

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics: 8.02

Problem Solving 10: Faraday's Law Solution

Objectives

1. To explore a particular situation that can lead to a changing magnetic flux through the open surface bounded by an electric circuit.
2. To calculate the rate of change of magnetic flux through the open surface bounded by that circuit in this situation.
3. To determine the sense of the induced current in the circuit from Lenz's Law.
4. To look at the forces on the current carrying wires in our circuit and determine the effects of these forces on the dynamics of the circuit.

Reference: Sections 10.1 – 10.4, 8.02 Course Notes.

Problem-Solving Strategy for Faraday's Law

In Chapter 10 of the 8.02 Course Notes, we have seen that a changing magnetic flux induces an emf:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

according to Faraday's law of induction. For a conductor that forms a closed loop, the emf sets up an induced current $I = |\mathcal{E}|/R$, where R is the resistance of the loop. To compute the induced current and its direction, we follow the procedure below:

- (1) For the closed loop of area A , define an area vector $\vec{A}$ and choose a positive direction. For convenience of applying the right-hand rule, let $\vec{A}$ point in the direction of your thumb. Compute the magnetic flux through the loop using

$$\Phi_B = \begin{cases} \vec{B} \cdot \vec{A} & (\vec{B} \text{ is uniform}) \\ \iint \vec{B} \cdot d\vec{A} & (\vec{B} \text{ is non-uniform}) \end{cases}$$

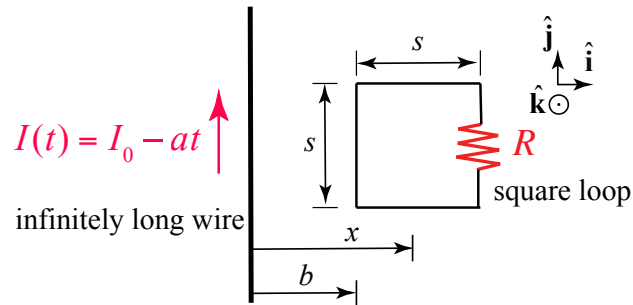
- (2) Evaluate the rate of change of magnetic flux $d\Phi_B/dt$. Keep in mind that the change could be caused by

- (i) changing the magnetic field $dB/dt \neq 0$,
- (ii) changing the loop area if the conductor is moving ($dA/dt \neq 0$), or
- (iii) changing the orientation of the loop with respect to the magnetic field ($d\theta/dt \neq 0$).

Determine the sign of $d\Phi_B/dt$ (is the flux increasing or decreasing)

(3) The sign of the induced emf is opposite the sign of $d\Phi_B/dt$. The direction of the induced current can be found by using the Lenz's law discussed in Section 10.4.

Problem 1: IC_W10D1_12 Moving Loop in a Non-uniform Magnetic Field Solution



A square loop with side of length s is at rest. A long straight wire carries a current that is decreasing in time according to $I(t) = I_0 - at$. The total resistance of the loop is R .

- (a) Find a vector expression for the magnetic field of the wire as a function of the distance x away from the wire (see figure). You may assume that the wire is infinitely long.
- (b) What is the magnetic flux through the rectangular loop at the instant when the left side of the square loop is a distance b away from the wire, as shown in the figure? You will need to calculate the flux integral. What should you use as your area element?
- (c) Find the induced emf and the corresponding induced current in the rectangular loop. Which direction does the induced current flow?