Section 1

MATHEMATICS BIBLIOGRAPHY; SI UNITS

CONTENTS

The third edition of the AIP Handbook, like the second, presents a bibliography of mathematical references in lieu of an assortment of actual mathematical tables. Selection of such tables necessarily would have been arbitrary; they would have been bound to duplicate many tables already easily available to most physicists; and, most important, including them would have necessitated the omission of significant physics material. The basic pattern of the third-edition bibliography is described at the beginning of Sec. 1a. For reasons outlined in the first paragraph of Sec. 1b, it was believed neither practicable nor desirable to attempt exclusive use of the International System of Units in this edition of the Handbook. Section 1b outlines the background of SI Units, and presents a portion of a National Bureau of Standards bulletin on their interpretation.

1a. Mathematics Bibliography

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Library of Congress

In view of the appearance of large compendiums and the increasing use of computers with built-in functions or function subroutines in their compilers, many of the tables of elementary functions have been omitted from this bibliography. An effort has been made to include a dictionary; indexes of mathematical and statistical tables; compendiums of general tables, series, integrals, transforms, and differential equations; and references to numerical methods, new tables, and new disciplines.

For algorithms covering a wide variety of subjects such as the evaluation of systems of linear equations, estimations of definite integrals, sorting of data, etc., reference should be made to the "Collected Algorithms from CACM" (Communications of the Association for Computing Machinery, Inc.).

1. Abramowitz, Milton, and Irene A. Stegun, eds.: "Handbook of Mathematical Functions, with Formulas, Graphs, and Mathematical Tables," Dover Publications, Inc., New York, 1965, 1046 pages (Republication of National Bureau of Standards, Applied mathematics series, 55. Government Printing Office, Washington, D.C., 1964):
   A compendium containing most of the tables that have previously appeared in the United States, including the National Bureau of Standards Mathematical Tables, Applied Mathematics, and Columbia University Press Series. Contains mathematical properties, interrelations, and numerical methods, as well as an updated bibliography of textbooks and tables.
2. Arfken, George Brown: "Mathematical Methods for Physicists," Academic Press, Inc., New York, 1968, 704 pages:
   Includes bibliographies.
3. Bierens de Haan, David: "Nouvelles Tables d'Intégrales Définies" (New tables of definite integrals). Corrected 1867 edition, with an English translation of the introduction by J. F. Ritt. Hafner Publishing Company, Inc., New York, 1965, 716 pages:
   A special collection of some 8,400 integrals.
4. British Association for the Advancement of Science: "Mathematical Tables." Prepared under the auspices of the Royal Society, Cambridge University Press, London, 1931–1958:
   Vol. 1: "Circular and Hyperbolic Functions," 3d ed, 1951.
   Vol. 2: "Emden Functions," 1932. New edition in preparation.
   Vol. 3: "Minimum Decompositions into Fifth Powers," 1933.
   Vol. 4: "Cycles of Reduced Ideals in Quadratic Fields," 1934.
   Vol. 5: "Factor Table." 1935.
   Vol. 6: "Bessel Functions," pt. 1, 1958.
   Vol. 7: "The Probability Integral," 1939.
   Vol. 8: "Number-divisor Tables," 1940.
   Vol. 9: "Table of Powers Giving Integral Powers of Integers," 1950.
   Vol. 10: "Bessel Functions," pt. 2, 1952.
   "Auxiliary Tables," nos. 1–2, 1946.
   Continued by the Royal Society mathematical tables.

