2h. Geodetic Data

HELLMUT H. SCHMID AND KARL R. KOCH

National Oceanic & Atmospheric Administration

2h-1. Introduction. The fundamental task of geodesy is the formulation of a three-dimensional mathematical model to which can be related uniquely:

1. The geometry of the physical surface of the earth which is truly the "shape" of the earth,
2. The mathematical description of the gravitational field associated with the earth's mass, where the detailed description of the equipotential surface representing mean sea level—the geoid—is of special interest, and
3. the Universal Time and the astronomical Right Ascension-Declination System.

Establishing the shape of the earth (1) requires the determination of three-dimensional coordinates for (ideally speaking) all points of the physical surface of the earth. Because of (2) it is convenient to establish the corresponding coordinate system in relation to the mass center of the earth. An expedient coordinate system is a geocentric equatorial cartesian (x,y,z) system, the origin of which coincides with the center of mass. In order to relate this system to both Universal Time and the astronomical reference system (3) it is necessary that the z axis coincide with the axis of rotation of the earth for a certain epoch, thus pointing toward a corresponding reference pole. The mean pole of the epoch 1900 to 1905, designated the Conventional International Origin (CIO), was adopted for this purpose by the International Association of Geodesy in 1968. The x axis points toward the meridian of Greenwich which is designated the null meridian for both the measurement of geographic longitude and Universal Time.

2h-2. Reference Ellipsoids. Mainly because of the uncertainty in the amounts of terrestial refraction (cf. Sec. 2h-3), geodetic surveys are generally based on horizontal angle measurements and projected onto reference surfaces. Ellipsoids of revolution, also called reference ellipsoids (in the United States sometimes reference spheroids) are used for the reduction of surveys covering extended continental areas. Portions of spheres or planes are introduced for more restricted surveys.

The method of triangulation for the purpose of surveying was introduced at the beginning of the seventeenth century. The horizontal angles in a triangle are measured with theodolites, and the size of the triangle—the scale of the triangulation—is determined by distance measurements. By connecting triangle to triangle, continents can be covered with triangulation nets. Chains of triangles along meridians and parallels were measured for determining the dimensions of the reference ellipsoids. Dimensions of reference ellipsoids are given in Table 2h-1.

The Clarke 1866 ellipsoid was adopted by the United States for the North American Datum 1927, while Hayford's ellipsoid of 1910 was accepted as International Ellipsoid by the International Association of Geodesy in 1924.

2h-3. Different Geodetic Systems. The conceptual approach to the establishment of a triangulation system begins with the selection of a datum point—ideally located near the center of the area under consideration. At this datum point the astronomical latitude, longitude, and azimuth of one side of a triangle, for which the datum point is a vertex, are determined. By setting these observed quantities equal to the corresponding ellipsoidal values, and with the additional assumption that the height above sea level is equal to the height above the reference ellipsoid, the surface of the ellipsoid provisionally becomes, neglecting the curvature of the plumb line, tangent to the geoid at this datum point. With the geodetic coordinates of one point thus fixed, the coordinates of other points in the triangulation net are then computed from the azimuths and lengths of the sides of the triangles.

Only horizontal angles are used in a triangulation, i.e., the angles measured in the plane perpendicular to the direction of local gravity, because they can be determined much more accurately than the typically small vertical angles which are distorted by

TABLE 2h-1. DIMENSIONS OF THE REFERENCE ELLIPSOID*
(a = semimajor axis, $f = (a - b)/a$ = flattening, b = semiminor axis)

Author	Year	a, meters	$1/f$
Bouguer, Maupertuis	1738	6,397,300	216.8
Delambre	1800	6,375,653	334.0
Walbeck	1819	6,376,896	302.8
Airy	1830	6,376,542	299.3
Bessel	1841	6,377,397	299.15
Clarke	1866	6,378,206	295.0
Hayford	1910	6,378,388	297.0
Krassowski	1938	6,378,245	298.3
IAU adopted†	1964	6,378,160	298.25
Anderle‡	1967	6,378,144	298.23

