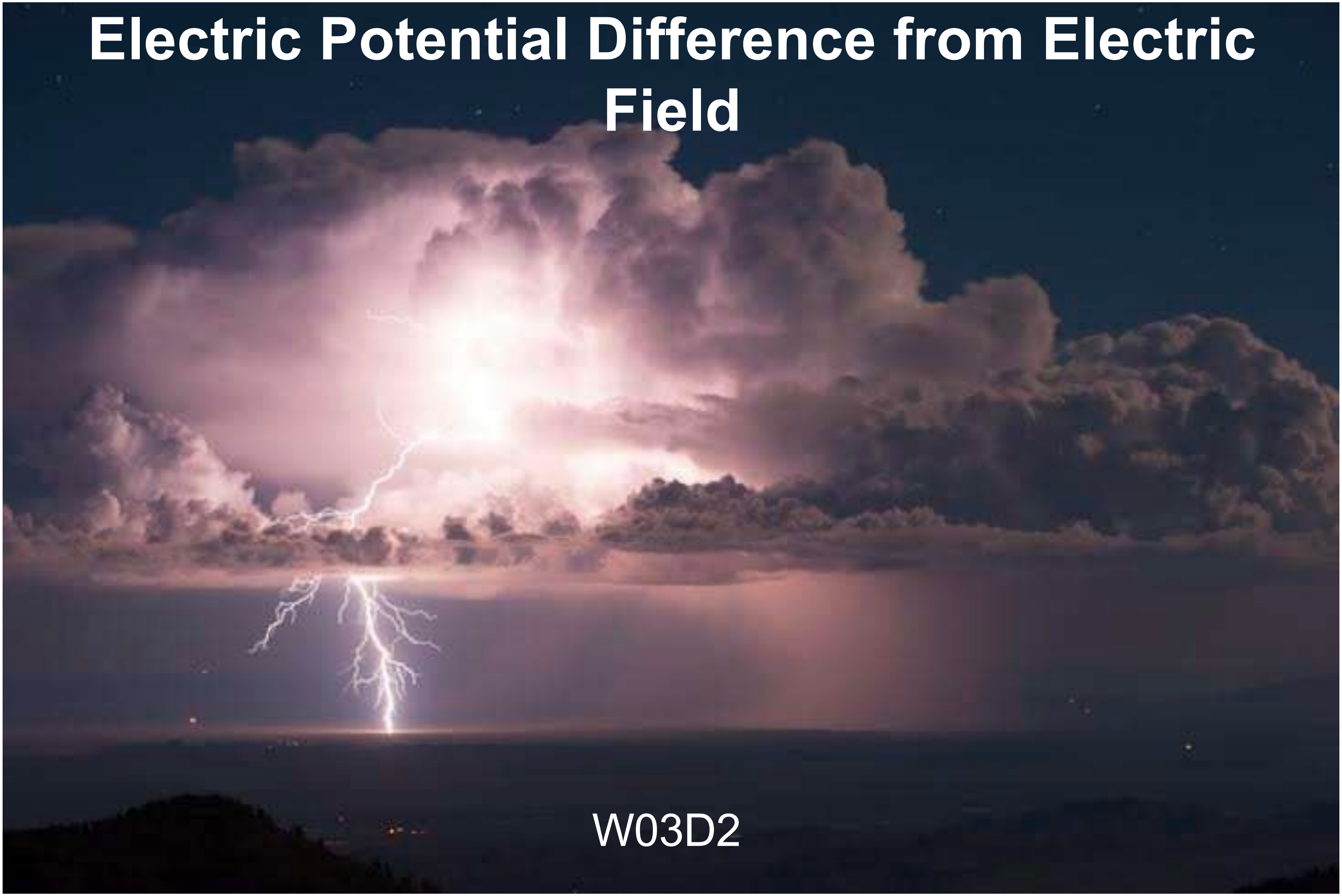
Electric Potential Difference from Electric Field



W03D2

Announcements

Week 3:

Problem Set 2 Due Wednesday at 11 pm

FPS week 3

Week 4:

LS 1 and LS 2 due Sunday at 11 pm

LS 3 and LS 4 due Tuesday at 11 pm

Problem Set 3 Due Wednesday at 11 pm

Outline

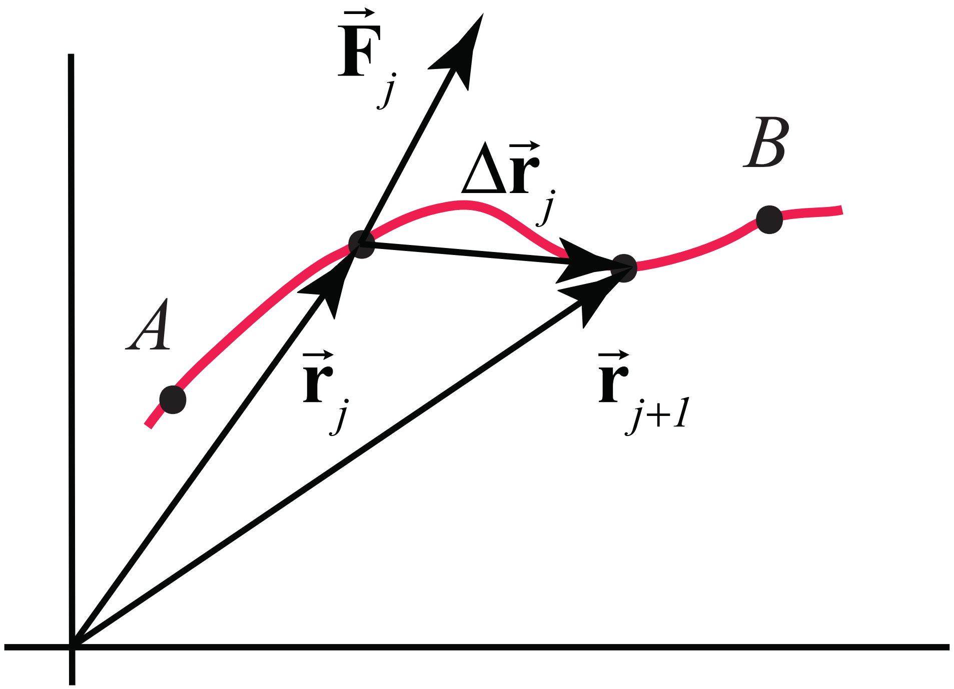
Work and Electric Potential Energy

Electric Potential Difference

Examples: Electric Potential Difference for Symmetric Sources of Charge

Conservation of Energy

Work Done Along an Arbitrary Path for a Non-uniform Force



Work done by force for small displacement

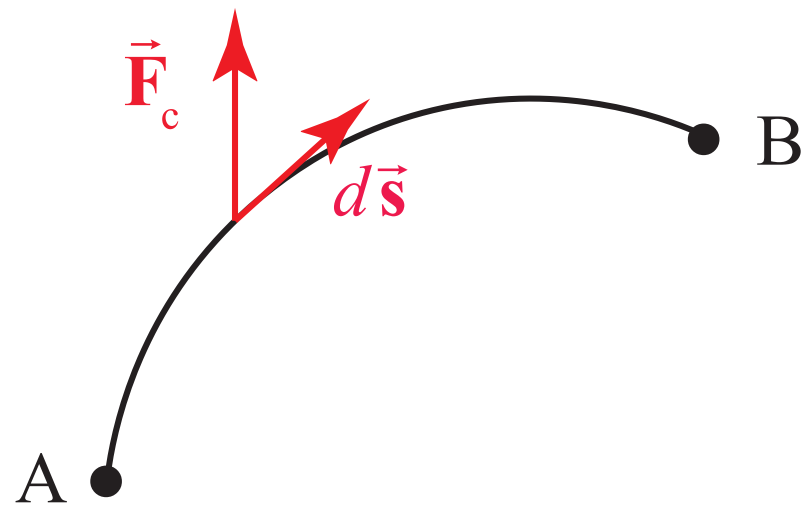
$$W_j = \vec{\mathbf{F}}_j \cdot \Delta\vec{\mathbf{r}}_j$$

Work done by force along path from *A* to *B*

$$W_{AB} = \lim_{N\to\infty} \sum_{j=1}^{j=N} \vec{\mathbf{F}}_j \cdot \Delta\vec{\mathbf{r}}_j \equiv \int_{A\ path}^{B} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \Delta K$$

Potential Energy Difference

Definition: *Potential Energy Difference between the points A and B* associated with a conservative force $\vec{\mathbf{F}}_c$ is the negative of the work done by the conservative force in moving the body along **any path** connecting the points A and B.



$$\Delta U \equiv -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{s}} = -W_c$$

Work, Potential Energy, Potential Difference

Conservative work on moving a test charged object (charge q_t) form A to B :

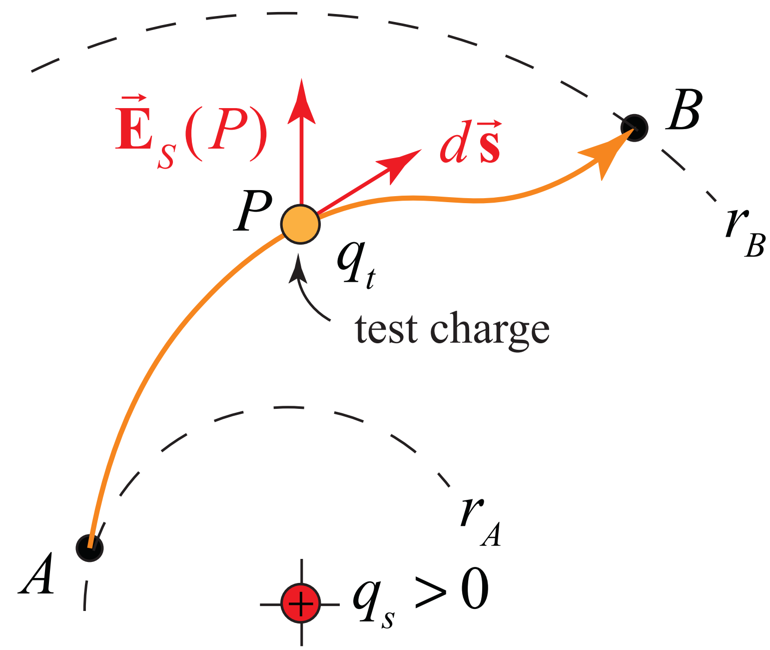
$$W_{AB} = \int_A^B \vec{\mathbf{F}}_{q_t} \cdot d\vec{\mathbf{r}} = \int_A^B q_t \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}}$$

The change in potential energy of the object is :

$$\Delta U_{A,B} = U(B) - U(A) = -W_{A,B}$$

Electric potential difference is **defined** to be the change in potential energy per charge in moving the test object (charge q_t) from A to B:

$$\Delta V_{AB} = \frac{-W_{AB}}{q_t} = \frac{\Delta U_{AB}}{q_t} = -\int_A^B \vec{\mathbf{E}}_s \cdot d\vec{\mathbf{s}}$$

