MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 | Fall 1997

# FINAL EXAMINATION

## Monday, December 15, 1997

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FAMILY (LAST) NAME

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GIVEN (FIRST) NAME

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|

STUDENT ID NUMBER

Your class (check one) $\Longrightarrow$

| | | | |
|---|---|---|---|
| Cl. 1 | MW 1:00 | R. Remillard | |
| Cl. 2 | MW 2:00 | R. Remillard | |
| Cl. 3 | MW 1:00 | A. Kerman | |
| Cl. 4 | MW 2:00 | A. Kerman | |
| Cl. 5 | TR 2:00 | W. Busza | |
| Cl. 6 | TR 3:00 | W. Busza | |
| Cl. 7 | TR 9:00 | S. Nahn | |
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| Cl. 9 | TR 2:00 | E. Lomon | |
| Cl. 10 | TR 3:00 | E. Lomon | |
| Cl. 11 | MW 12:00 | I. Pless | |
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| Cl. 13 | TR 10:00 | R. Hulsizer | |
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| Cl. 15 | MW 2:00 | H. Gao | |
| Cl. 16 | MW 3:00 | H. Gao | |
| Cl. 17 | MW 3:00 | M. Feld | |
| Cl. 18 | MW 4:00 | M. Feld | |
| Cl. 19 | TR 9:00 | L. Royden | |
| Cl. 20 | TR 10:00 | L. Royden | |
| Cl. 21 | TR 10:00 | W. Smith | |
| Cl. 22 | TR 11:00 | W. Smith | |
| Cl. 23 | TR 10:00 | R. Aggarwal | |
| Cl. 24 | TR 11:00 | R. Aggarwal | |
| Cl. 25 | TR 2:00 | P. Haridas | |
| Cl. 26 | TR 3:00 | P. Haridas | |

**Instructions:**

1. SHOW ALL WORK. All work is to be done in this booklet. Extra blank pages are provided.
2. This is a closed book test.
3. CALCULATORS, BOOKS, and NOTES are NOT ALLOWED.
4. Do all SEVEN (7) problems.
5. Print your name on each page of this booklet.
6. Exams will be collected 5 minutes before the hour.

| **Problem** | **Maximum** | **Score** | **Grader** |
|---|---|---|---|
| 1 | 10 | | |
| 2 | 10 | | |
| 3 | 15 | | |
| 4 | 15 | | |
| 5 | 17 | | |
| 6 | 15 | | |
| 7 | 18 | | |
| **TOTAL** | 100 | | |

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Fall 1997

# FINAL EXAMINATION ERRATA

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## 1) The exam as printed is too long!

The following parts are therefore ***deleted*** from the exam:

| | |
|---|---|
| Problem 3(e) | *@3 points* |
| Problem 4(e) | *@3 points* |
| Problem 5(d) | *@6 points* |
| Problem 6(d) | *@3 points* |
| Total: | *15 points* |

The maximum score on the exam, after the deletions, is therefore 85. The scores will be multiplied by 100/85 before they are averaged with the other components to compute your final course grade.

**Recommendation:** Put X's over the deleted problems before you start.

**Do not waste time** on the deleted parts, because they will **not** be graded.

## 2) Clarification for Problem 5(c):

Recall that $v_0$ is the orbital speed of the shuttle before the cannon is fired.

## 3) Clarification for Problem 7(d):

You may express your answer in terms of any of the quantities $m$, $v_0$, $h$, $R$, and $I$, where $I$ is the moment of inertia of the disk about the pivoted axis. Please leave your answer in terms of $I$, whether or not you evaluated it in part (b). Consider this a challenge problem, so don't feel frustrated if you can't get it.

**FORMULA SHEETS:** A five page "formula sheet" was handed out as part of this examination. It is available on the 8.01 website at
http://web.mit.edu/8.01/www/gen97/fif1l97.html.

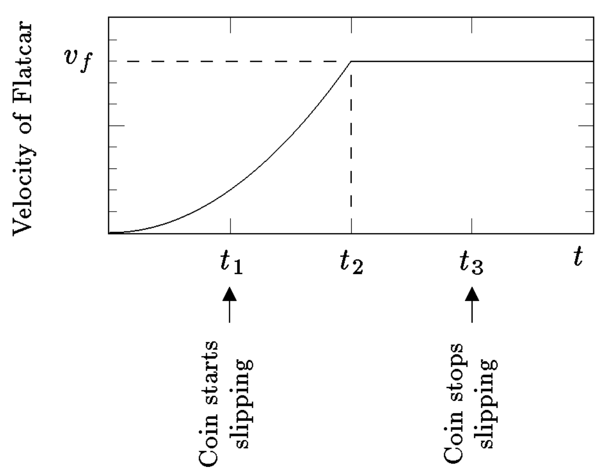
## Problem 1 (*10 points*):

*This problem is based on Problem 3 of the Unit 7 Quiz of this term.*

A rocket-propelled railroad flatcar begins at rest at time $t = 0$, and then accelerates along a straight track with a speed given by

$$v(t) = bt^2$$

where $b$ is a constant, for $0 < t < t_2$. Then the acceleration ends, and the flatcar continues at a constant speed of $v_f = bt_2^2$, as shown on the graph below. A coin is initially at rest on the floor of the flatcar. At $t = t_1$ the coin begins to slip, and it stops slipping at $t = t_3$. You may assume that $0 < t_1 < t_2 < t_3$, as shown in the graph. Gravity acts downward with an acceleration of magnitude $g$.



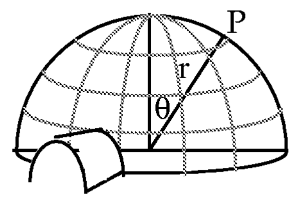

a) (*5 points*) What is the coefficient of static friction $\mu_s$ between the coin and the floor?

b) (*5 points*) What is the coefficient of kinetic friction $\mu_k$ between the coin and the floor? *(Hint: Note that between $t = t_1$ and $t = t_3$, the coin has a constant acceleration. Can you find this acceleration from some or all of the quantities $b$, $t_1$, $t_2$, $t_3$, and $v_f$?)*

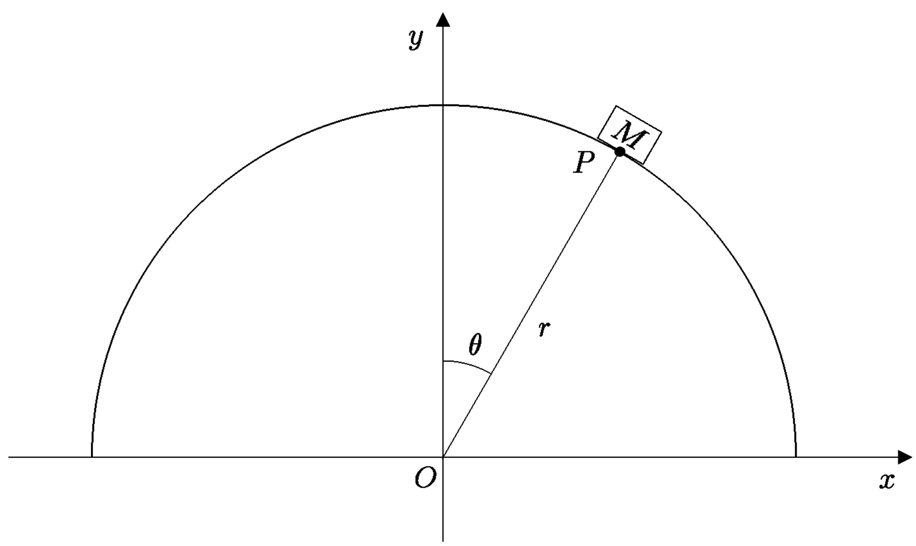
**Problem 2** (*10 points*):

*This problem is based on Problem 6.3 of the Study Guide.*

An Eskimo child of mass $M$ is using her parents' hemispherical igloo as a slide. She starts off from rest at the top and slides down under the influence of gravity. The surface of the igloo is effectively frictionless.




a) (*2 points, no partial credit*) What is her potential energy at point $P$ (see diagram)? Define the potential energy so that it is zero when the child is on the ground. (Note the sphere has radius $r$, and that a straight line between $P$ and $O$, the center of the sphere of which the igloo is a part, makes an angle $\theta$ with the vertical.)



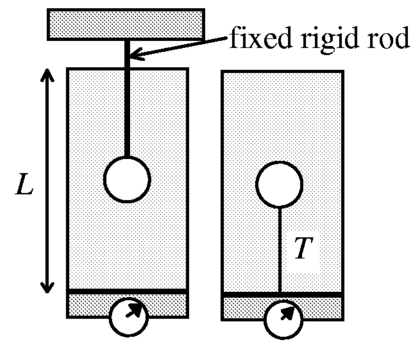

b) (*3 points*) On the diagram above, indicate (and clearly label) the forces acting on her at point $P$.

c) (*5 points*) Does she remain in contact with the igloo all the way to the ground? If not, at what angle $\theta$ does she lose contact?

**Problem 3** (*15 points*):

*This problem is based on Problem 13.13 of the Study Guide.*

A cylindrical container of length $L$ is full to the brim with a liquid which has mass density $\rho$. It is placed on a weigh-scale (which measures the downward force on the pan of the scale), and the scale reading is $W$. A light ball (which would float on the liquid if allowed to do so) of volume $V$ and mass $m$ is pushed gently down and held beneath the surface of the liquid with a rigid rod of negligible volume, as shown.




In each of the following parts, you can express your answer in terms of the given variables and/or the answers to the previous parts, whether or not you have correctly answered the previous parts.

a) (*3 points, no partial credit*) What is the mass $M$ of liquid which overflowed while the ball was being pushed into the container?

b) (*3 points, no partial credit*) What is the reading $R_1$ on the scale when the ball is fully immersed?

c) (*3 points, no partial credit*) If instead of being pushed down by a rod the ball is held in place by a fine string attached to the bottom of the container, what is the tension $T$ in the string?

d) (*3 points, no partial credit*) In part (c), what is the reading $R_2$ on the scale?

e) (*3 points, no partial credit*) If the string is cut, what will be the initial acceleration $a$ of the ball? Assume that viscosity effects are negligible.

**Problem 4** (*15 points*):

a) (*3 points, no partial credit*) A ball is thrown straight upward with an initial speed $v_0$. Denoting the magnitude of the acceleration of gravity as $g$, and neglecting friction, what will be the maximum height $h$ that the ball will reach?

b) (*3 points*) A ball of mass $M$ and velocity $\vec{\mathbf{v}}_1 = [v_M, 0, 0]$ collides with a ball of mass $m$ and velocity $\vec{\mathbf{v}}_2 = [0, 0, v_m]$. The two stick together. Ignoring friction, what is the speed of the combined mass after the collision?

c) (*3 points, no partial credit*) A block of mass $m$ slides down a frictionless hill, starting at a height $h$ and finishing at height zero. Let $g$ be the magnitude of the acceleration of gravity. What is the kinetic energy of the block at the bottom of the hill?

d) (*3 points, no partial credit*) A ball of radius $R$ and mass $m$ rolls without slipping down a hill, starting at a height $h$ and finishing at height zero. Again let $g$ be the magnitude of the acceleration of gravity. Neglecting all friction besides the force needed to keep the ball from slipping, what is the kinetic energy of the ball at the bottom of the hill?

e) (*3 points*) A compressed spring of negligible mass, which provides a fixed but uncalibrated force, is placed in contact with an unknown mass labeled $A$. When the spring is released, so that it pushes on the block, the initial acceleration of the block is measured to have magnitude $a_A$. In an identical experiment with the same spring compressed by the same amount, a block labeled $B$ is found to have an initial acceleration of magnitude $a_B$. If the two blocks are glued together (neglect the mass of the glue) and the identical experiment is carried out with the pair, what will be the magnitude of the initial acceleration?

**Problem 5** (*17 points*):

A space shuttle is in a circular orbit of radius $R$ about the Earth. The shuttle and its contents have mass $M$, and the Earth has mass $M_E$.

a) (*3 points*) In the frame of the Earth, which you may treat as an inertial frame, what is the orbital speed $v_0$ of the shuttle? You may express your answer in terms of any of the quantities $R$, $M$, $M_E$, and $G$ (Newton's constant), and you may assume that $M \ll M_E$.

b) (*4 points*) What is the gravitational potential energy of the shuttle, relative to the potential energy it would have at infinite distance from the Earth? Again you may express your answer in terms of any of the quantities $R$, $M$, $M_E$, and $G$ (Newton's constant).

c) (*4 points*) A cannon on the shuttle fires a probe of mass $m$ in the direction opposite to the shuttle's velocity. After the firing, the probe has a speed $\Delta v$ relative to the shuttle, where $\Delta v < v_0$. What is the speed $v_1$ of the probe, relative to the Earth, immediately after the firing? You may assume that no other mass is ejected in the firing of the probe. Do not assume, however, that $m$ is negligible compared to $M$. You may express your answer in terms of any of the quantities $R$, $m$, $M$, $M_E$, $G$, $v_0$, and $\Delta v$.

d) (*6 points*) After being fired from the cannon, the probe will follow an elliptical orbit. Assume that it remains far enough from the Earth so that friction with the atmosphere can be ignored. Write two equations which could be solved to determine $r_p$ and $v_p$, the radius and speed of the orbit at perigee, the nearest point to the center of the Earth. **Do not attempt to solve these equations.** Note that $r_p$ is to be measured from the center of the Earth. You may express your answer in terms of any of the quantities $v_1$, $R$, $m$, $M_E$, and $G$.

