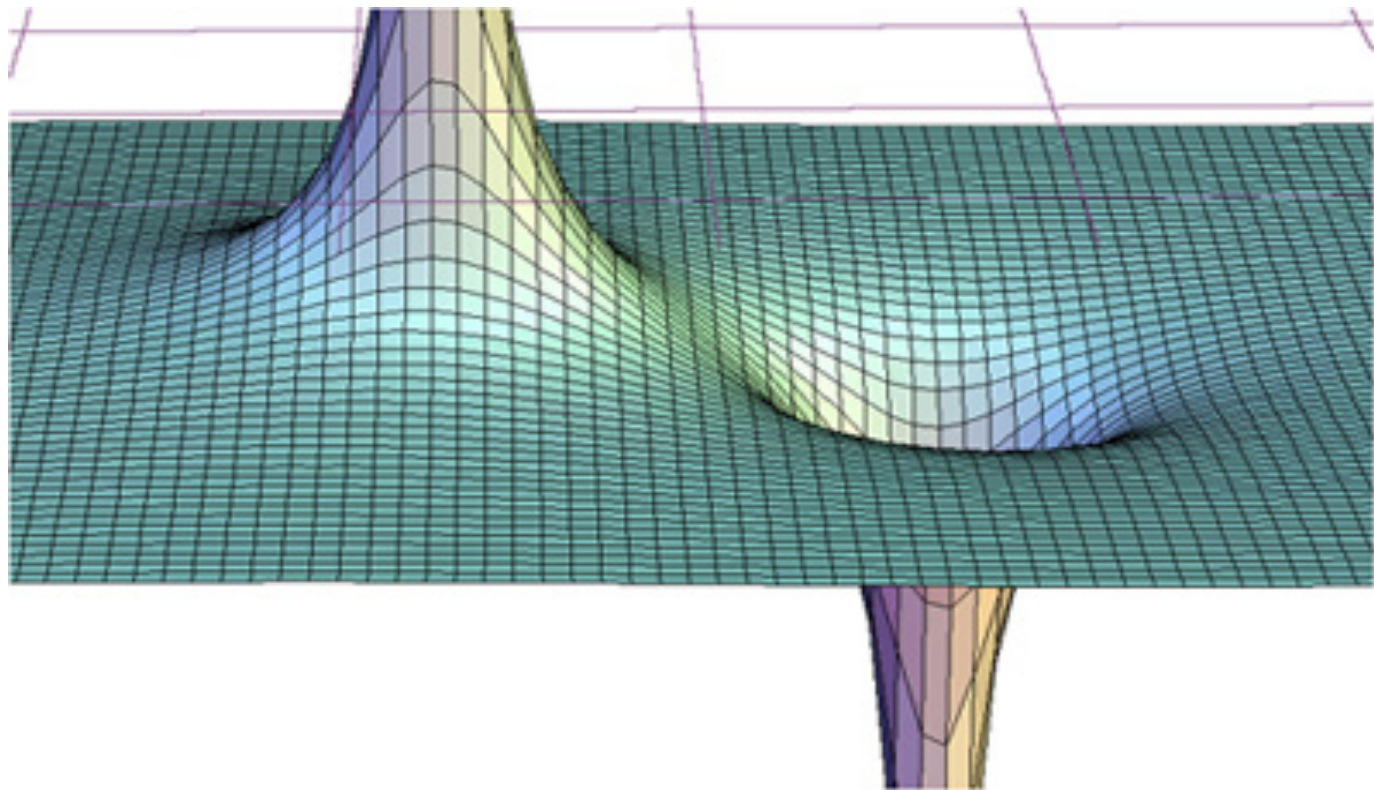


Equipotential Surfaces, Conductors and Insulators, Capacitors and Dielectrics



8.02

W04D2

Announcements

Week 4:

LS 3 and LS 4 due Tuesday at 11 pm

Problem Set 3 Due Wednesday at 11 pm

Friday: Special Friday Problem Solving

Week 5:

LS 1 and LS 2 due Sunday at 11 pm

LS 3 and LS 4 due Tuesday at 11 pm

Problem Set 4 Due Wednesday at 11 pm

Outline

Equipotentials

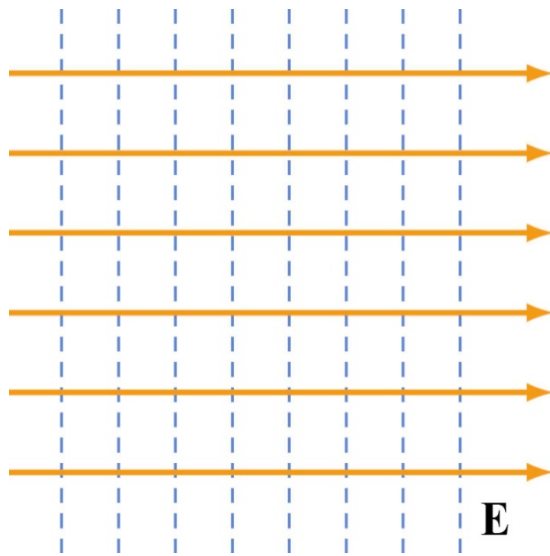
Properties of Conductors

Capacitance and Capacitors

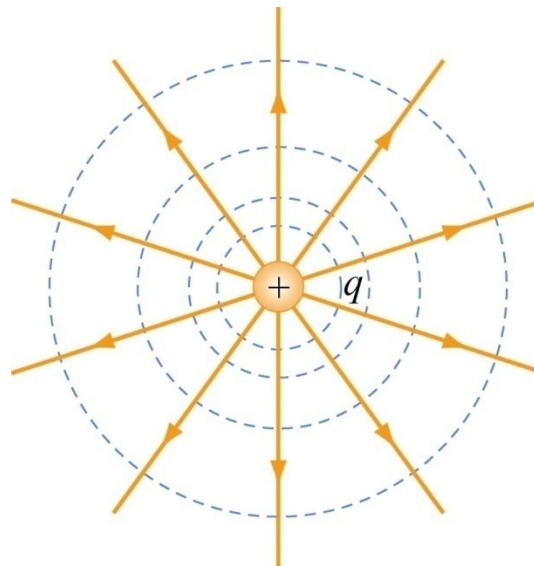
Dielectrics

Review: Equipotentials and Electric Fields

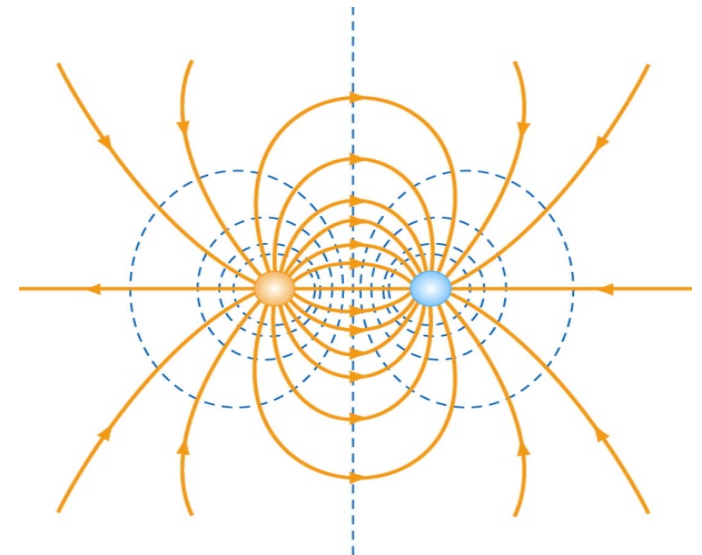
$$\vec{E} = -\vec{\nabla} V$$



Constant E field



Point Charge



Electric dipole

CQ: Properties of Equipotentials

Which of the following statements are true about electric field lines and equipotentials?

1. E field lines point from low to high potential.
2. E field lines are parallel to equipotentials.
3. E field lines are perpendicular to equipotentials.
4. The electric force does positive work on a charged particle as it accelerates along a path that coincides with an equipotential.

CQ Ans.: Properties of Equipotentials

Answer 3:

E field lines point from high to low potential.

E field lines are perpendicular to equipotentials.

Some non-electrostatic forces can do positive work on a charged particle as it accelerates along a path that coincides with an equipotential. The electric force cannot do positive (or negative) work on a charger particle accelerating on an equipotential surface because the electric force is always perpendicular to the equipotential.

Conductors and Insulators

Conductor:

Charges are free to move

Electrons weakly bound to atoms

Example: metals

Insulator:

Charges are NOT free to move

Electrons strongly bound to atoms

Examples: plastic, paper, wood

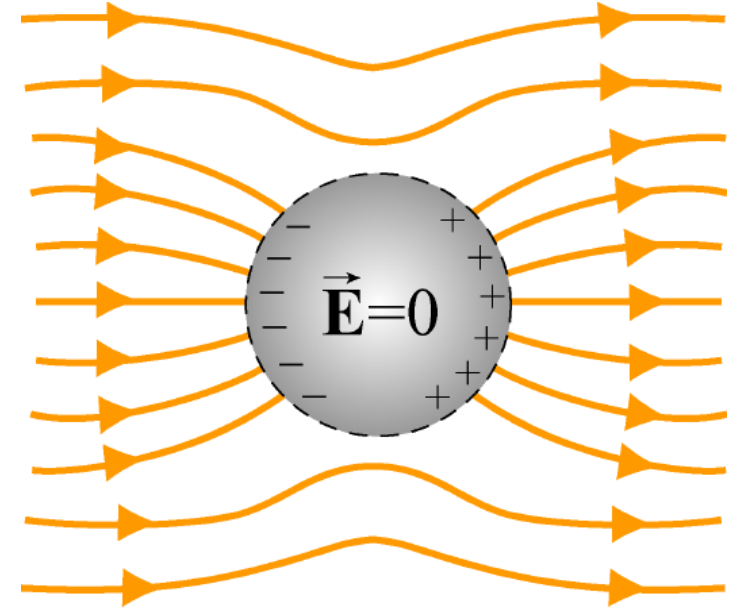
Demonstration: Bouncing Balloon Van de Graaff Generator D17

Why is the balloon attracted to the metal sphere?

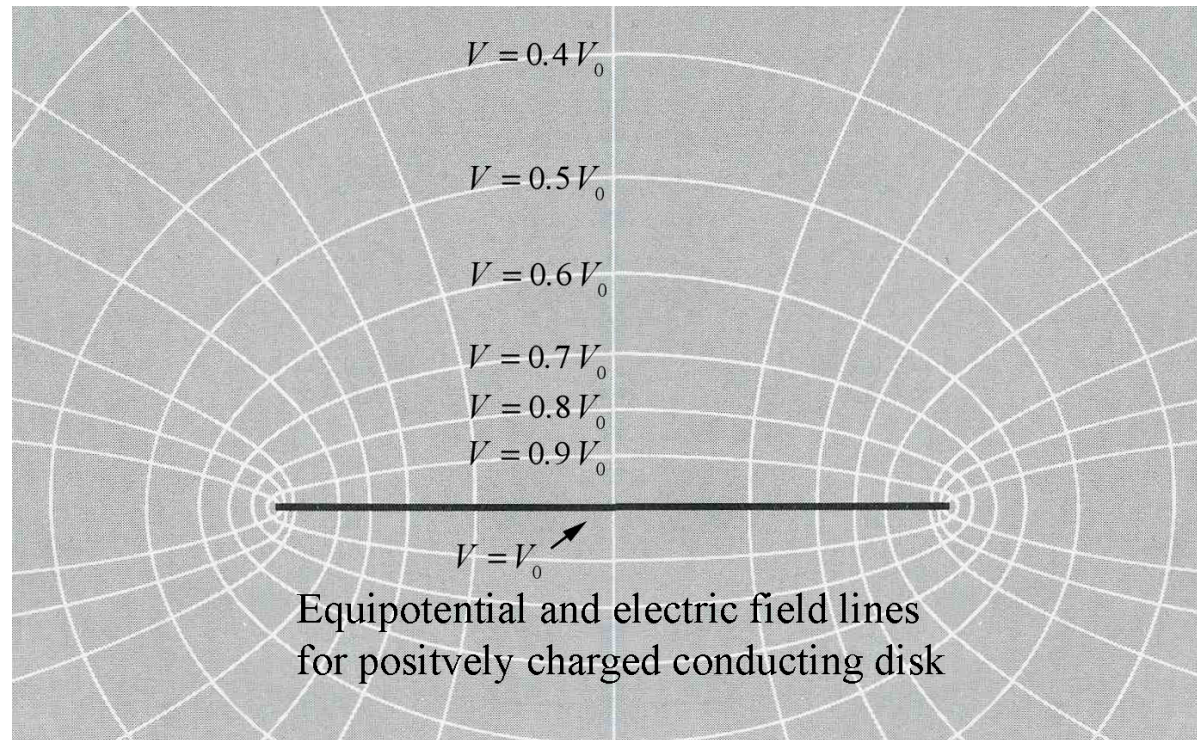
CQ: Conductors in Equilibrium

When a conductor is placed in external electric field: charges in the conductor will move and if static equilibrium is reached then

