

Current, Resistance, Ohm's Law, DC circuits



8.02
W05D2

Announcements

Week 5:

Week 5 LS 3 and LS 4 due Tuesday at 11 pm

Problem Set 4 Due Wednesday at 11 pm

Week 6:

Week 6 LS 1 and LS 2 due Sunday at 11 pm

Week 6 LS 3 and LS 4 due Tuesday at 11 pm

Exam One: Thursday March 13 from 7:30-9:30 pm.

Conflict Exam 1: Friday March 14 8-10 am and 9-11 am.

Conflict Exam One Information

Conflict Exam One:

If you have an academic conflict or a regular scheduled activity (like MITSO) during the scheduled time for the March 13 Exam One, please fill out the online conflict form

<https://www.tealmit.com/conflict>

There are two times for Conflict Quiz One.

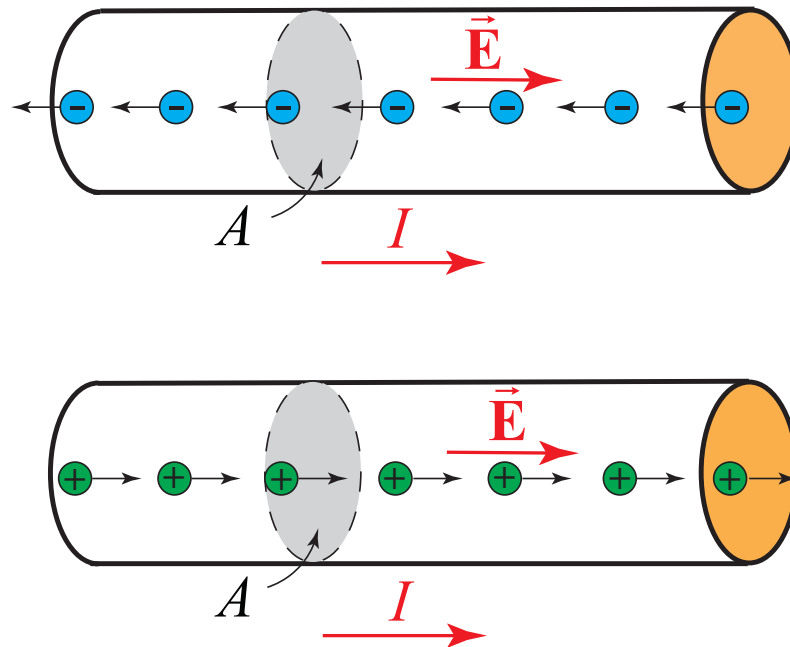
Friday March 14 from 8:00 -10:00 am

Friday March 4 from 9:00 -11:00 am

If you will not be at MIT on March 13 or March 14 due to an athletic event please fill out the conflict form and indicate what the conflict is and when you will return.

Current, Current Density, Resistance and Ohm's Law,

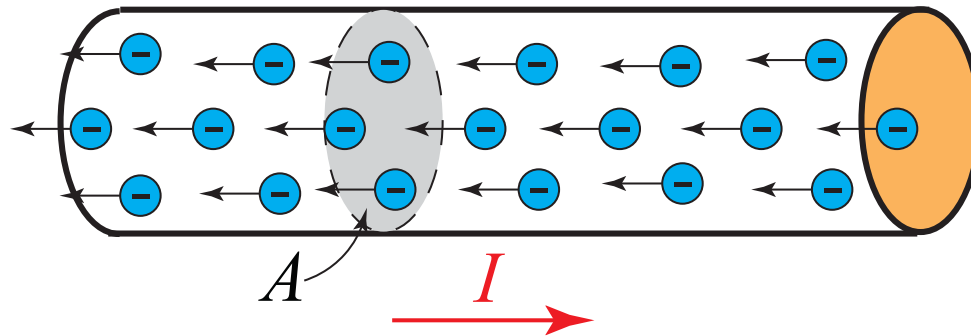
Why Does Charge Flow?



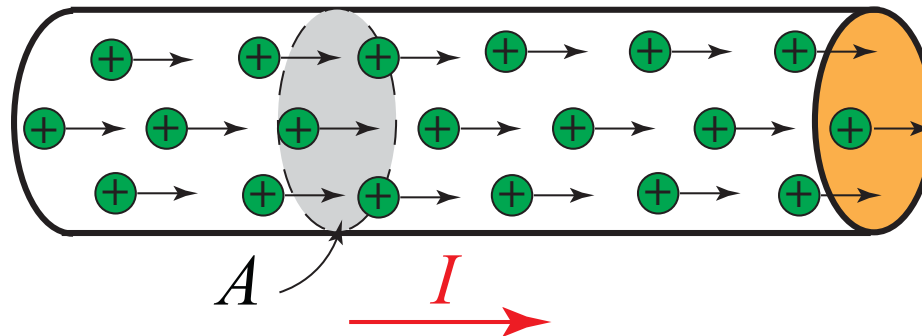
If an electric field is set up in a conducting wire, charge will experience a force and due to interference from the lattice of atoms that make up the wire, the charge will on average move at a constant speed called the ***drift velocity***.

Direction of the Current and Charge

Flow of negative charge per sec through area A :
opposite the direction of the current:



Flow of positive charge per sec through area A :
in the direction of the current



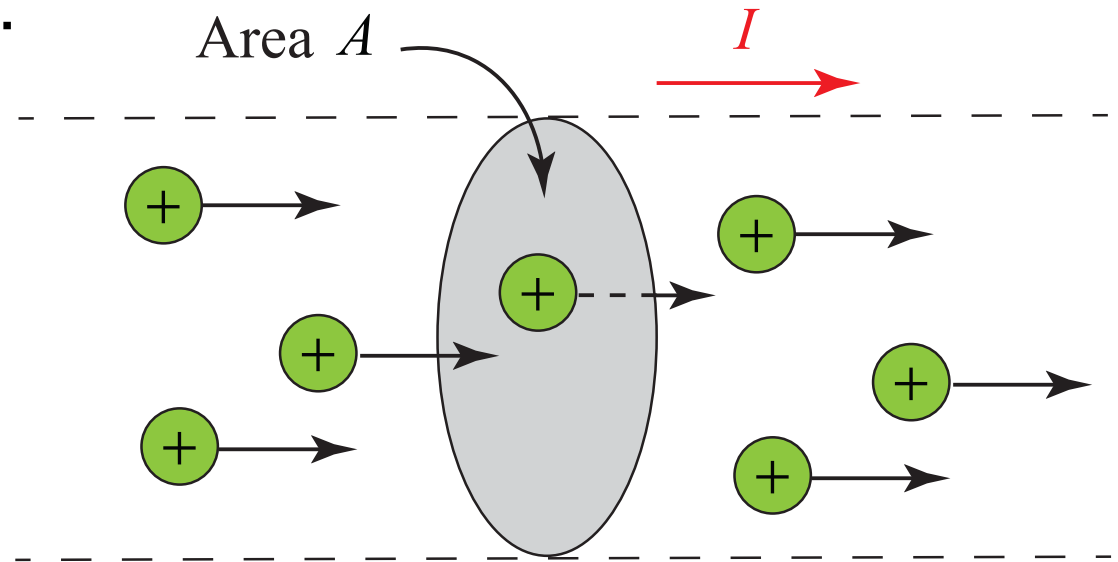
Current: Flow Of Charge

Average current I_{av} : Charge ΔQ
flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current:
differential limit of I_{av}

$$I = \frac{dQ}{dt}$$



Units of Current: Coulomb/second = Ampere

How Big is an Ampere?

