2f. Viscosity of Solids

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Symbols

- E elastic modulus
- t time
- T absolute temperature
- u' elastic strain rate
- u'' plastic strain rate
- δ logarithmic decrement
- ϵ' elastic strain
- ϵ'' plastic strain
- η viscosity
- σ stress

2f-1. Anelasticity. A perfectly elastic solid is truly an ideal material. Actual materials contain structural imperfections which prohibit them from behaving in a perfectly elastic manner. Even when the stresses are low enough to ensure that no perceptible permanent deformation takes place the total strain is made up of a purely elastic part that is directly proportional to the load and a time-dependent but fully recoverable part that will vary with the rate of loading and the duration of the load. The behavior associated with the time-dependent part of the strain has been called "anelasticity" by Zener,[1] who has endeavored to explain this behavior in terms of the atomic arrangement and the microstructure of the material.

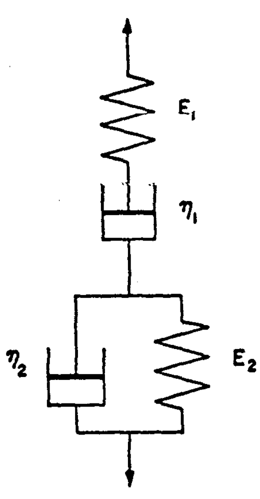

FIG. 2f-1. Mechanical model for demonstrating anelastic and creep behavior of solids.

Anelastic behavior is observed in many ways, depending upon the manner in which the material is loaded. Its effect may be referred to as elastic hysteresis, internal friction, elastic aftereffect, specific damping capacity, or dynamic and static moduli of elasticity. The fact that the term anelasticity has been limited to the region of no permanent deformation does not exclude the existence of such behavior at higher stresses. When a material deforms permanently, however, the anelastic effects are overshadowed by and engulfed in the plastic behavior.

In the realm of small deformations a metal or a plastic can be represented qualitatively by the mechanical model of springs and dashpots shown in Fig. 2f-1. For the anelastic

[1] C. Zener, "Elasticity and Anelasticity of Metals," University of Chicago Press, Chicago, 1948.

behavior at low stresses the viscosity η_1 of the upper dashpot can be considered as infinite. The spring with the elastic modulus E_1 contributes the purely elastic strain. The time-dependent part of the strain comes from the parallel arrangement of spring E_2 and dashpot η_2. This model will exhibit, though not in a quantitative manner, the various anelastic effects of solids.

If the unit is elongated at a slow rate, dashpot η_2 will have little effect in resisting the deformation of spring E_2. The static or isothermal modulus of elasticity will be that of springs E_1 and E_2 connected in series. If the unit is elongated rapidly dashpot η_2 will tend to act as a rigid mechanism. The dynamic or adiabatic modulus of elasticity will be that of spring E_1 acting alone.

If the unit is put through a constant-rate loading and unloading cycle a hysteresis loop will be traced out in the stress-strain diagram. The area of the loop will be proportional to the amount of energy dissipated in dashpot η_2.

If the unit is loaded slowly and then unloaded rapidly the strain will not immediately return to zero. What appears to be a permanent strain or elastic aftereffect will be observed. The strain will return to zero when the stress trapped in the spring E_2 by dashpot η_2 has been relaxed.

If the mass is attached to the lower end of the unit and the entire mechanism is allowed to vibrate freely the amplitude of vibration will decrease with each cycle. The decrease in amplitude of vibration is due to the dissipation of energy in dashpot η_2. If the springs are linear and elastic and the dashpot behaves in a perfectly viscous manner the ratio of the decrease in amplitude for any given cycle to the amplitude at the beginning of the cycle will be a constant. This constant is called the logarithmic decrement δ, and it is probably the most-used measure of the anelastic behavior of materials.

The logarithmic decrement of actual materials is relatively high for dielectric materials and low for metals. Since this quantity depends upon imperfections in the atomic structure it will vary with such factors as heat-treatment, grain size, or the amount of cold working, and it will be impossible to assign a value to a specific material such as steel. The values listed by Kimball[1] and shown in Table 2f-1 and those listed by Gemant[2] and shown in Table 2f-2 are to be considered as representative values which give the order of magnitude of the decrement or internal friction.