- 1) E is perpendicular to surface.
- 2) The induced surface charge distribution on surface of conductor is in general non-uniform
- 3) Every point in the conductor is at the same potential
- 4) All of the above.



Conductors are Equipotential Surfaces



When a conductor is placed in external electric field: charges in the conductor will move until the electric field is zero inside material that composes the conductor.. Then

- 1) All points in the conductors are at the same potential
- 2) The electric field is perpendicular to the surface of the conductor (edge effects are illustrated in above figure)

Demonstration:

Faraday Cage D33

<http://tsgphysics.mit.edu/front/?page=demo.php&letnum=D%2033&show=0>

Electric Field inside Cavity of Conductor

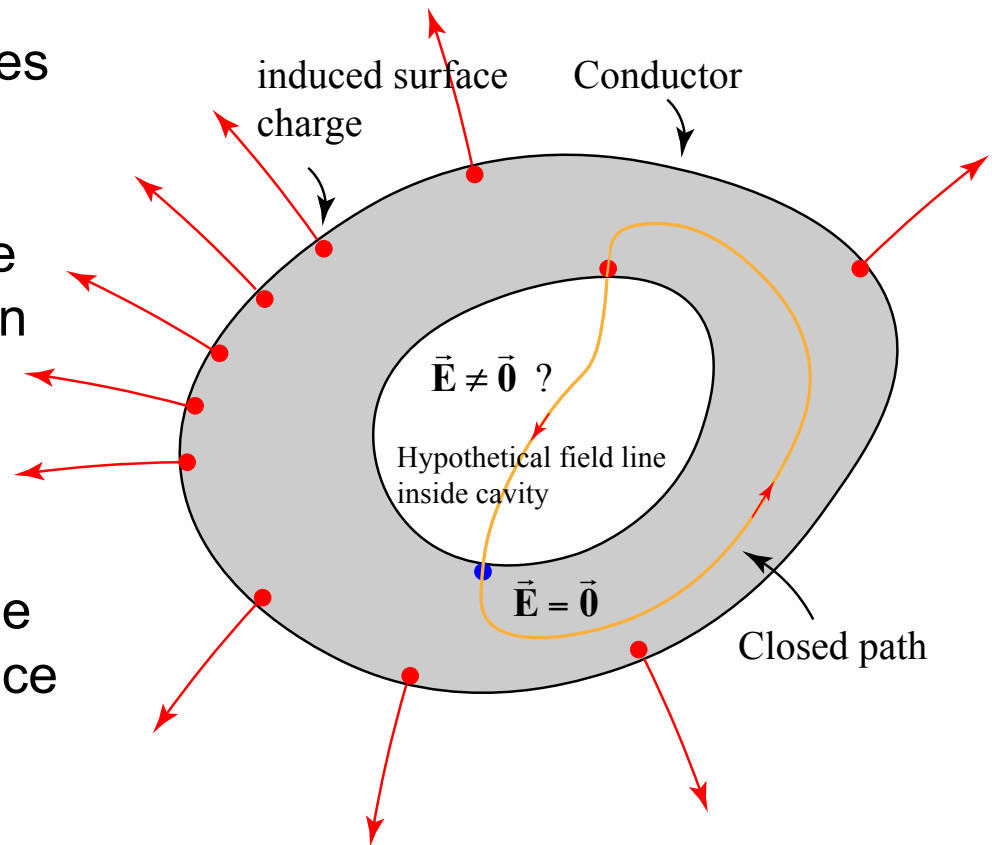
Assume non-zero electric field inside cavity beginning and ending on charges that lie on inner surface. Consider a closed path that follows the field line inside the cavity and returns inside the conductor where the field is zero. Then on closed path

$$\oint_{\text{closed path}} \vec{E} \cdot d\vec{s} > 0$$

which contradicts the assertion that the electrostatic field is conservative. Hence inside the cavity

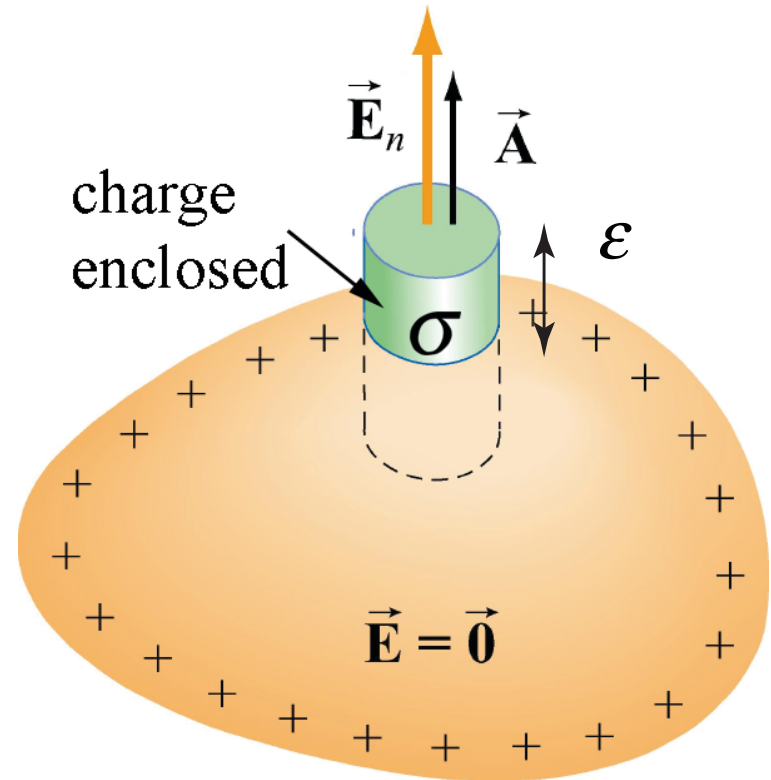
$$\vec{E} = \vec{0}$$

This also implies that there is no charge on the inner surface of the conductor.