- Household Electronics $\sim 1\text{ A}$
- Battery Powered $\sim 100\text{ mA}$ (1-10 A-Hr)
- Household Service 100 A
- Lightning Bolt 10 to 100 kA

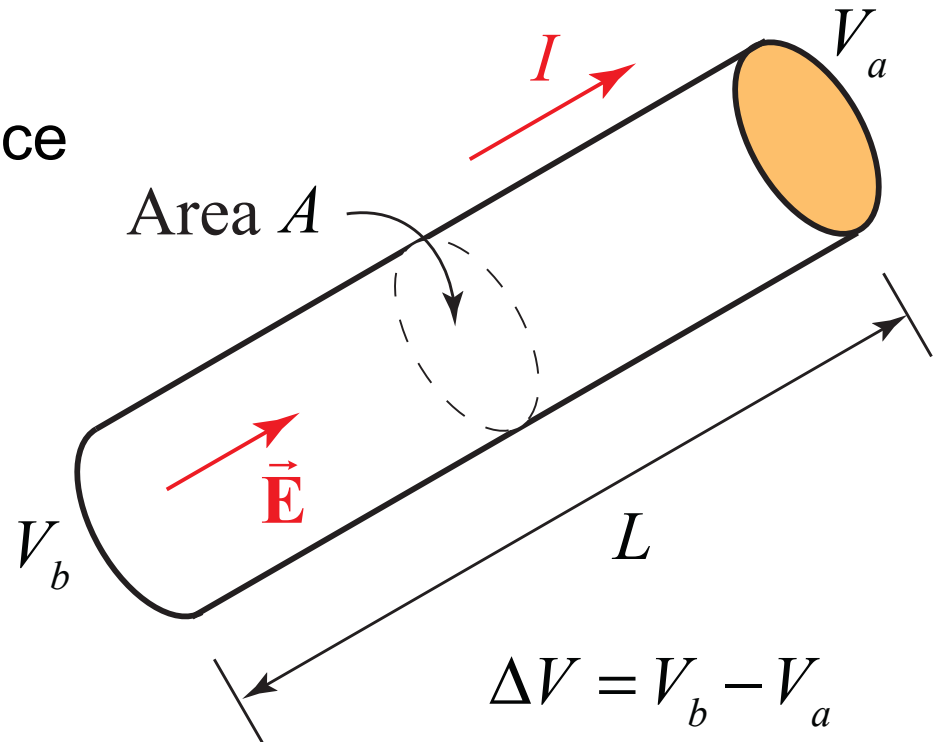
- To hurt you 40 (5) mA DC(AC)
- To throw you 60 (15) mA DC(AC)
- To kill you 0.5 (0.1) A DC(AC)

- Fuse/Circuit Breaker 15-30 A

Ohm's Law: Resistance

Instead of thinking of electric field, think of potential difference across the conductor

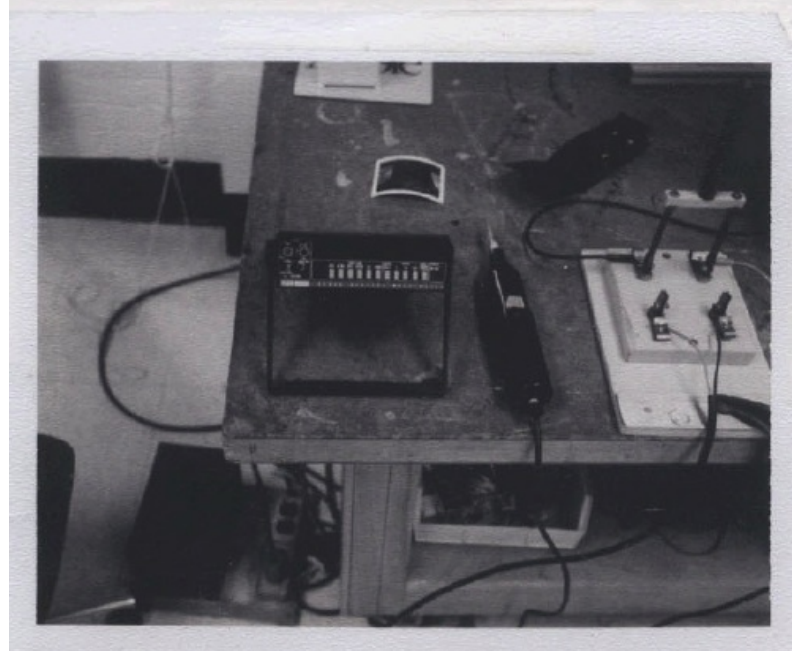
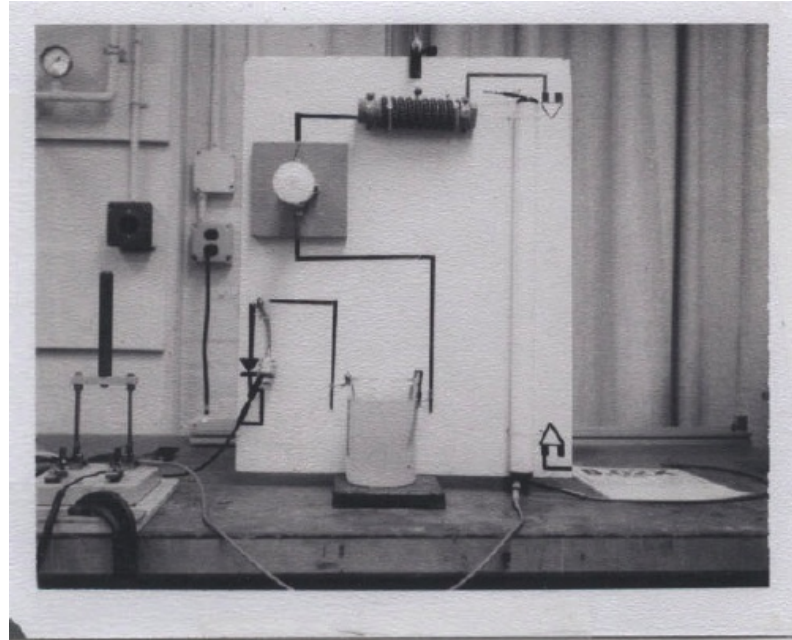
$$\Delta V = V_b - V_a = IR$$



For some materials, which are called ohmic, the electric potential difference is linearly proportional to the current, the constant of proportionality is called resistance and has SI units Ohms

$$[\text{Ohm}] = [\text{V} \cdot \text{A}^{-1}]$$

Demo: Five Different Types of Resistance F13



How Big is an Ohm?

- Short Copper Wire milliohms ($m\Omega$)
- Notebook paper (thru) $\sim 1\text{ G}\Omega$
- Typical resistors Ω to $100\text{ M}\Omega$

- You (when dry) $100\text{ k}\Omega$
- You (when wet) $1\text{ k}\Omega$
- Internally (hand to foot) 500Ω

Stick your wet fingers in an electrical socket:

$$I = V / R \sim 120\text{ V} / 1\text{k}\Omega \sim 0.1\text{ A} \quad \text{You're dead!}$$

Demo:

Temperature Effects on Resistance F4

<http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F%204&show=0>

Conducting Glass F1

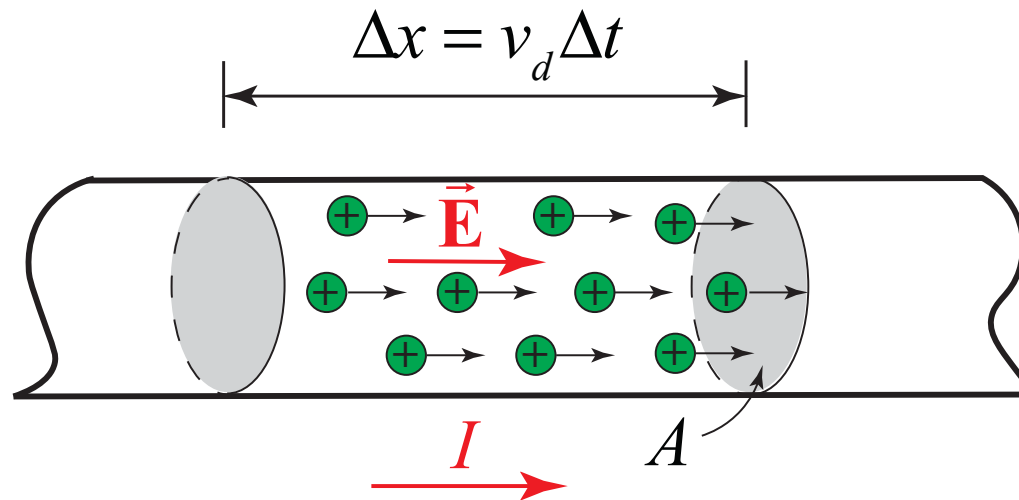
<http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F%201&show=0>

Conductivity of Ionizing Water F5

<http://tsgphysics.mit.edu/front/?page=demo.php&letnum=F%205&show=0>

Flow Tube for Charge

How much charge will pass through a cross section of a wire of area A in a time interval Δt ?



