W01D3-1 Problem 1 Dimensional Analysis: Speed of a Boat

The speed of a sail-boat or any other craft that does not plane is limited by the wave it makes – it can't climb uphill over the front of the wave. What is the maximum speed you'd expect?

a) Use dimensional analysis to find a proportional relationship between the speed of the boat v_{boat} in terms of the quantities: length l of the boat, the density ρ of the water, and the gravitational acceleration g. Hint: You may not need all these quantities and you can write

$$v_{boat} \sim l^x \rho^y g^z , \tag{0.1}$$

where you should find the values of $x, y,$ and z

b) The maximum speed of a single-hull displacement boat in SI units is given by

$$v_{boat} = 1.25 \, \mathrm{m}^{1/2} \cdot \mathrm{s}^{-1} \sqrt{l} \tag{0.2}$$

Based on your answer to part (a), set $v_{boat} = b \; l^x \rho^y g^z$, and determine the constant of proportionality $v_{boat} = b \; l^x \rho^y g^z$.