Group Problem: Electric Field on Surface of Conductor

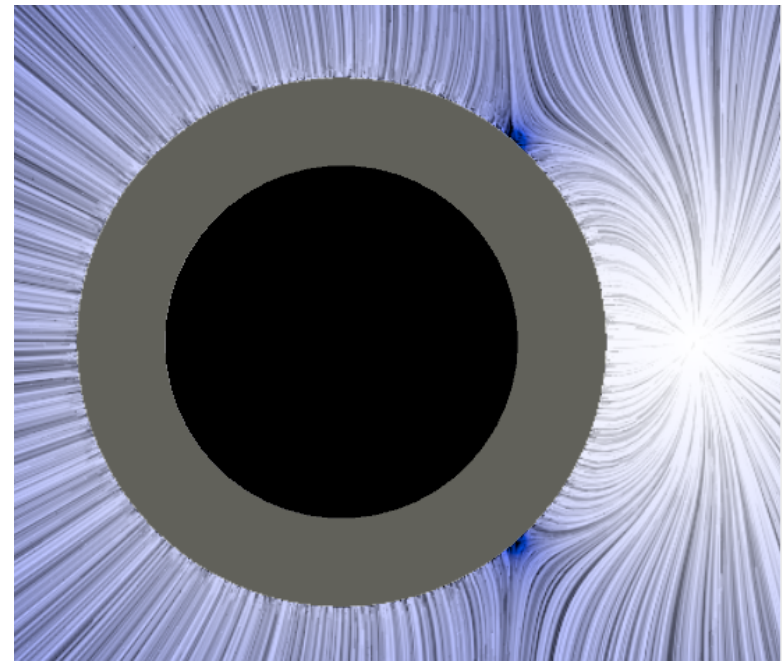
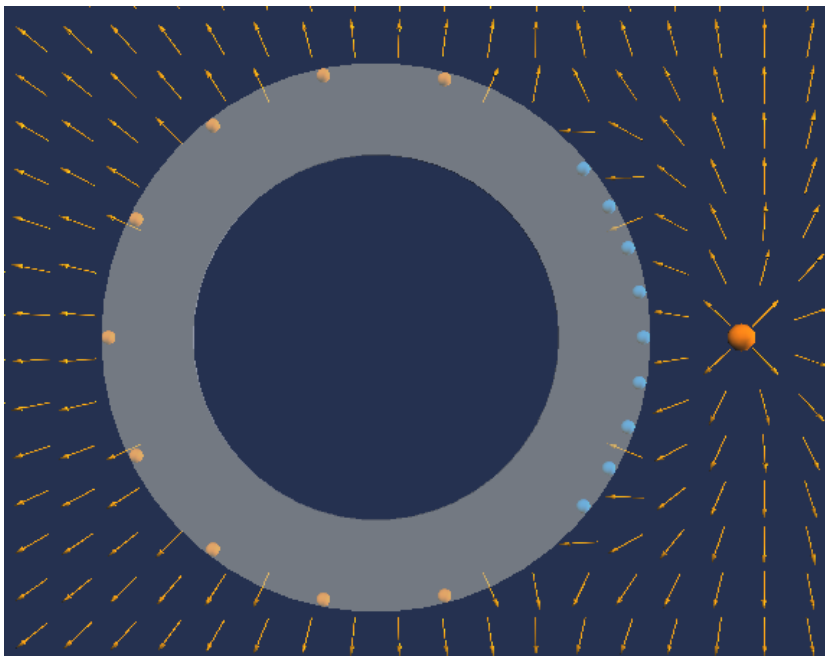
Consider a conductor with a positive non-uniform charge density $\sigma > 0$ on the outer surface. **Choose a very thin Gaussian surface, where one end-cap is just above the surface, and the other end-cap lies inside the material of the conductor, as shown in the figure.**



What is the direction and magnitude of the electric field on the surface?

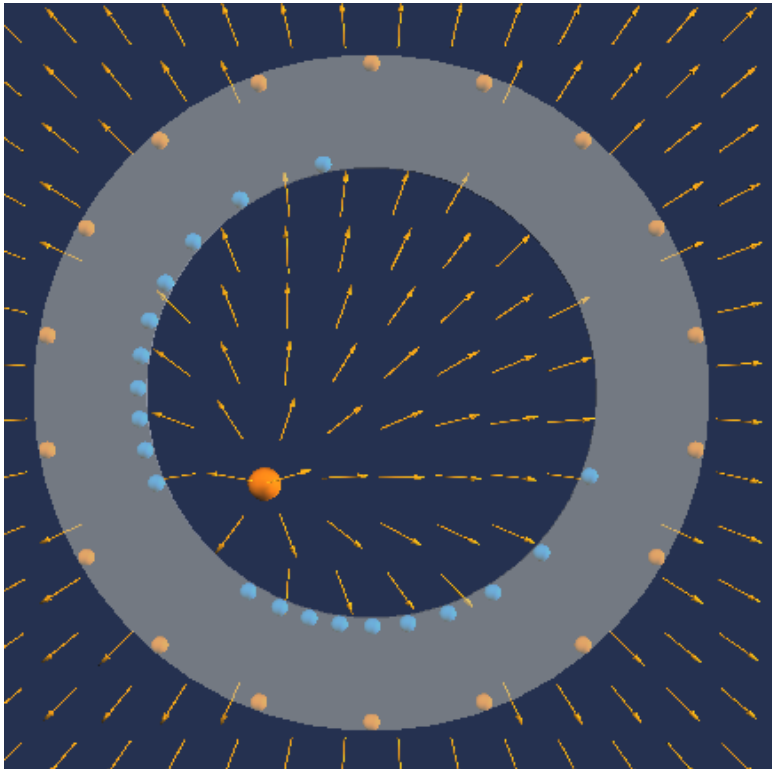
Charge Placed Outside Hollow Conductor

Charge placed outside spherical shell conductor induces charge separation on outer surface of conductor. Electric field is zero inside both conductor and region interior to conductor.