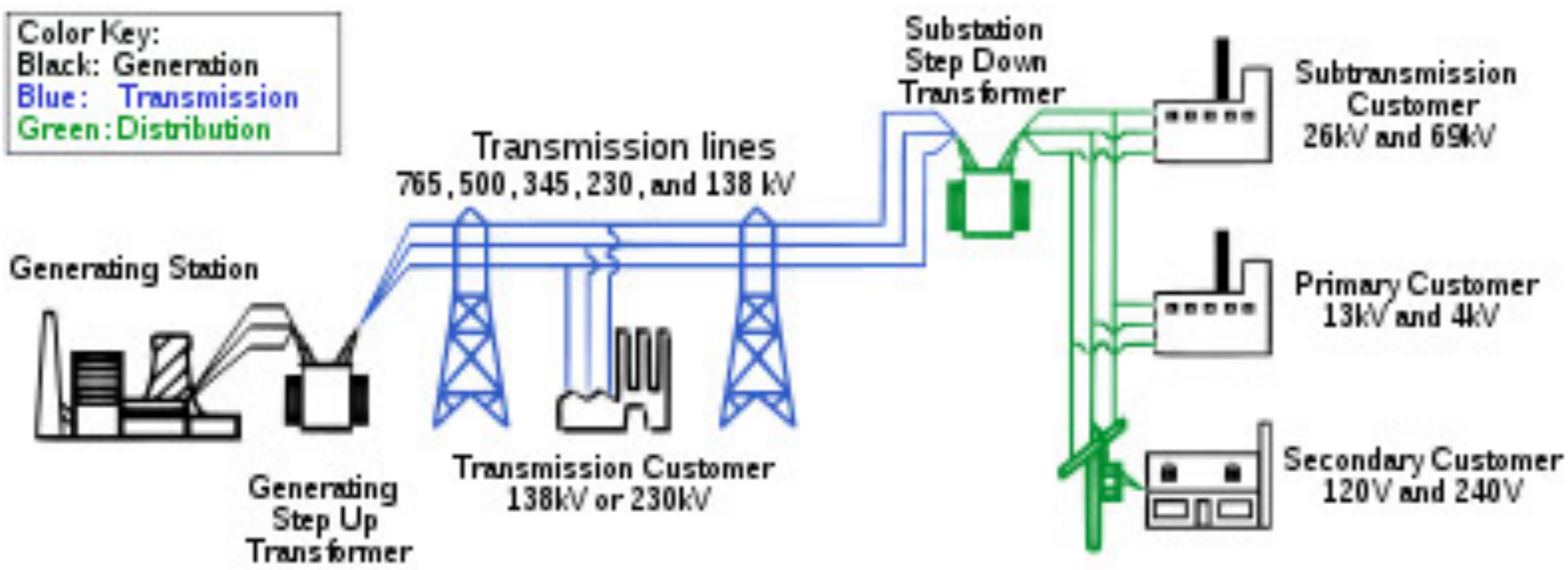


SI Units:

Joule/Coulomb = Volt

How Big is a Volt?

AA Batteries	1.5 V	High Voltage Transmission Lines	100 kV-700 kV
Car Batteries	12 V	Van der Graaf	300 kV
US Outlet (AC)	120 V	Tesla Coil	500 kV
Distribution Power Lines	120 V- 70 kV	Lightning	10-1000 MV



Electric Potential Energy and Electric Potential Difference

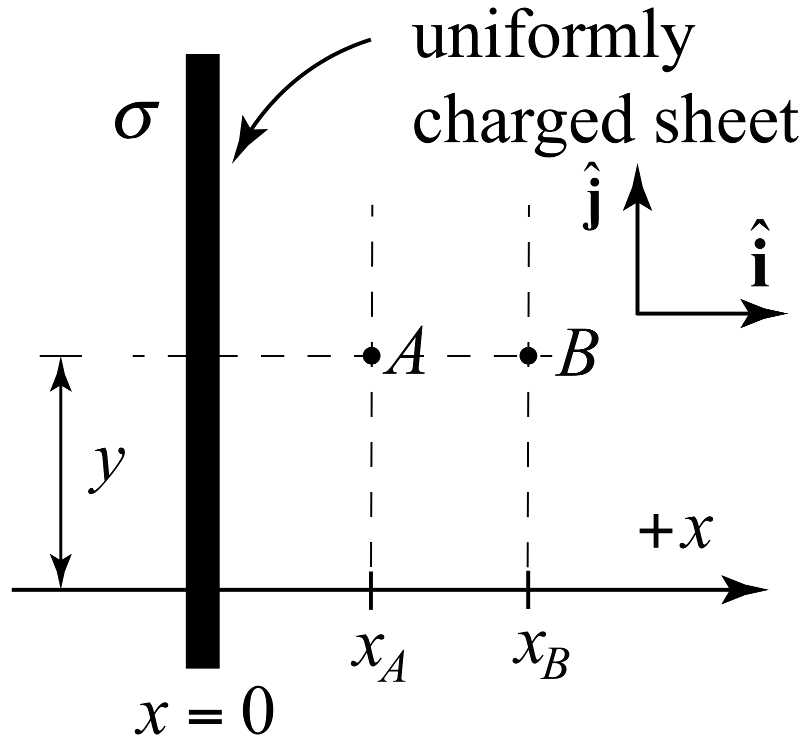
When a particle with charge q moves across a potential difference ΔV due to an external electric field $\vec{\mathbf{E}}_S$, the change in the potential energy of the particle is

$$\Delta U = q\Delta V$$

Note: At each point in space the force on the charged particle is

$$\vec{\mathbf{F}}_q = q\vec{\mathbf{E}}_S$$

Group Prob. Potential Difference Near a Uniformly Charged Sheet



A very thin long non-conducting plate, lying in the *yz*-plane, is positively charged with uniform surface charge density σ > 0.

a) Find a vector expression for the electric field for the region x > 0.

