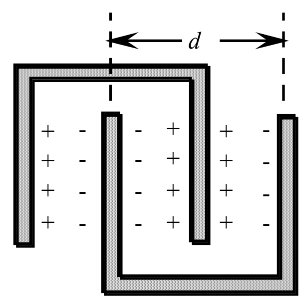
MASSACHUSETTS INSTINUTE OF TECHNOLOGY
ESG Physics

8.02 with Kai — Spring 2003

Problem Set 4 Solution

Problem 1: 26.10
A variable air capacitor used in tuning circuits is made of N semicircular plates each of radius R and positioned a distance d from each other. As shown in Figure P26.10, a second identical set of plates is enmeshed with its plates halfway between those of the first set. The second set can rotate as a unit. Determine the capacitance as a function of the angle of rotation θ, where $\theta = 0$ corresponds to the maximum capacitance.

Solution:
When $\theta = \pi$, the plates are out of touch and the overlap area is zero.



When $\theta = 0$, the overlap area is that of a semi-circle $\frac{\pi R^2}{2}$.

By proportion, the effective area of a single sheet of charge is $\frac{(\pi - \theta) R^2}{2}$.

Where there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is and the total capacitance is

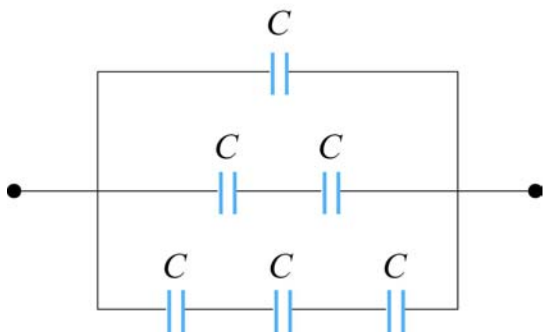
$$C = (2N-1)\frac{\varepsilon_0 A_{effective}}{r} = (2N-1)\frac{\varepsilon_0 (\pi - \theta)\frac{R^2}{2}}{\left(\frac{d}{2}\right)} \tag{1.1}$$

which gives

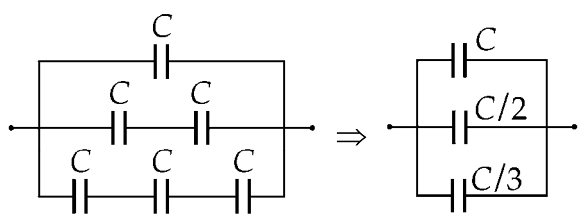
$$\boxed{C = \frac{(2N-1)\varepsilon_0 (\pi - \theta) R^2}{d}} \tag{1.2}$$

Problem 2: 26.22

Evaluate the equivalent capacitance of the configuration shown in Figure P26.22. All the capacitors are identical, and each has capacitance *C*.



Solution:



The circuit reduces first according to the rule for capacitors in series, as shown in the figure, then according to the rule for capacitors in parallel, $C_{eq} = C\left(1+\frac{1}{2}+\frac{1}{3}\right)$

Which gives

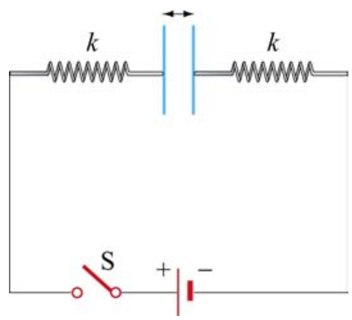
$$\boxed{C_{eq} = 1.83\text{ C}} \tag{2.1}$$

Problem 3: 26.25

The circuit below consists of two identical parallel metallic plates connected by identical metallic springs to a battery with emf V. With the switch open, the plates are uncharged, are separated by a distance d, and have a capacitance C. When the switch is closed, the distance between the plates decreases by a factor of 0.500.

(a) How much charge collects on each plate and

(b) What is the spring constant for each spring? (hint: use the result of Problem 35)



Solution:

(a) With the switch closed, the distance $d' = 0.500d$ and capacitance

$$C' = \frac{\varepsilon_0 A}{d'} = \frac{\varepsilon_0 A}{\left(\frac{d}{2}\right)} = 2C \tag{3.1}$$

therefore

$$Q = C'(\Delta V) = \boxed{2C(\Delta V)} \tag{3.2}$$

(b) From problem 35, we know that the force exerted on each plate by the other is

$$F = \frac{Q^2}{2\varepsilon_0 A} \tag{3.3}$$

Substitute the value that we get from equation (3.1), we have

$$F = \frac{Q^2}{2\varepsilon_0 A} = \frac{(2CV)^2}{2\varepsilon_0 A} = \frac{2C^2(\Delta V)^2}{\left(\frac{\varepsilon_0 A}{d}\right)d} = \frac{2C(\Delta V)^2}{d} \tag{3.4}$$