**Problem 6** (*15 points*):

A piston chamber of volume $V_0$ is filled with an ideal monatomic gas at temperature $T_0$ and pressure $P_0$. Denote Boltzmann's constant by $k$, and Avogadro's number by $N_A$.

a) (*2 points, no partial credit*) In terms of the given quantities, what is the number $N$ of atoms of the gas present in the chamber?

b) (*2 points, no partial credit*) What is the number $\mathcal{N}$ of moles of the gas present in the chamber?

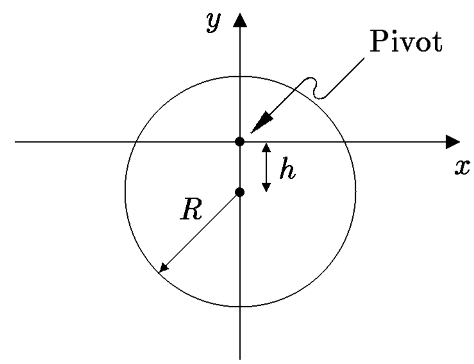
c) (*2 points, no partial credit*) The gas is allowed to expand, so the volume increases by an amount $\Delta V$. The gas is heated while it expands by just the right amount to keep it at constant pressure $P_0$. How much work $\Delta W$ does the gas do during this expansion?

d) (*3 points*) By what amount $\Delta U$ does the internal energy change during this expansion? Define $\Delta U$ so that a positive value denotes an increase in internal energy.

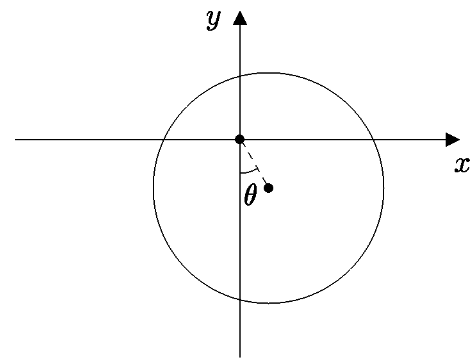
e) (*6 points*) Starting again from the initial values of $V = V_0$, $T = T_0$, and $P = P_0$, the gas is allowed to expand without the addition or emission of any heat. As the gas expands the values of $T$ and $P$ can both be measured as a function of $V$. Find an expression for $\frac{dP}{dV}$ that depends on no variables other than $P$ and $V$.

**Problem 7** (*18 points*):

A uniform disk of mass $M$ and radius $R$ is oriented in a vertical plane. The $y$ axis is vertical, and the $x$ axis is horizontal. The disk is pivoted about the origin of the coordinate system, with the center of the disk hanging a distance $h$ below the pivot, as shown. The disk is free to rotate without friction about the pivot in the $x$-$y$ plane. The magnitude of the acceleration of gravity is $g$, directed in the negative $y$ direction.



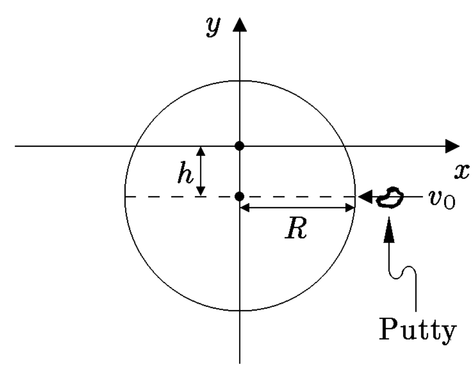

a) (*4 points*) If the disk is rotated by an angle $\theta$ from its equilibrium position, as shown, what is the magnitude of the torque about the pivot? (In this part, do not assume that the angle $\theta$ is necessarily small.)




b) (*4 points*) What is the moment of inertia $I$ of the disk about the pivoted axis?

c) (*4 points*) If the disk is allowed to oscillate about its equilibrium position, what will be the period of small oscillations? Your answer may be written in terms of $I$, whether or not you answered the previous part.

d) (*6 points*) A ball of putty of mass $m$ collides with the disk from the right, hitting it at the point $[R, -h, 0]$, as shown. At the moment just before the impact, the ball of putty has a velocity $\vec{\mathbf{v}} = [-v_0, 0, 0]$, and the disk is at rest. If the putty sticks to the side of the disk, what will be its velocity vector $\vec{\mathbf{v}}_f$ immediately after the collision?



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 Fall 1997

# FINAL EXAMINATION SOLUTIONS

**Monday, December 15, 1997**

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FAMILY (LAST) NAME

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GIVEN (FIRST) NAME

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STUDENT ID NUMBER

Your class (check one) $\Longrightarrow$

| | | | |
|---|---|---|---|
| Cl. 1 | MW 1:00 | R. Remillard | |
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|---|---|---|---|
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| 6 | 12 | | |
| 7 | 18 | | |
| **TOTAL** | 85 | | |

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Department of Physics

Physics 8.01 Fall 1997

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The following parts are therefore ***deleted*** from the exam:

| | |
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**Recommendation:** Put X's over the deleted problems before you start.

**Do not waste time** on the deleted parts, because they will **not** be graded.

## 2) Clarification for Problem 5(c):

Recall that $v_0$ is the orbital speed of the shuttle before the cannon is fired.

## 3) Clarification for Problem 7(d):

You may express your answer in terms of any of the quantities $m$, $v_0$, $h$, $R$, and $I$, where $I$ is the moment of inertia of the disk about the pivoted axis. Please leave your answer in terms of $I$, whether or not you evaluated it in part (b). Consider this a challenge problem, so don't feel frustrated if you can't get it.

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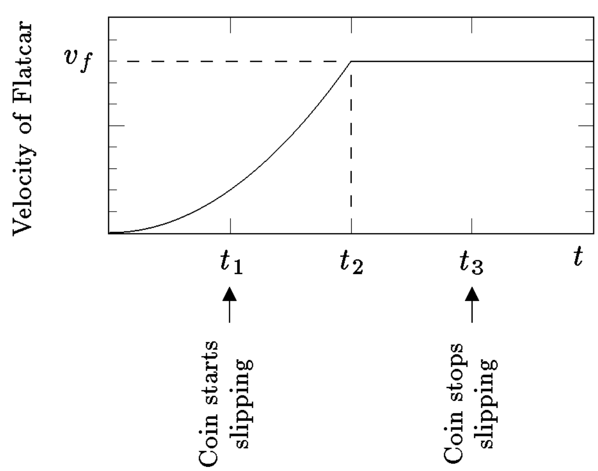
**Problem 1** (*10 points*):

*This problem is based on Problem 3 of the Unit 7 Quiz of this term.*

A rocket-propelled railroad flatcar begins at rest at time $t = 0$, and then accelerates along a straight track with a speed given by

$$v(t) = b\,t^2$$

where $b$ is a constant, for $0 < t < t_2$. Then the acceleration ends, and the flatcar continues at a constant speed of $v_f = bt_2^2$, as shown on the graph below. A coin is initially at rest on the floor of the flatcar. At $t = t_1$ the coin begins to slip, and it stops slipping at $t = t_3$. You may assume that $0 < t_1 < t_2 < t_3$, as shown in the graph. Gravity acts downward with an acceleration of magnitude $g$.

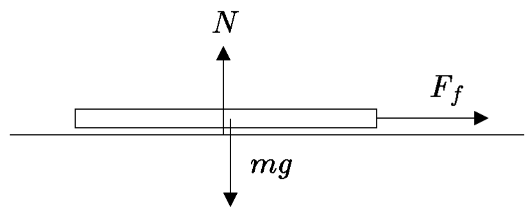



a) (*5 points*) What is the coefficient of static friction $\mu_s$ between the coin and the floor?

b) (*5 points*) What is the coefficient of kinetic friction $\mu_k$ between the coin and the floor? *(Hint: Note that between $t = t_1$ and $t = t_3$, the coin has a constant acceleration. Can you find this acceleration from some or all of the quantities $b$, $t_1$, $t_2$, $t_3$, and $v_f$?)*

**Solution:**

*(a) As the flatcar begins to accelerate, the coin is held at a fixed position relative to the car by static friction. The coefficient of static friction can be computed from the time at which it starts to slip, $t_1$, which is when static friction has reached its maximal force. The acceleration along the track during the period $0 \leq t \leq t_2$ is given by*




$$a = \frac{dv}{dt} = \frac{d}{dt} bt^2 = 2bt \ .$$

*It slips when*

$$ma = \mu_s N \ ,$$

*where m is the mass of the coin and $N = mg$ is the normal force. So*

$$m(2bt_1) = \mu_s mg \quad \Longrightarrow \quad \boxed{\mu_s = \frac{2bt_1}{g} \ .}$$

*(b) As the hint suggests, the coefficient of kinetic friction is found from acceleration during the time interval $t_1 < t < t_3$, while the coin is slipping. The acceleration along the track is given by*

$$a = \frac{F_f}{m} = \frac{\mu_k mg}{m} = \mu_k g \ .$$

*Knowing that the acceleration is constant, we can express its value in terms of the initial and final velocities:*

$$a = \frac{v(t_3) - v(t_1)}{t_3 - t_1} = \frac{b\left(t_2^2 - t_1^2\right)}{t_3 - t_1} \ .$$

*Equating the two expressions for a,*

$$\boxed{\mu_k = \frac{b\left(t_2^2 - t_1^2\right)}{(t_3 - t_1)g} \ ,}$$

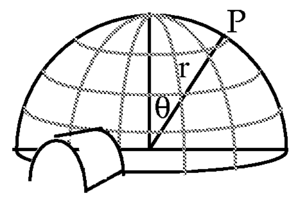
*or equivalently*

$$\boxed{\mu_k = \frac{v_f - bt_1^2}{(t_3 - t_1)g} \ .}$$

**Problem 2** (*10 points*):

*This problem is based on Problem 6.3 of the Study Guide.*

An Eskimo child of mass $M$ is using her parents' hemispherical igloo as a slide. She starts off from rest at the top and slides down under the influence of gravity. The surface of the igloo is effectively frictionless.




a) (*2 points, no partial credit*) What is her potential energy at point $P$ (see diagram)? Define the potential energy so that it is zero when the child is on the ground. (Note the sphere has radius $r$, and that a straight line between $P$ and $O$, the center of the sphere of which the igloo is a part, makes an angle $\theta$ with the vertical.)

**Solution:** *In general*

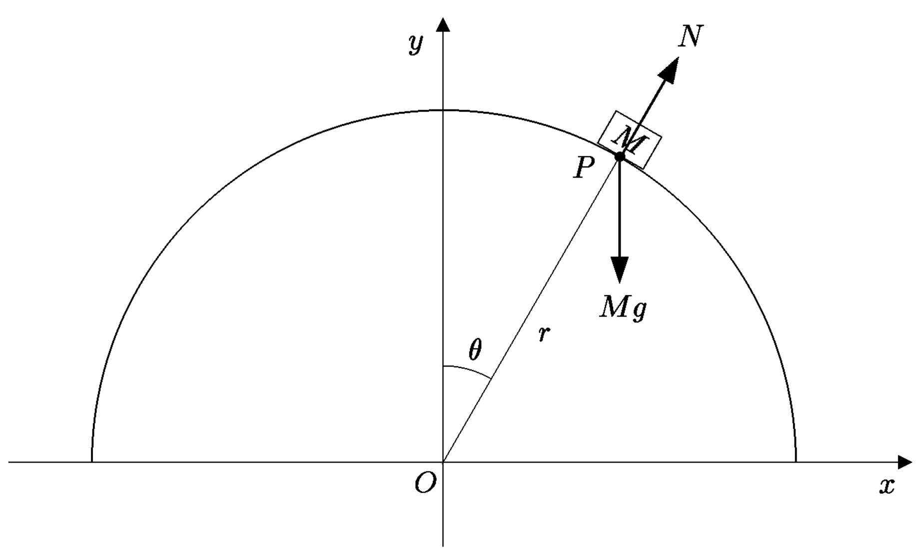
$$U = Mgh \ ,$$

*where $h$ is the height above the zero of potential, which in this case is the ground. Since*

$$h = r\cos\theta \ ,$$

*we have*

$$\boxed{U = Mgr\cos\theta \ .}$$




b) (*3 points*) On the diagram above, indicate (and clearly label) the forces acting on her at point $P$.

c) (*5 points*) Does she remain in contact with the igloo all the way to the ground? If not, at what angle $\theta$ does she lose contact?

**Solution:** *She will remain in contact as long as the normal force is positive, but she will lose contact if it ever falls to zero. To find the normal force, balance forces in the radial direction. To know the radial acceleration, we must know* $v$*, which can be found by energy conservation:*

$$Mgr\cos\theta + \frac{1}{2}Mv^2 = Mgr \quad \Longrightarrow \quad v^2 = 2gr(1-\cos\theta) \ .$$

*The radial component of the* $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$ *equation reads:*

$$N - Mg\cos\theta = -M\frac{v^2}{r} \ ,$$

*so*

$$\begin{aligned} N &= Mg\cos\theta - 2Mg(1-\cos\theta) \\ &= Mg(3\cos\theta - 2) \ . \end{aligned}$$

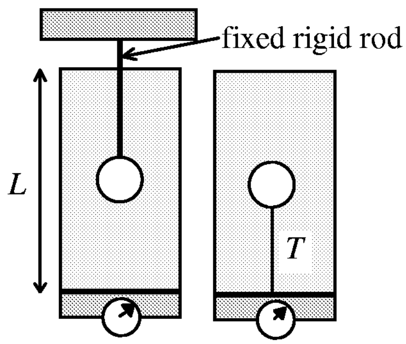
*So* $N = 0$ *when* $3\cos\theta - 2 = 0$*, at which point the child would lose contact. Finally,*

$$\boxed{\textit{Child loses contact at } \theta = \cos^{-1}\left(\frac{2}{3}\right) \ .}$$

**Problem 3** (*12 points, after deletion*):

*This problem is based on Problem 13.13 of the Study Guide.*

A cylindrical container of length $L$ is full to the brim with a liquid which has mass density $\rho$. It is placed on a weigh-scale (which measures the downward force on the pan of the scale), and the scale reading is $W$. A light ball (which would float on the liquid if allowed to do so) of volume $V$ and mass $m$ is pushed gently down and held beneath the surface of the liquid with a rigid rod of negligible volume, as shown.