The factors which affect the logarithmic decrement are discussed in detail by Zener and by Gemant. The decrement is influenced by such factors as frequency, temperature, amplitude, elastic modulus, grain size, annealing temperature, and aging time.

In general there is not much change in decrement with frequency. Gemant and Jackson[3] found slight increases in the decrement of ebonite and glass over rather narrow frequency ranges (Fig. 2f-2). Gemant shows a slight increase in the decrement for paraffin wax and a slight decrease in the decrement for steel (Fig. 2f-3). An exception to this rule was found by Rinehart,[4] who reported an appreciable increase in the decrement of Lucite at room temperature (Fig. 2f-4).

Certain materials show steep peaks in the log decrement vs. log frequency curve. These peaks are associated with frequencies that correspond to the reciprocal of some characteristic time for the material. Such a curve, taken from Gemant and based on the work of Zener and Bennewitz and Rötger,[5] is shown in Fig. 2f-5. In this case the peak in the internal-friction curve is due to the diffusion of heat from parts heated by compression to parts cooled by tensile stresses.

[1] A. L. Kimball, "Vibration Prevention in Engineering," John Wiley & Sons, Inc., New York, 1932.

[2] A. Gemant, "Frictional Phenomena," Chemical Publishing Company, Inc., New York, 1950.

[3] A. Gemant and W. Jackson, *Phil. Mag.* **23**, 960 (1937).

[4] J. S. Rinehart, *J. Appl. Phys.* **12**, 811 (1941).

[5] K. Bennewitz and H. Rötger, *Z. tech. Phys.* **19**, 521 (1938).

TABLE 2f-1. LOGARITHMIC DECREMENTS FOR VARIOUS MATERIALS*

Material	*Logarithmic Decrement* δ
Phosphor bronze, cold rolled	0.37×10^{-3}
Monel, cold rolled	1.43
Nickel steel, 3½% swaged	2.3
Nickel, cold rolled	3.2
Phosphor bronze, annealed	3.2
Aluminum, cold rolled	3.4
Brass, cold rolled	4.8
Mild steel, cold rolled	4.9
Copper, cold rolled	5.0
Glass	6.4
Molybdenum, swaged	6.9
Swedish iron, annealed	7.9
Tungsten, swaged	16.5
Zinc, swaged	20
Maple wood	22
Celluloid	45
Tin, swaged	129
Rubber, 90% pure	260

* A. L. Kimball, "Vibration Prevention in Engineering," John Wiley & Sons, Inc., New York, 1932.

TABLE 2f-2. LOGARITHMIC DECREMENT OF VARIOUS MATERIALS*

Material	*Logarithmic Decrement* δ
Steel	0.6×10^{-3}
Quartz	2.6
Copper	3.2
Lead glass	4.2
Wood	27
Polystyrene	48
Ebonite	85
Paraffin wax	150

* A. Gemant, "Frictional Phenomena," Chemical Publishing Company, Inc., New York, 1950.

The logarithmic decrement usually increases with increasing temperature. The viscous behavior changes more rapidly than the elastic properties with temperature, with the result that at higher temperatures more energy is dissipated in the dashpot.

The decrement does not vary greatly with amplitude when the amplitudes are small. The decrement increases at higher amplitudes. This is evidence that the viscosity of materials is not of a pure viscous nature. The rate of strain increases more rapidly at the higher stresses than the linear viscous law would predict.

Materials with high elastic moduli have lower decrements than those with low moduli. There is some evidence to show that the product of the elastic modulus and the decrement is nearly a constant value.

The damping capacity of a structure depends upon the stress distribution in the structural members and the energy absorption characteristics of the material from which the members are made. This energy absorption may be brought about by plastic flow, thermoelastic effect, magnetoelastic effect, and atomic diffusion. The relative importance of these effects will depend upon the magnitude of the vibratory stresses.[1]

2f-2. Creep. When a material is subjected to the proper combination of high stress and temperature, it will deform permanently. A representative behavior will be produced by the model shown in Fig. 2f-1 if the viscosity of both dashpots η_1 and η_2 is finite. The continuing deformation of a material under a constant load is called