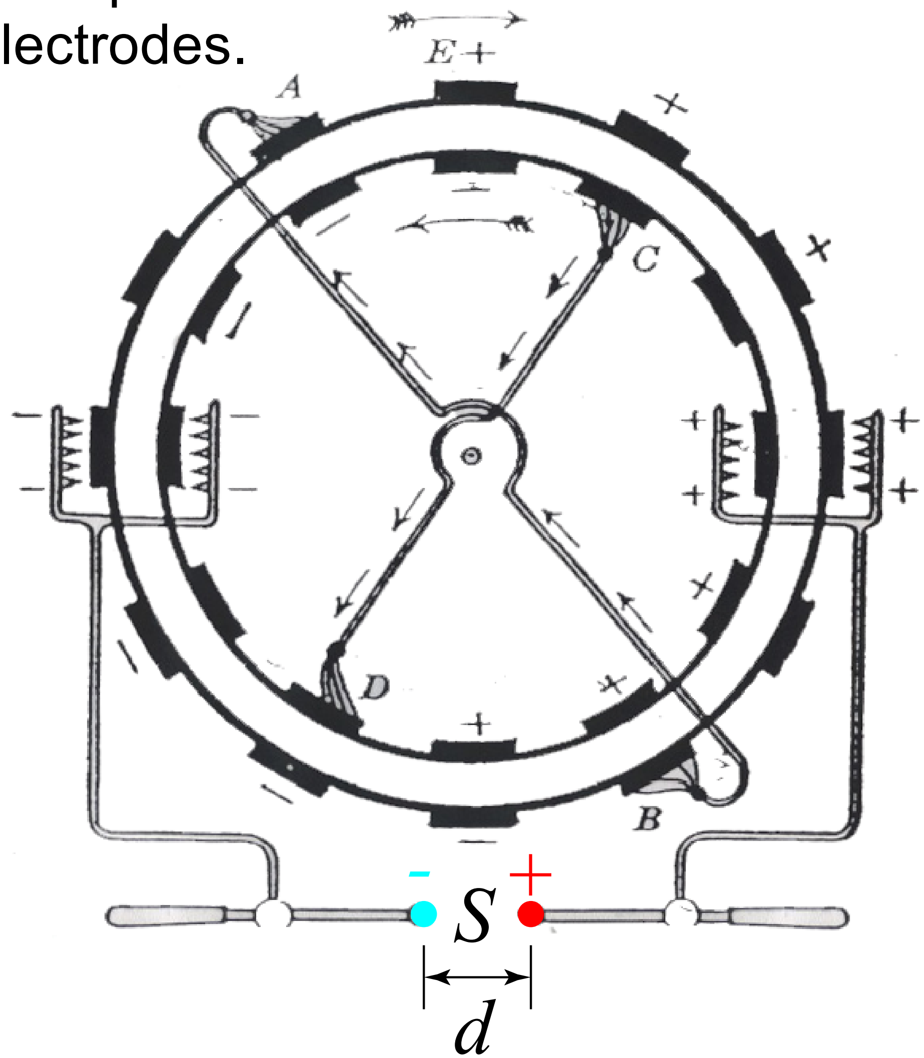
a) Find an expression for the electric potential difference between the points *B* and *A*:

$$V(B) - V(A) = -\int_A^B \vec{\mathbf{E}}_s \cdot d\vec{\mathbf{s}}.$$

The point *B* is located at ($x_{B,}$,*y*) and the point *A* is located at ($x_{A,}$,*y*) where $x_B > x_A > 0$.

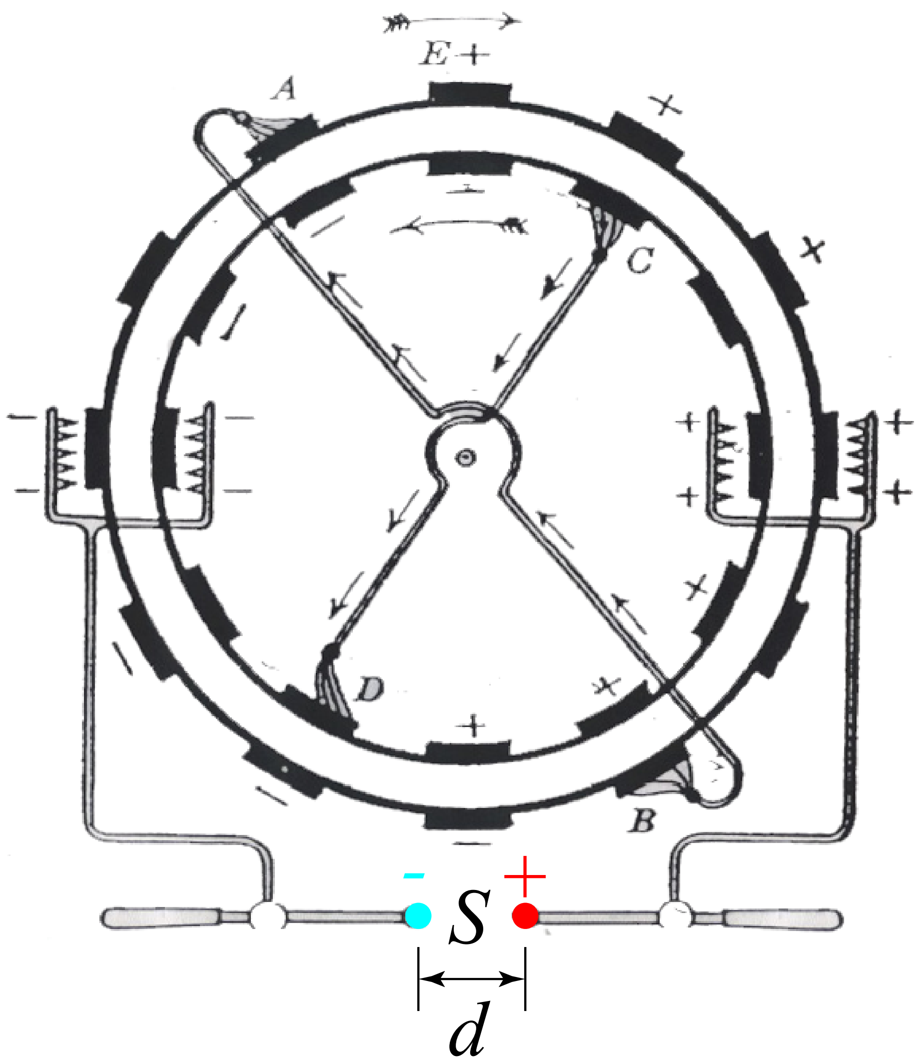
demo: Wimshurst Machine

A Wimshurst electrostatic generator, working on the principle of induction, generates high voltage differences and sparks between two movable electrodes.