In each of the following parts, you can express your answer in terms of the given variables and/or the answers to the previous parts, whether or not you have correctly answered the previous parts.

a) (*3 points, no partial credit*) What is the mass $M$ of liquid which overflowed while the ball was being pushed into the container?

**Solution:** *The displaced volume is $V$ and the density of the liquid is $\rho$, so the displaced mass is*

$$\boxed{M = \rho V\ .}$$

b) (*3 points, no partial credit*) What is the reading $R_1$ on the scale when the ball is fully immersed?

**Solution:** *The ball experiences a buoyant force upward with magnitude equal to the weight of the displaced liquid, $F_B = \rho V g$. By Newton's third law, the ball must exert a force on the liquid of equal magnitude, acting downward. Since the weight of liquid on the scale has been reduced from its initial value by $\rho V g$, the reading is*

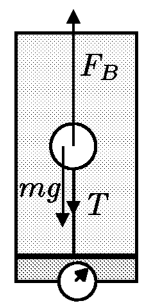
$$R_1 = W - \rho V g + F_B = \boxed{W\ .}$$

c) (*3 points, no partial credit*) If instead of being pushed down by a rod the ball is held in place by a fine string attached to the bottom of the container, what is the tension $T$ in the string?

**Solution:**

*The vertical component of the net force acting on the ball must be zero, so*

$$F_B - mg - T = 0 \ .$$




*But* $F_B = \rho V g$, *so*

$$\boxed{T = \rho V g - mg = (\rho V - m)g \ .}$$

d) (*3 points, no partial credit*) In part (c), what is the reading $R_2$ on the scale?

**Solution:** *There are no forces acting on the items on the scale other than the usual forces of gravity and the normal force of the scale acting upward. So, the scale simply weighs the items. The weight started as* $W$, *was reduced by the weight of the displaced liquid* $\rho V g$, *and increased by the weight of the ball* $mg$:

$$\boxed{\therefore \ R_2 = W - \rho V g + mg \ .}$$

e) (*3 points, no partial credit*) If the string is cut, what will be the initial acceleration $a$ of the ball? Assume that viscosity effects are negligible.

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**Problem 4** (*12 points, after deletion*):

a) (*3 points, no partial credit*) A ball is thrown straight upward with an initial speed $v_0$. Denoting the magnitude of the acceleration of gravity as $g$, and neglecting friction, what will be the maximum height $h$ that the ball will reach?

**Solution:** *By conservation of energy,*

$$\frac{1}{2}mv_0^2 = mgh \ ,$$

*where $m$ is the mass of the ball. So*

$$\boxed{h = \frac{v_0^2}{2g} \ .}$$

b) (*3 points*) A ball of mass $M$ and velocity $\vec{\mathbf{v}}_1 = [v_M, 0, 0]$ collides with a ball of mass $m$ and velocity $\vec{\mathbf{v}}_2 = [0, 0, v_m]$. The two stick together. Ignoring friction, what is the speed of the combined mass after the collision?

**Solution:** *By conservation of momentum,*

$$\vec{\mathbf{P}}_f = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = [Mv_M, 0, mv_m] = (M+m)\vec{\mathbf{v}}_f \ .$$

*So*

$$\vec{\mathbf{v}}_f = \frac{[Mv_M, 0, mv_m]}{M+m} \ ,$$

*and*

$$\boxed{|\vec{\mathbf{v}}_f| = \frac{\sqrt{M^2v_M^2 + m^2v_m^2}}{M+m} \ .}$$

c) (*3 points, no partial credit*) A block of mass $m$ slides down a frictionless hill, starting at a height $h$ and finishing at height zero. Let $g$ be the magnitude of the acceleration of gravity. What is the kinetic energy of the block at the bottom of the hill?

**Solution:** *Conservation of energy implies* $\boxed{E_{k,f} = mgh}$.

d) (*3 points, no partial credit*) A ball of radius $R$ and mass $m$ rolls without slipping down a hill, starting at a height $h$ and finishing at height zero. Again let $g$ be the magnitude of the acceleration of gravity. Neglecting all friction besides the force needed to keep the ball from slipping, what is the kinetic energy of the ball at the bottom of the hill?

> ***Solution:*** *Again, conservation of energy implies* $\boxed{E_{k,f} = mgh}$. [The energy will be divided between translational and rotational kinetic energy, but the total kinetic energy must equal the original potential energy.]

e) (*3 points*) A compressed spring of negligible mass, which provides a fixed but uncalibrated force, is placed in contact with an unknown mass labeled $A$. When the spring is released, so that it pushes on the block, the initial acceleration of the block is measured to have magnitude $a_A$. In an identical experiment with the same spring compressed by the same amount, a block labeled $B$ is found to have an initial acceleration of magnitude $a_B$. If the two blocks are glued together (neglect the mass of the glue) and the identical experiment is carried out with the pair, what will be the magnitude of the initial acceleration?

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**Problem 5** (*11 points, after deletion*):

A space shuttle is in a circular orbit of radius $R$ about the Earth. The shuttle and its contents have mass $M$, and the Earth has mass $M_E$.

a) (*3 points*) In the frame of the Earth, which you may treat as an inertial frame, what is the orbital speed $v_0$ of the shuttle? You may express your answer in terms of any of the quantities $R$, $M$, $M_E$, and $G$ (Newton's constant), and you may assume that $M \ll M_E$.

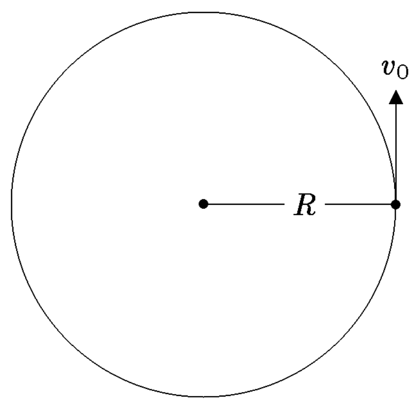
***Solution:***

*Writing the radial component of* $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$,

$$F_r = -\frac{GMM_E}{R^2} = -\frac{Mv_0^2}{R} ,$$

*which implies that*

$$v_0^2 = \frac{GM_E}{R} \quad \Longrightarrow \quad \boxed{v_0 = \sqrt{\frac{GM_E}{R}}} .$$




b) (*4 points*) What is the gravitational potential energy of the shuttle, relative to the potential energy it would have at infinite distance from the Earth? Again you may express your answer in terms of any of the quantities $R$, $M$, $M_E$, and $G$ (Newton's constant).

**Solution:** *In general, for point masses or spherical bodies,*

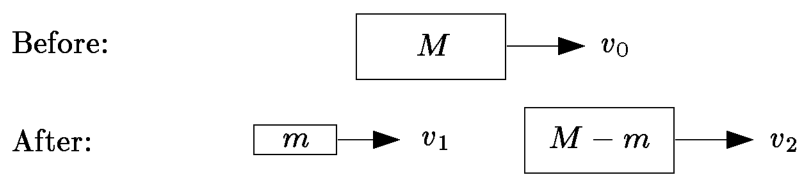
$$U_{\text{grav}} = -\frac{GM_1M_2}{R} .$$

*So, in this case*

$$U_{\text{grav}} = -\frac{GMM_E}{R} - \left[-\frac{GMM_E}{\infty}\right] = \boxed{-\frac{GMM_E}{R}} .$$

c) (*4 points*) A cannon on the shuttle fires a probe of mass $m$ in the direction opposite to the shuttle's velocity. After the firing, the probe has a speed $\Delta v$ relative to the shuttle, where $\Delta v < v_0$. What is the speed $v_1$ of the probe, relative to the Earth, immediately after the firing? You may assume that no other mass is ejected in the firing of the probe. Do not assume, however, that $m$ is negligible compared to $M$. You may express your answer in terms of any of the quantities $R$, $m$, $M$, $M_E$, $G$, $v_0$, and $\Delta v$.

***Solution:***




*Conservation of momentum implies that*

$$mv_1 + (M - m)v_2 = Mv_0 \ . \tag{1}$$

*The condition that the relative speed is $\Delta v$ implies that*

$$v_2 - v_1 = \Delta v \ . \tag{2}$$

*Thus we have two equations and two unknowns ($v_1$ and $v_2$), so the problem is reduced to algebra. From (2),*

$$v_2 = \Delta v + v_1 \ .$$

*Substituting into (1),*

$$mv_1 + (M - m)(\Delta v + v_1) = Mv_0 \ .$$

$$\therefore \ Mv_1 + (M - m)\Delta v = Mv_0$$

$$\boxed{\therefore \ v_1 = v_0 - \frac{(M - m)\Delta v}{M} \ .}$$

d) (*6 points*) After being fired from the cannon, the probe will follow an elliptical orbit. Assume that it remains far enough from the Earth so that friction with the atmosphere can be ignored. Write two equations which could be solved to determine $r_p$ and $v_p$, the radius and speed of the orbit at perigee, the nearest point to the center of the Earth. **Do not attempt to solve these equations.** Note that $r_p$ is to be measured from the center of the Earth. You may express your answer in terms of any of the quantities $v_1$, $R$, $m$, $M_E$, and $G$.

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**Problem 6** (*12 points, after deletion*):

A piston chamber of volume $V_0$ is filled with an ideal monatomic gas at temperature $T_0$ and pressure $P_0$. Denote Boltzmann's constant by $k$, and Avogadro's number by $N_A$.

a) (*2 points, no partial credit*) In terms of the given quantities, what is the number $N$ of atoms of the gas present in the chamber?

***Solution:***

$$PV = NkT \quad \Longrightarrow \quad \boxed{N = \frac{P_0 V_0}{kT_0}\ .}$$

b) (*2 points, no partial credit*) What is the number $\mathcal{N}$ of moles of the gas present in the chamber?

***Solution:***

$$\mathcal{N} = \frac{N}{N_A} \quad \Longrightarrow \quad \boxed{\mathcal{N} = \frac{P_0 V_0}{N_A k T_0}}$$

*or equivalently*

$$PV = \mathcal{N}RT \quad \Longrightarrow \quad \boxed{\mathcal{N} = \frac{P_0 V_0}{RT_0}\ .}$$

c) (*2 points, no partial credit*) The gas is allowed to expand, so the volume increases by an amount $\Delta V$. The gas is heated while it expands by just the right amount to keep it at constant pressure $P_0$. How much work $\Delta W$ does the gas do during this expansion?

***Solution:*** *In general $dW = P\,dV$, so if the pressure is constant we have*

$$\boxed{\Delta W = P_0\,\Delta V\ .}$$

d) (*3 points*) By what amount $\Delta U$ does the internal energy change during this expansion? Define $\Delta U$ so that a positive value denotes an increase in internal energy.

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e) (*6 points*) Starting again from the initial values of $V = V_0$, $T = T_0$, and $P = P_0$, the gas is allowed to expand without the addition or emission of any heat. As the gas expands the values of $T$ and $P$ can both be measured as a function of $V$. Find an expression for $\frac{dP}{dV}$ that depends on no variables other than $P$ and $V$.

**Solution:** *From the first law of thermodynamics,*

$$dU = dQ - P\,dV \ ,$$

*but in this case we are told that no heat is transferred, so* $dQ = 0$. *It follows that*

$$dU = -P\,dV, \ \textit{or} \ \frac{dU}{dV} = -P \ .$$

*For an ideal monatomic gas, we also know that*

$$U = N\left\langle \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}PV \ .$$

*Differentiating this expression gives*

$$\frac{dU}{dV} = \frac{3}{2}P + \frac{3}{2}V\frac{dP}{dV} \ .$$

*Equating the two expressions for* $\frac{dU}{dV}$,

$$\frac{3}{2}P + \frac{3}{2}V\frac{dP}{dV} = -P \ ,$$

*so*

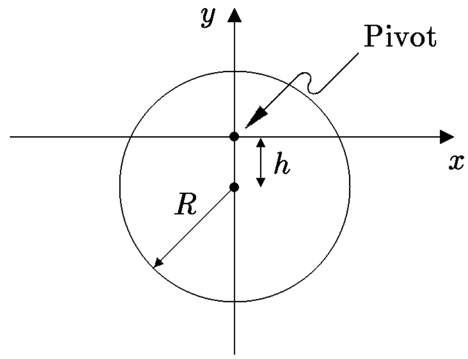
$$\frac{3}{2}V\frac{dP}{dV} = -\frac{5}{2}P \ ,$$

*and*

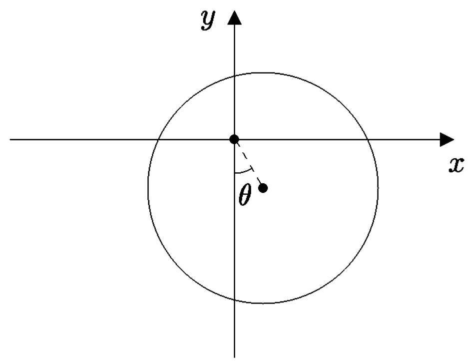
$$\boxed{\frac{dP}{dV} = -\frac{5}{3}\frac{P}{V} \ .}$$

**Problem 7** (*18 points*):

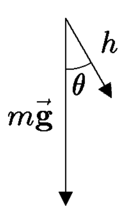
A uniform disk of mass $M$ and radius $R$ is oriented in a vertical plane. The $y$ axis is vertical, and the $x$ axis is horizontal. The disk is pivoted about the origin of the coordinate system, with the center of the disk hanging a distance $h$ below the pivot, as shown. The disk is free to rotate without friction about the pivot in the $x$-$y$ plane. The magnitude of the acceleration of gravity is $g$, directed in the negative $y$ direction.




a) (*4 points*) If the disk is rotated by an angle $\theta$ from its equilibrium position, as shown, what is the magnitude of the torque about the pivot? (In this part, do not assume that the angle $\theta$ is necessarily small.)