5. Burington, Richard S.: "Handbook of Mathematical Tables and Formulas," 4th ed., McGraw-Hill Book Company, New York, 1965, 448 pages:
A companion to the "Handbook of Probability and Statistics with Tables," by Richard S. Burington and Donald C. May, the 4th edition includes new sections on sets, relations, and functions; algebraic structures; Boolean algebra; number systems; matrices; and statistics. A table of derivatives and a comprehensive table of integrals have been included.
6. Burington, Richard S., and Donald C. May: "Handbook of Probability and Statistics with Tables," 2d ed., McGraw-Hill Book Company, New York, 1970, 450 pages.
7. Byerly, William E.: "An Elementary Treatise on Fourier's Series, and Spherical, Cylindrical, and Ellipsoidal Harmonics, with Applications to Problems in Mathematical Physics," Dover Publications, Inc., New York, 1959, 287 pages.
8. Byrd, Paul F., and Morris D. Friedman: "Handbook of Elliptic Integrals for Engineers and Physicists," (Die Grundlehren der mathematischen Wissenschaften, Band 67), Springer Verlag, Berlin, 1954, 355 pages:
A collection of over 3,000 integrals and formulas using Legendre's and Jacobi's notations.
9. Campbell, George A., and Ronald M. Foster: "Fourier Integrals for Practical Applications," D. Van Nostrand Company, Inc., Princeton, N.J., 1948, 177 pages: A large number of the known closed-form evaluations of Fourier integrals are compiled and tabulated in compact form for convenient use. Tables give coefficient pairs, admittances, and transient solutions.
10. "C. R. C. Standard Mathematical Tables," 16th ed, edited by Samuel Selby, Chemical Rubber Co., Cleveland, 1968, 692 pages:
An expanded, revised edition of a standard work. The sections involving mensuration, trigonometry, analytic geometry, curves and graphs, and the algebra of sets have been completely rewritten, and sections to cover determinants and matrices have been added. An extension to the octal decimal conversion table to include hexadecimal and decimal conversion increases the usefulness of the volume.
11. David, F. N., M. G. Kendall, and D. E. Barton: "Symmetric Function and Allied Tables," Cambridge University Press, London, 1966, 278 pages:
An elaborate set of 49 major tables accompanied by a detailed introduction of 63 pages, constituting a self-contained treatment of symmetric functions and their applications in statistics. A definitive compilation.
12. Davis, Harold T.: "The Summation of Series," Principia Press of Trinity University, San Antonio, Tex., 1962, 140 pages:
Special emphasis placed upon the case of finite limits.
13. Davis, Harold T., comp.: "Tables of the Higher Mathematical Functions," Principia Press, Bloomington, Ind., 1933–1935, 2 vols.:
Vol. I: Various tables of the gamma and psi functions as well as sections on classification and history of tables, interpolation and its uses, and interpolation tables. Vol. II: Tables of the polygamma functions (trigamma-hexagamma), the Bernoulli and Euler polynomials and numbers, gram polynomials, and polynomial approximation.
14. Davis, Philip J., and Philip Rabinowitz: "Numerical Integration," Blaisdell Publishing Company, Waltham, Mass., 1967, 230 pages:
Includes bibliographies.
15. Doetsch, Gustav: "Handbuch der Laplace-Transformation" (Handbook of Laplace transforms). Verlag Birkhäuser, Basel, 1950–1956, 3 vols. ("Lehrbücher und Monographien aus dom Gebiete der exakten Wissenschaften, Mathematische Reihe," vols. 14, 15, and 19):
Contents: Vol. 1, "Theory of Laplace Transforms"; vols. 2–3, "Applications of Laplace Transforms," including asymptotic expansions, convergent expansions, ordinary and partial differential equations, integral equations, and whole exponential functions.
16. Dwight, H. B.: "Tables of Integrals and Other Mathematical Data," 4th ed., The Macmillan Company, New York, 1961, 336 pages:
Contains derivatives and integrals, classified as algebraic, trigonometric, inverse trigonometric, and exponential functions; probability integrals; logarithmic, hyperbolic, inverse hyperbolic, elliptic, and Bessel functions; surface zonal harmonics; definite integrals; and differential equations. Appendixes: A, Tables of Numerical Values; B, Bibliography.
17. Erdélyi, Arthur, and others: "Higher Transcendental Functions" (Based, in part, on notes left by Harry Bateman and compiled by the staff of the Bateman Manuscript Project, California Institute of Technology), McGraw-Hill Book Company, New York, 1953–1955, 3 vols.:

An account of the principal properties of such functions as gamma, hypergeometric, Legendre, Bessel, elliptic, automorphic, and generating functions, with extensive lists of references at the end of each chapter.

18. Erdélyi, Arthur, and others: "Tables of Integral Transforms" (Based, in part, on notes left by Harry Bateman and compiled by the staff of the Bateman Manuscript Project, California Institute of Technology) McGraw-Hill Book Company, New York, 1954, 2 vols:

    Intended as a companion and sequel to "Higher Transcendental Functions." Contains Fourier, Laplace, and Mellin transforms and their inversions, as well as Hankel transforms. Also included are gamma, Legendre, Bessel, and hypergeometric functions. The entries are arranged in tabular form.

19. Fettis, Henry E., and James C. Caslin: "Elliptic Functions for Complex Arguments," Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio, 1967, 404 pages, ARL 67-0001 (Available from Clearinghouse for Federal Scientific and Technical Information, Springfield, Va. 22151):

    These unique tables consist of 5D values of the Jacobian elliptic functions $sn(w,k)$, $cn(w,k)$, and $dn(w,k)$, where $w = u + iv$, as functions of Jacobi's nome q, which equals $exp\ (-K'/K)$, where K and K' are the quarter-periods (the complete elliptic integrals of the first kind of modulus k and of complementary modulus k', respectively). The range of parameters in the table is: $q = 0.005(0.005)0.480$, $u/K = 0(0.1)1$, and $v/K' = 0(0.1)1$.

20. Fettis, Henry E., and James C. Caslin: "Ten-place Tables of the Jacobian Elliptic Functions," pt. 1, Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio, 1965, 562 pages, ARL 65-180 (Available from Clearinghouse for Federal Scientific and Technical Information, Springfield, Va. 22151):

    This report contains 10D tables of the Jacobi elliptic functions $am(u,k)$, $sn(u,k)$, $cn(u,k)$, and $dn(u,k)$, as well as the elliptic integral $E(am(u),k)$, $k^2 = 0(0.01)0.99$, $u = 0(0.01)K(k)$, and for $k^2 = 1$, $u = 0(0.01)3.69$.

21. Fettis, Henry E., and James C. Caslin: "An Extended Table of Zeros of Cross Products of Bessel Functions," Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio, 1966, 126 pages, ARL 66-0023 (Available from Clearinghouse for Federal Scientific and Technical Information, Springfield, Va. 22151):

    This report presents 10D tables of the first five roots of the equations:
    (*a*) $J_0(\alpha)Y_0(k\alpha) - Y_0(\alpha)J_0(k\alpha) = 0$, (*b*) $J_1(\alpha)Y_1(k\alpha) - Y_1(\alpha)J_1(k\alpha) = 0$,
    (*c*) $J_0(\alpha)Y_1(k\alpha) - Y_0(\alpha)J_1(k\alpha) = 0$.