Charge Placed Inside Hollow Conductor

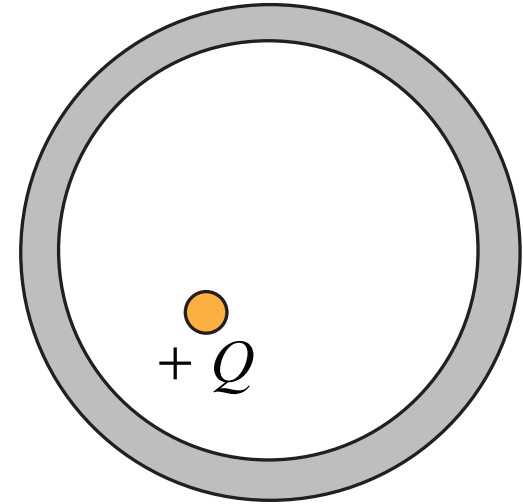
Charge placed inside interior of spherical shell conductor.
Electric field outside is field of point charge.



CQ: Charge inside cavity of Conductor I

spherical conducting shell

A point charge $+Q$ is placed inside a neutral, hollow, spherical conductor. As the charge is moved around *inside*, the **surface charge density on the outside**



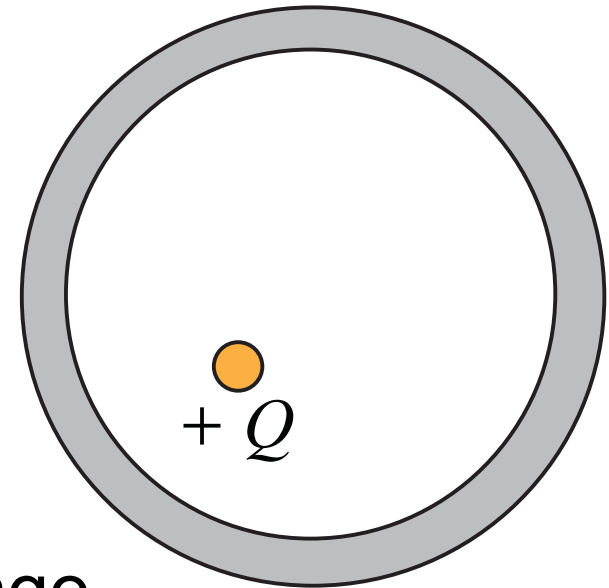
1. is initially uniform and does not change when the charge is moved.
2. is initially uniform but does become non-uniform when the charge is moved.
3. is initially non-uniform but does not change when the charge is moved.
4. is initially non-uniform but does change when the charge is moved.

CQ: Charge inside cavity of Conductor

II

A point charge $+Q$ is placed inside a neutral, hollow, spherical conductor. As the charge is moved around *inside*, the ***electric field outside***

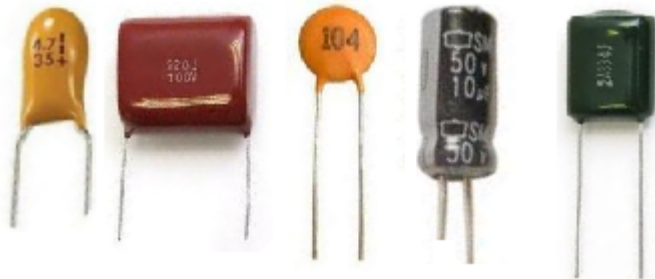
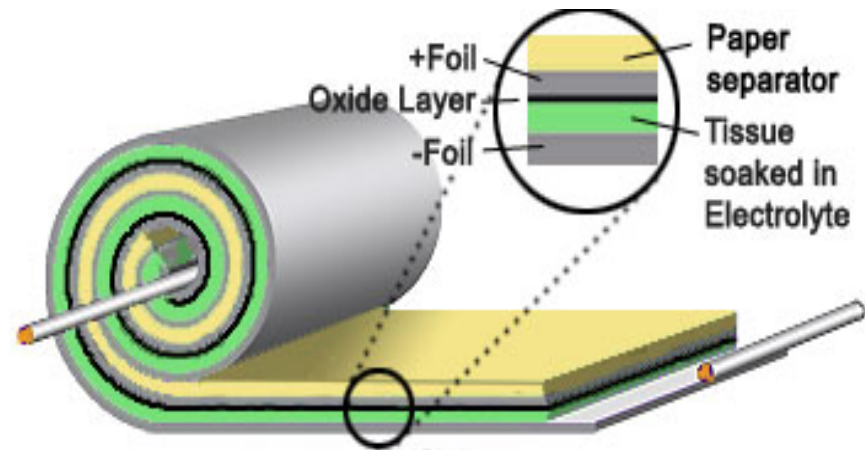
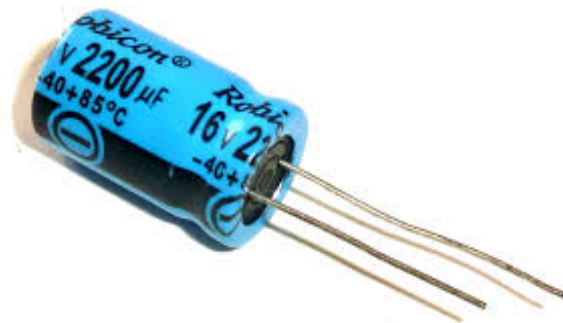
spherical conducting shell



1. is zero and does not change
2. is non-zero but does not change
3. is zero when centered but changes
4. is non-zero and changes

Capacitors and Capacitance

Our first of 3 standard electronics devices
(Capacitors, Resistors & Inductors)



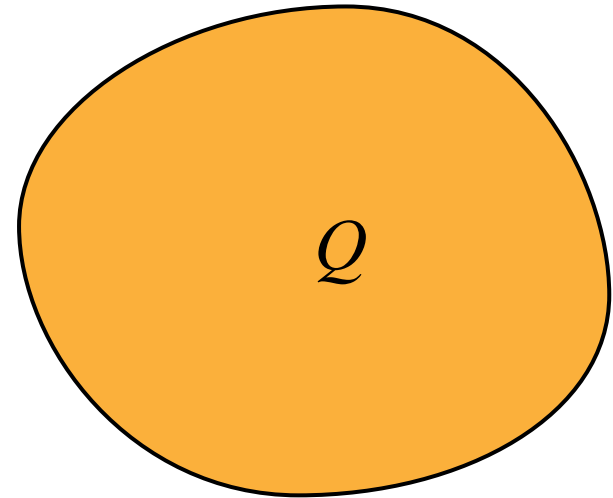
Capacitance

Consider an isolated conductor with total charge Q . The conductor is an equipotential surface, with a certain potential V , where $V = 0$ at infinity. The charge on the conductor Q is linearly proportional to the potential

$$Q = CV$$

The constant of proportionality C is called the capacitance and is only depends on the size and shape of the conductor.