***Solution:***




*In general,*

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \ .$$

*In this case the force is gravity, $Mg$ downward, which can be taken to act on the center of mass. From the diagram,*

$$\boxed{|\vec{\tau}| = Mgh \sin\theta \ ,}$$

*where by right-hand rule* *the direction is into the page.* *Expressed alternatively in components,*

$$\boxed{\vec{\tau} = [0, 0, -Mgh \sin\theta] \ .}$$

b) (*4 points*) What is the moment of inertia $I$ of the disk about the pivoted axis?

**Solution:** *This problem can be answered by using the parallel axis theorem. The moment of inertia of a uniform disk about its central axis is given in the table at the start of the exam as*

$$I_{\text{cm}} = \frac{1}{2}MR^2 \ .$$

*By the parallel axis theorem, the moment of inertia about a parallel axis separated by a distance h from the central axis is given by*

$$I = I_{\text{cm}} + Mh^2 \ ,$$

*so*

$$\boxed{I = M\left(\frac{1}{2}R^2 + h^2\right) \ .}$$

c) (*4 points*) If the disk is allowed to oscillate about its equilibrium position, what will be the period of small oscillations? Your answer may be written in terms of $I$, whether or not you answered the previous part.

**Solution:** *The disk is pivoted about the z axis, so the equation of motion is*

$$I\alpha = \tau_z = -Mgh\sin\theta \ .$$

*To discuss small oscillations, we approximate* $\sin\theta \approx \theta$, *so*

$$\alpha = \frac{d^2\theta}{dt^2} = -\frac{Mgh}{I}\theta \ .$$

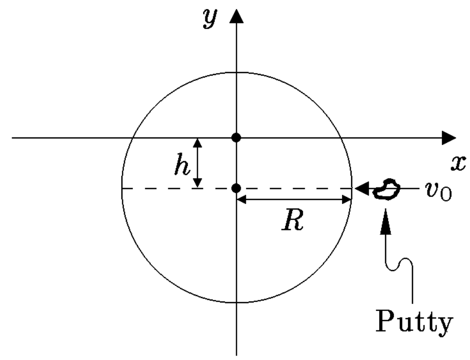
*To put this into the standard form we define*

$$\omega^2 \equiv \frac{Mgh}{I} \ , \ \text{so} \ \frac{d^2\theta}{dt^2} = -\omega^2\theta \ .$$

*As listed in the Formula Sheet, this equation has the solution* $\theta = A\sin\omega t$, *where* $A$ *is any constant. The sine function has a period of* $2\pi$, *so* $\theta$ *will go through one full cycle in a time* $T$, *where* $\omega T = 2\pi$. *So*

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{I}{Mgh}} \ .}$$

d) (*6 points*) A ball of putty of mass $m$ collides with the disk from the right, hitting it at the point $[R, -h, 0]$, as shown. At the moment just before the impact, the ball of putty has a velocity $\vec{\mathbf{v}} = [-v_0, 0, 0]$, and the disk is at rest. If the putty sticks to the side of the disk, what will be its velocity vector $\vec{\mathbf{v}}_f$ immediately after the collision?




***Solution:*** *The collision is inelastic, so mechanical energy is not conserved. The pivot will exert a force to prevent the axis of rotation of the disk from moving, so momentum is also not conserved. But nothing in the problem can exert a torque about the axis of rotation, so angular momentum about the axis is conserved.*

*The angular momentum of the putty is given by* $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$, *where* $\vec{\mathbf{r}}$ *is the displacement vector from the origin (the point about which the angular momentum is to be computed), and* $\vec{\mathbf{p}}$ *is the momentum. Right-hand rule shows that the direction is into the page.* $|\vec{\mathbf{L}}| = pr\sin\theta = pr^{\perp}$, *where* $r^{\perp} = r\sin\theta$ *is the component of* $\vec{\mathbf{r}}$ *in the direction perpendicular to* $\vec{\mathbf{p}}$. $r^{\perp}$ *can be described as the length of a line which extends from the point about which the torque is calculated to the line along which the object is moving, intersecting the line at a right angle. In this case, that line has length* $r^{\perp} = h$, *so*

$$\vec{\mathbf{L}} = [0, 0, -mv_0 h] \ .$$

*After the collision the putty and disk will move together. Since the putty is at a distance* $\sqrt{h^2 + R^2}$ *from the origin, the total moment of inertia will be*

$$I_{\text{tot}} = I + m(h^2 + R^2) \ .$$

*Conservation of angular momentum implies that the angular velocity just after the collision,* $\omega_f$, *can be found from*

$$L_z = I_{\text{tot}}\omega_f = -mv_0 h \quad \Longrightarrow \quad \omega_f = -\frac{mv_0 h}{I + m(h^2 + R^2)} \ .$$

*The velocity of the putty can be found from*

$$\vec{\mathbf{v}}_f = \vec{\boldsymbol{\omega}}_f \times \vec{\mathbf{r}} = \left[0, 0, -\frac{mv_0 h}{I + m(h^2 + R^2)}\right] \times [R, -h, 0] \ .$$

*Using the component definition of the cross product from the Formula Sheet, one has finally,*

$$\boxed{\vec{\mathbf{v}}_f = -\frac{mv_0 h}{I + m(h^2 + R^2)}[h, R, 0] \ .}$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 | Fall 2000

# FINAL EXAMINATION
# FORMULA SHEET
**Monday, December 18, 2000**

**Equations introduced in Chapter 1:**

$$\vec{v} = \frac{d\vec{r}}{dt}\,; \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}\,; \qquad \vec{r}(t_1) = \vec{r}_0 + \int_0^{t_1} \vec{v}\,dt\,; \qquad \vec{v}(t_1) = \vec{v}_0 + \int_0^{t_1} \vec{a}\,dt\,.$$

For *constant* acceleration $\vec{a}$, if $\vec{r} = \vec{r}_0$ and $\vec{v} = \vec{v}_0$ at time $t = 0$, then

$$\vec{v}(t) = \vec{v}_0 + \vec{a}t$$

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2\,.$$

For one-dimensional motion with constant acceleration $a$:

$$v^2 = v_0^2 + 2a(x - x_0)\,.$$

For circular motion at constant speed $v$:

$$a = \frac{v^2}{r}\,,$$

where $r$ is the radius of the circle, and the acceleration is directed towards the center of the circle.

If an object has position $\vec{r}$ and velocity $\vec{v}$, its position and velocity relative to an observer with position $\vec{r}_0$ and velocity $\vec{v}_0$ are given respectively by

$$\vec{r}\,' = \vec{r} - \vec{r}_0\,, \qquad \vec{v}\,' = \vec{v} - \vec{v}_0\,.$$

Average velocity and acceleration are given by

$$\vec{v}_{\text{average}} = \frac{\Delta\vec{r}}{\Delta t}\,, \qquad \vec{a}_{\text{average}} = \frac{\Delta\vec{v}}{\Delta t}\,.$$

**Equations introduced in Chapter 2:**

$\vec{F} = m\vec{a}$ (Newton's second law);

$\vec{F} = -\frac{GMm}{r^2}\hat{r}$ (the gravitational force between two particles);

$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}\hat{r}$ (the electrostatic force between two particles);

$F_x = -kx$ (Hooke's law);

$\frac{d^2x}{dt^2} = -\omega^2 x$ (for a particle near a point of stable equilibrium; equation leads to simple harmonic motion);

$x = A\sin\omega t$ (a solution to the above equation; any solution can be written this way if we choose $t = 0$ when $x = 0$);

$\omega = 2\pi f$ (relation between angular frequency and frequency);

$T = \frac{1}{f} = \frac{2\pi}{\omega}$ (period of an oscillator).

**Equations introduced in Chapter 4:**

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$ (scalar (or dot) product of two vectors)

$\quad = a_x b_x + a_y b_y + a_z b_z$

$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_0^2$ (kinetic & potential energy for projectile)

$\frac{1}{2}mv^2 + U(x) = \text{constant}$ (energy conservation)

$W = K_f - K_i$ (work-energy theorem)

$U = \frac{1}{2}kx^2$ (potential energy for spring force)

$U = mgh$ (gravitational potential energy, near Earth)

$U = -\frac{GMm}{r}$ (gravitational potential energy, spherical bodies)

| 1 Dimension | 3 Dimensions | Description |
|---|---|---|
| $W \equiv F\Delta x$ | $W \equiv \vec{F}\cdot\overrightarrow{\Delta r}$ | Work done by a constant force $\vec{F}$ |
| $W \equiv \int F(x)\,dx$ | $W \equiv \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot\overrightarrow{dr}$ | Work done by a varying force $\vec{F}$ |
| $U(x_p) \equiv U_0 - \int_{x_0}^{x_p} F\,dx$ | $U(\vec{r}_p) \equiv U_0 - \int_{\vec{r}_0}^{\vec{r}_p} \vec{F}\cdot\overrightarrow{dr}$ | Potential energy derived from force $\vec{F}$ |
| $F = -\frac{dU}{dx}$ | $\vec{F} = \left[-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial z}\right]$ | Force derived from potential energy |

**Equations introduced in Chapter 5:**

$\vec{F}_{AB} = -\vec{F}_{BA}$ (Newton's third law);

$\vec{p} = m\vec{v}$ (momentum);

$\frac{d\vec{P}_{\text{tot}}}{dt} = 0$ (conservation of momentum in absence of external force)

$\vec{F} = \frac{d\vec{p}}{dt}$ (Newton's second law in terms of momentum);

$\vec{r}_{\text{cm}} \equiv \frac{1}{M_{\text{tot}}}\sum_i m_i\vec{r}_i$ (position of center of mass);

$\vec{v}_{\text{cm}} \equiv \frac{d\vec{r}_{\text{cm}}}{dt} = \frac{1}{M_{\text{tot}}}\sum_i m_i\vec{v}_i$ (velocity of center of mass);

$\vec{F}^{\text{ext}}_{\text{tot}} = M_{\text{tot}}\vec{a}_{\text{cm}} = \frac{d\vec{P}_{\text{tot}}}{dt}$ (acceleration of a system of particles);

$\vec{P}_{\text{tot}} = \sum_i m_i\vec{v}_i = M_{\text{tot}}\vec{v}_{\text{cm}}$ (momentum of a system of particles);

$K_{\text{tot}} = \frac{1}{2}M_{\text{tot}}v_{\text{cm}}^2 + \sum_i \frac{1}{2}m_i\left(\vec{v}_i - \vec{v}_{\text{cm}}\right)^2$ (K.E. of a system of particles);

$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\,dt = \vec{p}_2 - \vec{p}_1$ (impulse-momentum theorem).

**Equations introduced in Chapter 6:**

$$\left|\vec{\mathbf{F}}_k\right| = \mu_k \left|\vec{\mathbf{N}}\right| \qquad \text{(kinetic friction);}$$

$$\left|\vec{\mathbf{F}}_s\right| \le \mu_s \left|\vec{\mathbf{N}}\right| \qquad \text{(static friction);}$$

$$\vec{\mathbf{F}}_{\text{fict}} = -m\vec{\mathbf{a}}(t) \qquad \text{(fictitious force in linearly accelerating frame).}$$

**Equations introduced in Chapter 8:**

Most of the equations in this chapter are most easily remembered in the context of the analogous equations for linear motion in one dimension:

| TRANSLATION (one dimension) | | ROTATION (about fixed axis) | |
|---|---|---|---|
| Name | Symbol | Name | Symbol |
| Position | $x$ | Orientation | $\theta$ |
| Velocity | $v = \dfrac{dx}{dt}$ | Angular velocity | $\omega = \dfrac{d\theta}{dt}$ |
| Acceleration | $a = \dfrac{dv}{dt}$ | Angular acceleration | $\alpha = \dfrac{d\omega}{dt}$ |
| Mass | $M = \sum_i m_i$ | Moment of inertia | $I = \sum_i m_i R_i^2$ |
| Force | $F$ | Torque | $\tau = F_\perp R$ $= \pm\lvert\vec{\mathbf{F}}\rvert R_\perp$ |
| Force equation | $\sum_i \vec{\mathbf{F}}^{\text{ext}} = M\vec{\mathbf{a}}_{\text{cm}}$ | Torque equation | $\sum_i \boldsymbol{\tau}^{\text{ext}} = I\alpha$ |
| Momentum | $p = Mv$ | Angular momentum | $L = I\omega$ |
| Kinetic energy | $\frac{1}{2}Mv^2$ | Kinetic energy | $\frac{1}{2}I\omega^2$ |
| Work done | $\vec{\mathbf{F}} \cdot \overrightarrow{\Delta r}$ | Work done | $\boldsymbol{\tau}\,\Delta\theta$ |