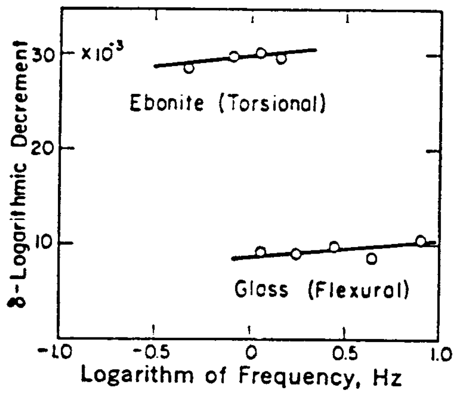

FIG. 2f-2. Logarithmic decrement vs. logarithm of frequency for ebonite and glass. (*Gemant and Jackson.*)

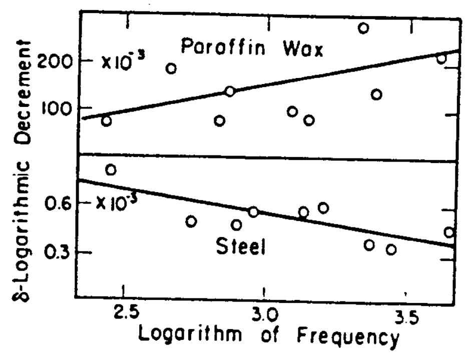

FIG. 2f-3. Logarithmic decrement vs. frequency at room temperature for steel and paraffin wax. (*Gemant.*)

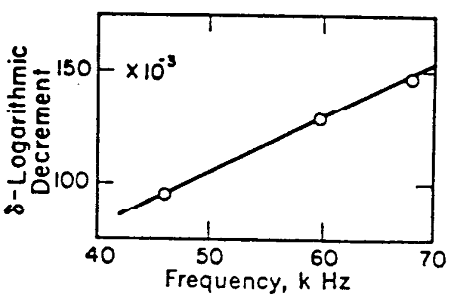

FIG. 2f-4. Logarithmic decrement vs. frequency for Lucite at 26°C. (*Rinehart.*)

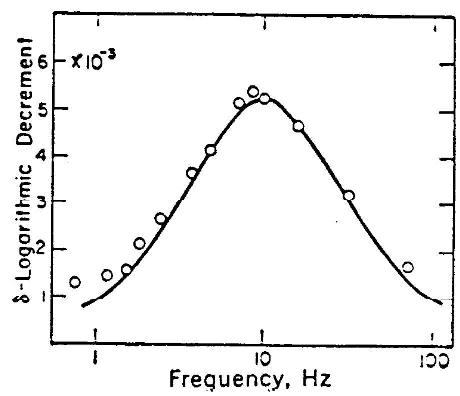

FIG. 2f-5. Logarithmic decrement vs. frequency for German silver. (*Measured points after Bennemitz and Rötger; theoretical curve after Zener.*)

"creep." If the model is loaded with a given load at $t = 0$, there will be an instantaneous elastic deflection ϵ' of spring E_1, dashpot η_1 will deform at some constant rate u_0'', and dashpot η_2 will deform at a decreasing rate.[2] The rate of strain in dashpot η_2

[1] This problem was discussed in detail during the early 1950s. See, for example, the following papers and their reference lists: B. J. Lazan, *J. Appl. Mech., Trans. ASME* **75** (1953); A. W. Cochardt, *J. Appl. Mech., Trans. ASME* **76** (1954).

[2] A prime (′) on a strain or strain rate indicates elastic deformation; a double prime (″) indicates plastic or permanent strain. The total strain, or strain rate, is the sum of the elastic and the plastic parts; i.e.,

$$\epsilon = \epsilon' + \epsilon'' \quad \text{or} \quad u = u' + u''$$

decreases because the load is gradually transferred to spring E_2 as the deformation takes place, and this part of the deformation stops at a strain ϵ_0'' when the spring E_2 carries the complete load. The creep curve for the model and for materials which are not stressed high enough to cause fracture will have the form shown in Fig. 2f-6 (the elastic strain ϵ' is not shown). The plastic strain starts at a rapid rate but approaches the asymptotic value given by

$$\epsilon'' = \epsilon_0'' + u_0''t \tag{2f-1}$$

The shape of the initial part of the creep curve or the manner in which the curve approaches the asymptote has been studied by Andrade[1] and by McVetty.[2] Andrade