Assume each charge q is moving with drift speed v_d . Consider a cylinder of length $\Delta x = v_d \Delta t$ and cross sectional area A . In a time interval Δt , all the moving charge inside that cylinder, $\Delta Q = nqA\Delta x$ will pass through the cross sectional area, where n is the number of moving charges per volume. Therefore the current is

$$I = \Delta Q / \Delta t = nqA\Delta x / \Delta t = nqv_d A$$

Current Density

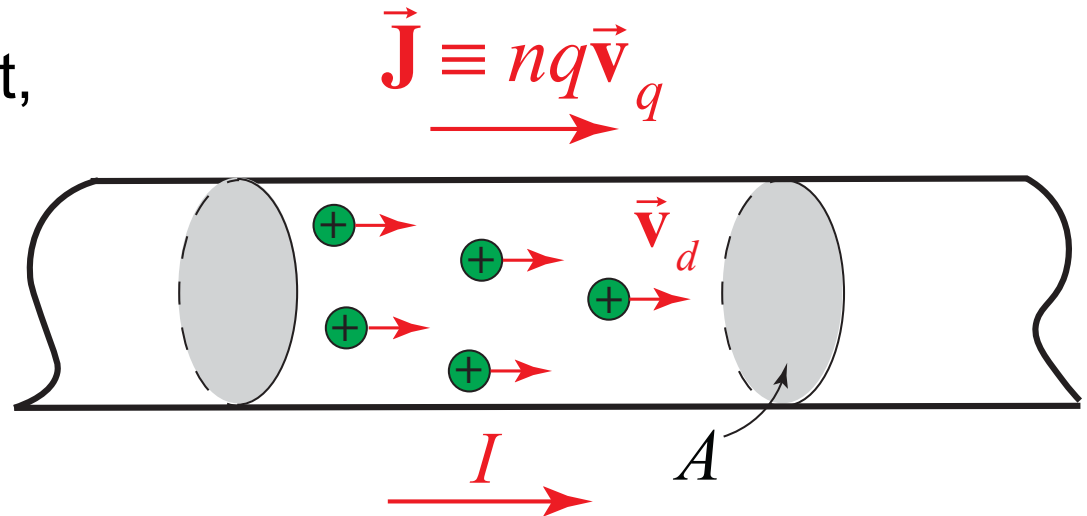
Let n = number of charged objects per unit volume,

q = charge of object,

$\vec{v}_q$ = drift velocity of object,

The current density,
(current per unit area) is

$$\vec{J} \equiv nq\vec{v}_q$$



Generalization for many charged moving objects

$$\vec{J} \equiv \sum_i n_i q_i \vec{v}_{q_i}$$

Current and Current Density

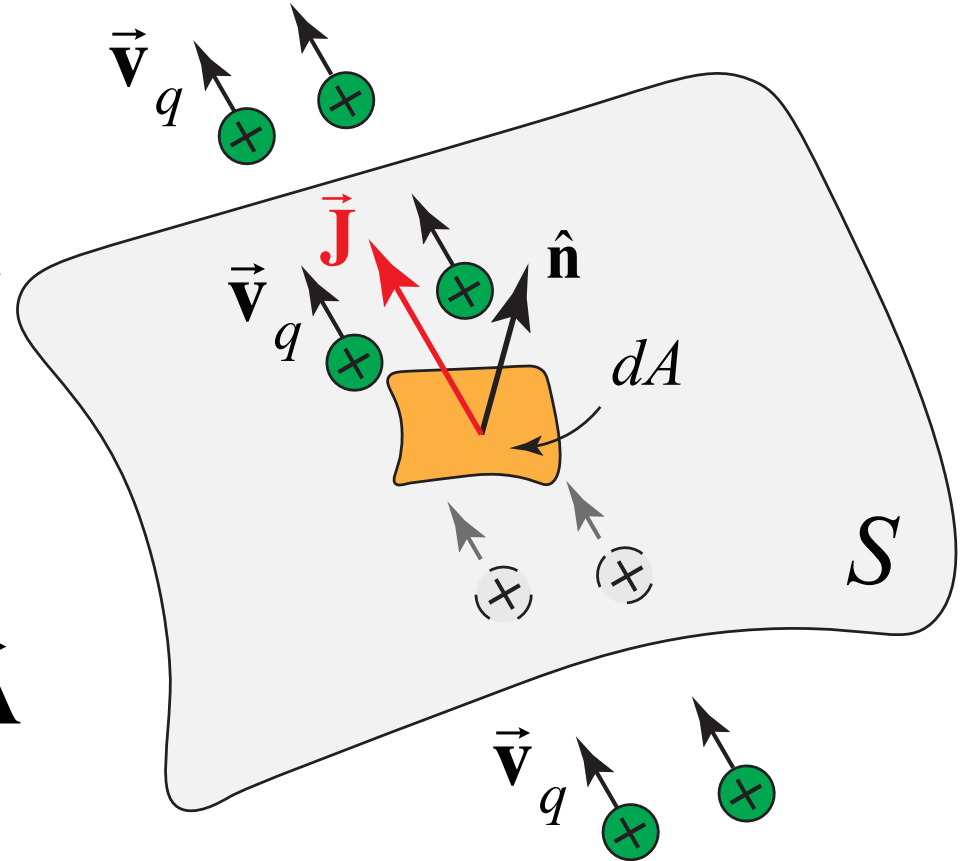
$$\vec{\mathbf{J}} \equiv nq\vec{\mathbf{v}}_q$$

Current is the flow (flux) of current density through an open surface

$$I = \int_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}} dA = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

Special case: $\vec{\mathbf{J}}$ uniform and perpendicular to surface

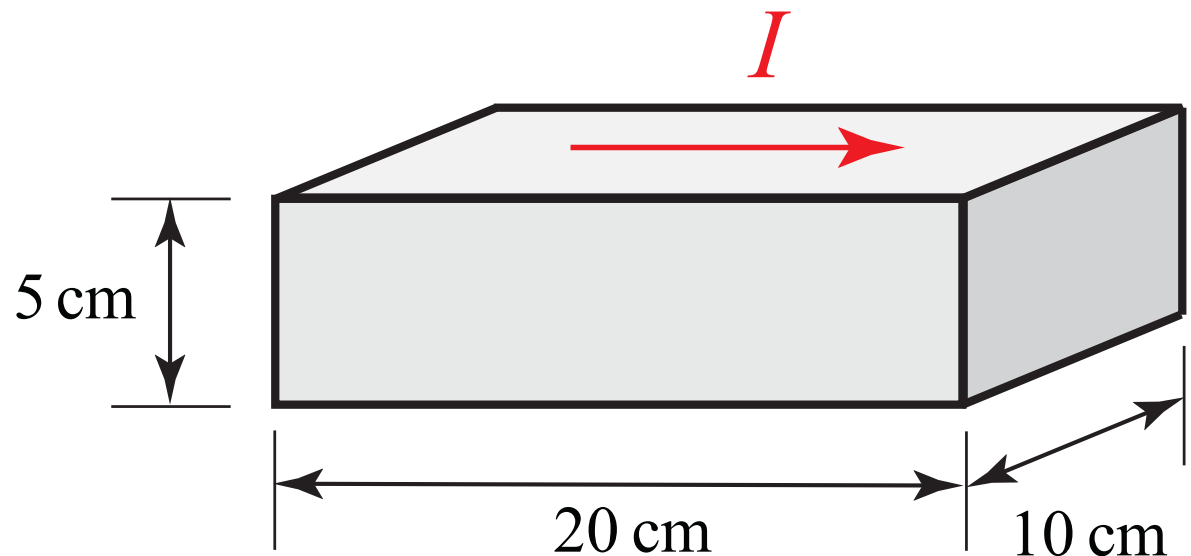
$$I = \pm JA$$



CQ: Current Density

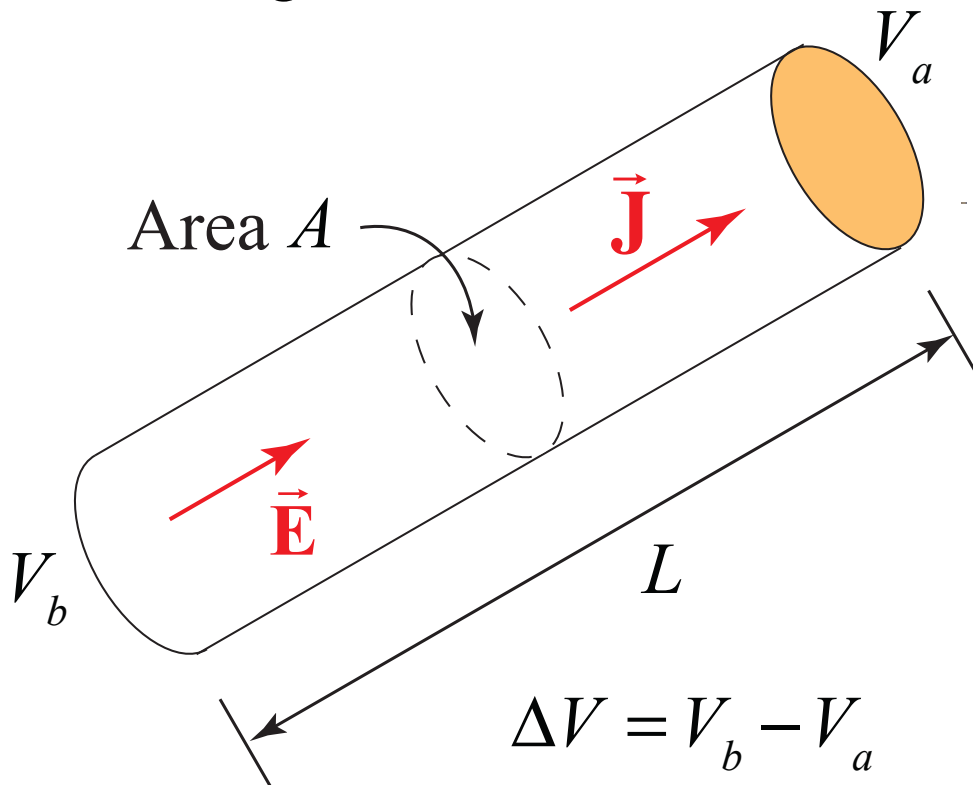
A current $I = 200$ mA flows in the conductor shown below. What is the magnitude of the current density J ?