Other equations introduced in this chapter:

$$v_r = 0\ ; \quad v_\perp = R\omega \qquad \text{(velocity of point on rotating body);}$$

$$a_r = -\frac{v^2}{R} = -R\omega^2\ ; \quad a_\perp = R\alpha \qquad \text{(acceleration of point on rotating body);}$$

$$v = \pm R|\omega| \qquad \text{(rolling without slipping);}$$

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{d\vec{\mathbf{p}}}{dt} \\ \sum \boldsymbol{\tau}^{\text{ext}} &= I_{\text{cm}}\alpha = \frac{dL}{dt} \end{aligned}\right\} \qquad \text{(combined translational and rotational motion);}$$

$$K_{\text{tot}} = \tfrac{1}{2}Mv_{\text{cm}}^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2 \qquad \text{(kinetic energy for combined translational and rotational motion);}$$

$$I_{\parallel} = I_{\text{cm}} + Md^2 \qquad \text{(parallel-axis theorem);}$$

$$I_z = I_x + I_y \qquad \text{(perpendicular-axis theorem).}$$

3

**TABLE OF STANDARD MOMENTS OF INERTIA:**

| | |
|---|---|
| Slender uniform rod of length $\ell$, axis through center and perpendicular to axis of rod | $\frac{1}{12}m\ell^2$ |
| Rectangular plate with dimensions $a \times b$, axis along one of the $b$ edges | $\frac{1}{3}ma^2$ |
| Thin-walled hollow cylinder of radius R, axis along axis of cylinder | $mR^2$ |
| Uniform solid cylinder of radius R, axis along axis of cylinder | $\frac{1}{2}mR^2$ |
| Thin-walled hollow sphere of radius $R$, axis through center | $\frac{2}{3}mR^2$ |
| Solid uniform sphere of radius $R$, axis through center | $\frac{2}{5}mR^2$ |

**Equations introduced in Chapter 9:**

$$\begin{aligned} c_x &= a_y b_z - a_z b_y\ ; \\ c_y &= a_z b_x - a_x b_z\ ; \\ c_z &= a_x b_y - a_y b_x\ . \end{aligned} \qquad \text{(vector cross product, component form);}$$

$$|\vec{\mathbf{c}}| = |\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta \qquad \text{(magnitude of vector cross product);}$$

$$\vec{\mathbf{v}} = \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}} \qquad \text{(velocity of atom in rotating body with a fixed point);}$$

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_P + \vec{\boldsymbol{\omega}} \times (\vec{\mathbf{r}} - \vec{\mathbf{r}}_P) \qquad \text{(velocity of atom in rotating body, general case);}$$

$$\vec{\mathbf{L}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i \qquad \text{(angular momentum, as vector product);}$$

$$\vec{\boldsymbol{\tau}} = \sum_i \vec{\mathbf{r}}_i \times \vec{\mathbf{F}}_i \qquad \text{(vector torque, as vector product);}$$

$$\vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt} \qquad \text{(torque equation);}$$

$$\left.\begin{aligned} \sum \vec{\mathbf{F}}^{\text{ext}} &= M\vec{\mathbf{a}}_{\text{cm}} = \frac{d\vec{\mathbf{p}}}{dt} \\ \sum \vec{\boldsymbol{\tau}}^{\text{ext}} &= \frac{d\vec{\mathbf{L}}_{\text{cm}}}{dt} \end{aligned}\right\} \qquad \text{(combined translational and rotational motion);}$$

$$\begin{aligned} \vec{\mathbf{L}} &= \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{p}}_{\text{tot}} \\ &\quad + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times m_i \vec{\mathbf{v}}_{\text{rel},i} \end{aligned} \qquad \text{(angular momentum decomposition);}$$

$$\begin{aligned} \vec{\boldsymbol{\tau}} &= \vec{\mathbf{r}}_{\text{cm}} \times \vec{\mathbf{F}}_{\text{tot}} \\ &\quad + \sum_i \vec{\mathbf{r}}_{\text{rel},i} \times \vec{\mathbf{F}}_i \end{aligned} \qquad \text{(torque decomposition).}$$

4

**Equations introduced in Chapter 11:**

$$PV = \frac{2}{3}N\left\langle\frac{1}{2}mv^2\right\rangle$$ (pressure of an ideal gas);

$$\left\langle\frac{1}{2}mv^2\right\rangle = \frac{3}{2}kT$$ (definition of kinetic temperature);

$$PV = NkT = \mathcal{N}RT$$ (ideal-gas law);

$$\Delta U = Q - W$$ (first law of thermodynamics);

$$\Delta W = P\,\Delta V$$ (work done by expanding gas).

* Useful constants:

$R = 8.3$ J/mol · K (gas constant);

$k = 1.38 \times 10^{-23}$ J/K (Boltzmann's constant);

$N_A = 6.02 \times 10^{23}$ molecules/mole (Avogadro's number);

1 u $= 1.66 \times 10^{-27}$ kg (atomic mass unit).

* Temperature scales:

$$T(\text{K}) = T(^\circ\text{C}) + 273.15\ \text{K}$$ (Centigrade to Kelvin)

$$T(^\circ\text{F}) = 32^\circ\text{F} + \frac{9}{5}T(^\circ\text{C})$$ (Centigrade to Fahrenheit)

$$T(^\circ\text{C}) = \frac{5}{9}\left(T(^\circ\text{F}) - 32^\circ\text{F}\right)$$ (Fahrenheit to Centigrade)

The triple point of water corresponds to 273.16 K, or 0.01°C, by definition.



**Equations introduced in Chapter 12:**

$$P_2 - P_1 = -\rho g(y_2 - y_1)$$ (Pascal's Law: pressure in a liquid as a function of height, for a stationary liquid);

$$A_2 v_2 = A_1 v_1$$ (equation of continuity for steady flow);

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$ (Bernoulli's equation);

$$\gamma = \frac{F}{\ell} = \frac{U}{A}$$ (surface tension).

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

Physics 8.01 Fall 2000

# FINAL EXAM
## MONDAY, December 18, 2000

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GIVEN (First) NAME

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Student ID Number

**Your Recitation (check one) →**

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|---|---|---|---|
| R01 | MW 1:00 | W. Bertozzi | |
| R02 | MW 2:00 | W. Bertozzi | |
| R03 | MW 3:00 | W. Bertozzi | |
| R12 | TR 1:00 | A. Bolton | |
| R18 | TR 9:00 | B. Burke | |
| R19 | TR 10:00 | B. Burke | |
| R20 | TR 11:00 | B. Burke | |
| R21 | TR 2:00 | M. Evans | |
| R22 | TR 3:00 | M. Evans | |
| R06 | MW 2:00 | M. Feld | |
| R07 | MW 3:00 | M. Feld | |
| R08 | MW 4:00 | M. Feld | |
| R16 | TR 11:00 | D. Fernie | |
| R15 | TR 10:00 | A.Guth | |
| R23 | TR 11:00 | J. Hager | |
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| R09 | MW 1:00 | P. Joss | |
| R10 | MW 2:00 | P. Joss | |
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| R26(M) | TR 3:00 | McBride/Bove | |
| R17 | TR 12:00 | TA- Ribeiro | |
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| R14 | TR 3:00 | J. Shelton | |
| R04 | MW 1:00 | G. Stephans | |
| R05 | MW 2:00 | G. Stephans | |

Instructions:

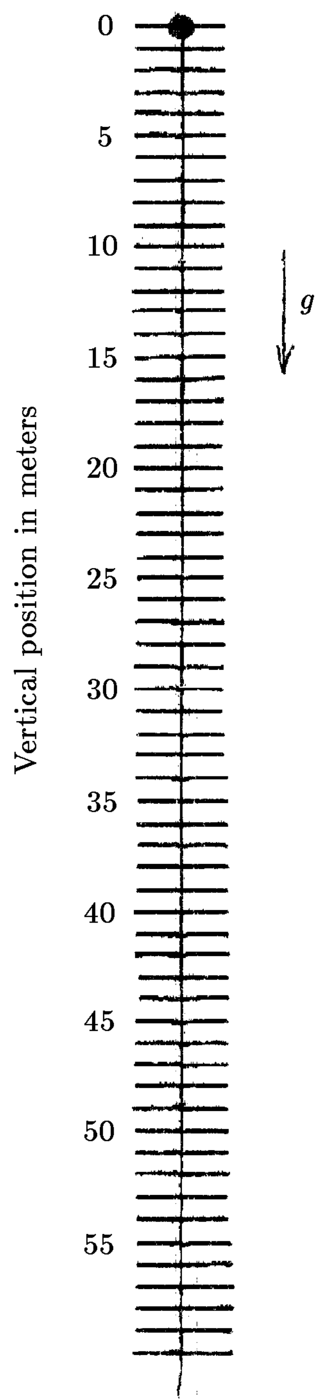
1. SHOW ALL WORK. All work must be done in this booklet. Extra blank pages are provided.
2. This is a closed book exam.
3. <u>CALCULATORS</u>, BOOKS, and NOTES are NOT ALLOWED.
4. Do all TEN (10) problems.

**Yellow Formula Sheets for this exam will be handed out separately.**

| Problem | Maximum | Score | Grader |
|---|---|---|---|
| 1 | 10 | | |
| 2 | 10 | | |
| 3 | 10 | | |
| 4 | 10 | | |
| 5 | 10 | | |
| 6 | 10 | | |
| 7 | 10 | | |
| 8 | 10 | | |
| 9 | 10 | | |
| 10 | 10 | | |
| **TOTAL** | 100 | | |

**Problem 1** (10 points, no partial credit)

At a location where the acceleration due to gravity is 10 $m/s^2$, a 2 kg ball is dropped from rest in vacuum at $t = 0$. On the scale below, indicate the vertical position of the ball at one second intervals after the ball is released (i.e., at $t = 1\,s, 2\,s, 3\,s, \ldots$) until it falls off of the scale.



**Problem 2** (10 points)

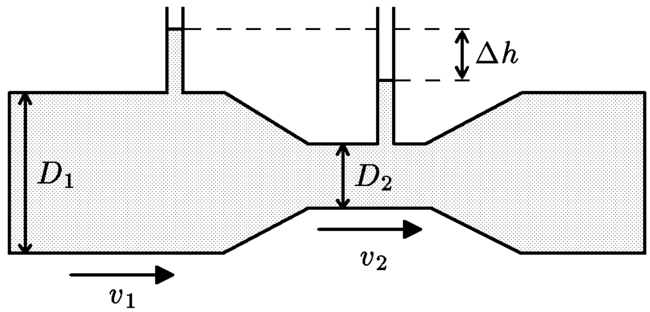
You are walking on a horizontal road. At some instant of time you accelerate forward. Your acceleration has magnitude $a$. Your mass is $M$.

a) In words, state what forces are acting on you and which force causes the acceleration.

b) What is the magnitude of that force?

**Problem 3** (10 points)

The diagram shows a Venturi meter installed in a water main. The pipe has a circular cross section at all points, with diameter $D_1$ in the first segment and $D_2$ in the second segment, with $D_2 < D_1$. The mass density of the water is $\rho$, and the acceleration of gravity is $g$ ($g > 0$). If the water in the pipe is flowing at volume flow rate $R$ (measured, for example, in $m^3/s$), what is:




a) the speed of flow $v_1$ in the first section of pipe (of diameter $D_1$), and the speed of flow $v_2$ in the second section of pipe (of diameter $D_2$)?

b) the difference in the water level $\Delta h$ in the two tubes?

**Problem 4** (10 points)

A monatomic ideal gas, originally at a pressure $P$, volume $V$ and temperature $T$, is compressed to one half of its initial volume.

A) If the compression is isothermal (i.e., at constant temperature)

a) The final pressure is:

i) $P$ ii) $2P$ iii) $3P$ iv) $4P$ v) $5P$

vi) $P/2$ vii) $P/3$ viii) $P/4$ ix) $P/5$

b) The work done ***by the gas*** during the compression is:

i) $-PV \ln 2$ ii) $PV \ln 2$ iii) $-\frac{P}{V} \ln 2$ iv) $\frac{P}{V} \ln 2$ v) $-2PV$

vi) $2PV$ vii) $-2\frac{PV}{T}$ viii) $2\frac{PV}{T}$ ix) $-\frac{PV}{2T}$

ix) $\frac{PV}{2T}$

B) If the compression is isobaric (i.e., at constant pressure)

a) The final temperature is:

i) $T/2$ ii) $2T$ iii) $T/4$ iv) $4T$ v) $T \ln 2$

vi) $T/\ln 2$ vii) $T \ln 4$ viii) $T/\ln 4$

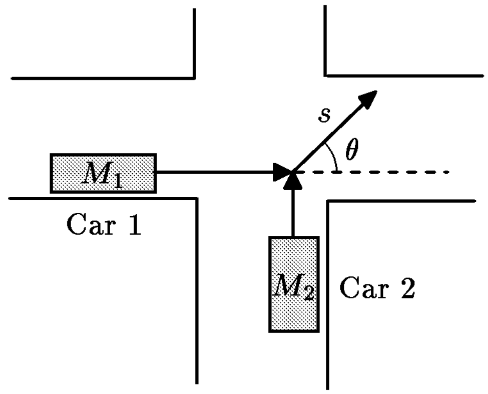
b) The amount of heat supplied ***to the gas*** during the compression is :

i) $-\frac{1}{4}PV$ ii) $\frac{1}{4}PV$ iii) $-\frac{1}{2}PV$ iv) $\frac{1}{2}PV$ v) $-\frac{3}{4}PV$

vi) $\frac{3}{4}PV$ vii) $-\frac{5}{4}PV$ viii) $\frac{5}{4}PV$ ix) $-\frac{11}{4}PV$

ix) $\frac{11}{4}PV$

**Problem 5** (10 points)



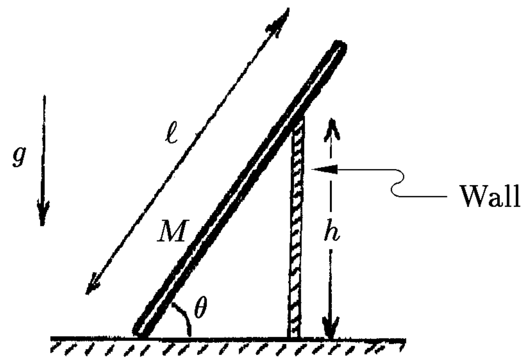

Two cars collide at an intersection. They remain locked together after the collision and travel a distance $s$, at an angle $\theta$ to car 1's original direction. Car 1 has mass $M_1$ and car 2 has mass $M_2$. The accident happened in conditions when the coefficient of kinetic friction between rubber and the road is $\mu_k$. What were the speeds of the two cars immediately before the collision?