* W. A. Heiskanen and F. A. Vening Meinesz, "The Earth and Its Gravity Field," p. 230, McGraw-Hill Book Company, New York, 1958.
† Cf. Sec. 2h-7.
‡ Cf. Sec. 2h-5.

refraction of the light path. As a consequence the triangulation computations must be based on a two-dimensional solution on the surface of a suitable reference ellipsoid. Once the observations have been made on points of the physical surface of the earth, the necessity arises to reduce these observations to the chosen reference ellipsoid. This reduction requires the deflection of the vertical (the small angle between the ellipsoid normal and the direction of gravity) and the height of the triangulation station above the ellipsoid. Some of these reduction corrections, being of small magnitude, were neglected in older triangulations but are at present considered significant in meeting modern accuracy requirements. The deflection at a triangulation station is obtained by observing astronomical latitude and longitude and comparing these data with the corresponding geodetic coordinates computed on the ellipsoid. Integrating the deflections along the path between two points and adding the difference in mean sea level elevations gives the difference in height above the reference ellipsoid between these two points. This approach is known as the *method of astrogeodetic deflections.*[1]

Since the deflections of the vertical are needed in the reduction of the observations to the ellipsoid, it is necessary that the geodetic coordinates—latitude and longitude—

[1] W. A. Heiskanen and H. Moritz: "Physical Geodesy," W. H. Freeman and Co., San Francisco, 1967.

be available, which can be computed only after the necessary reductions on the observations are made. Therefore an iterative procedure becomes necessary, an approach which is typical for the solution of many classical geodetic problems. In the course of such iterative steps, certain a priori assumptions are progressively modified. For example the condition of tangency of geoid and ellipsoid at the datum point may be relaxed and, at least in principle, the parameters of the originally chosen reference ellipsoid can be improved. However, despite the application of complex theoretical reduction methods, classical geodetic triangulation systems cannot establish ties between continents. Consequently triangulation systems on different continents have only partially related coordinate systems on, usually, different reference ellipsoids. Approximately a hundred different datums have been established in various parts of the earth, approximately eight of them being designated as major datums. One of these is the North American Datum (NAD 1927).

The quantities needed for reducing observations in triangulation nets can also be computed as functions of gravity anomalies. Furthermore, the gravimetric method (cf. Sec. 2h-6) also provides, at least in principle, a means for establishing a worldwide geodetic system by determining absolute geoidal undulations and deflections of the vertical with respect to a mass-centered reference ellipsoid.[1] Because of lack of sufficient observations over the oceans the usefulness of this method is impaired, particularly when considering modern accuracy requirements.

With the use of man-made satellites in geodesy the limitations of classical geodetic methods can be surmounted. In a strictly geometric method satellites serve as highly elevated target points for a three-dimensional triangulation of ground-based observation stations (cf. Sec. 2h-4). A dynamic interpretation of the observed satellite orbits leads to a simultaneous solution for the mass-center-referenced station coordinates, parameters of the orbital model, and certain gravitational parameters. Theoretical limitations arise from the necessary assumption that the effect of higher-order terms in the gravitational field is negligibly small. Practical difficulties result from the large number of unknowns solved for simultaneously, including, in addition to the geodetic parameters, nongravitational parameters for instance, for the air drag. The ultimate geodetic solution can therefore be expected when, in the foreseeable future, both the geometrically and dynamically obtained solutions are combined in a statistically significant result.

2h-4. Satellite Triangulation. The main objective of geometric satellite geodesy is the establishment of three-dimensional positions of a selected number of points on the physical surface of the earth. The significance of geometric satellite geodesy rests on the fact that, for the first time, such a spatial triangulation can be established on a worldwide basis with a minimum of a priori hypothesis; specifically without reference to either the direction or magnitude of the force of gravity. By simultaneously interpolating the satellite position, as seen from at least two observing stations, into the star background, the spatial directions are not only determined directly in terms of the astronomical system (cf. Sec. 2h-1) but are also interpolated in a physical sense into the astronomical refraction effect, thus providing a method essentially free of bias errors. This method—sometimes referred to as *stellar triangulation*[2,3]—is presently being applied in establishing a worldwide reference frame including some 40 stations, and, among other applications, in providing a precise spatial triangulation framework in the area of the North American Datum. Positional accuracy of one part per million

[1] *Ibid.*

[2] Y. Väisälä, An Astronomical Method of Triangulation, Helskinki, *Sitzber. Finn. Akad. Wiss.* **1947**.