Charge up the two electrodes (located at S in the figure) that are separated by a distance d. If the electric field is strong enough they will discharge.

CQ: Wimshurst Machine



Charge up the two electrodes (located at S in the figure) that are separated by a distance d. They discharge when the potential difference between them is V.

The electrodes are then moved further apart, separated now by a distance $d_1 > d$ and allowed to charge up and then discharge when the potential difference is V_1.

Which of the following is true?

1. $V_1 < V$
2. $V_1 = V$
3. $V_1 > V$

Conservation of Energy

When a charged particle (charge *q*) is moved across an electric potential difference then the change in potential energy is

$$\Delta U = q\Delta V = q(V_B - V_A)$$

Assuming no other energy changes then the change in energy of the charged particle is

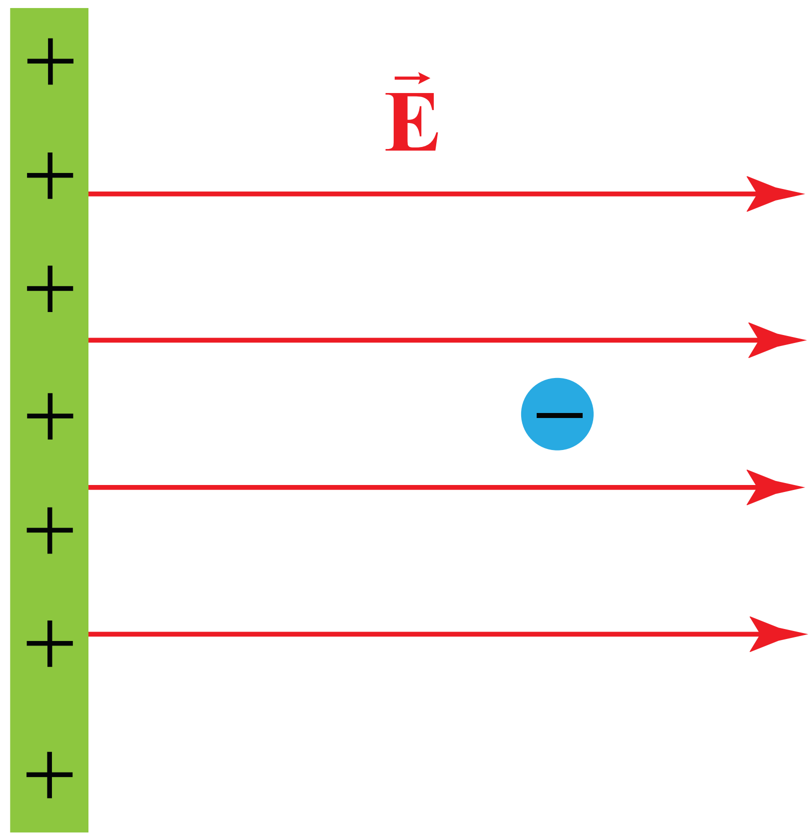
$$\Delta E = \Delta K + q\Delta V = 0 \Rightarrow$$

$$(1/2)m({v_B}^2 - {v_A}^2) = -q(V_B - V_A)$$

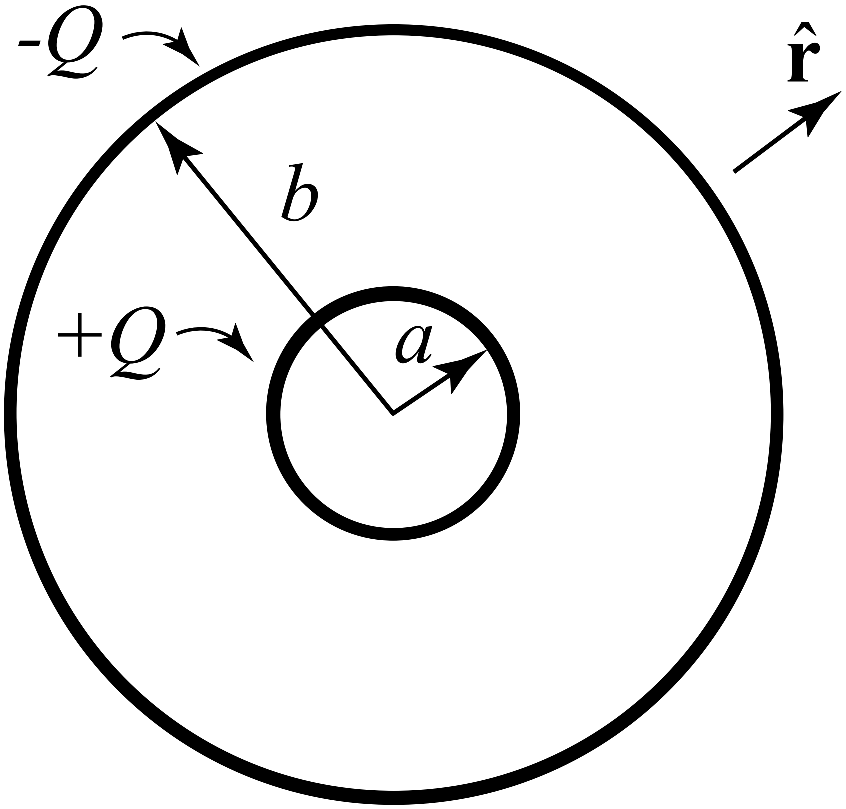
CQ: Motion of Negative Charged Object in External Electric Field

If a negatively charged particle is released from rest in an electric field, the charge will move

1. from higher to lower electric potential resulting in an increase in potential energy.
2. from higher to lower electric potential resulting in a decrease in potential energy.
3. from lower to higher electric potential resulting in an increase in potential energy.
4. from lower to higher electric potential resulting in a decrease in potential energy.