Since one spring stretches by a distance $x = \frac{d}{4}$, we have

$$F = kx = k\frac{d}{4} = \frac{2C(\Delta V)^2}{d} \tag{3.5}$$

which gives

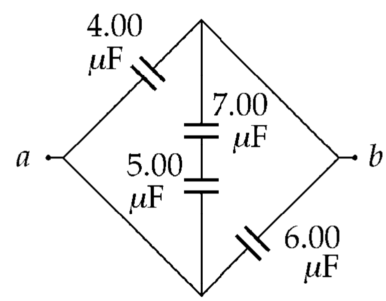
$$\boxed{k = \frac{8C(\Delta V)^2}{d^2}} \tag{3.6}$$

Problem 4: 26.30
Find the equivalent capacitance between points *a* and *b* in the combination of capacitors shown in Figure P26.30

Solution:

$$C_s = \frac{1}{\frac{1}{5.00} + \frac{1}{7.00}} = 2.92\ \mu\text{F} \tag{4.1}$$

$$C_P = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}} \tag{4.2}$$



Problem 5: 26.36

Plate *a* of a parallel-plate, air filled capacitor is connected to a spring having force constant *k*, and plate *b* is fixed. They rest on a table top as shown (top view) in Figure P26.36. If a charge $+Q$ is placed on plate *a* and a charge $-Q$ is placed on plate *b*, by how much does the spring expand?

Solution:

Denote F_s to be the spring force and F_e to be the electric force that plate *a* experience, thus

$$F_s = -kx\,\vec{\mathbf{i}} \tag{5.1}$$

and

$$F_e = QE\,\vec{\mathbf{i}} \tag{5.2}$$

The electric force is due to the electric field created by plate *b* according to the equation

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0} \tag{5.3}$$

where *A* is the area of one plate. Therefore

$$kx = Q\frac{Q}{2A\varepsilon_0} \tag{5.4}$$

which gives

$$\boxed{x = \frac{Q^2}{2A\varepsilon_0 k}} \tag{5.5}$$

Problem 6: 26.47

A conducting spherical shell has inner radius a and outer radius c. The space between gthese two surfaces is filed with a dielectric for which the dielectric constant is κ_1 between a and b, and κ_2 between b and c (Fig. P26.47). Determine the capacitance of this system.

Solution:

We can consider this as capacitance in series. Therefore

$$\frac{1}{C_{eq}} = \frac{1}{\left(\frac{\kappa_1 ab}{k_e(b-a)}\right)} + \frac{1}{\left(\frac{\kappa_2 bc}{k_e(c-b)}\right)} = k_e \frac{(b-a)(\kappa_2 c)+(c-b)(\kappa_1 a)}{\kappa_1\kappa_2 abc} \tag{6.1}$$

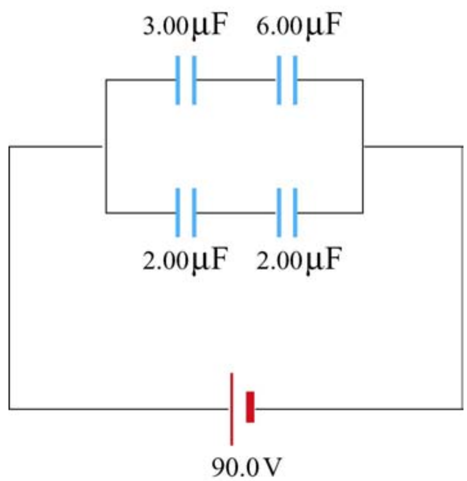
Therefore

$$\boxed{C_{eq} = \frac{1}{k_e}\frac{\kappa_1\kappa_2 abc}{\kappa_2(bc-ac)+\kappa_1(ac-ab)}} \tag{6.2}$$

Problem 7: 26.54

For the system of capacitors shown in Figure P26.54, find

(a) the equivalent capacitance of the system,
(b) the charge on each capacitor,
(c) the potential difference across each capacitor, and
(d) the total energy stored by the group



Solution:

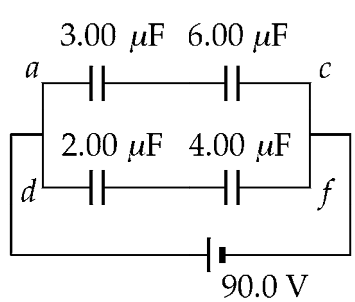
(a) The equivalence capacitance is

$$C = \frac{1}{\frac{1}{3.00} + \frac{1}{6.00}} + \frac{1}{\frac{1}{2.00} + \frac{1}{4.00}} \tag{6.3}$$

which gives

$$\boxed{C = 3.33\ \mu\text{F}} \tag{6.4}$$

(b)



$$Q_{ac} = C_{ac}\left(\Delta V_{ac}\right) = \left(2.00\ \mu\text{F}\right)\left(90.0\,\text{V}\right) = 180\ \mu\text{C} \tag{6.5}$$

Therefore

$$\boxed{Q_3 = Q_6 = 180\ \mu\text{C}} \tag{6.6}$$

Similarly,

$$Q_{df} = C_{df}\left(\Delta V_{df}\right) = (1.33\ \mu\text{F})(90.0\ \text{V}) = 120\ \mu\text{C} \tag{6.7}$$

Therefore

$$\boxed{Q_2 = Q_4 = 120\ \mu\text{C}} \tag{6.8}$$

(c)

Capacitance (C)	Charge (Q)	Potential Difference $\left(\Delta V = \frac{Q}{C}\right)$
3.00 μF	180 μC	60.0 V
6.00 μF	180 μC	30.0 V
2.00 μF	120 μC	60.0 V
4.00 μF	120 μC	30.0 V

(d)

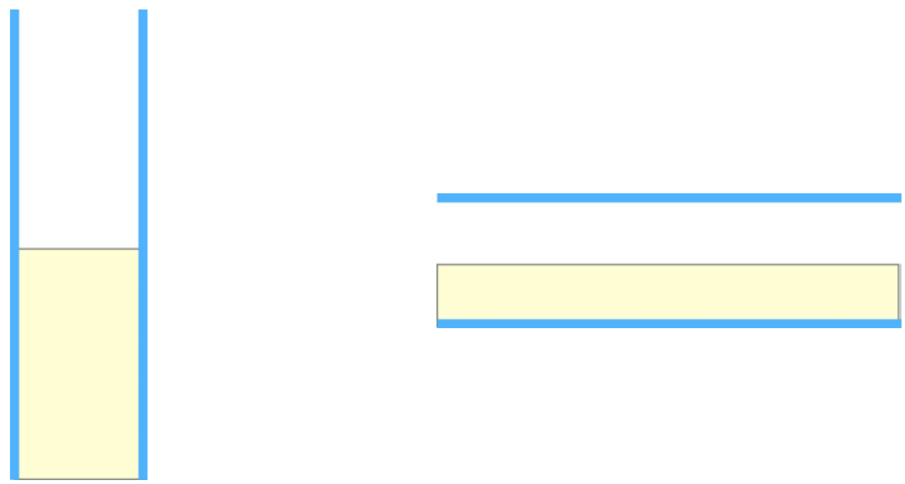
$$U_T = \frac{1}{2}C_{eq}\left(\Delta V\right)^2 = \frac{1}{2}\left(3.33\times10^{-6}\right)\left(90.0\right)^2 \tag{6.9}$$

which gives

$$\boxed{U_T = 13.4\ \mu\text{J}} \tag{6.10}$$

Problem 8: 26.71

A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is κ. When this capacitor is positioned horizontally, what fraction of it should be filled with the same dielectric so that the two capacitors have equal capacitance?



Solution:

The vertical orientation sets up two capacitors in parallel, with equivalent capacitance

$$C_P = \frac{\varepsilon_0\left(\frac{A}{2}\right)}{d} + \frac{\kappa\varepsilon_0\left(\frac{A}{2}\right)}{d} = \frac{\kappa+1}{2}\frac{\varepsilon_0 A}{d} \tag{6.11}$$

where A is the area of either plate and d is the separation of the plates. The horizontal orientation produces two capacitors in series. If f is the fraction of the horizontal capacitor filled with dielectric, the equivalent capacitance is

$$\frac{1}{C_s} = \frac{fd}{\kappa\varepsilon_0 A} + \frac{(1-f)d}{\varepsilon_0 A} = \frac{f+\kappa(1-f)}{\kappa}\frac{d}{\varepsilon_0 A} \tag{6.12}$$

which gives

$$C_s = \frac{\kappa}{f+\kappa(1-f)}\frac{\varepsilon_0 A}{d} \tag{6.13}$$

Requiring that $C_p = C_s$, we have

$$\frac{\kappa+1}{2} = \frac{\kappa}{f+\kappa(1-f)} \tag{6.14}$$

which gives

$$\boxed{f = \frac{\kappa-\kappa^2}{1-\kappa^2} = \frac{\kappa}{1+\kappa}} \tag{6.15}$$