1. $J = 40$ mA/cm
2. $J = 20$ mA/cm
3. $J = 10$ mA/cm
4. $J = 1$ mA/cm²
5. $J = 2$ mA/cm²
6. $J = 4$ mA/cm²



Microscopic Ohm's Law: Conductivity

Current density is proportional to the electric field.
Ability of current to flow depends on density of charges & rate of scattering.



$$\vec{J} = -ne\vec{v}_{drift} = \left(\frac{ne^2\tau}{m_e}\right)\vec{E}$$

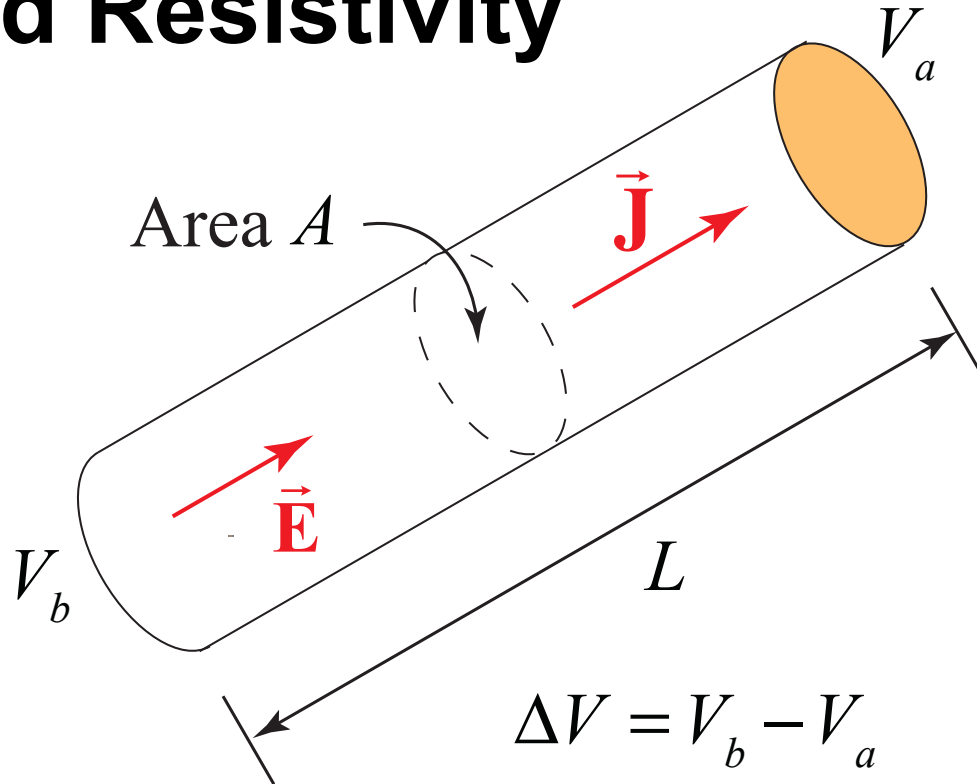
$$\vec{J} = \sigma_c \vec{E} \Rightarrow \sigma_c = \frac{ne^2\tau}{m_e}$$

σ_c = conductivity

Microscopic Ohm's Law: Conductivity and Resistivity

$$\vec{\mathbf{J}} = \sigma_c \vec{\mathbf{E}}$$
$$\vec{\mathbf{E}} = \rho_r \vec{\mathbf{J}}$$

$$\rho_r \equiv \frac{1}{\sigma_c}$$

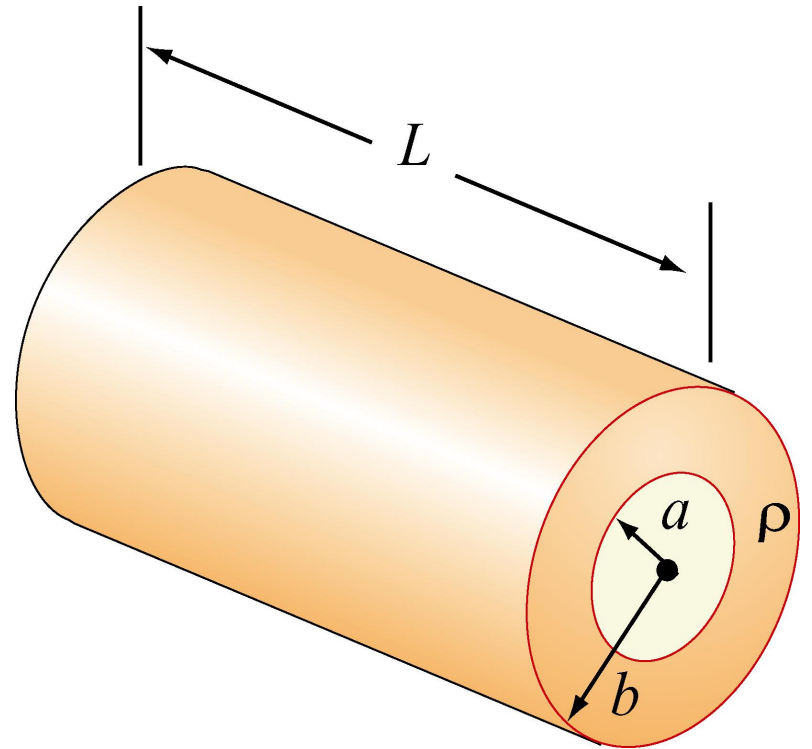


The **conductivity** σ_c only depends on the microscopic properties of the conductor, and not on its geometric shape. The inverse of the conductivity σ_c is defined to be the **resistivity** ρ_r .

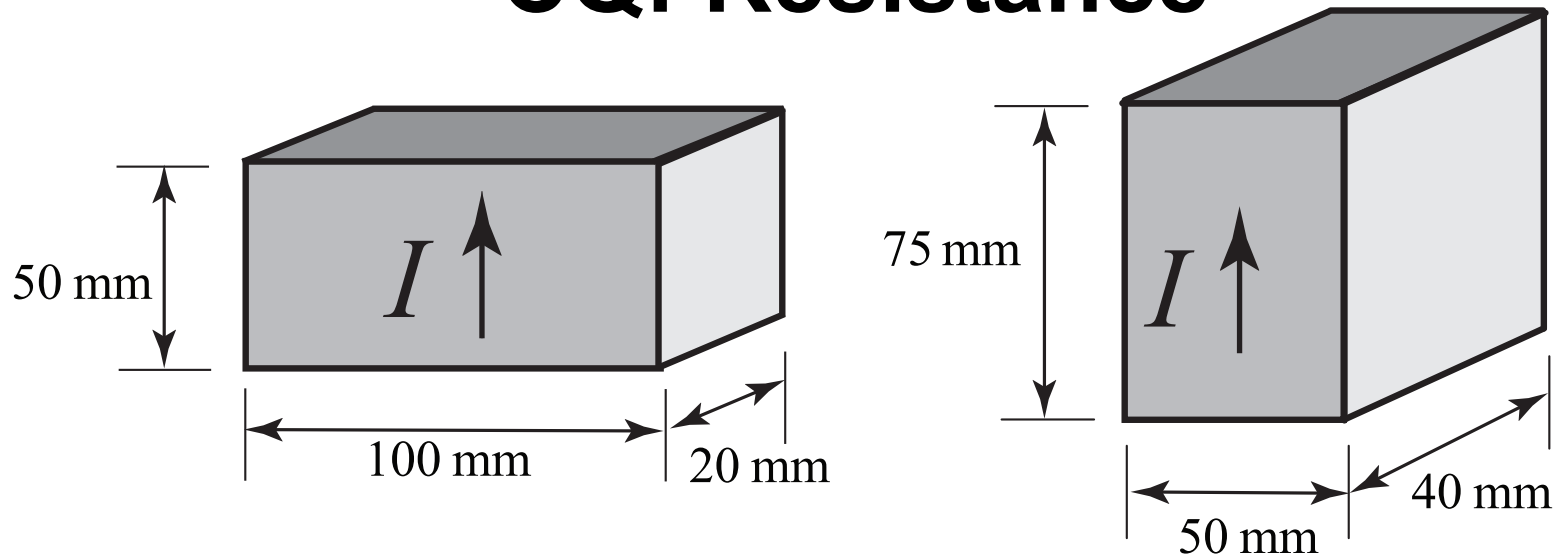
Group Problem: Calculating Resistance

Consider a hollow cylinder of length L and inner radius a and outer radius b . The material has resistivity ρ_r .