22. Fletcher, Alan, and others, eds.: "An Index of Mathematical Tables," 2d ed., Addison-Wesley Publishing Company, Inc., Reading, Mass., for Scientific Computing Service, Ltd., 1962, 2 vols.:

    The second edition is more than double the size of the 1946 edition, and includes, as a new feature, a list of errors found in published tables. Contains an index according to function, giving for each table the range, tabular interval, number of significant figures in the values, whether or not tables of proportional parts are given, what orders of differences are shown, etc. Also includes an alphabetical list of references by author and publication year. Considered an important index to well-known tables of functions and to other less-known tables appearing in books and periodicals.

23. Forsythe, G. E., and P. C. Rosenbloom: "Numerical Analysis and Partial Differential Equations," John Wiley & Sons, Inc., New York, 1958, 204 pages.

24. Frazer, Robert A., W. J. Duncan, and A. R. Collar: "Elementary Matrices and Some Applications to Dynamics and Differential Equations," The Macmillan Company, New York, 1946, 416 pages.

25. Great Britain, National Physical Laboratory: "Mathematical Tables," H. M. Stationery Office, London, 1956——

    Vol. 1: "The Use and Construction of Mathematical Tables," by L. Fox, 1956.
    Vol. 2: "Tables of Everett Interpolation Coefficients," by L. Fox, 1956.
    Vol. 3: "Tables of Generalized Exponential Integrals," by G. F. Miller, 1960.
    Vol. 4: "Tables of Weber Parabolic Cylinder Functions and Other Functions for Large Arguments," by L. Fox, 1961.
    Vol. 5: "Chebyshev Series for Mathematical Functions," 1962.
    Vol. 6: "Tables for Bessel Functions of Moderate or Large Order," 1962.
    Vol 7: "Tables of Jacobian Elliptic Functions Whose Arguments Are Rational Fractions of the Quarter Period," 1964.
    This series contains tables of mathematical functions which may not come within the range of the more fundamental tables.

26. Greenwood, Joseph A., and H. O. Hartley: "Guide to Tables in Mathematical Statistics." Princeton University Press, Princeton, N.J., 1962, 1014 pages.

27. Gröbner, Wolfgang, and N. Hofreiter: "Integraltafel (Integral table), pt. I, "Indefinite Integrals"; pt. II, "Definite Integrals," 2d improved ed., Springer Verlag, Vienna, 1965–1966, 2 vols., 166 and 204 pages:
An extensive collection of integrals including a brief survey of methods of evaluation and transformation of integrals.
28. Hart, John F., and others: "Computer Approximations," John Wiley & Sons, Inc., New York, 1968, 343 pages:
Extensive in its range in accuracy from a few digits up to 25D in the approximations; its wide selection of functions includes square root and cube root, exponential and hyperbolic, logarithm, trigonometric and inverse trigonometric functions, gamma function and its logarithm, error function, Bessel functions, and complete elliptic integrals; and in the range of methods, the book describes and compares them from the general methods of subroutine design to procedures for the design of maximum efficiency programs for commonly needed functions.
29. Hartree, Douglas R.: "Numerical Analysis," Oxford University Press, London, 1952, 287 pages:
Includes interpolation and numerical integration formulas, finite differences, harmonic analysis, smoothing.
30. Harvard University Computation Laboratory: "Tables of the Bessel Functions of the First Kind," Harvard University Press, Cambridge, Mass., 1947–1951, 12 vols.: $J_n(x)$, $0 \leq x \leq 100$, 18D for $n = 1$; 10D for $n = 2$ through 135.
31. Harvard University Computation Laboratory: "Tables of the Cumulative Binomial Probability Distribution," Harvard University Press, Cambridge, Mass., 1955, 503 pages ("Annals of the Computation Laboratory of Harvard University," vol. 35).
32. Harvard University Computation Laboratory: "Tables of the Function arc sin z," Harvard University Press, Cambridge, Mass., 1956, 586 pages ("Annals of the Computation Laboratory of Harvard University," vol. 40).
33. Householder, Alston S.: "Principles of Numerical Analysis," McGraw-Hill Book Company, New York, 1953, 274 pages.
34. Jahnke, Eugene, Fritz Emde, and Friedrich Lösch: "Tables of Higher Functions," 7th ed, B. G. Teubner, Stuttgart, 1966, 322 pages:
Text in German and English. Essentially a corrected version of the sixth edition, containing Bessel functions, circular and hyperbolic functions of a complex variable, cubic equations, miscellaneous conversion tables, Planck's radiation function, powers (2d to 15th), probability integral and related functions, reciprocals and square roots of complex numbers, Riemann zeta function, theta functions, transcendental equations, vector addition, and sine, cosine, and logarithmic integral.
35. James, Glenn, and Robert C. James: "Mathematics Dictionary," 3d ed., D. Van Nostrand Company, Inc., Princeton, N.J., 1968, 448 pages:
Correlated condensation of mathematical concepts designed for time saving reference work.
36. Jolley, Leonard B. W.: "Summation of Series," Dover Publications, Inc., New York, 1961, 251 pages.
37. Kamke, Erich: "Differentialgleichungen. Lösungsmethoden und Lösungen (Differential equations, methods of solution, and solutions), 6th improved ed., Akademische Verlagsgesellschaft Geest & Portig, Leipzig, 1959, 666 pages ("Mathematik und ihre Anwendungen in Physik und Technik," Ser. A, vol. 18):
A reference work containing general methods of solution and properties of solution, boundary-, and characteristic-value problems, and a dictionary of some 1,600 equations in lexicographical order with solutions, techniques for solving, and references.
38. Knuth, Donald E.: "The Art of Computer Programming," Addison-Wesley Publishing Company, Inc., Reading, Mass., 1968–1969, 2 vols.
39. Korn, Granino A., and Theresa M. Korn.: "Mathematical Handbook for Scientists and Engineers: Definitions, Theorems, and Formulas for Reference and Review," 2d, enlarged and rev. ed., McGraw-Hill Book Company, New York, 1968, 1129 pages.
40. Lehmer, Derrick H.: "Guides to Tables in the Theory of Numbers," National Academy of Sciences–National Research Council, Washington, D.C., 1941, 177 pages (National Research Council Bulletin 105).
41. Lehmer, Derrick Norman: "Factor Table for the First Ten Millions Containing the Smallest Factor of Every Number not Divisible by 2, 3, 5, or 7 between the Limits 0 and 10,017,000." Hafner Publishing Company, Inc., New York, 1956, 476 pages (Carnegie Institution of Washington Publ. 105):
Introduction includes a list of errors in former tables by other authors.
42. Lehmer, Derrick Norman: "List of Prime Numbers from 1 to 10,006,721," Hafner Publishing Company, Inc., New York, 1956, 133 pages (Carnegie Institution of Washington Publ. 165):

