For 8.02T - *Babinet's Principle* for a Single Slit

The explanation for diffraction around an object, as given in the PowerPoint slide #13 on Monday May 10 and Tuesday May 11 is not convincing; nor is it consistent with other presentations. What I'd like to do here is attempt a reconciliation of different expositions of *Babinet's Principle* as applied to the situation encountered in Experiment 13.

The sources I have at hand are:

- *Electromagnetic Fields and Waves*, Bekefi & Barrett, MIT Press 1987, Pages 559-577. Referred to in these notes as "B&B." I spoke to at least one student who will be using B&B in 8.03 next term.
- *Optics*, Eugene Hecht, Addison-Wesley 1990, Pages 457-459. A full understanding of this presentation requires understanding of the entire chapter, including Fresnel integrals.
- *Classical Electrodynamics*, J. D. Jackson, John Wiley & Sons 1975, Pages 432-441. This is an older edition and the page references might be off. Jackson is not at all illuminating for freshmen, but the notation is rigorous and useful.

In different ways, with different degrees of rigor, all three of these sources state *Babinet's Principle* in a way that may be paraphrased as:

> "If the light from a source is diffracted through an aperture and then diffracted by an obstruction which is complementary to that aperture, the sum of the respective electric fields observed on the forward side of the aperture will be that which would be observed in the absence of aperture or obstruction."

This makes much more intuitive sense than PP slide#13; the sum of the two observed fields (of course, the illumination due to these fields) is that observed if there is *no aperture or obstruction* as opposed to *completely obstructed*.

To use a simplified version of Jackson's notation, if S_a is a set of apertures and obstructions and S_b is the complement of S_b in that their union is a completely obstructing surface S, and if $\mathbf{E}_0$ is the electric field on one side of the surface S, then

$$\mathbf{E}_0 = \mathbf{E}_a + \mathbf{E}_b.$$

Okay, that's Jackson for you. His form of the Kirchoff integrals is the result of a boundary-value problem, hence an integral over more or less arbitrary surfaces. The integral and result will not be reproduced here.

From this formulation, however, we can see why the explanation on PP slide#13 cannot be correct. In the limit as S_a is *no obstruction* (the light passes through unobstructed by any aperture or object), the complementary set S_b is a *complete obstruction* (no light passes through at all), $\mathbf{E}_a = \mathbf{E}_0$, whereas PP slide#13 would imply that $\mathbf{E}_a = \mathbf{0}$.

Next, B&B introduce *Babinet's Principle* in the context of a uniform incident plane wave on a circular aperture in order to find the pattern due to an obstructing disk. For that case, it is shown by calculation that

$$E_{\text{hole}} = E_{\text{plane wave}} + E_{\text{another term}},$$

leading to a fairly simple form for E_{disk} if *illuminated by an ideal uniform plane wave.*

Fine, but not at all useful for our lab, in which the laser source is demonstrably finite in extent perpendicular to the direction of propagation. More applicable is Hecht's exposition, essentially the scalar version of Jackson's, further simplified here as

$$E_0 = E_1 + E_2. \qquad \text{Hecht's Eq. } \textit{(10.110)}$$

Then follows a somewhat cryptic statement "The principle implies that when $E_0 = 0$, $E_1 = -E_2 \ldots$" and this is indeed consistent with what is presented on PP slide#13, "$E_{\text{filling}} = -E_{\text{slit}}$."

Later in the same paragraph, Hecht explains what is meant, in that it is assumed that the incident unobstructed source is concentrated on the screen, and that $E_0 = 0$ not identically, but for most of the screen, where any diffraction pattern is observed. Hecht also qualifies the principle to apply only to the Fraunhofer region.

How this applies to Experiment 13:

In our experiment, we can model the laser beam, after passing through the large hole, as creating a pattern on the screen (a card in our case) that is concentrated in a small circle, with the rest of the screen being dark ($E_0 = 0$ for most of the screen). When the slide with the single slit is in place, the part of the slit in the path of the beam will be uniformly illuminated across its width.

When the hair is in place, ideally the hair will go through the middle of the beam, and the complement of this obstruction would indeed be a single slit with the width of the hair. Schematically,

$$E_0(\text{concentrated, localized bright spot}) = E_1(\text{single slit}) + E_2(\text{hair obstacle}),$$

$$E_2(\text{hair obstacle}) = E_0(\text{concentrated, localized bright spot}) - E_1(\text{single slit}).$$

In the limit that the intensity of the spot is much greater than that of any part of the single slit pattern,

$$I_2(\text{hair obstacle}) = I_0(\text{concentrated, localized bright spot}) + I_1(\text{single slit}).$$

Why this matters:

What we observe is indeed a bright spot superimposed on a diffraction pattern with a central maximum which is much narrower than that found from the single-slit slide. This makes measurement of the width of the central maximum difficult, and we need to emphasize that precise results are not important for this part of the experiment.