[3] H. Schmid, Precision and Accuracy Considerations for the Execution of Geometric Satellite Triangulation, *Proc. 2d Intern. Symp. on Use of Artificial Satellites for Geodesy* II, Athens, Greece (1965).

is obtained for the worldwide triangulation net, and accuracies of ± 2 m are obtained for continental densification nets, where distances between stations are typically on the order of 1,000 to 1,500 km.

In common with all strictly geometric methods, satellite triangulation—executed as stellar triangulation or as a kind of three-dimensional trilateration, based on optical-electronic ranging—can only provide positions relative to an arbitrarily chosen origin. To obtain positions relative to the center of mass requires recourse either to potential theory, in the case of the gravimetric method, or to celestial mechanics in the dynamic method of satellite geodesy (cf. Sec. 2h-5).

2h-5. Dynamical Methods in Satellite Geodesy. The equations of motion of an artificial earth satellite are given by

$$\ddot{\mathbf{r}} = \mathbf{F}$$

where $\mathbf{r}$ is the position vector of the satellite in the geocentric equatorial coordinate system and $\mathbf{F}$ the force vector. $\mathbf{F}$ is a combination of individual terms

$$\mathbf{F} = \mathbf{F}_E + \mathbf{F}_{SM} + \mathbf{F}_D + \mathbf{F}_R$$

where $\mathbf{F}_E$ is the earth's gravitational effect, $\mathbf{F}_{SM}$ is the sun's and the moon's gravitational effects, $\mathbf{F}_D$ is the atmospheric drag effect, and $\mathbf{F}_R$ is the effect, due to solar radiation pressure.

Generally for earth satellites the terms $\mathbf{F}_{SM}$, $\mathbf{F}_D$, and $\mathbf{F}_R$ are small in comparison to $\mathbf{F}_E$ and can be computed from solar and lunar ephemerides and from models for the air drag and the radiation pressure.

The force term $\mathbf{F}_E$ is obtained as

$$\mathbf{F}_E = \text{grad } V$$

where V is the potential of gravitation of the earth, usually given by an expansion into spherical harmonics

$$V = \frac{GM}{r}\left[1 + \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left(\frac{a}{r}\right)^l \bar{P}_{lm}(\sin\varphi)(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda)\right]$$

G is the gravitational constant; M is the mass of the earth, r, φ, λ are polar coordinates in the geocentric equatorial coordinate system; a is the equatorial radius of the earth; $\bar{C}_{lM}$ and $\bar{S}_{lm}$ are normalized harmonic coefficients of degree l and order m; $\bar{P}_{lm}(\sin\varphi)$ is the associated Legendre functions, usually normalized in such a manner that

$$\frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\left[\bar{P}_{lm}(\sin\varphi)\begin{Bmatrix}\cos m\lambda \\ \sin m\lambda\end{Bmatrix}\right]^2 \cos\varphi \, d\varphi \, d\lambda = 1$$

If the harmonic coefficients in the expression for V are known, orbits of earth satellites can be computed by numerical integration of the equations of motion or by perturbation theories, provided the initial position and velocity of the satellite are given. If, on the other hand, satellite orbits are observed by means of photographic cameras or electronic tracking devices, the initial positions and velocities of satellites and the harmonic coefficients in the expression for V can be determined.[1,2] Because of the restricted number and accuracy of the observations, only the harmonic coefficients of

[1] I. I. Mueller, "Introduction to Satellite Geodesy," Frederick Ungar Publishing Co., New York, 1964.