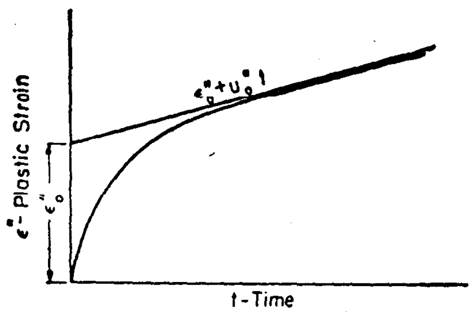

FIG. 2f-6. Typical creep curve.

found that the increase of strain during the first part of the test was proportional to the cube root of the time.

$$\epsilon'' = \beta t^{\frac{1}{3}} \tag{2f-2}$$

McVetty used an exponential relationship to describe the initial deformation.

$$\epsilon'' = \epsilon_0''(1 - e^{-\alpha t}) + u_0''t \tag{2f-3}$$

When creep tests are made to obtain design data for equipment having long service life, and most of the early creep tests were made under these conditions, the major part of the strain is accounted for by the $u_0''t$ term in Eq. (2f-1). The important relationship to be established, then, is that between the minimum creep rate u_0'' and the stress σ, and this is the only information reported by many investigations.

If shorter service times are considered, the initial part of the creep curve becomes more important, and it becomes desirable to know the relationship between the plastic intercept ϵ_0'' and the stress σ. McVetty shows a plot of this relationship for the lower stress range where a power function or hyperbolic sine relationship would be suitable.

$$\epsilon_0'' = A\sigma^n \qquad \text{or} \qquad \epsilon_0'' = B \sinh \frac{\sigma}{\sigma_0} \tag{2f-4}$$

Such relationships indicate that, if the model of Fig. 2f-1 is to represent actual materials, spring E_2 must be nonlinear. At higher stresses these relationships do not hold.

[1] E. N. da C. Andrade, *Proc. Roy. Soc. (London)*, ser. A, **84**, 1 (1911); **90**, 329 (1914).
[2] P. G. McVetty, *Mech. Eng.* **56**, 149 (March, 1934).

TABLE 2f-3. CREEP RATES FOR VARIOUS MATERIALS*

Material and composition	Condition	Temp °C	Temp °F	Stress for 0.001 strain in 1,000 hr, psi
Aluminum copper alloy, Cu 4.25, Mn 0.63, Mg 0.44, Fe 0.52, Si 0.25	$\frac{5}{8}$ diam rod, wrought, aged	150	302	22,000
		250	482	5,700
		350	662	1,500
Aluminum silicon alloy, Si 13.18, Ni 3.08, Cu 2.96, Mg 1.04, Fe 0.53	Wrought	205	400	8,800
		315	600	950
Electrocopper	Fully annealed	205	400	6,700
Deoxidized copper	$\frac{3}{4}$ diam rod, cold drawn, annealed	205	400	20,500
Copper nickel alloy, Ni 20.0, Zn 5.08, Mn 0.69	$\frac{3}{4}$ diam rod, cold drawn, annealed at 1200°F	315	600	27,800
Copper tin alloy, Sn 5.99, Zn 5.10, Pb 2.33, Ni 0.23, Fe 0.06	Cast	260	500	10,000
		315	600	3,000
Copper zinc alloy, Cu 96.43, Pb 0.05, Fe 0.01, Zn remainder	$\frac{1}{8}$ diam wire, drawn, fine-grained	149	300	50,000
		205	400	3,500
		260	500	700
Carbon steel, C 0.15, Mn 0.46, Si 0.28 (basic open hearth)	1 in. diam bar, wrought, annealed at 1500°F, grain size 5–6 ASTM	427	800	17,200
		538	1000	3,300
		648	1200	540
Carbon steel, C 0.15, Mn 0.50, Si 0.23 (basic electric furnace)	1 in. diam bar, wrought, annealed at 1550°F, grain size 4–5 ASTM	427	800	26,800
		482	900	16,900
		538	1000	5,750
		593	1100	1,800
		648	1200	620
Chromium steel, C 0.10, Cr 5.09, Mo 0.55, Mn 0.45, Si 0.18	1 in. diam bar, wrought, annealed at 1550°F, grain size 4–5 ASTM	482	900	15,200
		538	1000	10,100
		593	1100	5,850
		648	1200	2,800
Molybdenum steel, C 0.22, Mo 1.06, Mn 0.50, Si 0.13 (induction furnace)	Bar $1\frac{1}{4}$ sq. cast, annealed at 1650°F, grain size 7	427	800	28,000
		482	900	20,800
		538	1000	11,200
Nickel steel, C 0.36, Ni 1.19, Mn 0.58, Cr 0.51, Mo 0.51, Si 0.22 (induction furnace)	1 in. diam bar, hot rolled, normalized at 1600°F, tempered 3 hr at 1250°F	454	850	40,000
		538	1000	12,300
		593	1100	3,600
		648	1200	1,600
Lead	Grade 2	43	110	320
Magnesium alloy, Al 3, Zn 1	Sand cast, $\frac{1}{2}$ diam rods	150	302	4,900†
Nickel alloy, Cu 28.46, Fe 1.24, Mn 0.94, C 0.18, Si 0.10	Wrought	427	800	30,000
		482	900	23,000
		538	1000	3,700
		593	1100	1,300
		648	1200	450