Group Problem: Spherical Shells



These two spherical conducting shells have equal but opposite charge.

a) Find a vector expression for the electric field as a function of the distance r from the center, for the region $a \leq r \leq b$.

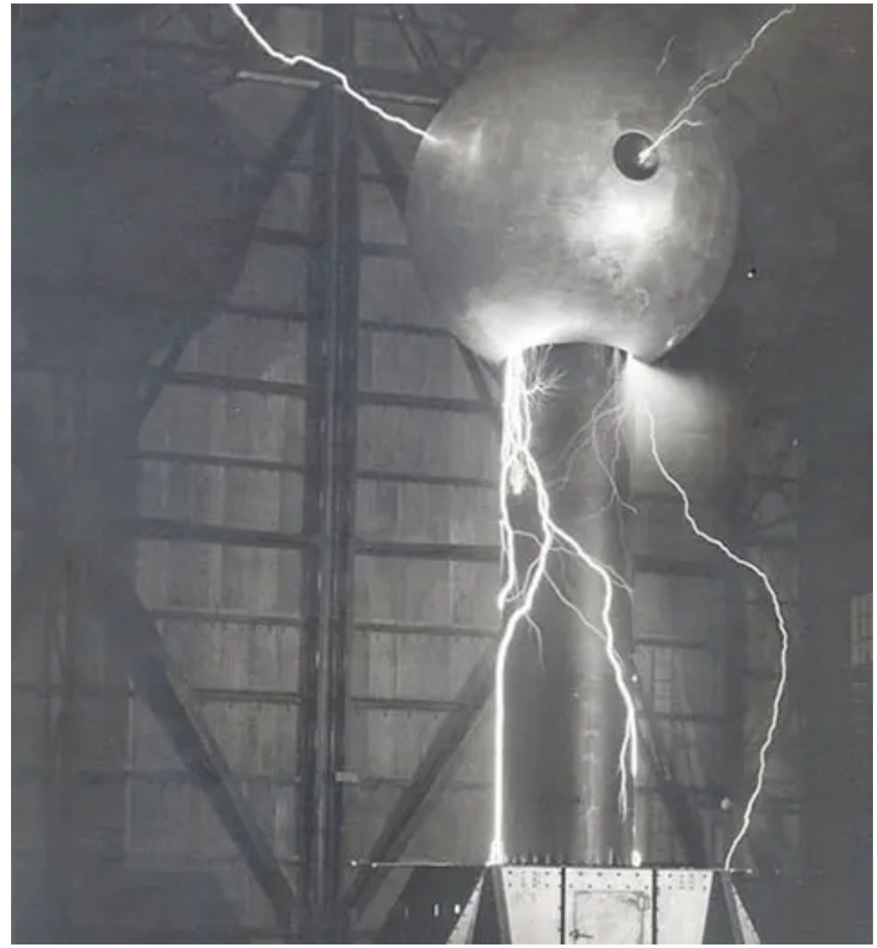
b) Find the potential difference between the shells

$$V(b) - V(a) = -\int_{r=a}^{r=b} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

c) Check your answer: do you expect it to be positive or negative?

demo: Van de Graaff Generator

Photo of generators on Round Hill estate, generator donated to Museum of Science in 1956