Suppose a potential difference is applied between the ends of the cylinder and produces a current flowing parallel to the axis. What is the resistance measured?



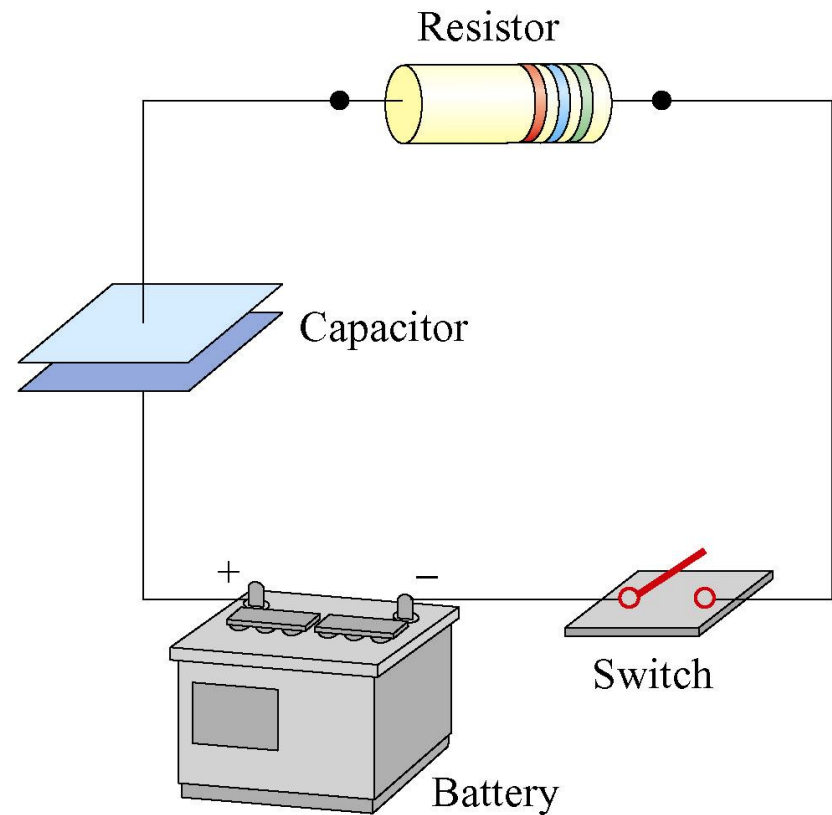
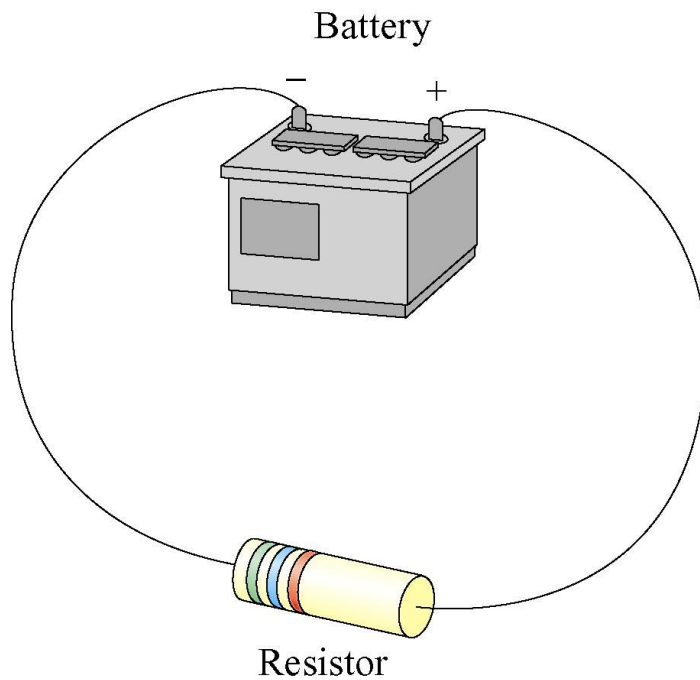
CQ: Resistance



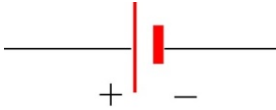
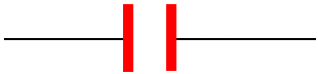
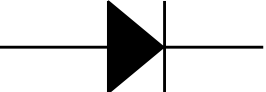
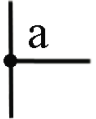
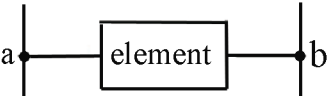
Two resistors are made out of the same material, but have different dimensions, as shown in the figures above. The current through these two resistors is in the directions shown. If the resistor **on the left** has a resistance of 1.00 ohm, the resistor **on the right** will have a resistance of

- | | | | |
|----|----------|----|----------|
| 1. | 1.50 ohm | 2. | 2.00 ohm |
| 3. | 0.50 ohm | 4. | 0.75 ohm |
| 5. | 1.00 ohm | 6. | 0.67 ohm |

Examples of DC Circuits



Symbols for Circuit Elements

Battery	ε	
Resistor	R	
Capacitor	C	
Inductor	L	
Diode	D	
Switch	S	
Equipotential		
Junction		
Branch		

Electromotive Force (EMF)

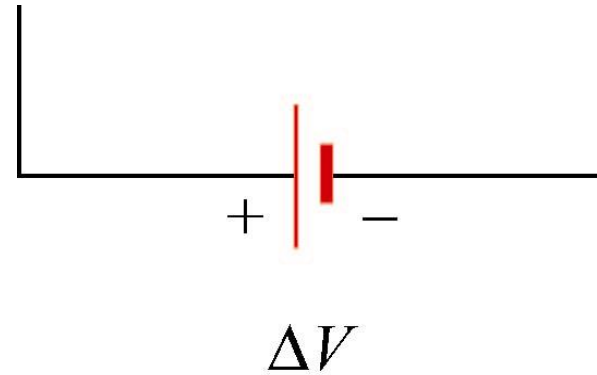
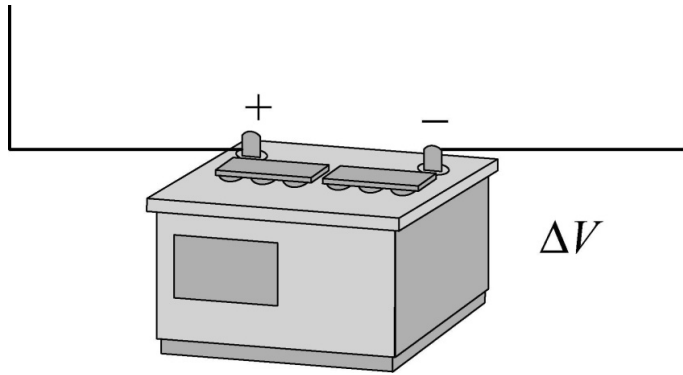
The work per unit charge around a closed path done is called electromotive force EMF. **This is a bad name because it is not a force.** Let $\vec{\mathbf{f}}_s$ denote the force per unit charge, then the EMF is

$$\mathcal{E} = \oint_{\text{closed path}} \vec{\mathbf{f}}_s \cdot d\vec{\mathbf{s}}$$

If a conducting closed path is present then

$$\mathcal{E} = IR$$

Ideal Battery: emf



- 1) Inside Battery: non-zero chemical force $\vec{f}_s$ inside battery (zero outside), moves charges through a region in which static electric field $\vec{E}_{static}$ opposes motion $\vec{E}_{static} = -\vec{f}_s$
- 2) Ideal battery: net force on charges is zero
- 3) Potential difference between its terminals

$$V(+)-V(-)=-\int_{-}^{+}\vec{E}_{static}\cdot d\vec{s}=\int_{-}^{+}\vec{f}_s\cdot d\vec{s}$$

- 4) Extend path through external circuit where $\vec{f}_s = \vec{0}$

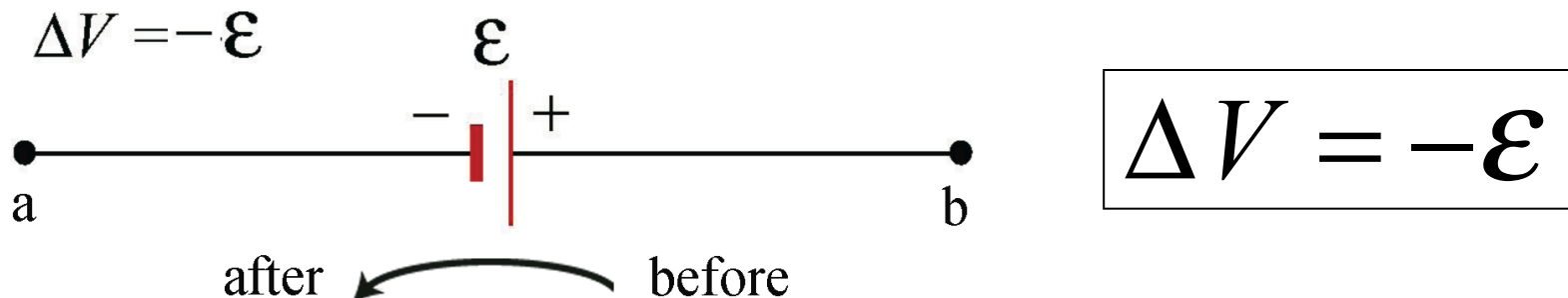
$$V(+)-V(-)=\int_{-}^{+}\vec{f}_s\cdot d\vec{s}=\oint\vec{f}_s\cdot d\vec{s}=\varepsilon$$