[2] W. M. Kaula, "Theory of Satellite Geodesy," Blaisdell Publishing Co., a division of Ginn and Company, Waltham, Mass., 1966.

low degrees are computed. To diminish the correlation between the coefficients, satellite orbits with different orbital parameters are used in the solution. At the present time the most complete set of harmonic coefficients is published by the Smithsonian Institution and given in Table 2h-2. More complete sets exist, but are unpublished.

TABLE 2h-2. HARMONIC COEFFICIENTS IN THE EARTH'S GRAVITATIONAL POTENTIAL*

l	m	$\bar{C} \times 10^6$	$\bar{S} \times 10^6$
2	0	−484.1735	0
2	2	2.379	−1.351
3	0	0.9623	0
3	1	1.936	0.266
3	2	0.734	−0.538
3	3	0.561	1.620
4	0	0.5497	0
4	1	−0.572	−0.469
4	2	0.330	0.661
4	3	0.851	−0.190
4	4	−0.053	0.230
5	0	0.0633	0
5	1	−0.079	−0.103
5	2	0.631	−0.232
5	3	−0.520	0.007
5	4	−0.265	0.064
5	5	0.156	−0.592
6	0	−0.1792	0
6	1	−0.047	−0.027
6	2	0.069	−0.366
6	3	−0.054	0.031
6	4	−0.044	−0.518
6	5	−0.313	−0.458
6	6	−0.040	−0.155
7	0	0.0860	0
7	1	0.197	0.156
7	2	0.364	0.163
7	3	0.250	0.018
7	4	−0.152	−0.102
7	5	0.076	0.054
7	6	−0.209	0.063
7	7	0.055	0.096
8	0	0.0655	0
8	1	−0.075	0.065
8	2	0.026	0.039
8	3	−0.037	0.004
8	4	−0.212	−0.012
8	5	−0.053	0.118
8	6	−0.017	0.318
8	7	−0.0087	0.031
8	8	−0.248	0.102
9	0	0.0122	0
9	1	0.117	0.012
9	2	−0.0040	0.035
10	00	0.0118	0
10	01	0.105	−0.126
10	02	−0.105	−0.042
10	03	−0.065	0.030
10	04	−0.074	−0.111
11	00	−0.0630	0
11	01	−0.053	0.015
12	00	0.0714	0
12	01	−0.163	−0.071
12	02	−0.103	−0.0051
12	12	−0.031	0.0008
13	00	0.0219	0
13	12	−0.059	0.050
13	13	−0.059	0.077
14	00	−0.0332	0
14	01	−0.015	0.0053
14	11	0.0002	−0.0001
14	12	−0.094	−0.028
14	14	−0.014	−0.003
15	09	−0.0009	−0.0018
15	12	−0.0619	0.0578
15	13	−0.058	−0.046
15	14	0.0043	−0.0211

* *Smithsonian Astrophys. Observatory Spec. Rept.* 200, p. 2, Cambridge, Mass., 1966.

Satellite observations are not only a function of the initial position and velocity of the satellite, and the harmonic coefficients in the expansion of the earth's potential, but also a function of the coordinates of the tracking stations. Hence, together with the harmonic coefficients, these coordinates can be determined in the geocentric equatorial coordinate system used to formulate the equation of motion of the satellite.

With the coordinates of the tracking stations the dimensions of the reference ellipsoid and datum shifts are obtained. These shifts are needed to transform the coordinates of the various datums to the geocentric equatorial system. With the value

$$GM = 398{,}601 \text{ km}^3/\text{sec}^2$$

as determined by lunar probes, it was found[1] that

$$a = 6{,}378{,}144 \text{ m} \qquad \text{and} \qquad 1/f = 298.23$$

and the datum shifts are given in Table 2h-3.

2h-6. Physical Geodesy. Another method for determining the earth's gravity potential is given by the solution of the geodetic boundary-value problem. The earth's potential W, consisting of the potential V of gravitation and the potential of the centrifugal force, is separated into the potential U of a given reference ellipsoid, whose surface is an equipotential surface, and the disturbing potential T:

TABLE 2h-3. DATUM SHIFTS*

Datum	Rectangular coordinate shifts		
	Δx meters	Δy meters	Δz meters
North American 1927	−23	159	185
European	−81	−99	−118
Tokyo	−147	530	676
Old Hawaiian	52	−262	−183

* Anderle et al., *op. cit.*, p. 13.

$$W = U + T$$

T can be regarded as a harmonic function if the reference ellipsoid closely approximates the geoid and if the rotational axes of the ellipsoid and the earth and their angular velocities are identical.