The standard list of primes. Arranged in such a way that it is easy to find the nth prime for a given n.

43. Lieberman, Gerald J., and D. B. Owen: "Tables of the Hypergeometric Probability Distribution," Stanford University Press, Stanford, Calif., 1961, 726 pages (Stanford studies in mathematics and statistics no. 3):
In addition to the following tables of the hypergeometric probability distribution: $N = 2$, $n = 1$ through $N = 100$, $n = 50$; $N = 1000$, $n = 500$; $k = n - 1$, n; $n = N/2$: $N = 100$, $n = 50$ through $N = 2000$, $n = 1000$. the theory, rationale, and specific applications of the hypergeometric probability are discussed.
44. Luke, Yudell: "Integrals of Bessel Functions," McGraw-Hill Book Company, New York, 1962, 424 pages:
Designed to provide the research worker with basic information dealing with definite and indefinite integrals involving Bessel functions.
45. Madelung, Erwin: "Die Mathematischen Hilfsmittel des Physikers" (Mathematical tools for the physicist), 6th rev. ed., Springer Verlag, Berlin, 1957, 535 pages ("Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen," vol. IV):
Comprehensive collection of formulas used in mathematical physics. Included are numbers, functions and operators, series, algebra, transformations, and statistics.
46. Magnus, Wilhelm, Fritz Oberhettinger, and Raj Pal Soni: "Formulas and Theorems for the Special Functions of Mathematical Physics," 3d enlarged ed., Springer Verlag, Berlin, 1966, 508 pages ("Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen," Band 52):
Survey of the properties of a number of special functions including the following: gamma, hypergeometric, Bessel, Legendre, theta, and elliptic, as well as spherical harmonics, orthogonal polynomials, integral transforms and inversions, and coordinate transforms.
47. Mangulis, V.: "Handbook of Series for Scientists and Engineers," Academic Press, Inc., New York, 1965, 134 pages.
48. Margenau, Henry, and George M. Murphy: "The Mathematics of Physics and Chemistry, 2d ed., D. Van Nostrand Company, Inc., Princeton, N.J., 1956, 2 vols.
49. "Mathematics of Computation" (formerly: "Mathematical Tables and other Aids to Computation"), National Academy of Sciences National Research Council, quarterly, Washington, D.C.:
A journal devoted to advances in numerical analysis, the application of computational methods, mathematical tables, high-speed calculators, and other aids to computation.
50. Meyer zur Capellen, Walther: "Integraltafeln. Sammlung unbestimmter Integrale elementarer Funktionen" (Tables of integrals; Collection of indefinite integrals of elemetary functions), Springer Verlag, Berlin, 1950, 292 pages:
Lists some 3,000 integrals of algebraic and transcendental functions, as well as products of algebraic and transcendental functions. Tabulation permits use for differentiation purposes.
51. Morse, Philip M., and Herman Feshbach: "Methods of Theoretical Physics," McGraw-Hill Book Company, New York, 1953, 2 vols.
52. Oberhettinger, Fritz, "Tabellen zur Fourier Transformation" (Tables of Fourier transforms), Springer Verlag, Berlin, 1957, 213 pages.
53. Parke, Nathan Grier: "Guide to the Literature of Mathematics and Physics including Related Works on Engineering Science," 2d rev. ed., Dover Publications, Inc., New York, 1958, 436 pages:
A useful handbook comprising chapters on principles of reading and study, searching the literature, types of materials, library usage, etc.; includes an annotated bibliography of some 5,000 titles arranged by subject with author and subject indexes.
54. Pearson, Karl: "Tables of the Incomplete Beta-function," 2d ed., with a new introduction by E. S. Pearson and N. L. Johnson, published for the Biometrika Trustees by the Cambridge University Press, Cambridge, Mass., 1968, 505 pages:
Gives $I(u,p)$ with the argument u proceeding by increments of 0.1 from 0 up to that value of u which gives $I(u,p) = 1.0000000$ to the seventh decimal place. The argument p advances from -1.0 by increments of 0.05, from 1.0 to 5.0 by increments of 0.1, and from 5.0 to 50.00 by intervals of 0.2. Two new tables give some additional values to the integral computed a number of years ago but not hitherto published, and a list of references has been added.
55. Peirce, Benjamin O.: "A Short Table of Integrals," 4th ed., rev. by Ronald M. Foster, Ginn and Company, Boston, 1956, 189 pages:
Fourth revision of Peirce's tables consisting of indefinite integrals, definite integrals, auxiliary formulas, and numerical tables, including common algebraic expressions; functions of angles in radians; differential equations; exponential functions; hyperbolic-function formulas; elliptic integrals; natural logs; tables of logs of numbers, logs of sines, cosines, etc.; probability integral and trigonometric formulas.