Sign Conventions - Battery

Circulating from the negative to positive terminal of a battery **increases** your potential



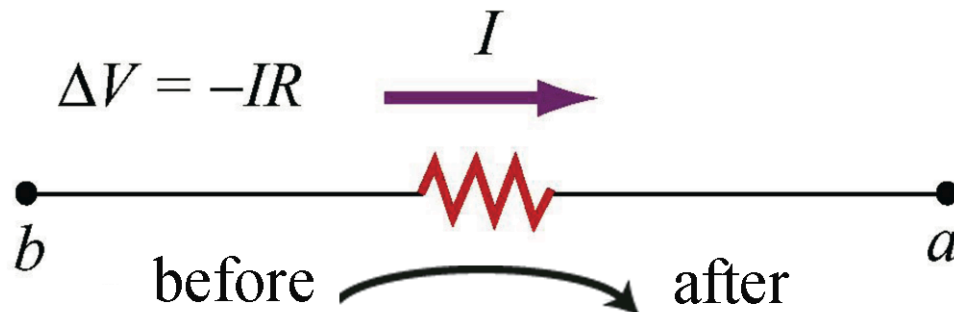
Circulating from the positive to negative terminal of a battery **decreases** your potential



Sign Conventions - Resistor

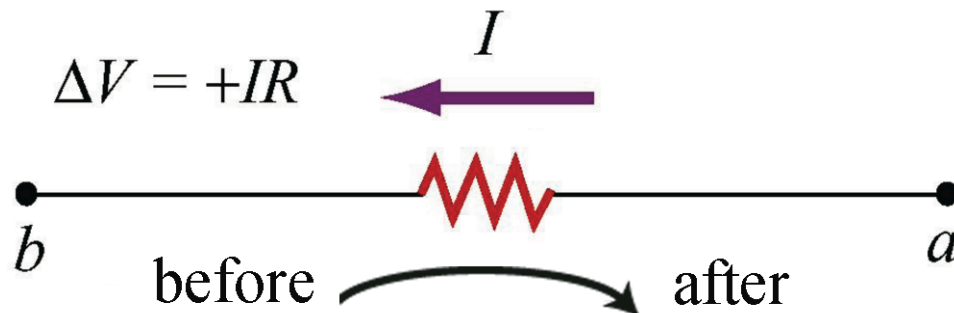
Direction of current is the same as direction of electric field which points to lower electric potential

Circulating across a resistor in the direction of current **decreases** the potential



$$\Delta V = -IR$$

Circulating across a resistor in the opposite direction of current **increases** your potential

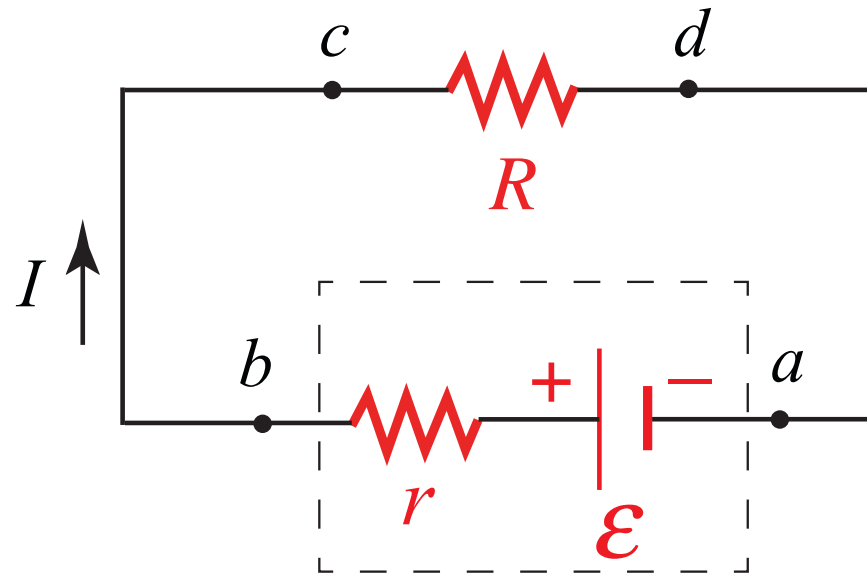


$$\Delta V = +IR$$

Internal Resistance

Real batteries have an internal resistance, r , which is small but non-zero

Circulate clockwise:



Terminal voltage:

$$\Delta V = V_b - V_a = \mathcal{E} - I r$$

(Even if you short the leads you don't get infinite current)

Reference System for Circuit Analysis

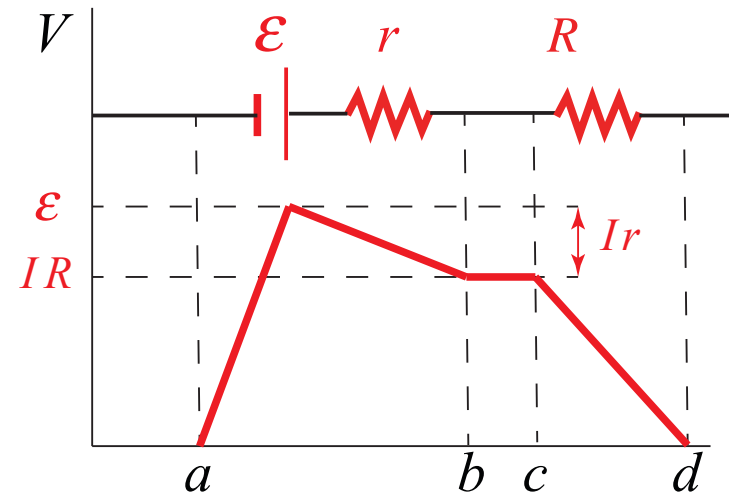
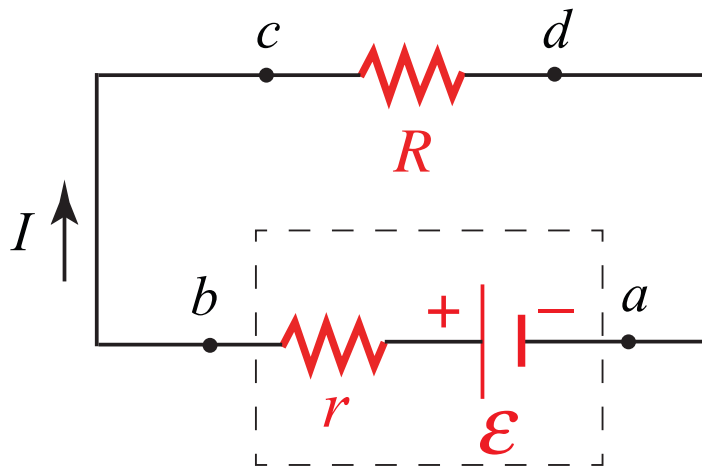
1. Simplify resistors in series/parallel.
2. Assign current and current direction in each branch
3. Assign circulation direction for each loop.
4. Circulation direction across each element defines “before” and “after” side of element.
5. Define electric potential difference across a circuit element by

$$\Delta V \equiv V(\text{after}) - V(\text{before}) \equiv V(a) - V(b)$$

Sum of Potential Differences Around a Closed Path

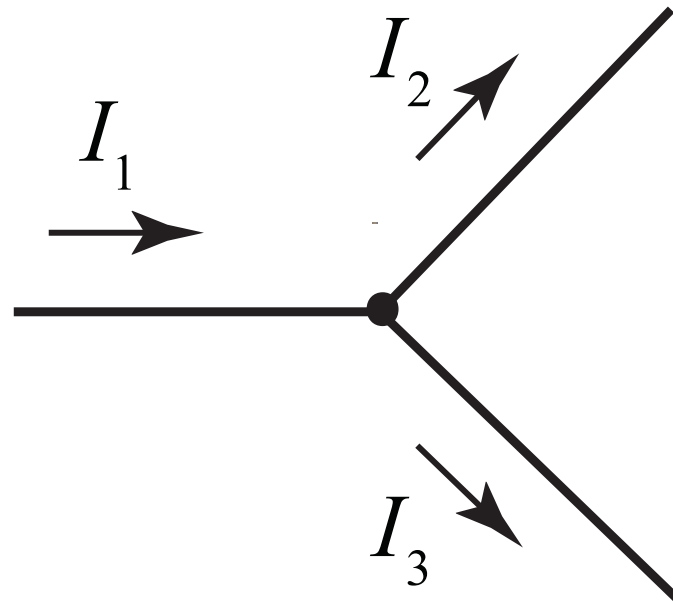
Sum of potential differences across all elements around any closed circuit loop must be zero.

$$\sum_i \Delta V_i = - \oint_{\text{Closed Path}} \vec{\mathbf{E}}_{\text{static}} \cdot d\vec{\mathbf{s}} = 0$$



Current Conservation

Sum of currents entering any junction in a circuit must equal sum of currents leaving that junction.