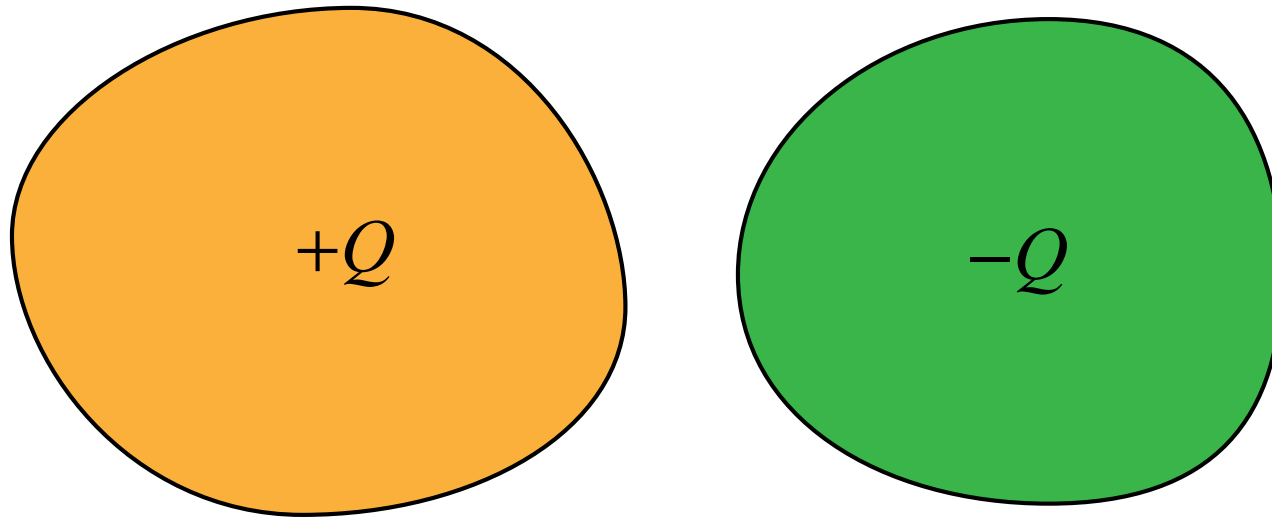
conductor with charge Q ,



conductor at potential V ,
($V = 0$ at infinity)

SI Units: 1 coulomb/volt = 1 farad

Capacitor: Two Conductors



Capacitor: Two isolated conductors with equal and opposite charges $\pm Q$. Potential difference ΔV between the two conductors. The capacitance of the two conducting surfaces is

$$C = Q / |\Delta V|$$

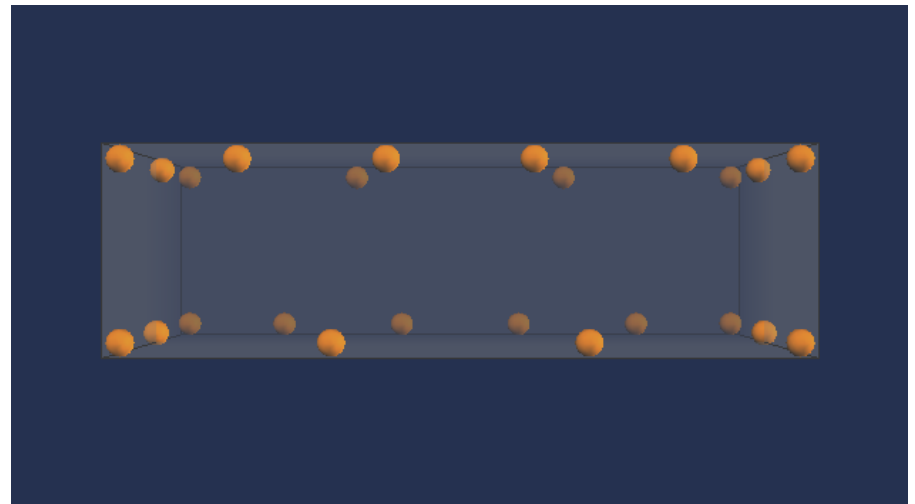
C is always a positive constant, depending on the size and shape of the conductors, and the permittivity.

Induced Charges on Conductors

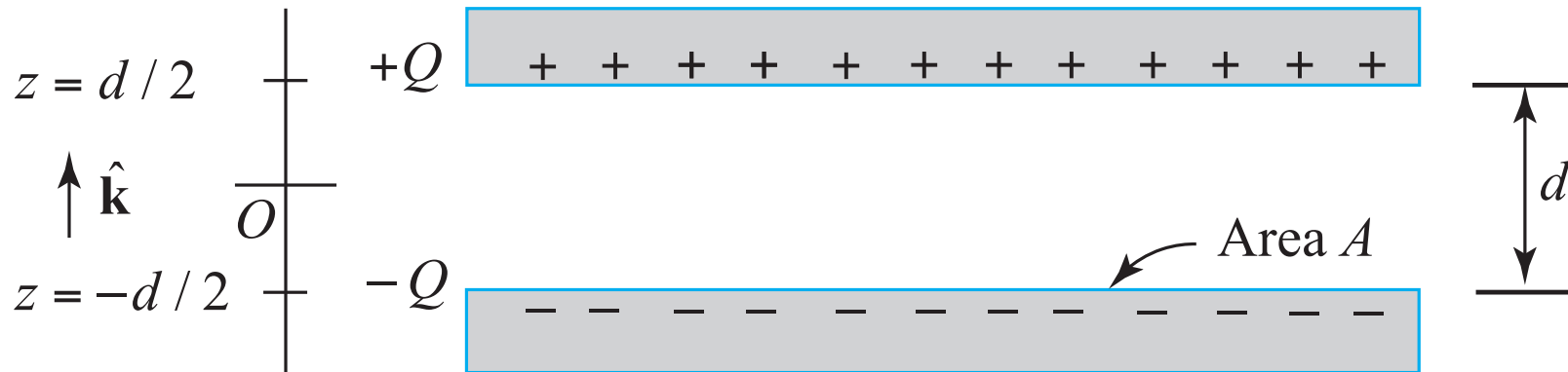
Non-zero charge placed inside a metal slab (conductor): charges move to surface (move as far apart as possible)