56. Riordan, John: "An Introduction to Combinatorial Analysis," John Wiley & Sons, Inc., New York, 1958, 244 pages.
57. Roberts, G. E., and H. Kaufman: "Table of Laplace Transforms," W. B. Saunders Company, Philadelphia, 1966, 367 pages:
A comprehensive reference of Laplace transforms and their inverses which should prove useful to pure and applied mathematicians. The volume is in two parts—the first devoted to direct transforms and the second to inverse transforms.
58. Royal Society of London: "Royal Society Mathematical Tables," Cambridge University Press, London, 1950——.
Vol. 1: "Farey Series of Order 1025," 1950.
Vol. 2: "Rectangular-polar Conversion Tables," 1956.
Vol. 3: "Tables of Binomial Coefficients," 1954.
Vol. 4: "Tables of Partitions," 1958.
Vol. 5: "Representations of Primes by Quadratic Forms," 1960.
Vol. 6: "Tables of the Riemann Zeta Function," 1960.
Vol. 7: "Bessel Functions," pt. 3, "Zeros and Associated Values," 1960.
Vol. 8: "Tables of Natural and Common Logarithms to 110 Decimals," 1964.
Vol. 9: "Tables of Indices and Primitive Roots," 1968.
Vol. 10: "Bessel Functions," pt. 4, "Kelvin Functions," 1964.
Vol. 11: "Coulomb Wave Functions," 1964.
59. Ryzhik, Iosif M., and I. S. Gradshteyn: "Table of Integrals, Series, and Products," translated from the 4th Russian ed., Academic Press Inc., New York, 1965, 1086 pages:
An inclusive compilation, the work is advertised as the most comprehensive table of integrals ever published. New material on Mathieu, Struve, Lommel, as well as other special functions, has been added.
60. Slater, L. J.: "Confluent Hypergeometric Functions," Cambridge University Press, New York, 1960, 247 pages.
61. Smithsonian Institution: "Smithsonian Mathematical Formulae and Tables of Elliptic Functions," 3d reprinting, Washington, D.C., 1957, 314 pages.
62. Stroud, A. H., and D. Secrest: "Gaussian Quadrature Formulas," Prentice Hall, Inc., Englewood Cliffs, N.J., 1966, 374 pages:
Valuable reference book for use and application of Gaussian quadrature formulas. Text is divided into five parts. Fortran programs to compute the abscissas and weights for quadrature formulas based on classical Jacobi, Laguerre, and Hermite polynomials are presented. Chapter 5 summarizes the tables of quadrature formulas found in the literature.
63. Todd, John, ed.: "Survey of Numerical Analysis," McGraw-Hill Book Company, New York, 1962, 608 pages.
64. U.S. National Bureau of Standards: "Basic Theorems in Matrix Theory," Marvin Marcus, Government Printing Office, Washington, D.C., 1960, 27 pages (Applied mathematics series, 57).
65. ——: "Experimental Statistics," Mary Gibbons Natrella, Government Printing Office, Washington, D.C., 1963, 1 vol. (various pagings) (Handbook 91):
A collection of statistical procedures useful in the design, development, and testing of materials; the evaluation of equipment performance; and the conduct and interpretation of scientific experiments.
66. ——: "Guide to Tables of the Normal Probability Integral," Government Printing Office, Washington, D.C., 1952, 16 pages (Applied mathematics series, 21):
A ready desk reference to the normal probability integral tabulated in standard statistical textbooks and other important sources. Provides a list of available tables as well as the form of the function tabulated.
67. ——: "Matrix Representations of Groups," Morris Newman, Government Printing Office, Washington, D.C., 1968, 79 pages (Applied mathematics series, 60).
68. ——: "Probability Tables for the Analysis of Extreme-value Data," Government Printing Office, Washington, D.C., 1953, 32 pages (Applied mathematics series, 22):
Introduction outlines the theory and application of extreme values and describes nature, use, accuracy, and method of computation of tables. There are six tables for the asymptotic (cumulative) distribution of the largest value $\Phi = \exp(-e^{-y})$, its inverse, the corresponding density function, probability points for the asymptotic distribution of the mth largest values up to $m = 50$, and the asymptotic cumulative and density functions of the range.
69. ——: "Table for Conversion of X-ray Diffraction Angles to Interplanar Spacing," Government Printing Office, Washington, D.C., 1950, 159 pages (Applied mathematics series, no. 10):
Tables of spacing values, $\theta = 0(0.01)90°$, 5S, calculated by using the $K\alpha_1$ wavelengths for X-ray targets of molybdenum, copper, nickel, cobalt, iron, and chro-