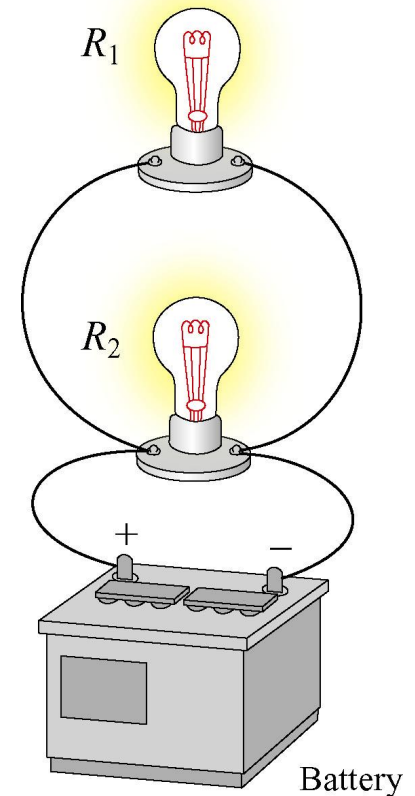


$$I_1 = I_2 + I_3$$

CQ: Bulbs & Batteries

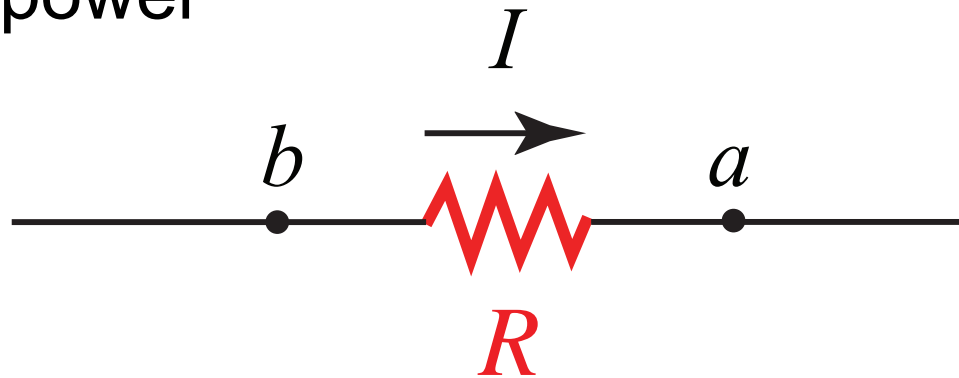
An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected

1. is higher.
2. is lower.
3. is the same.



Power – Resistor (Joule Heating)

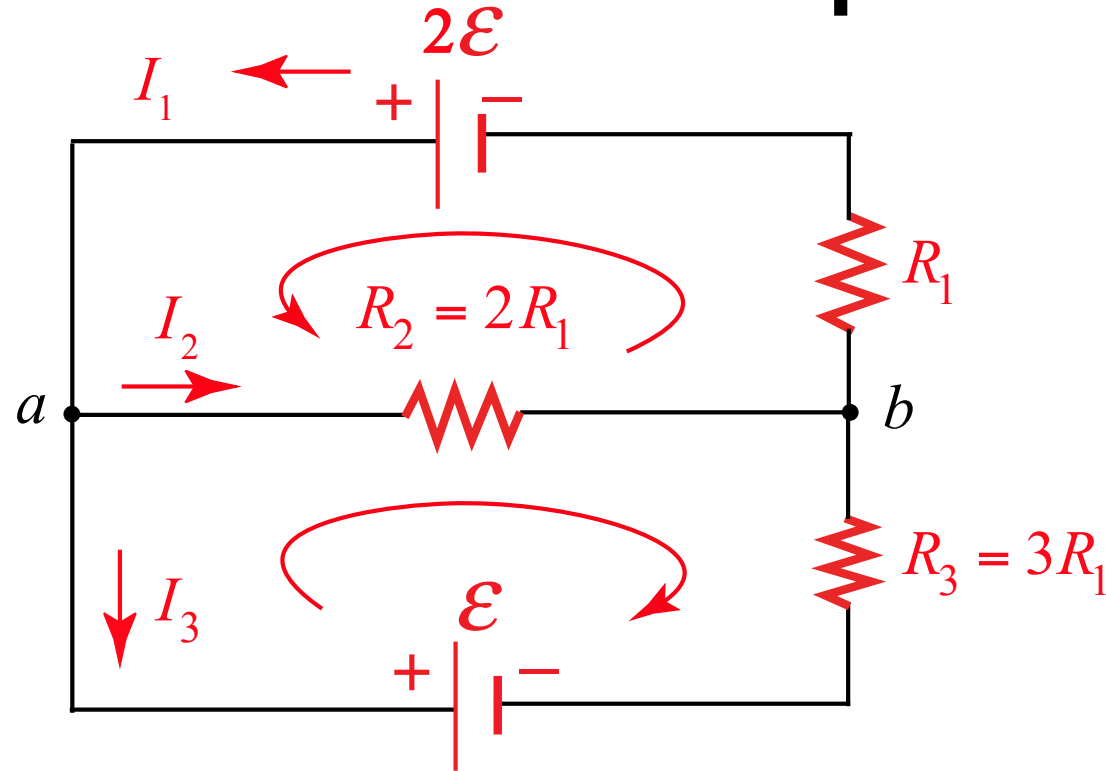
Moving across a resistor in the direction of current **decreases** your potential. Resistors **always** **dissipate** power



$$V(b) - V(a) = IR$$

$$P_{\text{dissipated}} = I|\Delta V| = I^2 R = \frac{\Delta V^2}{R}$$

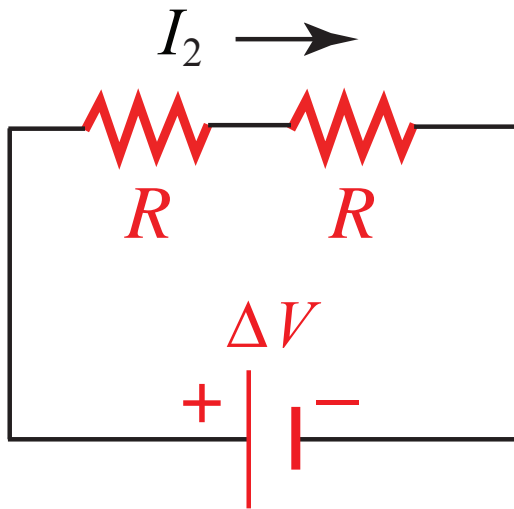
Group Problem: Two Loop Circuit



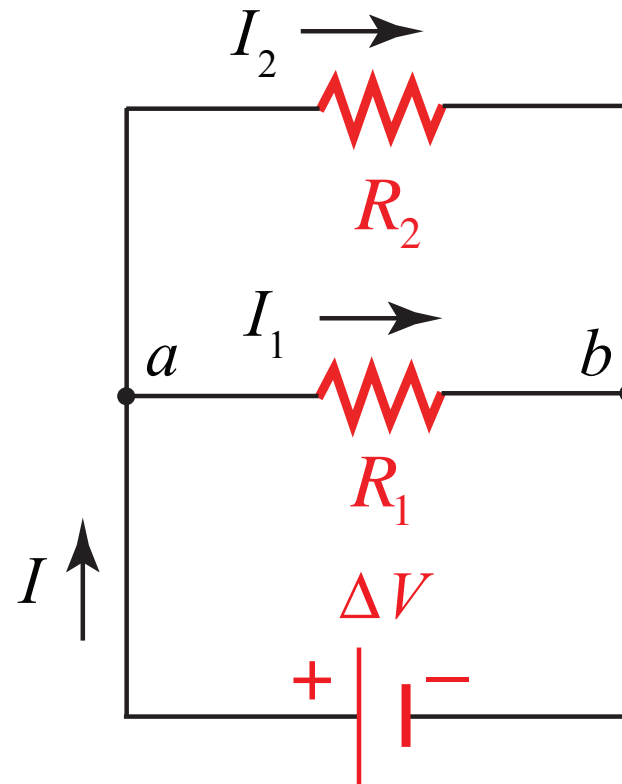
- Use the loop rule and current conservation to find three independent equations for the currents I_1 , I_2 and I_3 in each branch
- If you have time solve these equations for the three currents.