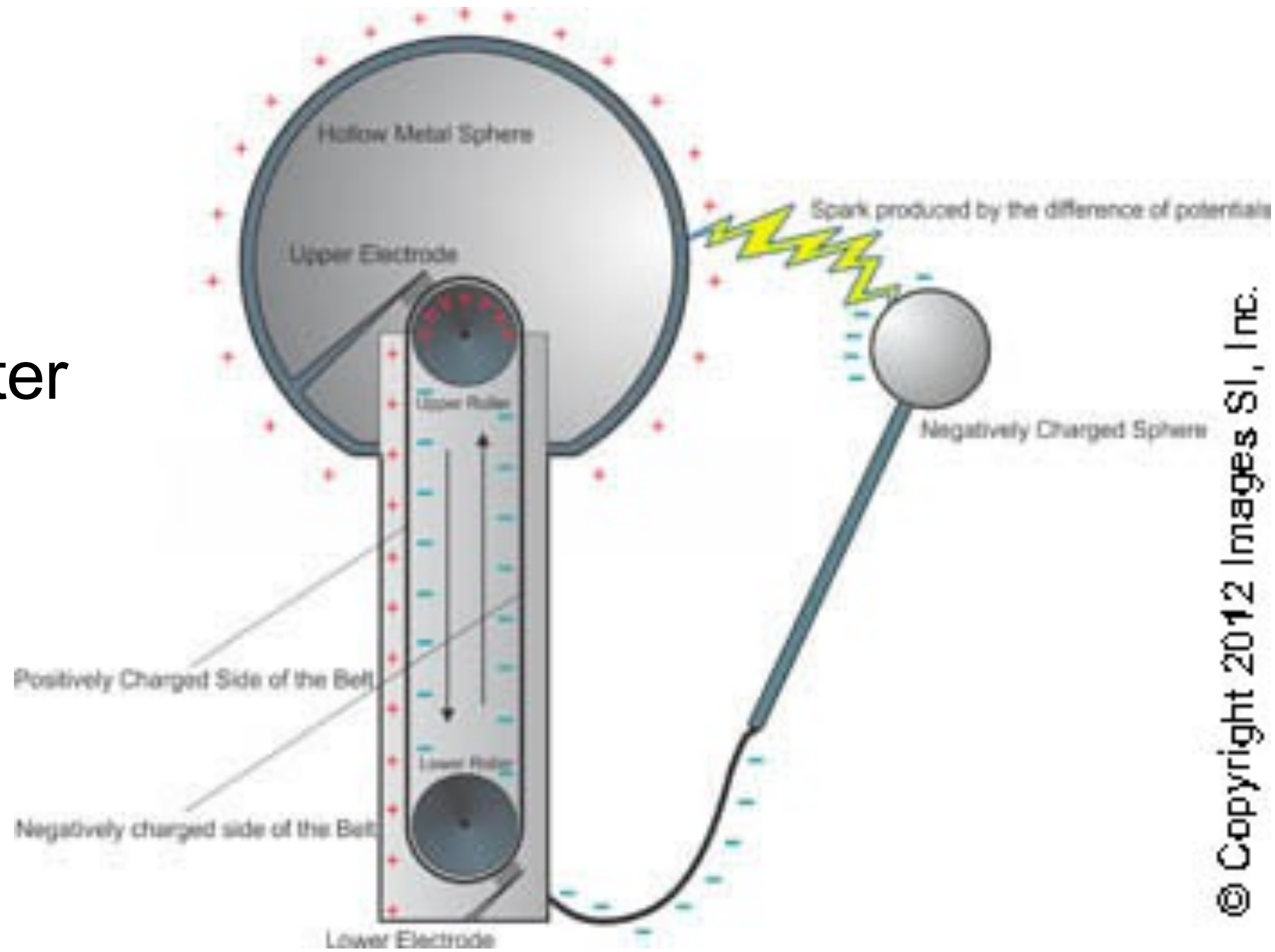
W03D2

CQ: Van de Graaff Generator

Consider a spherical shell of radius and that has been charged with a charge Q, resulting in a potential difference V(a)-V(∞)=V. The magnitude of the electric field on the surface of the sphere is

1. $E = a / V$ 2. $E = Va$ 3. $E = V / a$ 4. $E = V = 0$

Hint: Start with your result from the group problem, Spherical Shells, where the outer shell has radius b = infinity, and the inner shell radius a.



demo: Van de Graaff D29

For the potential difference between two shells; set b = infinity, a = 11.5 cm. Breakdown of dry air occurs when the magnitude of the electric field is E(a) = 33 kV/cm. Then potential at discharge is approximately

$$V(a) = \frac{kQ}{a}; \quad \vec{\mathbf{E}}(a) = \frac{kQ}{a^2}\hat{\mathbf{r}} \Rightarrow V(a) = E(a)a$$

$$V(a) = (33\,\mathrm{kV}\cdot\mathrm{cm}^{-1})(11.5\,\mathrm{cm}) = 3.8\times10^5\ \mathrm{V}$$

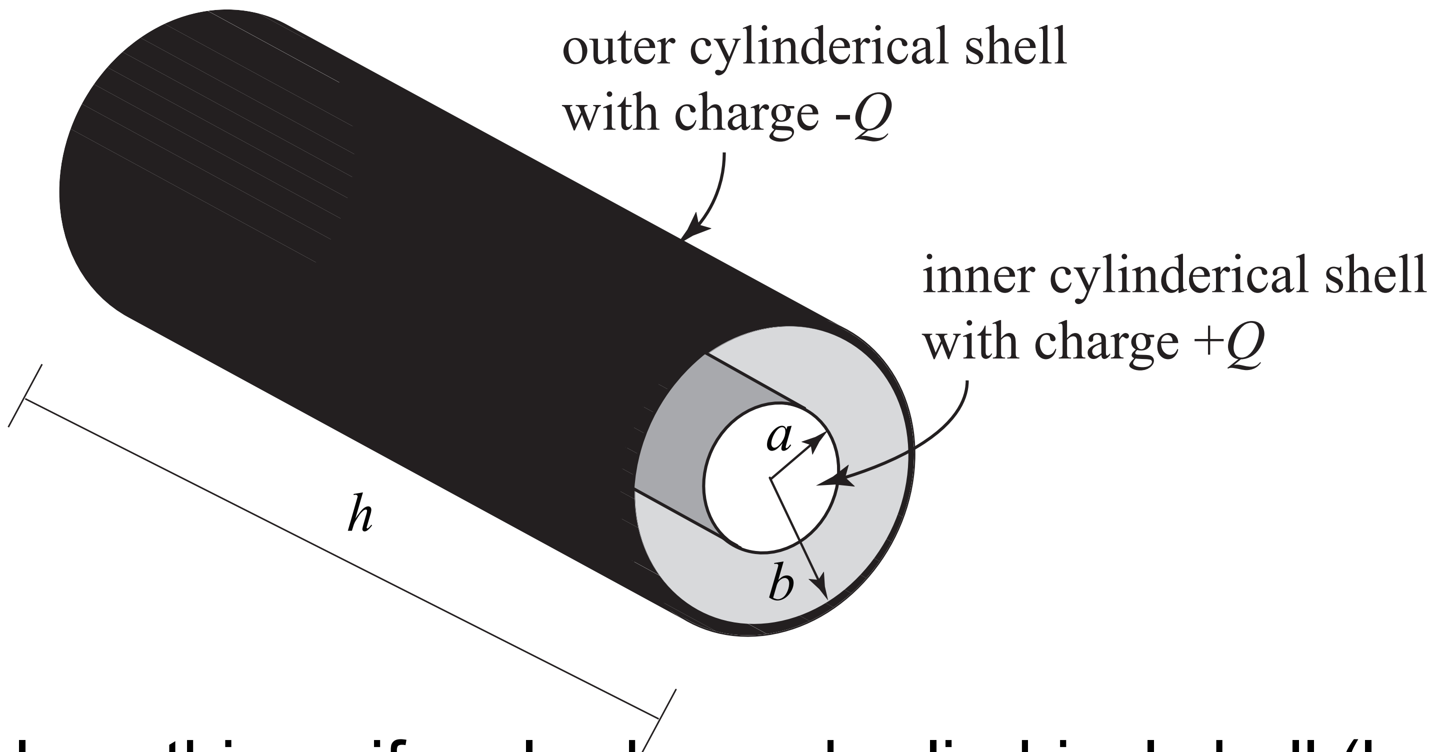
$$Q = (3.3\times10^6\ \mathrm{N}\cdot\mathrm{C}^{-1})(11.5\times10^{-2}\mathrm{m})^2 / (9.0\times10^9\ \mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{C}^{-2})$$

$$\simeq 5\times10^{-6}\ \mathrm{C}$$

which corresponds to about 3 x 10^{13} electrons.