mium. The wavelengths are those adopted at the International Conference sponsored by the British Institute of Physics in 1946.

70. ——: "Tables for the Analysis of Beta Spectra," Government Printing Office, Washington, D.C., 1952, 61 pages (Applied mathematics series, 13):
Tables of the values of the so-called Fermi function

$$F(Z,\eta) = \eta^{2+2S}e^{\pm\pi\delta}\cdot|\Gamma(i+S+i\delta)|^2$$

where the upper and lower signs of exp $(\pm\pi\delta)$ apply to the spectra of negative and positive electrons, respectively; η = momentum of the electron after its ejection from the atom, in units of megaHertz; $S = \sqrt{1-Z^2/137^2}-1$; Z = atomic number: $\delta = Z\sqrt{1+\eta^2}/137\eta$.

71. Wheelon, Albert D.: "Tables of Summable Series and Integrals Involving Bessel Functions," Holden-Day, Inc., Publisher, San Francisco, 1968, 125 pages.

1b. SI Units

With the continuing expansion of international cooperation and communication in science, the need for uniform usage of units and symbols has become increasingly critical. The culmination of a number of efforts in this direction is the International System of Units (abbreviated to SI for Système International) which was defined and given official status by the 11th General Conference on Weights and Measures, held in Paris in 1960. But achievement of widespread, let alone universal, acceptance and implementation of any international agreement takes time and requires the solution of a host of problems—some physical, some psychological. The SI units system has proved no exception. Many organizations have adopted SI officially; many have not. As one might expect, viewpoints of individual physicists within both groups vary from enthusiastic support through indifference to strong opposition. One British scientist has written, for example: "The reaction of a physicist asked to adopt the SI units is liable to be that which one could imagine from an abstemious early Christian anchorite asked by a Salvation Army tract bearer to sign the pledge . . . . Physicists have thought themselves particularly intelligent and virtuous about units, and they count the guardians of the units among their own number" [1]. Thus, even though the Paris conference occurred almost a decade ago, science in the United States is still very much in a period of transition with regard to the use of SI units. For this reason, the editors of the AIP Handbook did not think it practical to expect the more than 100 contributors to this edition to use SI units exclusively. On the other hand, it was clear that this new system could not be ignored. Inclusion of this descriptive subsection represents the compromise upon which the editors agreed.

In 1964, the National Bureau of Standards adopted SI units for use by its staff. Then, recognizing that there would have to be a transitional period, the Bureau appointed a Units and Usage Committee, and directed it to recommend interim practices for the NBS staff to follow. The rest of this subsection is quoted from a Bureau Technical News Bulletin titled NBS Interprets Policy on SI Units [2].

1b-1. Statement of Policy. Numerical data are used in NBS publications in two distinct ways: as descriptive data and as essential data. NBS policy for the transition period accepts different treatment of these two classes of data, although they may be presented in the same textbook, each with appropriate units. For example, it is acceptable to write "the interferometer mirror mounted on 1-in. rod, was advanced in 10-nanometer increments," or "a 200-in. telescope of 0.497 m effective aperture."

Descriptive data describe arrangements, environments, noncritical dimensions and shapes of apparatus, and similar measurements not entering into calculations or expression of results.

Essential data express, lead up to, or help to interpret the quantitative results of the activity that is being reported.

NBS policy also recognizes that communication of *scientific* results, via scientific papers, calls for more rigorous standards of units usage than does communication of *technological* results in technological papers.

Descriptive Data. Descriptive data should be expressed in the most useful and convenient manner. Forced translation into SI is not required. The units best understood by the expected audience are the most appropriate. Where non-SI units are used, the author may add SI equivalents in parentheses at his own discretion, but usage within a paper should be consistent on this point. Commercial gage designations or other standard nomenclatures, e.g., drill sizes, are acceptable. As SI units become more commonly used for commercial products, use of SI units in descriptive data should conform.

Essential Data. In technological papers the essential data may be expressed in the units customarily used in the relevant field of technology. SI equivalents should be added in parentheses, or in parallel columns in tables. If graphs are used as the primary or sole means of presenting essential data, the coordinates may be divided according to customary usage, but a secondary set of coordinate markings in SI units should be included. The top and right-hand sides of the graph are often appropriate for this purpose. If graphs are used only to indicate trends, or as supplements to tables, units customary in the field are adequate without SI translation. NBS authors should, however, use the SI as soon as the level of SI usage in the related field of technology renders it an efficient communication device. Familiarity with SI units (see Arts. 1.1 and 1.2 of the Appendix, Sec. 1b-2) is recommended to all NBS authors and all NBS staff.