The Charged Metal Slab Applet:

<http://web.mit.edu/viz/EM/visualizations/electrostatics/CapacitorsAndCondcutors/chargedmetalslab/chargedmetalslab.htm>



Group Problem: Parallel Plate Capacitor:

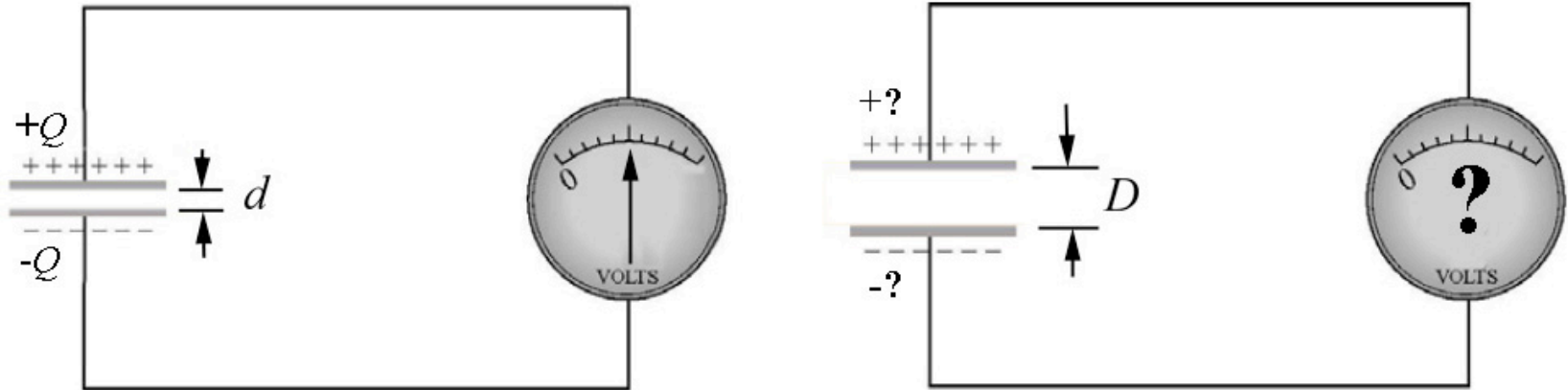


cross sectional view of two very long and wide parallel plates

A parallel plate capacitor consists of two long parallel conducting plates each of area A that are separated by a distance d . The inner surface of the upper plate is positively charged with charge $+Q$ and the inner surface of the lower plate is negatively charged with charge $-Q$. **You may assume that the charge distributions on the inner surfaces are uniform and neglect edge effects.**

Find an expression for the capacitance in terms of ϵ_0 , d , and A

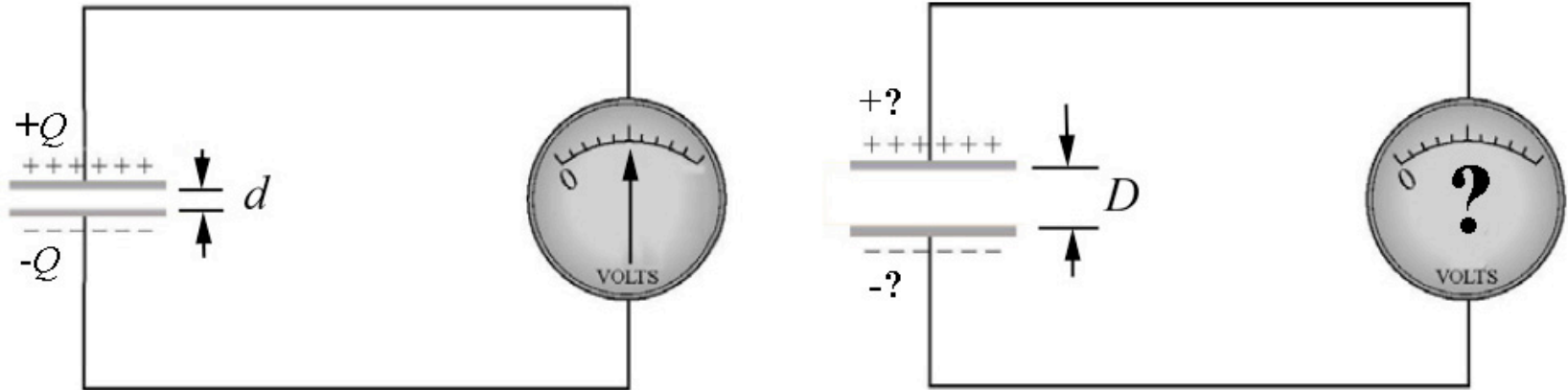
CQ: Changing Dimensions



A parallel-plate capacitor is charged until the plates have equal and opposite charges $\pm Q$, separated by a distance d , and **then disconnected from the charging source (battery)**. The plates are pulled apart to a distance $D > d$. What happens to the magnitude of the charge on the plates?

1. Q increases.
2. Q is the same.
3. Q decreases.

CQ: Changing Dimensions



A parallel-plate capacitor is charged until the plates have equal and opposite charges $\pm Q$, separated by a distance d , and **then disconnected from the charging source (battery)**. The plates are pulled apart to a distance $D > d$. What happens to the magnitude of the potential difference V ?

