MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Final Exam Equation Sheet

Maxwell's Equations:

Gauss's Law:

$$\underset{\text{closed surface S}}{\oiint} \vec{\mathbf{E}}\cdot\hat{\mathbf{n}}\,da = \frac{1}{\varepsilon_0}\int_{\substack{\text{volume}\\ \text{enclosed by S}}} \rho\, dV_{vol}$$

Faraday's Law:

$$\oint_{\substack{\text{closed}\\ \text{path C}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_{\substack{\text{any surface}\\ \text{enclosed by C}}} \vec{\mathbf{B}}\cdot\hat{\mathbf{n}}\,da$$

Gauss's Law for Magnetism:

$$\underset{\text{closed surface}}{\oiint} \vec{\mathbf{B}}\cdot\hat{\mathbf{n}}\,da = 0$$

Maxwell-Ampere's Law:

$$\oint_{\substack{\text{closed}\\ \text{path C}}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0\iint_S \vec{\mathbf{J}}\cdot\hat{\mathbf{n}}\,da + \mu_0\varepsilon_0\frac{d}{dt}\iint_S \vec{\mathbf{E}}\cdot\hat{\mathbf{n}}\,da$$

Lorentz Force Law:

$$\vec{\mathbf{F}}_q = q(\vec{\mathbf{E}}_{ext} + \vec{\mathbf{v}}_q \times \vec{\mathbf{B}}_{ext})$$

Current Density and Current:

$$I = \iint_{\text{open surface}} \vec{\mathbf{J}}\cdot\hat{\mathbf{n}}\,da$$

Force on Current Carrying Wire:

$$\vec{\mathbf{F}} = \int_{wire} I\,d\vec{\mathbf{s}}'\times\vec{\mathbf{B}}_{ext}$$

Source Equations:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{\text{source}}\frac{dq'}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\int_{\text{source}}\frac{dq'(\vec{\mathbf{r}}-\vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right|^3}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int_{source}\frac{Id\vec{\mathbf{s}}'\times\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi}\int_{source}\frac{Id\vec{\mathbf{s}}'\times(\vec{\mathbf{r}}-\vec{\mathbf{r}}')}{\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right|^3}$$

Electrostatic Potential Difference:

$$\Delta V = V_b - V_a \equiv -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{E}} = -\vec{\nabla}V$$

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{\text{source}}\frac{dq}{\left|\vec{\mathbf{r}}-\vec{\mathbf{r}}'\right|}$$

Potential Energy (electrostatics):

$$\Delta U = q\Delta V$$

Energy Stored In Charge Configuration:

$$U_{\text{stored}} = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_i q_j}{r_{ij}};\ U(\infty) = 0$$

Energy Density Stored in Fields:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2, \qquad u_B = B^2/2\mu_0,$$

Electric Dipole:

Electric Dipole Moment:

$$\vec{\mathbf{p}}_A = \sum_{i=1}^{N} q_i\vec{\mathbf{r}}_{A,i},$$

$\vec{\mathbf{r}}_{A,i}$ vector from point A to the ith charge

Torque: $\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}}\times\vec{\mathbf{E}}_{\text{ext}}$

Force: $\vec{\mathbf{F}} = \vec{\nabla}(\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}_{\text{ext}})$

Potential Energy $U_E = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}_{\text{ext}}$

Magnetic Dipole:

Magnetic Dipole Moment: $\vec{\boldsymbol{\mu}} = IA\hat{\mathbf{n}}_{RHR}$

Torque: $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}_{ext}$

Force on a Magnetic Dipole:

$$\vec{\mathbf{F}} = \vec{\nabla}(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}_{ext})$$

$$F_z = \mu_z\frac{\partial B_{z,ext}}{\partial z}\ \text{(special case)}$$

Potential Energy: $U_B = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}_{\text{ext}}$

Capacitance:

$C = \frac{Q}{|\Delta V|}$, $U_E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C|\Delta V|^2$

Conductivity and Resistivity:

$\vec{\mathbf{J}} = \sigma_c \vec{\mathbf{E}}$ where σ_c is the conductivity

$\vec{\mathbf{E}} = \rho_r \vec{\mathbf{J}}$ where ρ_r is the resistivity

Ohm's Law and Resistance: $\Delta V = I\ R$

Power Dissipated in Resistor:

$P_{\text{Joule}} = I^2 R = \Delta V^2 / R$

DC Circuit Laws:

$\sum_{i=1}^{N} \Delta V_i = 0$, $I_{\text{in}} = I_{\text{out}}$

Power: $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Power Voltage Source: $P = I\Delta V$

Power Dissipated: $P = I^2 R$

Inductance:

$L = \frac{\Phi_B}{I}$, $U_B = \frac{1}{2}LI^2$, $\varepsilon = -L\frac{dI}{dt}$

Undriven LC Circuit:

$\omega_0 = 1/\sqrt{LC}$

Driven RLC Circuit:

$V(t) = V_0 \sin(\omega t)$

$I(t) = I_0 \sin(\omega t - \phi)$

$I_0 = V_0 / (R^2 + (\omega L - 1/\omega C)^2)^{1/2}$

$\tan\phi = (\omega L - 1/\omega C)/R$

Wave Equations:

Plane Linearized Polarized Wave

$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$, $\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \varepsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$

$B_0 = E_0 / c$

Ex: traveling in the $\pm x$-direction

$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$, $-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}$

$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2}$ $\quad$ $\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$

Properties of Waves:

$f = 1/T$ $\quad$ $\omega = 2\pi f$ $\quad$ $k = 2\pi/\lambda$

$c = \lambda/T = \lambda f = \omega/k$ $\quad$ $c = 1/\sqrt{\mu_0 \varepsilon_0}$

Time Averaging:

$\langle \sin^2(\omega t + \phi) \rangle = \frac{1}{T}\int_0^T \sin^2(\omega t + \phi)dt = \frac{1}{2}$

Poynting Vector:

$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$, $\quad$ $\text{Power} = \iint \vec{\mathbf{S}} \cdot \hat{\mathbf{n}}\ da$

Radiation Pressure:

$P^{abs}_{pressure} = \frac{1}{c}\langle |\vec{\mathbf{S}}| \rangle$, perfectly absorbing

$P^{ref}_{pressure} = 2\frac{1}{c}\langle |\vec{\mathbf{S}}| \rangle$, perfectly reflecting.

Pressure and Force: $P_{pressure} = F/A$

Stefan-Boltzmann Law

$\text{Power} = \sigma\varepsilon A T^4$, ε = emissivity

$\sigma = 5.67 \times 10^{-8}\ \text{W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

Constants:

$c = 3.0 \times 10^8\ \ \text{m} \cdot \text{s}^{-1}$

$\mu_0 \equiv 4\pi \times 10^{-7}\ \text{T} \cdot \text{m} \cdot \text{A}^{-1}$

$\varepsilon_0 \equiv 1/\mu_0 c^2 \simeq 8.85 \times 10^{-12}\ \text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}$

$k_e = 1/4\pi\varepsilon_0 \simeq 9.0 \times 10^9\ \text{N} \cdot \text{m}^2 \cdot \text{C}^{-2}$

Circumferences, Areas, Volumes

1. The area of a circle of radius r is πr^2. Its circumference is $2\pi r$.
2. The surface area of a sphere of radius r is $4\pi r^2$. Its volume is $(4/3)\pi r^3$.
3. The area of the side of a cylinder of radius r and height h is $2\pi rh$. Its volume is $\pi r^2 h$