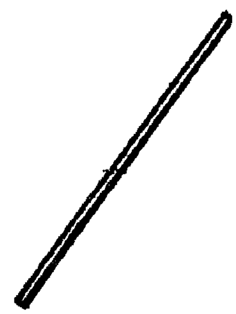
You may assume that the acceleration due to gravity is $g$, and that the force of the collision causes the wheels of the cars to immediately lock, so that the rotation of wheels can be ignored.

**Problem 6** (10 points)

A uniform plank of wood with mass $M$ and length $\ell$ rests against the top of a free standing wall which has height $h$ and a frictionless top. The plank makes an angle $\theta$ with the horizontal.

a) On the picture on the right, draw a free body diagram for the plank.



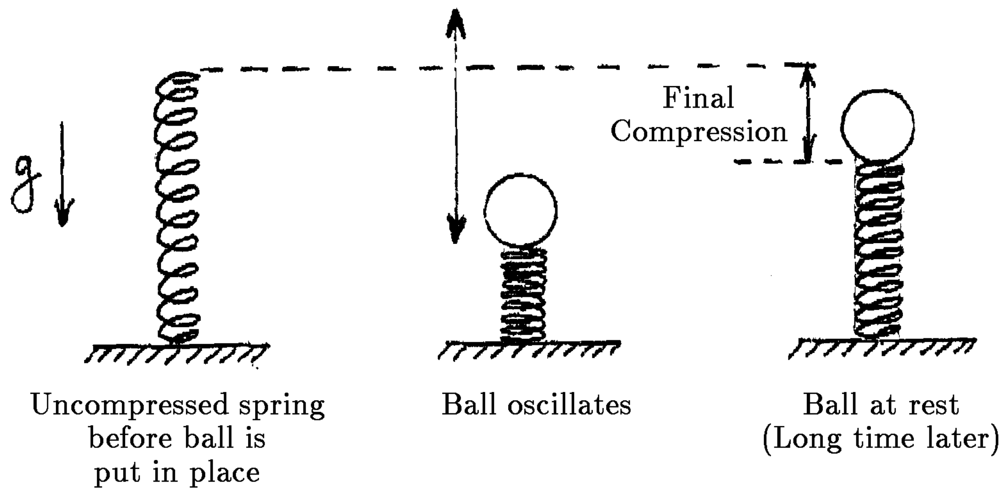
b) In the boxes below, write a complete set of independent equations which when solved give the minimum value of $\theta$ for which the plank will not slip. Express your equations in terms of only $M$, $\ell$, $h$, $g$, and $\mu_s$, the coefficient of static friction between the plank and the floor. Do not solve the equations.

Note: The number of equations you write could depend on how you have defined your variables, so some correct answers will not fill all boxes.

**Problem 7** (10 points)

A ball is placed on a vertical massless spring which obeys Hooke's Law and which initially has its natural uncompressed length. It is observed that at first the ball makes vertical simple harmonic oscillations with period $T$.

After a very large number of oscillations the ball comes to rest because of air resistance and losses in the spring. What is the final compression of the spring in terms of ***only*** $T$ and $g$.



**Problem 8** (10 points)

You have been given a nugget which you are told is a mixture of gold and zinc. You want to find out how much gold you have been given. Being an MIT student you make the following observations:

1. You put a cup partly full of water on an electronic (weight) scale and observe that it reads $M_1$, meaning that the force on the scale is equivalent to the gravitational force of a mass $M_1$.
2. You attach the nugget to a very thin stiff piece of wire and hold the nugget in the water fully submerged but not touching the bottom of the cup. The water does not overflow. You observe that the scale now reads $M_2$.
3. You remove the wire and drop the nugget into the cup. No water is spilled. The scale now reads $M_3$.
4. In a reference book you find that gold has a density $\rho_{\mathrm{Au}}$, zinc $\rho_{\mathrm{Zn}}$, and water $\rho_{\mathrm{H_2O}}$.

Using these observations determine

a) the volume of the nugget

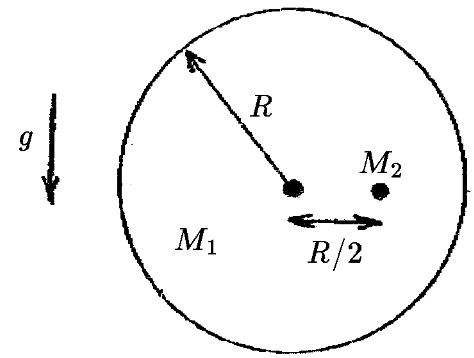
b) the mass of the nugget

c) the mass of the gold in the nugget.

**Problem 9** (10 points)

A uniform disk of mass $M_1$ and radius $R$ is pivoted on a frictionless horizontal axle through its center.

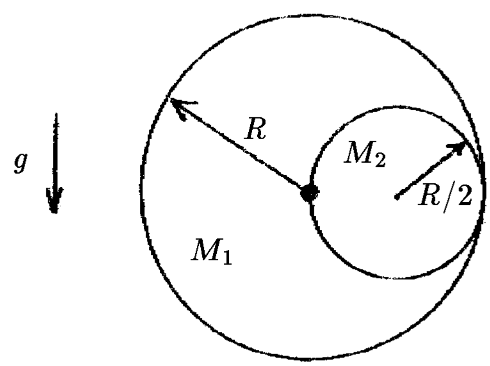
a) A small mass $M_2$ is attached to the disk at radius $R/2$, at the same height as the axle. If this system is released from rest:



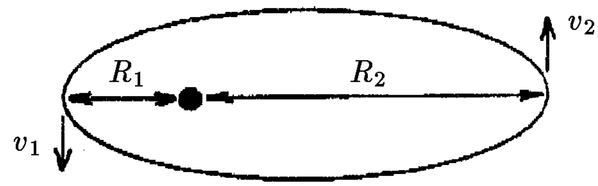

i) What is the angular acceleration of the disk immediately after it is released?

ii) What will be the magnitude of the maximum angular velocity that the disk will reach?

b) Now consider the situation if the mass $M_2$ is a disk of radius $R/2$ located with its center at the same place where $M_2$ is located in part (a). For this case, find the angular acceleration immediately after the system is released from rest.



**Problem 10** (10 points)




A satellite follows an elliptical orbit. Its closest approach to the earth is $R_1$, at which point it has speed $v_1$, and the furthest point is $R_2$, at which point it has speed $v_2$. Both distances are measured from the center of the earth. At the surface of the earth the acceleration due to gravity is $g$ and the earth's radius is $R$.

What is the magnitude of $v_1$ in terms of ***only*** $R_1$, $R_2$, $R$ and $g$?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Physics

Physics 8.01 Fall 2000

# FINAL EXAM SOLUTIONS

## MONDAY, December 18, 2000

| | | | | | | | | | | | | | | | |
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FAMILY (Last) NAME

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GIVEN (First) NAME

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Student ID Number

*Solutions written by Alan Guth*

**Your Recitation (check one) →**

| | | | |
|---|---|---|---|
| R01 | MW 1:00 | W. Bertozzi | |
| R02 | MW 2:00 | W. Bertozzi | |
| R03 | MW 3:00 | W. Bertozzi | |
| R12 | TR 1:00 | A. Bolton | |
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| R04 | MW 1:00 | G. Stephans | |
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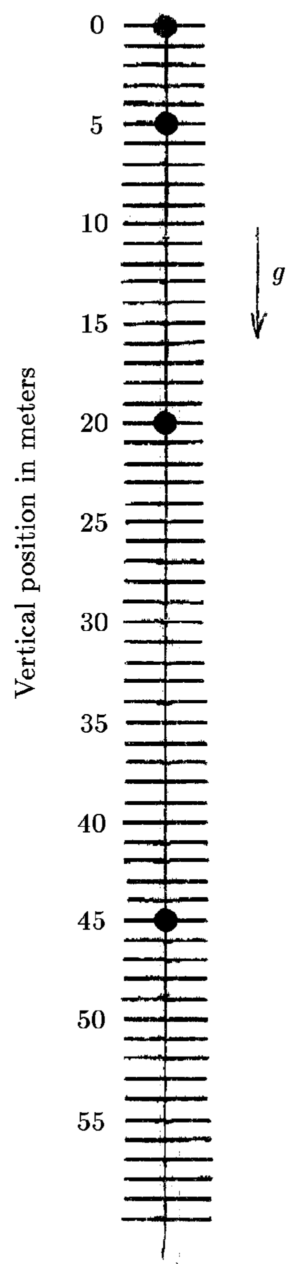
| Problem | Maximum | Score | Grader |
|---|---|---|---|
| 1 | 10 | | |
| 2 | 10 | | |
| 3 | 10 | | |
| 4 | 10 | | |
| 5 | 10 | | |
| 6 | 10 | | |
| 7 | 10 | | |
| 8 | 10 | | |
| 9 | 10 | | |
| 10 | 10 | | |
| **TOTAL** | 100 | | |

*Note: this exam included a 6-page formula sheet, which can be downloaded separately.*

**Problem 1** (10 points, no partial credit)

At a location where the acceleration due to gravity is 10 m/s$^2$, a 2 kg ball is dropped from rest in vacuum at $t = 0$. On the scale below, indicate the vertical position of the ball at one second intervals after the ball is released (i.e., at $t = 1\,\mathrm{s}, 2\,\mathrm{s}, 3\,\mathrm{s}, \ldots$) until it falls off of the scale.




$$s = \tfrac{1}{2}gt^2 = 5\,\mathrm{m}\,(t/\mathrm{s})^2$$

| $t$ | $s$ |
|---|---|
| 1 s | 5 m |
| 2 s | 20 m |
| 3 s | 45 m |
| 4 s | 80 m |

**Problem 2** (10 points)

You are walking on a horizontal road. At some instant of time you accelerate forward. Your acceleration has magnitude $a$. Your mass is $M$.

a) In words, state what forces are acting on you and which force causes the acceleration.

b) What is the magnitude of that force?

***Solution:***

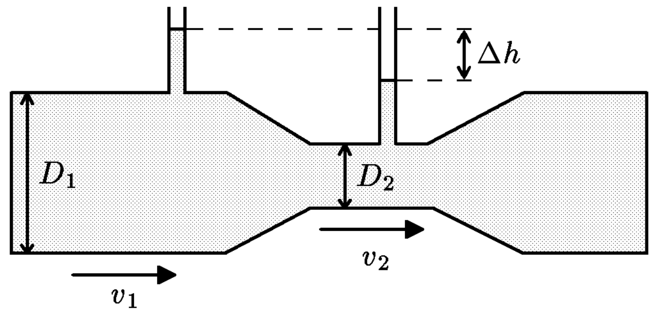
*a) The forces are that of gravity acting downward, the normal force of the road acting upward, and the force of friction acting forward. It is the force of friction that causes the acceleration.*

*b) Since* $\vec{\mathbf{F}} = M\vec{\mathbf{a}}$, *the magnitude of the frictional force must be*

$$\boxed{\left|\vec{\mathbf{F}}_{\text{friction}}\right| = Ma\ .}$$

**Problem 3** (10 points)

The diagram shows a Venturi meter installed in a water main. The pipe has a circular cross section at all points, with diameter $D_1$ in the first segment and $D_2$ in the second segment, with $D_2 < D_1$. The mass density of the water is $\rho$, and the acceleration of gravity is $g$ ($g > 0$). If the water in the pipe is flowing at volume flow rate $R$ (measured, for example, in $\text{m}^3/\text{s}$), what is:




a) the speed of flow $v_1$ in the first section of pipe (of diameter $D_1$), and the speed of flow $v_2$ in the second section of pipe (of diameter $D_2$)?

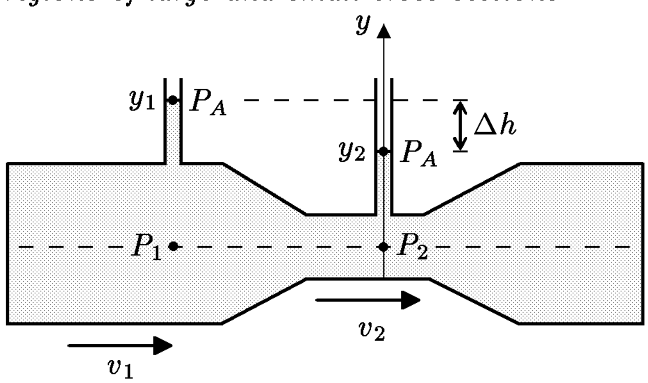
b) the difference in the water level $\Delta h$ in the two tubes?

