a small change in pressure, differentiate this and put $V_0 = \frac{4}{3}\pi r^3$ to get

$$\frac{\Delta r}{r} \approx -\frac{\Delta P}{3P}.$$

12D.4 An acceptable answer would be:

"The molecules making up a liquid exert attractive forces on their neighbors. A molecule at the surface, which has many more neighbors in the dense liquid below it than in the gas above it, therefore feels a net inward force. The sum of all such forces acts to minimize the liquid's surface area—this is surface tension."

ANSWERS TO ALL PROBLEMS

12A.1 b

12A.2 See complete solution.

12A.3 10.3 m; use overpressure at the bottom of the vertical section instead of reduced pressure at the top, and/or have holding tanks at various levels and pump in stages.

12A.4 See complete solution.

12A.5 See complete solution.

12A.6 5.05×10^6 Pa; zero.

First design preferable if the vessel makes frequent return trips to the surface and no-one leaves it while it is submerged. Second is better if vessel is to serve as a base for divers and will seldom return to the surface.

12B.1 d

12B.2 See complete solution.

12B.3 See complete solution.

12B.4 The base of the block is 2.9 cm below the interface.

To two significant figures, 320 and 1100 Pa above atmospheric, respectively. (To three significant figures, 325 and 1060 Pa above atmospheric.)

12C.1 c

12C.2 See complete solution.

12C.3 0.031 m/s (wide section); 0.637 m/s (narrow section); 2.1 cm.

12C.4 Applying Bernoulli's equation, we find that the pressure inside the tube (where the air is moving) is less than that outside the tube (where the air is stationary). The liquid in the bottle will therefore rise in the vertical tube to a height h, where $\rho_{\text{liquid}} g h = \frac{1}{2}\rho_{\text{air}} v^2$. If the design of the spray ensures that this value of h is greater than the actual height of the spray tube above the liquid level, liquid will enter the stream of moving air and be sprayed out.

$$v = \sqrt{\frac{2gh\rho_{\text{liquid}}}{\rho_{\text{air}}}}.$$

A constriction in the tube increases the speed of the airstream at that point, as can be calculated using the equation of continuity.

12C.5 See complete solution.

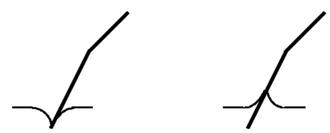
12C.6 The streamlines deflected upwards by the object are more crowded than those which pass below it undisturbed, and therefore the speed of the fluid above is higher. This results in a lower pressure above the object, and therefore a net upward force.

This object is approximately the same shape as the cross-section of an airplane wing, and thus the same arguments apply if air can be considered incompressible. (In this case the airfoil moves through stationary air, as seen from the ground, but if we consider the reference frame of the airplane we have a stationary object and a moving fluid. If the airplane is moving at a steady speed the results in the two frames must be equivalent, so we are allowed to do this.)

In reality, the lift of an airplane wing is much more complicated than this, because turbulent flow plays an important role.

12C.7 The ping-pong ball remains in the center of the airstream rather than falling out (even if the air is not directed vertically), because the lower velocities on each side lead to a net pressure difference pushing the ball towards the center.

12D.1 Because the water does not wet the insect's legs, the surface curves ***down*** where the insect's leg presses on it. Hence the net surface tension (acting to minimize surface area) is upwards and supports the insect. If water did wet the leg, the surface would curve upwards, and the surface tension would act to pull the insect under.



Non-wetting Wetting

12D.2 See complete solution.

12D.3 4 Pa.

It expands, because the decrease in external pressure creates a net outward force.

The radius of the bubble will increase by an undetectable 4.1×10^{-5} cm. The pressure in the bubble decreases as it expands because there is a fixed mass of air inside, so the bubble is not (as one might think) unstable against small pressure changes.