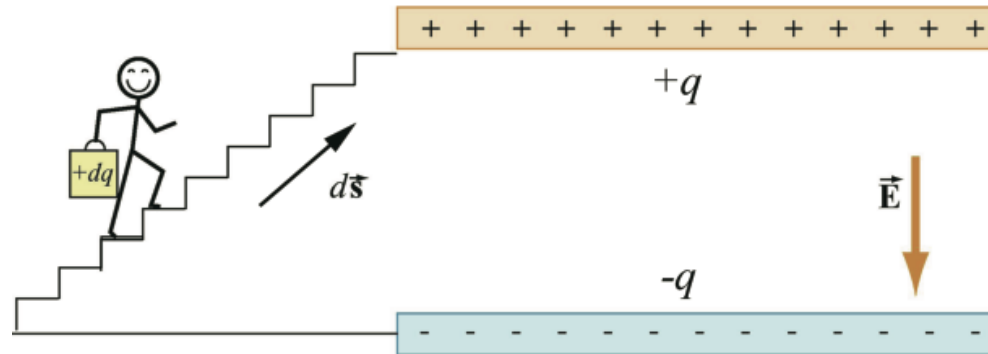
1. V increases.
2. V is the same.
3. V decreases.

Demonstration:

Changing Distance Between Circular Capacitor Plates E4

<http://tsgphysics.mit.edu/front/?page=demo.php&letnum=E%204&show=0>

Stored Energy in Charging Capacitor



Start charging capacitor by bringing charge from bottom plate to top plate. At some point top plate has $+q$, bottom has $-q$. Potential difference is $\Delta V = q / C$.

Change in stored energy done lifting another dq is $dU = (dq)(\Delta V) = (dq)(q/C)$.

Total energy needed to charge to $+Q$ on upper plate, $-Q$ on lower plate, with final potential difference $V_f = Q/C$

$$U = \int dU = \frac{1}{C} \int_0^Q q dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C V_f^2$$

Energy Stored in Capacitor

For parallel plate capacitor with charge Q , area A and distance between plates d : the capacitance is

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{Ed} = \frac{\epsilon_0 A}{d}$$

The stored energy is

$$U = \frac{Q^2}{2C} = \frac{1}{2} C |\Delta V|^2 = \frac{1}{2} \epsilon_0 E^2 A d = (u_E)(Volume)$$

Electric field energy density: $u_E = \frac{1}{2} \epsilon_0 E^2$

Generalization: $U = \int_{all\ space} \frac{1}{2} \epsilon_0 E^2 (d^3x)$

Demonstration: Charging Up a Capacitor

A 100 microfarad oil-filled capacitor is charged to 4 KV and discharged through a wire

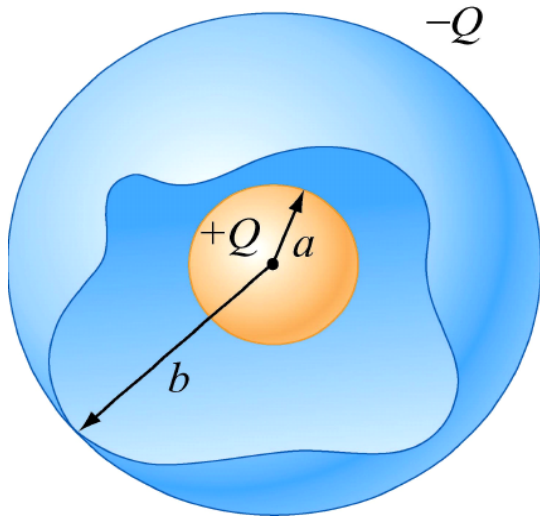
Stored Energy:

$$U = \frac{1}{2} C V^2 = \frac{1}{2} (1 \times 10^{-4} \text{ F}) (4 \times 10^3 \text{ V})^2 = 800 \text{ J}$$



Extra Problem

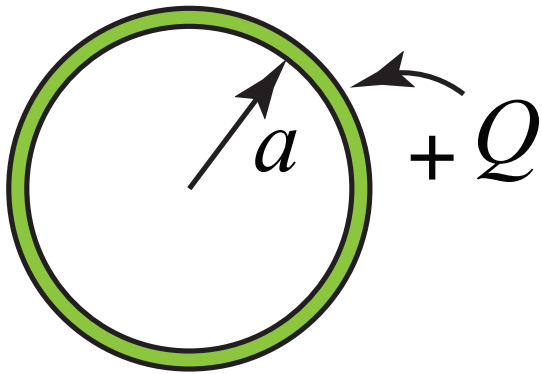
Group Problem: Capacitance of Concentric Spherical Shells



A capacitor consists of two concentric spherical shells. The outer radius of the inner shell is a and the inner radius of the outer shell is b . Suppose the inner shell has charge $+Q$ and the outer shell has charge $-Q$.

- a) What is the electric field between the two shells?
- b) What is the electric potential difference $V(a)-V(b)$?
- c) What is the capacitance C of this capacitor?
- d) How much energy is stored in this capacitor?

CQ: Capacitance of an Isolated Spherical Conducting Shell



What is the capacitance of an isolated spherical conduction shell of radius a ?

1. $C = 0$
2. $C = \infty$
3. $C = 4\pi\epsilon_0 a$
4. $C = \frac{4\pi\epsilon_0}{a}$

Capacitance of Human Being

Model human being as a spherical conductor of radius $a \approx 1$ m:

$$C = 4\pi\epsilon_0 a \qquad \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$C \simeq 100 \times 10^{-12} \text{ F} \simeq 100 \text{ pF}$$

You can measure your capacitance in a demonstration outside the Edgerton Center on the 4th floor of the infinite corridor.

Feedback: Learning Seq. W4LS3, W4LS4

1. **Behavior of charges on the surface of a conductor:** How does the presence of the external charges affect the distribution of charges on the surfaces of conductors?
2. **Equipotential lines and electric field lines:** Highlight relationship between two types of field lines
3. **Properties of insulators:** How do insulators differ from conductors in terms of charge distribution and the presence of mobile charge carriers?
4. **Sign of potential difference:** How does the direction of the electric field affect the sign of the potential difference between two points?
5. **Charge distribution in a conductor with a point charge inside:** How does the presence of a charge inside a conductor affect the distribution of charges on the outer surface?
6. **Converting units:** Factors of ten errors.