***Solution:***

*a) The volume flow rate $R$ is constant throughout the pipe and is given by the product of the cross sectional area $A$ of the pipe and the speed of the flow $v$. Hence*

$$v_1 = \frac{R}{A_1} = \boxed{\frac{4R}{\pi D_1^2}}\,, \qquad v_2 = \boxed{\frac{4R}{\pi D_2^2}}\,.$$

*b) First, we use Bernoulli's equation to relate the pressures $P_1$ and $P_2$ at the center of the pipe in the regions of large and small cross sections*



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad \Longrightarrow \quad P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \ .$$

*We have defined the vertical y-coordinate to be zero at the center of the pipe. Now we use Pascal's law to relate the pressures $P_1$ and $P_2$ to the pressure $P_A$ at the heights $y_1$ and $y_2$ of the water levels in the two tubes. Since the two tubes are in contact with the surrounding air, the pressure at the top of either column of liquid is just the ambient air pressure $P_A$. We find*

$$P_1 = P_A + \rho g y_1, \ P_2 = P_A + \rho g y_2 \quad \Longrightarrow$$

$$\Delta h = y_1 - y_2 = \frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g} = \boxed{\frac{8R^2}{\pi^2 g}\left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right)} \ .$$

*Note on subtle point:* In this problem one has to be careful about where to apply Bernoulli's equation, and where to use Pascal's law. The correct solution uses Bernoulli's equation to find the pressure differences along the flow line through the center of the pipe, but Pascal's law must be used to find how the pressure varies with height.

Along the $y$-axis, for example, Pascal's law says that the pressure should vary according to

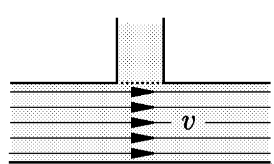
$$P(y) = P_2 - \rho g y \ ,$$

where $\rho$ is the density of water and $g$ is the acceleration of gravity. Note that Bernoulli's equation would give a different result, since it would imply that

$$P(y) + \frac{1}{2}\rho v^2(y) + \rho g y \underset{\substack{\text{(If Bernoulli's eq} \\ \text{were valid)}}}{=} P_2 + \frac{1}{2}\rho v_2^2 \ .$$

The two equations agree when $v(y) = v_2$, a relation which holds inside the horizontal pipe but not in the vertical pipes (where $v \approx 0$).

To understand which equation is valid, we need to examine the behavior of the water where its velocity $v$ changes, at the interface of the horizontal and vertical pipes. While the actual flow of water at such an interface can be complicated, for our purposes we can approximate the change in the water velocity as happening discontinuously along a horizontal line:



Recall that the derivation of Bernoulli's equation showed that the Bernoulli quantity is constant along flow lines. Since there are no flow lines that cross the dotted line along the velocity discontinuity, we can see that there is no reason to believe that the Bernoulli quantity has the same value on both sides. Pascal's equation, on the other hand, was derived by examining the forces on the water in the vertical direction. Since the vertical acceleration of the water is zero both above and below the dotted line, the derivation of Pascal's equation remains valid. The pressure varies continuously across the dotted line, while the velocity and the Bernoulli quantity undergo a jump at the dotted line.

Note, however, that Bernoulli's equation does describe the pressure variation along the flow lines of a pipe, even when those flow lines are vertical. In that case the derivation of Pascal's equation can break down, since the vertically flowing liquid can undergo acceleration in the vertical direction, if the pipe changes diameter.

**Problem 4** (10 points)

A monatomic ideal gas, originally at a pressure $P$, volume $V$ and temperature $T$, is compressed to one half of its initial volume.

A) If the compression is isothermal (i.e., at constant temperature)

a) The final pressure is:

i) $P$ ii) $2P$ iii) $3P$ iv) $4P$ v) $5P$

vi) $P/2$ vii) $P/3$ viii) $P/4$ ix) $P/5$

b) The work done ***by the gas*** during the compression is:

i) $-PV\ln 2$ ii) $PV\ln 2$ iii) $-\frac{P}{V}\ln 2$ iv) $\frac{P}{V}\ln 2$ v) $-2PV$

vi) $2PV$ vii) $-2\frac{PV}{T}$ viii) $2\frac{PV}{T}$ ix) $-\frac{PV}{2T}$

ix) $\frac{PV}{2T}$

B) If the compression is isobaric (i.e., at constant pressure)

a) The final temperature is:

i) $T/2$ ii) $2T$ iii) $T/4$ iv) $4T$ v) $T\ln 2$

vi) $T/\ln 2$ vii) $T\ln 4$ viii) $T/\ln 4$

b) The amount of heat supplied ***to the gas*** during the compression is:

i) $-\frac{1}{4}PV$ ii) $\frac{1}{4}PV$ iii) $-\frac{1}{2}PV$ iv) $\frac{1}{2}PV$ v) $-\frac{3}{4}PV$

vi) $\frac{3}{4}PV$ vii) $-\frac{5}{4}PV$ viii) $\frac{5}{4}PV$ ix) $-\frac{11}{4}PV$

ix) $\frac{11}{4}PV$

***Solution:***

*A) a) Since $PV = NkT$, constant temperature implies that $P \propto 1/V$. So if $V$ is halved, $P$ is doubled. The correct answer is* $\boxed{(ii)\ 2P.}$

*b) Since the pressure is changing, we must integrate to find the total work done:*

$$W = \int P\,\mathrm{d}V\ .$$

*Since* $P \propto 1/V$, *we can write* $P = P_0(V_0/V)$, *where* $P_0$ *and* $V_0$ *denote the original pressure and volume. So*

$$W = P_0 V_0 \int_{V_0}^{\frac{1}{2}V_0} \frac{\mathrm{d}V}{V} = P_0 V_0 \left[\ln\left(\frac{1}{2}V_0\right) - \ln V_0\right] = -P_0 V_0 \ln 2 \ .$$

*Since the problem called the initial values of pressure and volume* $P$ *and* $V$, *respectively, the right answer is* **(i)** $-PV \ln 2$.

B) a) *For isobaric expansion (constant pressure),* $PV = NkT$ *implies that* $T \propto V$. *So, if the volume is halved, then the temperature must be halved, and the correct answer is* **(i)** $T/2$.

b) *First we must calculate the work done by the gas. Since the pressure is constant, this is simply*

$$W = P\,\Delta V = -\frac{1}{2}PV \ .$$

*Next we must calculate the change in the internal energy of the gas. For a monatomic ideal gas, the internal energy is given by*

$$U = N\left\langle \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}NkT = \frac{3}{2}PV \ .$$

*During the compression the temperature falls by a factor of 2, so the internal energy also falls by a factor of 2, and therefore*

$$\Delta U = -\frac{3}{4}NkT = -\frac{3}{4}PV \ .$$

*By conservation of energy,*

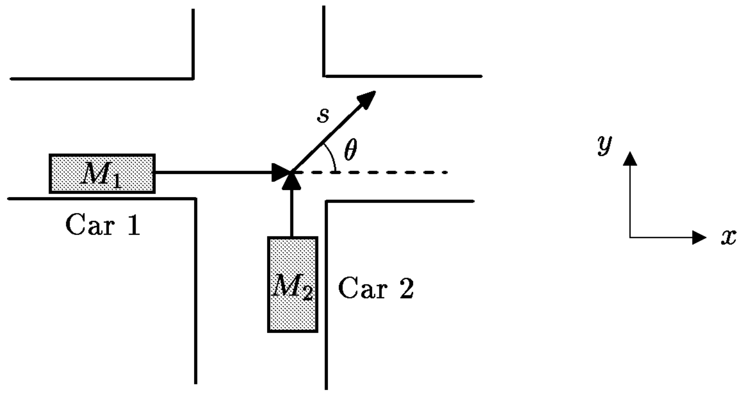
$$\Delta U = Q - W \ ,$$

*so the heat* $Q$ *supplied to the gas is given by*

$$Q = \Delta U + W = -\frac{5}{4}PV \ .$$

*The correct answer is therefore* **(vii)** $-\frac{5}{4}PV$.

**Problem 5** (10 points)




Two cars collide at an intersection. They remain locked together after the collision and travel a distance $s$, at an angle $\theta$ to car 1's original direction. Car 1 has mass $M_1$ and car 2 has mass $M_2$. The accident happened in conditions when the coefficient of kinetic friction between rubber and the road is $\mu_k$. What were the speeds of the two cars immediately before the collision?

You may assume that the acceleration due to gravity is $g$, and that the force of the collision causes the wheels of the cars to immediately lock, so that the rotation of wheels can be ignored.

**Solution:** *We treat the sequence of events as an instantaneous collision followed by a period of skidding. During the skidding phase, the only horizontal force acting is that of kinetic friction, which has a magnitude* $F_f = \mu_k(M_1 + M_2)g$. *This force directly opposes the motion, so the work done by friction is* $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{r}} = -\mu_k(M_1 + M_2)gs$. *By the work-energy theorem this must equal the change in the kinetic energy of the wreckage. Since the final kinetic energy is zero, the kinetic energy at the start of the skidding phase must be* $E_k = \mu_k(M_1 + M_2)gs$. *Thus the speed at the start of the skidding phase is given by*

$$\frac{1}{2}(M_1 + M_2)v_s^2 = \mu_k(M_1 + M_2)gs \quad \Longrightarrow \quad v_s = \sqrt{2\mu_k gs}\ .$$

*This is the speed of the wreckage just after the collision.*

*The collision is inelastic, since the cars stick together, so kinetic energy is not conserved. Momentum is conserved, however, as long as there are no external forces. (Note that the downward force of gravity is canceled by the upward normal force, but the force of friction can act horizontally during the collision. We use the approximation, however, that the collision happens during a very short length of time, so the* *change in momentum due to friction during the collision is negligible.) If we adopt a coordinate system as shown above, conservation of momentum can be written as*

$$M_1 v_1 = (M_1 + M_2) v_s \cos\theta \qquad (x\text{-component})$$
$$M_2 v_2 = (M_1 + M_2) v_s \sin\theta \qquad (y\text{-component}) \ ,$$

*where $v_1$ and $v_2$ are the speeds of the two cars, respectively, before the collision. Thus*

$$\boxed{v_1 = \frac{M_1 + M_2}{M_1}\sqrt{2\mu_k g s}\,\cos\theta \ ,}$$

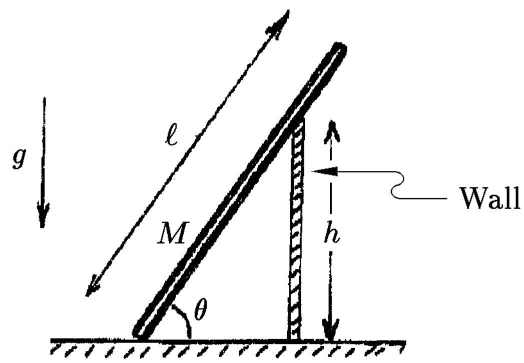
*and*

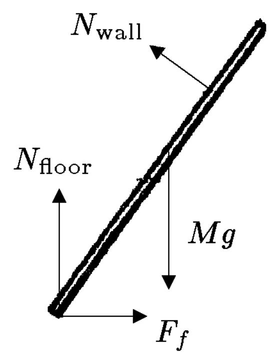
$$\boxed{v_2 = \frac{M_1 + M_2}{M_2}\sqrt{2\mu_k g s}\,\sin\theta \ .}$$

**Problem 6** (10 points)

A uniform plank of wood with mass $M$ and length $\ell$ rests against the top of a free standing wall which has height $h$ and a frictionless top. The plank makes an angle $\theta$ with the horizontal.

a) On the picture on the right, draw a free body diagram for the plank.




b) In the boxes below, write a complete set of independent equations which when solved give the minimum value of $\theta$ for which the plank will not slip, in terms of only $M$, $\ell$, $h$, $g$, and $\mu_s$, the coefficient of static friction between the plank and the floor. Do not solve the equations.

$F_x$: $F_f - N_{\text{wall}} \sin\theta = 0$

$F_y$: $N_{\text{floor}} - Mg + N_{\text{wall}} \cos\theta = 0$

$\tau$ (about contact with floor): $-Mg\dfrac{\ell}{2}\cos\theta + \dfrac{N_{\text{wall}} h}{\sin\theta} = 0$

About to slip: $F_f = \mu_s N_{\text{floor}}$

Note: The number of equations you write could depend on how you have defined your variables, so some correct answers will not fill all boxes.

*Alternatively, you could have calculated the torque about different points:*

*About center of plank:* $-N_{\text{floor}}\dfrac{\ell}{2}\cos\theta + F_f\dfrac{\ell}{2}\sin\theta + N_{\text{wall}}\left(\dfrac{h}{\sin\theta} - \dfrac{\ell}{2}\right) = 0$

*About contact with wall:* $F_f h - \dfrac{N_f h}{\tan\theta} + Mg\left(\dfrac{h}{\tan\theta} - \dfrac{\ell}{2}\cos\theta\right) = 0$

*Extension of solution:* You were not asked to solve these equations, but now that the exam is over you might be interested in trying. After the unknowns $N_{\text{wall}}$, $N_{\text{floor}}$, and $F_f$ are eliminated, one is left with one equation to determine $\theta$:

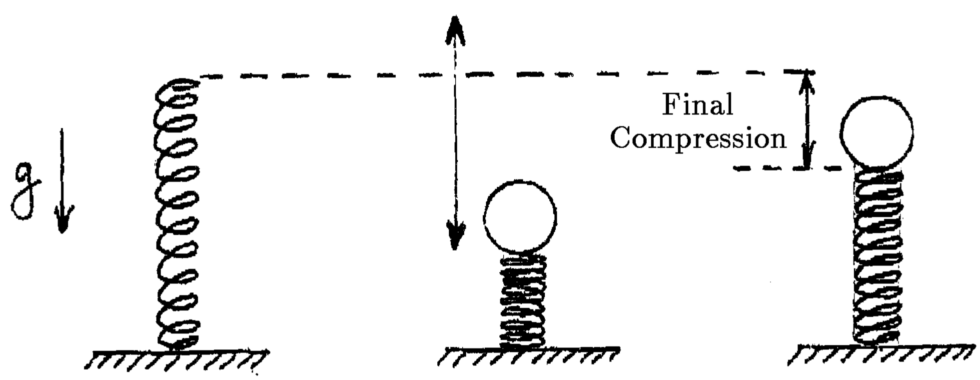
$$\sin\theta\,\cos\theta(\sin\theta + \mu_s\cos\theta) = \frac{2\mu_s h}{\ell}\ .$$

If one solves this equation numerically, one finds that, depending on $\mu_s$ and the ratio $h/\ell$, it might have zero, one, or two solutions in the allowed range, where the allowed range extends from the case where the tip of the plank makes contact with the wall ($\theta = \sin^{-1}(h/\ell)$) to the case where the plank is vertical ($\theta = \pi/2$). You might want to think about how the number of solutions is related to the description of the circumstances under which the plank will or will not slip.