Appendix

Series vs. Parallel



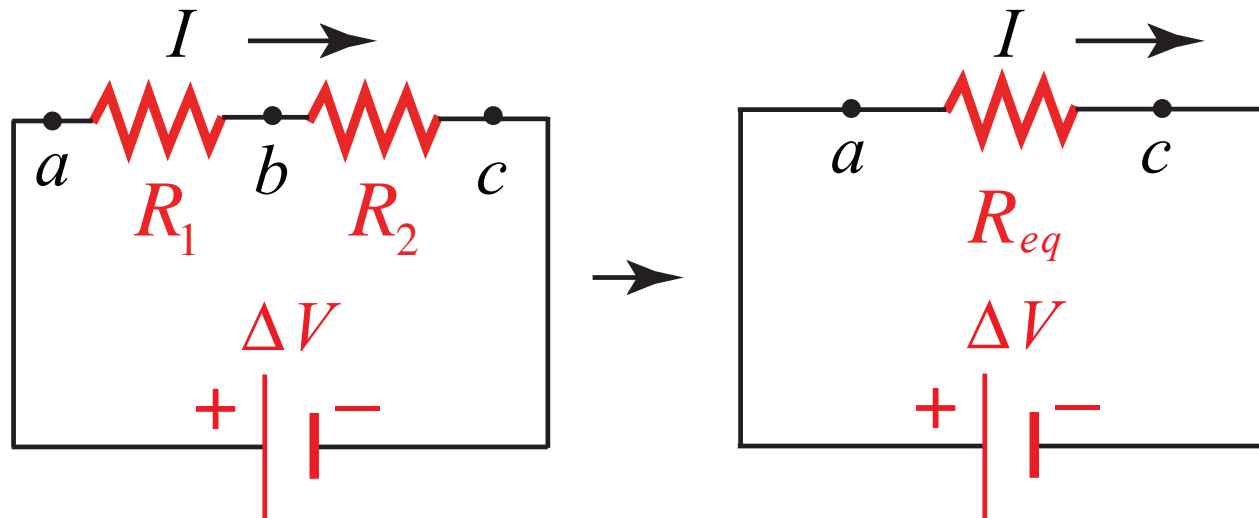
(a) resistors connected in series



(b) resistors connected in parallel

Resistors In Series

The same current I must flow through both resistors

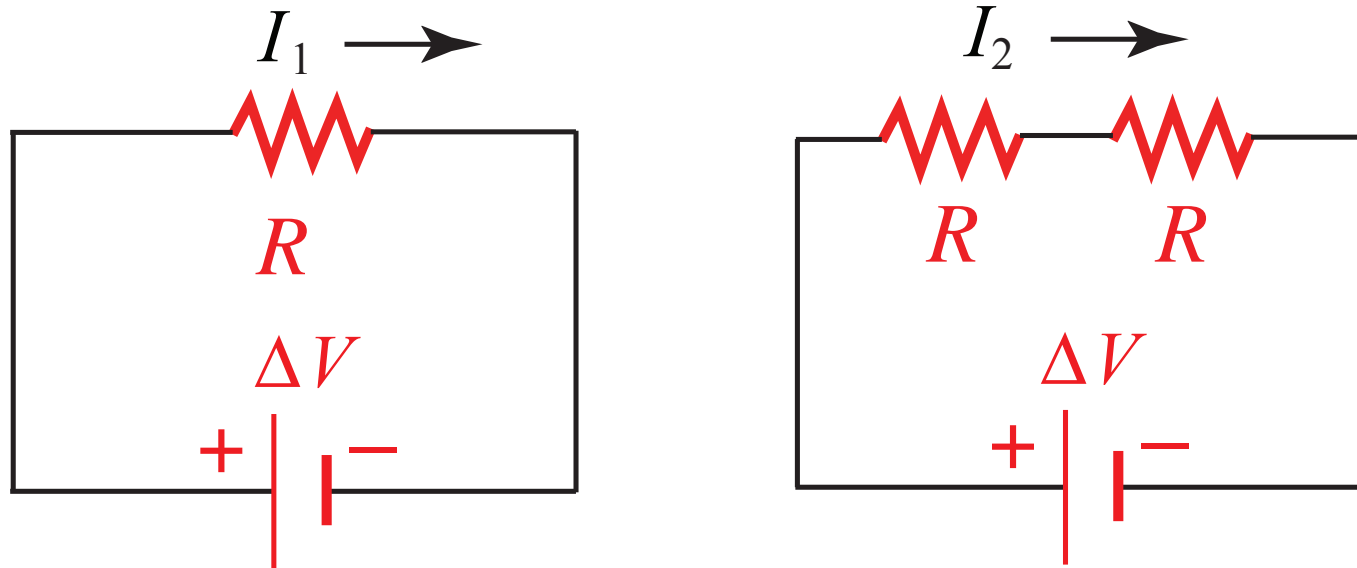


Circulate Clockwise:

$$\Delta V - I R_1 - I R_2 = 0 \Rightarrow \Delta V = I(R_1 + R_2) = I R_{eq}$$

$$R_{eq} = R_1 + R_2$$

CQ: Resistors in Series

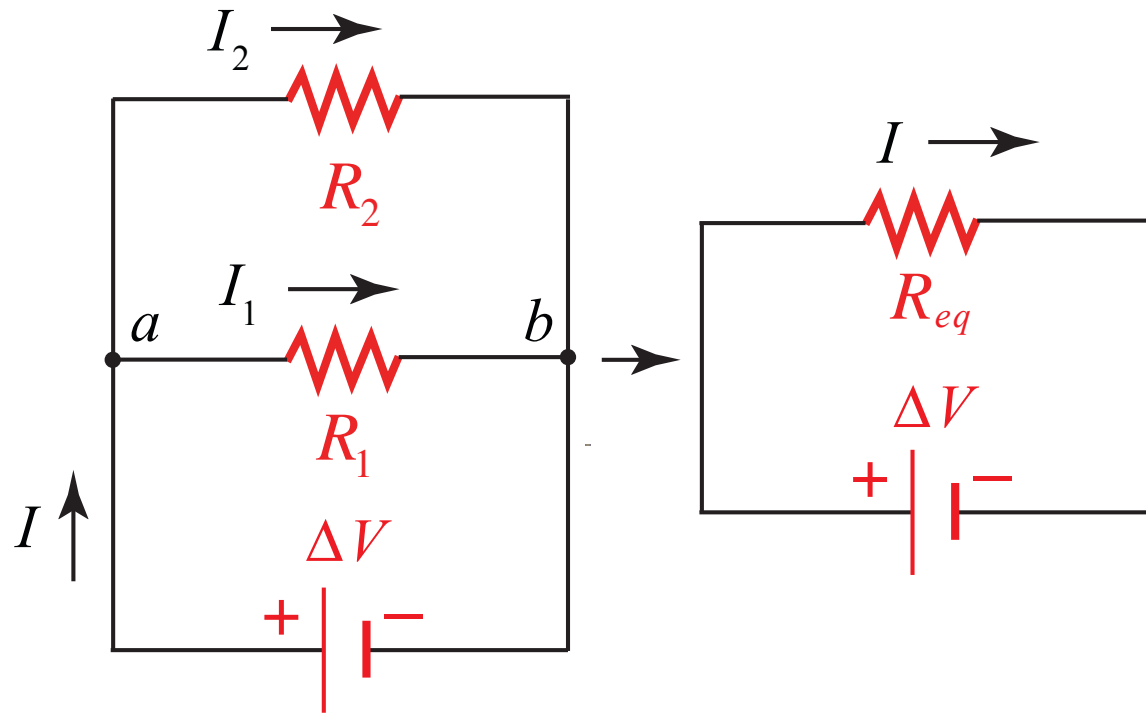


A circuit consists of a battery connected to a resistor with resistance R and carries a current I_1 . A second circuit consists of an identical battery connected in series to two resistors each with resistance R and carries a current I_2 . Which of the following statements is true?

1. $I_1 > I_2$.
2. $I_1 < I_2$.
3. $I_1 = I_2$.

Resistors In Parallel

Voltage drop across the resistors must be the same



$$\Delta V = \Delta V_1 = \Delta V_2 = I_1 R_1 = I_2 R_2 = I R_{eq}$$

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

CQ: Resistors In Parallel

Suppose that $R_1 \gg R_2$.
Then the equivalent
resistance

1. $R_{eq} \simeq R_2$

2. $R_{eq} \simeq R_1$

3. $R_{eq} \simeq R_2 / 2$

4. $R_{eq} \simeq R_1 / 2$