* Mechanical Properties of Metals and Alloys, *Natl. Bur. Standards (U.S.) Circ.* C447, 1943.
† Stress for 0.005 strain in 1,000 hr.

TABLE 2f-3. CREEP RATES FOR VARIOUS MATERIALS (*Continued*)

Material and composition	Condition	Temp °C	Temp °F	Stress for 0.001 strain in 1,000 hr, psi
Zinc alloy, Cd 0.3, Pb 0.3	Rolled, soft, tested parallel to rolling direction	20	68	10,100
		40	104	8,000
		60	140	6,300
Zinc alloy, Cd 0.3, Pb 0.3	Rolled, soft, tested perpendicular to rolling direction	20	68	15,400
		40	104	12,100
		60	140	8,000

As the stress is increased, a maximum value is reached above which the value of ϵ_0'' decreases with increasing stress.

In the range of strain rates that can be tolerated in reasonable testing times the minimum creep rate u_0'' vs. stress σ curve can be approximated by a straight line on either a double-log or a semilog plot.

$$u_0'' = D\sigma^m \qquad \text{or} \qquad u_0'' = u_1'' \sinh \frac{\sigma}{\sigma_0} \tag{2f-5}$$

The hyperbolic sine relationship has been shown by Kauzmann[1] to have some theoretical foundation in terms of the "chemical rate theory." The power-function relationship has the advantage of being more workable from a mathematical point of view, but it suffers somewhat from the illogical conclusion that the viscosity of dashpot η_1 should approach infinity as the stress approaches zero. Creep properties, like anelastic properties, vary with many factors, and compilation of creep data means very little unless heat-treatment, grain size, and amount of cold working are also specified. A few representative values of the stress required for a creep rate of 10^{-6} per hour, taken from the 1943 compilation of the National Bureau of Standards,[2] are given in Table 2f-3.[3]

Materials held under constant load during long-time creep tests recover part of their plastic strain when the load is removed. According to the model of Fig. 2f-1 the recoverable strain should be equal to ϵ_0''. In actual practice, however, the recovery is usually much less than ϵ_0'' and is generally less than the elastic strain of unloading. If after the first unloading and subsequent recovery the specimen is loaded and unloaded the new plastic intercept ϵ_0'' and the recoverable strain are approximately equal.

Both constants in either of the expressions of Eqs. (2f-5) vary with temperature. According to the chemical rate theory of Kauzmann and the various theories based on

[1] W. Kauzmann, *Trans. AIME* **143**, 57–83 (1941).

[2] Mechanical Properties of Metals and Alloys, *Natl. Bur. Standards* (*U.S.*) *Circ.* C447, 1943.

[3] Recent compilations of creep test data are published by the American Society for Testing and Materials in their Data Publication Series.

diffusion phenomena the constants D and u_1 should decrease with increasing temperature according to an exponential expression

$$u_0'' = C_1 e^{-(C_2/T)} \tag{2f-6}$$

This has been checked experimentally over reasonably wide temperature ranges. The constant σ_0, in the lower stress range, usually decreases slightly with increasing temperature. If the constant m changes with temperature caution must be observed in extrapolating toward regions where the curves for two different temperatures would cross.