In purely scientific papers, the essential data *shall* be expressed in SI units, or in units approved for use with the SI. The General Conference on Weights and Measures (CGPM) has designated names and symbols for many of the SI units. These and other names and units approved for NBS use but not yet acted upon by the General Conference are given in Arts. 1 to 6 of the Appendix. Values in other units may be added, in parentheses, where it is felt that this will improve the communication between authors and readers.

Reference data used generally in both science and technology should be expressed in SI units with appropriate indication of conversion factors into technological units or with parallel columns of converted values. Standard reference data applicable primarily to scientific interests (e.g., tables of X-ray atomic energy levels) should be expressed in SI units. Non-SI units may be included as parallel entries.

Where the general usage in any field does not recognize SI units, NBS authors should employ units comprehensible to their readers, but should try to increase reader familiarity with SI units as rapidly as possible. Again, the device of parallel columns (familiar plus SI units) is recommended.

In the lists in the Appendix, the short names for compound units, such as "coulomb" for "ampere-second," exist for convenience, and their use is not compulsory. For example, communication sometimes benefits if the author expresses magnetic flux in volt-seconds, instead of using the synonym webers, because of the descriptive value implicit in the former unit phrase.

The analysis, interpretation, or application of essential data may involve angles and the related values of natural functions (sine, tangent, log sin, etc.). In these cases, the angles may be expressed in degrees rather than in radians.

1b-2. Appendix. Articles 1 to 6 present those units that should be used by authors of *scientific* papers for *essential* data. Article 7 presents units that should *not* be

used by authors of scientific papers for essential data. Authors of *technological* papers are urged to become familiar with these guidelines, and to follow them as soon as their intended readers are ready to accept and understand them.

1.1. *Official SI Units Names and Symbols*

Unit	Symbol	Unit	Symbol
meter	m	watt	W
kilogram	kg	coulomb	C
second	s	volt	V
ampere	A	ohm	Ω
kelvin[1]	K	farad	F
candela	cd	weber	Wb
radian	rad	henry	H
steradian	sr	tesla	T
hertz	Hz	lumen	lm
newton	N	lux	lx
joule	J		

1.2. *Additional Names and Symbols Approved for NBS Use*

curie[2]	Ci
degree Celsius[3]	°C
gram	g
mho	mho
mole	mol
siemens[4]	S

1.3. *Official Prefixes Indicating Decimal Multiples and Submultiples*

Multiples and submultiples	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Note. Compound prefixes (e.g., millimicro) are not to be used.

[1] The same name and symbol are used for thermodynamic temperature and temperature interval. (Adopted by the 13th General Conference on Weights and Measures, 1967.)
[2] Accepted by the General Conference on Weights and Measures for use with the SI.
[3] For expressing "Celsius temperature"; may also be used for a temperature interval.
[4] Adopted by IEC and ISO.

2. *Decimal multiples of SI units,* bearing coined names, are acceptable in their special fields only.[1] Use of SI units, however, is recommended. Examples include:

Unit	Symbol	SI equivalent
angstrom	Å	$= 10^{-10}$ m
bar	bar	$= 10^{5}$ N/m^2
barn	b	$= 10^{-28}$ m^2
kayser[2]	K	$= \text{cm}^{-1} = 100\ \text{m}^{-1}$
liter[3]	l	$= 10^{-3}$ m^3
poise	P	$= 10^{-1}$ N · s/m^2
rad	rd	$= 10^{-2}$ J/kg
stokes	St	$= 10^{-4}$ m^2/s

3. *"Natural Units."* Natural units are acceptable. These are units tied directly to the fundamental Lorentz invariant constants of nature as well as to the properties of the microscopic constituents of matter. Although it is recognized that in casual conversation, dimensionally incorrect units are used, dimensionally correct units shall be used in published work. (Examples of conversational "shorthand" include: temperature in eV, mass in eV, momentum in fm^{-1}. These usages arise in expressing quantities by the value of *related* quantities: e.g., energy of a particle, wave number associated with momentum.) Acceptable natural units include:

electronic charge	e
electron mass	m_e
proton mass	m_p
speed of light	c
electron-volt	eV
Planck's constant	h or $\hbar$
Bohr radius	a_0
Bohr magneton	μ_B
nuclear magneton	μ_N
electron radius	r_e
Compton wavelength of electron	λ_C
atomic mass unit	u
Faraday	F

3a. *The term "X-unit"* is ambiguous, differing in American and European usage. It is acceptable, *provided* that its use is accompanied by its definition in terms of the $K\ \alpha$ line of molybdenum or of tungsten. When accurate conversions to the SI become available, the usage of X-unit should be discontinued.

4. *Special cgs-esu multiple,* acceptable pending CGPM naming of a suitable replacement:

debye (10^{-18} statcoulomb-centimeter) = 3.33564×10^{-30} coulomb-meter

5. *Acceptable logarithmic measures:*

pH
decibel (dB)
neper (Np)

[1] The 13th General Conference canceled the name *micron* and its old symbol, μ; use *micrometer,* μm.

[2] Note the conflict with K for kelvin, which is an official symbol. It is felt that context will preclude confusion.

