

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Problem Solving Session 1: Electric Dipoles and Torque

Section _____

Table (if applicable) _____

Group Members _____

Introduction: In the first problem you will learn to apply Coulomb's Law to find the electric field of an electric dipole. You will investigate the properties of the electric field associated with an electric dipole. You will also explore what happens to an electric dipole when it is placed in an uniform electric field.

Readings: Course Notes 8.02: Chapter 2 Coulomb's Law Section 2.7-2.8

Introduction Electric Dipole

An electric dipole consists of two equal but oppositely charged point-like objects, $+q$ and $-q$, separated by a distance $2a$, as shown in Figure 1.

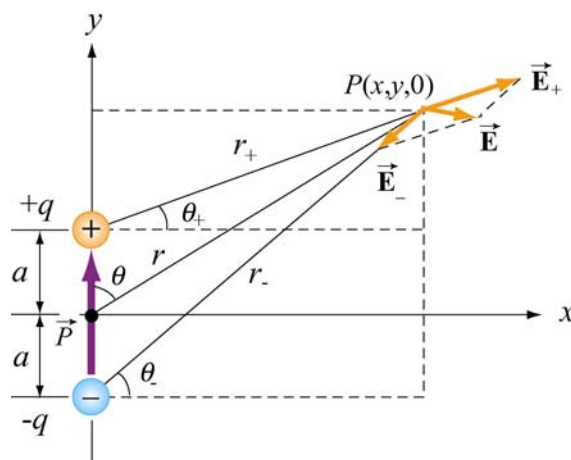


Figure 1 Electric dipole

The dipole moment vector $\vec{\mathbf{p}}$ which points from $-q$ to $+q$ (in the $+y$ -direction) is given by

$$\vec{\mathbf{p}} = 2qa \hat{\mathbf{j}} \quad (1)$$

The magnitude of the electric dipole is $p = 2qa$, where $q > 0$. For an overall charge-neutral system having N charged objects, the electric dipole vector $\vec{\mathbf{p}}$ is defined as

$$\vec{\mathbf{p}} \equiv \sum_{i=1}^{i=N} q_i \vec{\mathbf{r}}_i \quad (2)$$

where $\vec{\mathbf{r}}_i$ is the position vector of the charged object with charge q_i .

Problem 1 Electric field of a Dipole

Question 1: Consider the electric dipole moment shown in Figure 1. Find the x- and y-components of the electric field at a point with coordinates $(x, y, 0)$, using

$$\vec{\mathbf{E}}_+ = k_e \frac{q_+}{r_+^3} \vec{\mathbf{r}}_+ \quad \text{and} \quad \vec{\mathbf{E}}_- = k_e \frac{q_-}{r_-^3} \vec{\mathbf{r}}_- .$$

We can show that the electric field of the dipole in the limit where $r \gg a$ is

$$E_x = \frac{k_e 3p}{r^3} \sin \theta \cos \theta, \quad E_y = \frac{k_e p}{r^3} (3 \cos^2 \theta - 1) \quad (3)$$

where $\sin \theta = x / r$ and $\cos \theta = y / r$. **See Problem Set 1 Problem 4.**

Question 2: By what power of distance does the strength of electric field fall off? How does this compare to a single point charge? Briefly explain a reason for the difference between these two cases.

Question 3 Electric Dipole Animation

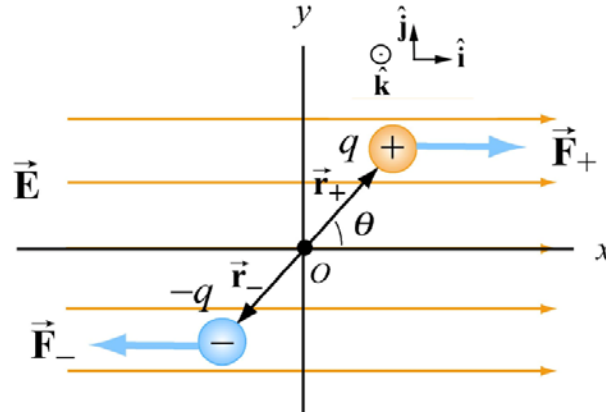
Open up the applet, Two Point Charges,

<http://web.mit.edu/viz/EM/visualizations/electrostatics/ElectricFieldConfigurations/pcharges/pcharges.htm>

The units for charge are μC . Create an electric dipole with charges $q_1 = 3.0 \mu\text{C}$ and $q_2 = -3.0 \mu\text{C}$. Click on the Electric Fields: Grass Seeds bar to see a representation of the electric field. Note that this applet can show the three different representations we use for vector fields: (1) vector field grid of arrows, (2) field lines, and (3) grass seeds.

Problem 2 Electric Dipole in a Uniform Electric Field

Place an electric dipole in a uniform field $\vec{E} = E \hat{i}$, with the dipole moment vector $\vec{p}$ making an angle θ with the x-axis.



Question 1: What is a vector expression for the dipole moment $\vec{p}$? Give your answer in the form $\vec{p} = p_x \hat{i} + p_y \hat{j}$ where you determine the components (p_x, p_y) .

Question 2: Find a vector expression for the torque on the dipole. What point did you compute the torque about? Is your answer independent of that choice of point?

Question 3: What type of motion does the dipole undergo if it is released from its position and is free to move?

Question 4: Show that $\vec{\tau} = \vec{p} \times \vec{E}$.

Question 5: Torque on an Electric Dipole Animation

Open up the applet, Torque on an Electric Dipole

<http://web.mit.edu/viz/EM/visualizations/electrostatics/ForcesTorquesOnDipoles/torqueondipolee/torqueondipolee.htm>

The units for the dipole moment are $10^{-6} \text{ m} \cdot \text{C}$ and the units for electric field strength are $10^{-3} \text{ N} \cdot \text{C}^{-1}$. Set the dipole moment $p = 0.05 \times 10^{-6} \text{ m} \cdot \text{C}$, the electric field strength $E = 0.1 \times 10^{-3} \text{ N} \cdot \text{C}^{-1}$, and the damping to zero. What approximately is the period of oscillation? Click on the Electric Fields: Grass Seeds bar to see a representation of the electric field at various moments in the oscillation. At what point in the cycle are the field lines most contorted? At what point are the field lines nearly straight (except close to the dipole).

Summary

- Two equal but opposite point-like charged objects form an electric field that far from the charged objects is an electric dipole. The electric dipole moment vector $\vec{p}$ points from the negative point-like charged object to the positive point-like charged object, and has a magnitude

$$p = 2aq$$

- The **torque** acting on an electric dipole $\vec{p}$ placed in a uniform electric field $\vec{E}$ is

$$\vec{\tau} = \vec{p} \times \vec{E}$$