file:///Users/padour/Documents/Coursework/8.02%20Coursework/8.02_Spring2010_web/802TEAL3D/visualizations/electrostatics/vdgdischarge/VanDeGraffSparkFulll_smooth.mpg

Group Problem: Coaxial Cylinders

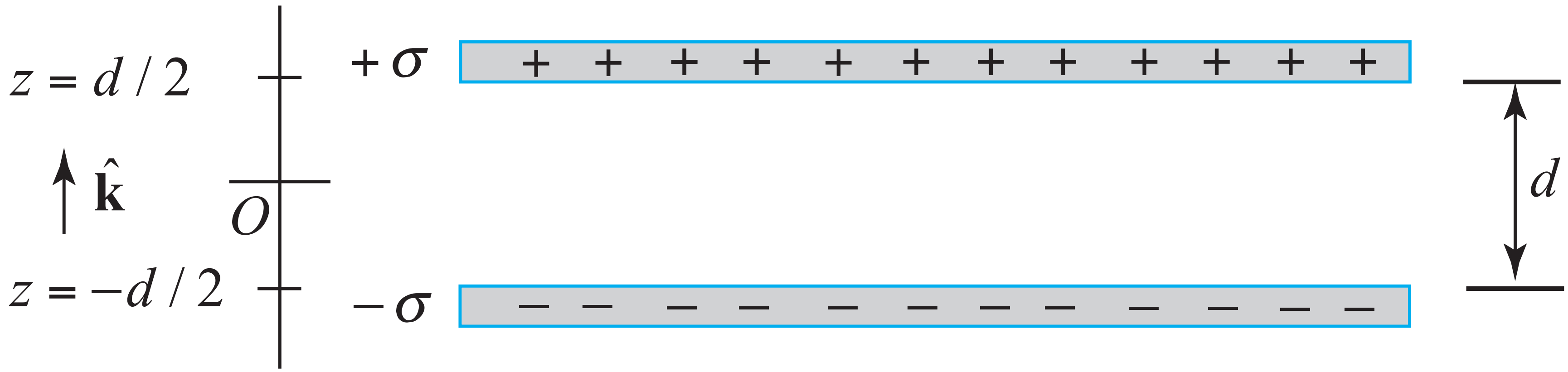


A very long thin uniformly charged cylindrical shell (length h and radius a carrying a positive charge +Q is surrounded by a thin uniformly charged cylindrical shell (length h and radius b) with negative charge -Q , as shown in the figure. You may ignore edge effects.

Find V(a) – V(b).

Appendix

Group Problem: Potential Difference of Two Oppositely Charged Plates

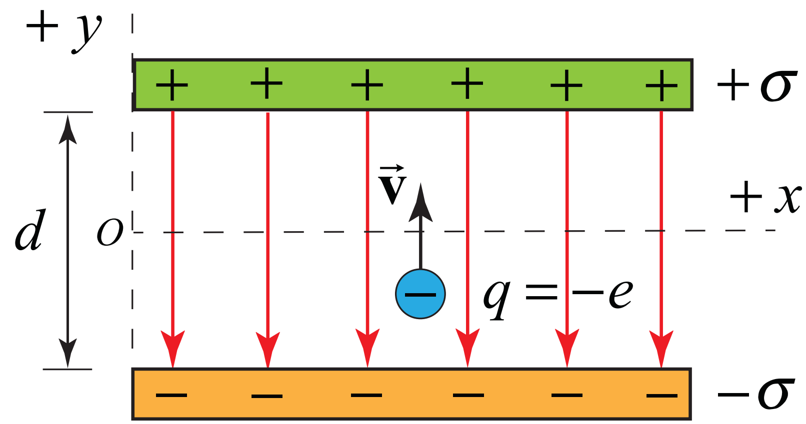


Two parallel very thin and long non-conducting plates, lying in the *xy*-plane, are separated by a distance d. The upper plate, located at z = d/2, is positively charged with uniform surface charge density +σ. The lower plate located at z = -d/2 is negatively charged with uniform surface charge density -σ. You may neglect any edge effects.

a) Find a vector expression for the electric field between the plates. (Don't forget you Gaussian surface!)

b) Find an expression for the electric potential difference $V(d/2) - V(-d/2)$

CQ.: Motion of an Electron in a Uniform Electric Field



Consider two oppositely uniform charged thin plates separated by a distance *d*. The surface charge densities $\pm\sigma$ on the plates are uniform and equal in magnitude.

During the class, you calculated the potential difference between the plates.

$$V(d/2) - V(-d/2) = \frac{\sigma d}{\varepsilon_0}$$

An electron with charge –e and mass m is released from rest at the negative plate and moves to the positive plate. What is the speed of the electron when it reaches the positive plate?

1. $v = \sqrt{m\varepsilon_0 / 2e\sigma d}$
2. $v = m\varepsilon_0 / 2e\sigma d$
3. $v = \sqrt{2e\sigma d / m\varepsilon_0}$
4. $v = 2e\sigma d / m\varepsilon_0$

Feedback from Learning Sequences W3LS1, W3LS2

1. **Units:** What are the correct units for electric potential energy and how should they be related to joules?

2. **Electric field and potential differnece**: How does the presence or absence of an electric field affect the potential difference between two points?

3. **Force and work:** How does the direction of force and work relate to the direction of the particle's motion and the electric field?

4. **Electric potential energy and electric potential:** How should students differentiate between these two concepts and their respective units?