**Problem 7** (10 points)

A ball is placed on a vertical massless spring which obeys Hooke's Law and which initially has its natural uncompressed length. It is observed that at first the ball makes vertical simple harmonic oscillations with period $T$.

After a very large number of oscillations the ball comes to rest because of air resistance and losses in the spring. What is the final compression of the spring in terms of ***only*** $T$ and $g$.




***Solution:*** *The first step is to relate the period $T$ to the spring constant $k$. Let $y$ equal the vertical coordinate of the wall, with $y = 0$ the position for which the spring is at its uncompressed length. Then*

$$M \frac{d^2 y}{dt^2} = -ky - Mg \ ,$$

*where $M$ is the mass of the ball. The equilibrium point is where the force vanishes, so*

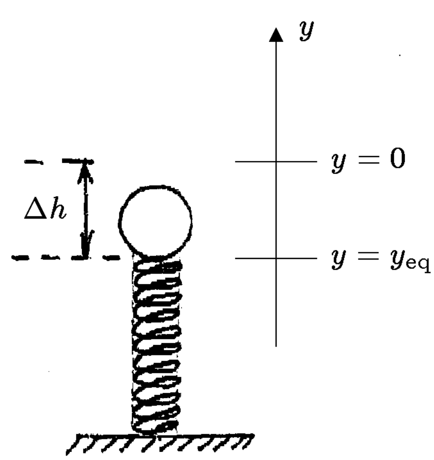
$$-ky_{eq} - Mg = 0 \quad \Longrightarrow \quad y_{eq} = -\frac{Mg}{k} \ .$$

*The differential equation simplifies if we define a new coordinate $\tilde{y}$ which measures the vertical displacement relative to the equilibrium point:*

$$\tilde{y} \equiv y - y_{eq} \ .$$

*Since $y_{eq}$ is independent of time,*

$$\frac{d^2 \tilde{y}}{dt^2} = \frac{d^2 y}{dt^2} \ ,$$



*so*

$$M\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}t^2} = -k\tilde{y} \ .$$

*This equation can be cast into the standard simple-harmonic-motion form by writing*

$$\frac{\mathrm{d}^2\tilde{y}}{\mathrm{d}t^2} = -\omega^2\tilde{y} \ ,$$

*where* $\omega = \sqrt{k/M}$. *A solution to this differential equation can be written as*

$$\tilde{y}(t) = A\sin\omega t \ ,$$

*where* $A$ *is a constant. The period* $T$ *is the time it takes for the argument of the sine function to change by* $2\pi$*, so*

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{k}} \ .$$

*The amount of compression* $\Delta h$ *is equal to* $-y_{\mathrm{eq}}$*, so*

$$\Delta h = \frac{Mg}{k} = \boxed{\ g\left(\frac{T}{2\pi}\right)^2 \ .}$$

**Problem 8** (10 points)

You have been given a nugget which you are told is a mixture of gold and zinc. You want to find out how much gold you have been given. Being an MIT student you make the following observations:

1. You put a cup partly full of water on an electronic (weight) scale and observe that it reads $M_1$, meaning that the force on the scale is equivalent to the gravitational force of a mass $M_1$.
2. You attach the nugget to a very thin stiff piece of wire and hold the nugget in the water fully submerged but not touching the bottom of the cup. The water does not overflow. You observe that the scale now reads $M_2$.
3. You remove the wire and drop the nugget into the cup. No water is spilled. The scale now reads $M_3$.
4. In a reference book you find that gold has a density $\rho_{\rm Au}$, zinc $\rho_{\rm Zn}$, and water $\rho_{\rm H_2O}$.

Using these observations determine

a) the volume of the nugget

b) the mass of the nugget

c) the mass of the gold in the nugget.

***Solution:***

*a) the volume of the nugget: This can be determined by comparing the results of observations 1 and 2. From observation 1, we know that the mass of the beaker plus the water in it is $M_1$. When observation 2 is made, the forces acting on the beaker-plus-water system are:*

*1) The force of gravity $M_1 g$ downward.*

*2) The bouyant force $F_b$ downward that the nugget exerts on the water. By Newton's 3rd law this is equal in magnitude to the bouyant force that the water exerts on the nugget, which by Archimedes' law is equal to $\rho_{\rm H_2O} V g$, where $V$ is the volume of the nugget.*

*3) The normal force of the scale acting upward on the beaker. Since the scale reads $M_2$, this normal force is $M_2 g$.*

*Since the system is in equilibrium the total force must be zero, so*

$$-M_1 g - \rho_{\rm H_2O} V g + M_2 g = 0 \quad \Longrightarrow \quad \boxed{V = \frac{M_2 - M_1}{\rho_{\rm H_2O}}}\ .$$

*b) the mass of the nugget: This can be determined by comparing the results of observations 3 and 1. The extra mass when the nugget is added to the scale is just the mass of the nugget, so*

$$\boxed{M_{\text{nugget}} = M_3 - M_1\ .}$$

*c) the mass of the gold in the nugget: By knowing the mass and volume of the nugget, and the relevant densities, the mass of gold can be found. We need to assume that when metals are mixed the resulting volume is equal to the sum of the original volumes, which is certainly an accurate assumption. If we let $M_{\text{Au}}$ and $M_{\text{Zn}}$ denote the mass of gold and zinc in the nugget, respectively, then*

$$M_{\text{Au}} + M_{\text{Zn}} = M_{\text{nugget}} = M_3 - M_1\ .$$

*The volume of gold and zinc are then given by $M_{\text{Au}}/\rho_{\text{Au}}$ and $M_{\text{Zn}}/\rho_{\text{Zn}}$, respectively, so we can write*

$$\frac{M_{\text{Au}}}{\rho_{\text{Au}}} + \frac{M_{\text{Zn}}}{\rho_{\text{Zn}}} = V = \frac{(M_2 - M_1)}{\rho_{\text{H}_2\text{O}}}\ .$$

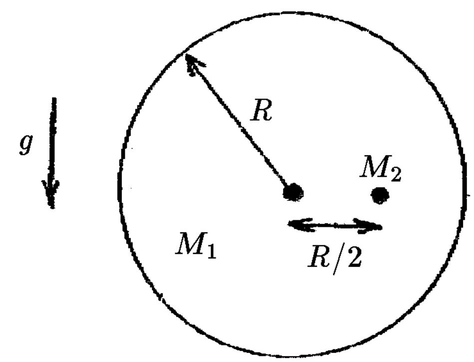
*The two equations above can then be solved for the two unknowns ($M_{\text{Au}}$ and $M_{\text{Zn}}$). After some algebra, one finds*

$$\boxed{M_{\text{Au}} = \frac{\rho_{\text{Au}}\rho_{\text{Zn}}}{\rho_{\text{Au}} - \rho_{\text{Zn}}}\left[\frac{(M_3 - M_1)}{\rho_{\text{Zn}}} - \frac{(M_2 - M_1)}{\rho_{\text{H}_2\text{O}}}\right]\ .}$$

**Problem 9** (10 points)

A uniform disk of mass $M_1$ and radius $R$ is pivoted on a frictionless horizontal axle through its center.

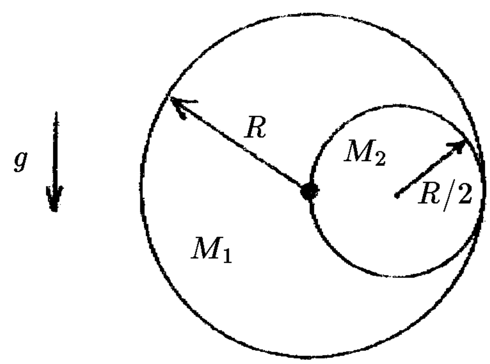
a) A small mass $M_2$ is attached to the disk at radius $R/2$, at the same height as the axle. If this system is released from rest:




i) What is the angular acceleration of the disk immediately after it is released?

ii) What will be the magnitude of the maximum angular velocity that the disk will reach?

b) Now consider the situation if the mass $M_2$ is a disk of radius $R/2$ located with its center at the same place where $M_2$ is located in part (a). For this case, find the angular acceleration immediately after the system is released from rest. (You may assume that the two disks are fused together to make one rigid body.)




***Solution:***

*a) i) Since the axle goes through the center of mass of the disk of mass $M_1$, the gravitational force on this disk does not result in any torque about the axle. But there is a torque caused by the gravitational force on $M_2$, given by*

$$\tau = -R_{\perp}F = -\frac{1}{2}M_2 g R \ .$$

*The moment of inertia of the combined system about the axle is that of the disk $M_1$ plus the mass $M_2$, so*

$$I = \frac{1}{2}M_1 R^2 + M_2\left(\frac{R}{2}\right)^2 = \frac{1}{4}(2M_1 + M_2)R^2 \ ,$$

*where the moment of inertia of the disk is taken directly from the table in the formula sheets. The angular acceleration immediately after release is therefore*

$$\boxed{\alpha = \frac{\tau}{I} = -\frac{2M_2 g}{(2M_1 + M_2)R}\ .}$$

*ii) The maximum angular velocity will be attained when $M_2$ is at the bottom of its motion. The value of the angular velocity can be determined by using the conservation of energy. The potential energy of the disk $M_1$ does not change, since its center of mass does not move, so the only potential energy that needs to be considered is that of $M_2$. This potential energy can be written $U = M_2 g y$, where $y$ is the vertical coordinate, measured from an arbitrary origin. I will take that origin as the height of the axle. Thus $U_{\text{initial}} = 0$, and $U_{\text{final}}$ (at the bottom of the motion) is $-M_2 gR/2$. Then*

$$E_{\text{initial}} = 0$$

$$E_{\text{final}} = \frac{1}{2} I \omega_f^2 - \frac{1}{2} M_2 g R$$

$$E_{\text{final}} = E_{\text{initial}} \quad \Longrightarrow \quad \boxed{\omega_f = \sqrt{\frac{M_2 g R}{I}} = \sqrt{\frac{4M_2 g}{(2M_1 + M_2)R}}\ .}$$

*b) The only difference between this case and the previous one is the moment of inertia of the disk of mass $M_2$. According to the table, the moment of inertia of this disk about its own center is $\frac{1}{2}M_2(R/2)^2$. But we need the moment of inertia about the center of the larger disk, for which we have to use the parallel axis theorem:*

$$I_{||} = I_{\text{cm}} + Md^2 = \frac{1}{2}M_2\left(\frac{R}{2}\right)^2 + M_2\left(\frac{R}{2}\right)^2 = \frac{3}{8}M_2 R^2\ .$$

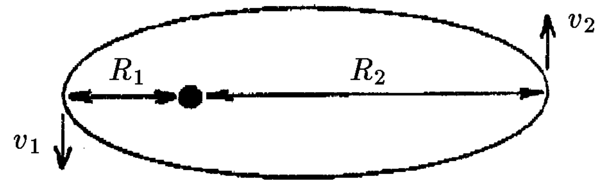
*So,*

$$I = \frac{1}{2}M_1 R^2 + \frac{3}{8}M_2 R^2 = \frac{1}{8}(8M_1 + 3M_2)R^2\ .$$

*The torque is the same as in part (a)(i), since the torque due to the gravitational force on $M_2$ can be calculated as if the entire force acted on the center of mass. Thus,*

$$\boxed{\alpha = \frac{\tau}{I} = -\frac{4M_2 g}{(8M_1 + 3M_2)R}\ .}$$

**Problem 10** (10 points)




A satellite follows an elliptical orbit. Its closest approach to the earth is $R_1$, at which point it has speed $v_1$, and the furthest point is $R_2$, at which point it has speed $v_2$. Both distances are measured from the center of the earth. At the surface of the earth the acceleration due to gravity is $g$ and the earth's radius is $R$.

What is the magnitude of $v_1$ in terms of ***only*** $R_1$, $R_2$, $R$ and $g$?

***Solution:*** *By conservation of angular momentum about the center of the earth,*

$$|\vec{\mathbf{r}} \times \vec{\mathbf{p}}|_1 = |\vec{\mathbf{r}} \times \vec{\mathbf{p}}|_2 \ ,$$

*or*

$$mv_1R_1 = mv_2R_2 \ ,$$

*where $m$ is the mass of the satellite. Similarly, conservation of energy implies that*

$$\frac{1}{2}mv_1^2 - \frac{GMm}{R_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{R_2} \ ,$$

*where $M$ is the mass of the earth. These two equations can be solved for $v_1$, giving*

$$v_1 = \sqrt{\frac{2GMR_2}{R_1(R_1+R_2)}} \ .$$

*We are not given $G$ or $M$, so this is not the final answer. However, we are allowed to use $g$ in our answer, where $g$ is the acceleration caused by gravity at the surface of the earth. Considering the gravitational force on an object of mass $\tilde{m}$ at the surface of the earth, we can write*

$$\tilde{m}g = \frac{GM\tilde{m}}{R^2} \ ,$$

*where $R$ is the radius of the earth. So*

$$GM = R^2g \ ,$$

*and*

$$\boxed{v_1 = \sqrt{\frac{2R^2R_2g}{R_1(R_1+R_2)}} \ .}$$