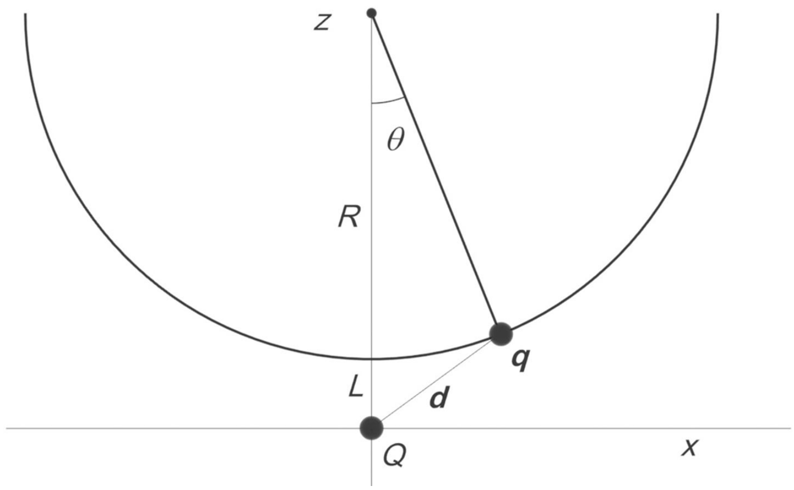
The Electrostatic Pendulum

For equations use *MathType 7.8.0.0*.



$$\frac{1}{2}mv^2 + mgz + \frac{qQ}{4\pi\varepsilon_o d} = \text{constant} \tag{1.1}$$

A charge q is located at (x,z) in the presence of a stationary charge of the same sign Q sitting at the origin. The charge q swings on a pendulum of radius R under gravity. When the charge q is at the bottom its swing it is a distance L just above the charge Q.

The charge q is released at rest from the position $(-R, R+L)$. It ends up at rest just above the stationary charge Q.

(a) Use conservation of total energy to determine the ratio q/Q for this to happen, in terms of the parameters of the problem. Write your (dimensionless) answer in terms of the dimensionless ratio $mgR/\left(Q^2/4\pi\varepsilon_o L\right)$ and the dimensionless ratio L/R .

Answer: $$\frac{q}{Q} = \frac{mgR}{\left(Q^2/4\pi\varepsilon_o L\right)\left(1 - \frac{L/R}{\sqrt{1+\left(1+L/R\right)^2}}\right)}$$

Solution to (a):

Conservation of energy:

$$\frac{1}{2}mv^2 + mgz + \frac{qQ}{4\pi\varepsilon_o d} = \text{constant} \tag{1.2}$$

Because it is located on a circle of radius R whose center is at $(0, R+L)$, we must have

$$x^2 + \left(R + L - z\right)^2 = R^2 \tag{1.3}$$

$$d^2 = x^2 + z^2 = R^2 - \left(R + L - z\right)^2 + z^2 \tag{1.4}$$

So, conservation of energy becomes:

$$\frac{1}{2}mv^2 + mgz + \frac{qQ}{4\pi\varepsilon_o\sqrt{R^2 - \left(R + L - z\right)^2 + z^2}} = \text{constant} \tag{1.5}$$

We evaluate the constant at *t = 0*, when the charge is at $(-R, R+L)$ with zero velocity:

$$0 + mg(R+L) + \frac{qQ}{4\pi\varepsilon_o\sqrt{R^2 + \left(R + L\right)^2}} = \text{constant} \tag{1.6}$$

So we can now rewrite equation (1.4) as

$$\frac{1}{2}mv^2 + mg\left(z - R - L\right) + \left(\frac{qQ}{4\pi\varepsilon_o}\right)\left(\frac{1}{\sqrt{R^2 - \left(R + L - z\right)^2 + z^2}} - \frac{1}{\sqrt{R^2 + \left(R + L\right)^2}}\right) = 0 \tag{1.7}$$

For the velocity to be zero at $z = L$, we must have therefore

$$-mgR + \left(\frac{qQ}{4\pi\varepsilon_o}\right)\left(\frac{1}{L} - \frac{1}{\sqrt{R^2 + \left(R + L\right)^2}}\right) = 0 \tag{1.8}$$

$$+\frac{q}{Q} = \frac{4\pi\varepsilon_o mgR}{Q^2\left(\frac{1}{L} - \frac{1}{\sqrt{R^2 + \left(R + L\right)^2}}\right)} \tag{1.9}$$

$$+\frac{q}{Q} = \frac{mgR}{\left(Q^2 / 4\pi\varepsilon_o L\right)\left(1 - \frac{L/R}{\sqrt{1 + \left(1 + L/R\right)^2}}\right)} \tag{1.10}$$

(b) For the purposes of our problem, assume that the height of the support is 25 cm, and the length of the pendulum is 20 cm, so the distance *L* is 5 cm. The mass of the bob with the swinging magnet is 5 grams. The fixed charge is 0.211 μC. Using equation 1.10, calculate the value of the ratio *q/Q* needed to bring the swinging charge to rest just above the fixed charge.

Answer: 1.45

Solution to (b):

$$\frac{q}{Q} = \frac{mgR}{\left(Q^2 / 4\pi\varepsilon_o L\right)\left(1 - \frac{L/R}{\sqrt{1+\left(1+L/R\right)^2}}\right)}$$

$$\left(1 - \frac{L/R}{\sqrt{1+\left(1+L/R\right)^2}}\right) = \left(1 - \frac{0.25}{1.60078}\right) = 0.843826$$

$$\left(Q^2 / 4\pi\varepsilon_o L\right) = (2.11\times10^{-7})^2 / (12.566 * 2.975\times10^{-6} * (.05)) = 0.008009$$

$$\frac{q}{Q} = \frac{mgR}{\left(Q^2 / 4\pi\varepsilon_o L\right)\left(1 - \frac{L/R}{\sqrt{1+\left(1+L/R\right)^2}}\right)} = \frac{(.005)(9.8)(.20)}{\left(0.008009\right)\left(0.843826\right)} = 1.45$$