[3] To be used only for expressing volumes of gases and liquids; otherwise use cubic meter, etc.

6. *Acceptable units for essential data in expression of angles* in relation to their natural functions, and for naturally occurring geometrical relationships:

degree	°
minute	′
second	″

7. *Units NOT acceptable for expressing essential data* in scientific papers (see *Comments* below):

7a. *Unnecessary coined names:*

gamma	(for nanotesla)
gamma	(for microgram)
fermi	(for femtometer, fm)

7b. *Coined names for cgs units* (and therefore not compatible with SI):

gauss	gal
erg	stilb
dyne	

7c. *Units not compatible with the SI*, nor any other metric system:

calorie	millimeter of mercury
British thermal unit	hour (time)[1]
entropy unit	minute (time and angle)[1]
roentgen	degree (angle)[2]
atmosphere	second (angle)[2]
torr	

Comments. The CGPM has not yet adopted a unit for "quantity of matter." Other organizations (ISO and IUPAC) have adopted a mole based on 0.012 kg of carbon-12; this is equivalent to the familiar gram-mole (symbol mol).

USASI and international usage require that letter symbols deriving from proper names be capitalized, although the unit names themselves are lowercase.

"Mho" and "siemens" are widely used coined names of derived SI units to which no name has yet been assigned by CGPM. They are, therefore, acceptable until CGPM acts to assign names.

The name "nit" (symbol nt) has been recommended by the International Commission on Illumination for the SI unit of luminance, "candela per square meter" (symbol cd/m^2). Although the name nit has not been assigned by CGPM and has not received extensive use, it may be used where the official name is felt to be awkward.

"Poise" and "stokes" are coined cgs names; there are no short names in use for the SI units of "viscosity" and "kinematic viscosity." (The compound names "newton-second per square meter" and "kilogram per meter-second" are long and awkward; a coined short name is needed. "Square meter per second" is also sometimes considered awkward.)

The lists of approved units given in Arts. 1 to 6 are not closed-ended. Other units *compatible with the SI* can and will be added on the basis of:

1. Actions taken by subsequent General Conferences

2. Clear indication of need in specialized fields, following approval of the NBS Editorial Review Boards and the NBS Units and Usage Committee

Artificial creations such as kiloangstrom and cubic angstrom are, for the most part, unnecessary and do more harm than good. They should be replaced by SI units as rapidly as possible.

Certain units based on natural constants, e.g., electron-volt, are both meaningful and convenient. These natural units are orders of magnitude outside the range of SI units with prefix.

Most tables of natural functions of angles give the angle in degrees, minutes, and

[1] Allowable when necessary for expression of extended time intervals.
[2] See Art. 6 for acceptable use.

seconds, or in degrees and decimals. Wherever the mathematical or expository treatment or application of essential data requires that an angle and its natural functions be related, degrees may be used for expressing the angle. Likewise, wherever naturally occurring geometrical phenomena (crystal data, bond angles, declination of the sun, etc.) are expressed quantitatively, degrees may be used.

The units in Art. 7*c* are convenient and acceptable for descriptive use, but *not* for essential data. For example, the torr is widely used for describing an environment that does not enter into the calculation.

The transition from calories and kilocalories to joules will of course involve some distress. NBS authors are urged to recognize their responsibilities for taking the lead in the acceptance of the joule, while still providing convenient communication to users, by parenthetical equivalents or parallel columns.

The use of "cycles per second" in place of hertz is deprecated. It tends to perpetuate the common misuse of cycle by itself for the frequency unit. When cycle is correctly used by itself, in the same paper as cycles per second, journal editors are prone to "correct" it!

Self-explanatory combinations of prefixed units are acceptable, e.g., milliampere per square centimeter, cubic decimeter, milligram.

The inclusion of the prefix "kilo" in the name of the base unit of mass creates an awkward situation. Logically, the "gram" should be called the "millikilogram"! On recommendation of the International Committee on Weights and Measures, the names of multiples of the kilogram are formed by adding prefixes to the word "gram."

Expression of extended time intervals (essential data) may sometimes be more readily comprehended if years, hours, and minutes are given in parentheses, as well as, e.g., kiloseconds. However, as the author will recognize, this usage borders on descriptive data. In any case, computations will always require use of seconds only.

Editorial Notes. Words and symbols should *not* be mixed; if mathematical operations are indicated, only symbols should be used. For example, one may write joules per mole, J/mol, $J \cdot mol^{-1}$, but *not* joules/mole, joules mol^{-1}, etc.

Note that exponents operate also on prefixes, as in cm^2, mm^3 which are *not* $10^{-2}m^2$, $10^{-3}m^3$.

In combinations such as meterkelvin, use of the product dot ($m \cdot K$) avoids confusion with millikelvin. It is good practice to indicate *all* unit products with multiplication dots, since some unit symbols consist of more than one letter, e.g., Wb for weber, *versus* $W \cdot b$ for watt-barn.

When a compound unit is formed by division of one unit by another, its symbol consists of the symbols for the separate units either separated by a solidus or multiplied by using negative powers (for example, m/s or $m \cdot s^{-1}$ for meter per second). In simple cases use of the solidus is preferred, but in no case should more than one solidus be included in a combination unless parentheses are inserted to avoid ambiguity. In complicated cases, negative powers should be used.

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