The unknown potential T is connected with the gravity anomalies Δg, measured at the surface of the earth, by the boundary condition, which is given here with the relative error of the flattening of the earth:

$$\Delta g = -\frac{\partial T}{\partial H} - \frac{2T}{R} - \frac{2}{R}(U_0 - W_0)$$

H is the height (i.e., the normal height), R the mean radius of the earth, U_0 the potential at the surface of the ellipsoid, and W_0 the potential of the earth at mean sea level. If the mass of the reference ellipsoid equals the earth's mass,

$$U_0 - W_0 = -\frac{R}{8\pi}\iint \Delta g \cos\phi \, d\phi \, d\lambda$$

[1] R. J. Anderle, and S. J. Smith, "NWL-8 Geodetic Parameters Based on Doppler Satellite Observations," p. 7, U.S. Naval Weapons Laboratory, Dahlgren, Va., 1967.

The gravity anomalies Δg are computed by subtracting the gravity of the reference ellipsoid at height H above the ellipsoid from the gravity measured at the earth's surface at height H above sea level.

Usually gravity anomalies are referred to the International Ellipsoid whose gravity at its surface is defined by the International Gravity Formula adopted in 1930 by the International Association of Geodesy:

$$\gamma = 978.0490(1 + 0.005,2884 \sin^2 B - 0.000,0059 \sin^2 2B) \qquad \text{cm/sec}^2$$

γ is called the normal gravity, and B is the ellipsoidal latitude. Table 2h-4 shows the normal gravity from the equator to the pole. The value γ_H at height H above the ellipsoid is computed approximately by

$$\gamma_H = \gamma - 0.3086H \text{ cm/sec}^2 \qquad \text{with } H \text{ in km}$$

The units of gravity anomalies are usually milligals, abbreviated mgal $= 10^{-3}$ cm/sec^2.

The easiest way to obtain the disturbing potential T is by expressing T as the potential of a simple layer distributed over the surface of the earth. If T in the boundary condition is replaced by this expression, an integral equation is obtained, with the density of the surface layer as sought function and the gravity anomalies as absolute values. This integral equation has been derived by Molodenskii[1] who solved it by successive approximations. The first approximation T_0 of T is the well-known formula of Stokes,[2]

$$T_0 = \frac{R}{4\pi} \iint \Delta g\, S(\psi) \cos\phi\, d\phi\, d\lambda$$

$$\text{with } S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin\frac{\psi}{2} + 1 - 5\cos\psi - 3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)$$

This formula holds if the mass of the earth equals the mass of the reference ellipsoid and if both centers of mass coincide. ψ is the spherical distance between the fixed point where T_0 is computed and the variable point at the surface of the sphere with radius R on which the anomalies Δg are assumed to be given. Hence, by means of gravity anomalies the earth's gravitational potential can be computed.

The value of T/γ_H, where T is the value of the disturbing potential at the earth's surface, approximately equals the geoid undulation, i.e., the distance between the surface of the reference ellipsoid and the equipotential surface W_0 = const at mean sea level, the geoid (cf. Sec. 2h-1). The deflection of the vertical is found by differentiating the disturbing potential T in the horizontal direction. The horizontal derivative of Stokes' formula is known as Vening Meinesz' formula. Thus, by means of gravity anomalies we are able to compute geoid undulations and deflections of the vertical with respect to an ellipsoid whose mass is identical with the mass of the earth and whose center coincides with the mass center of the earth. By knowing the undulations and the deflections of the vertical for the different datums of the world, all datums can be shifted into one common system.

To determine the earth's potential from the solution of the geodetic boundary-value problem requires that the earth's surface be covered with gravity measurements. At present, huge parts of the earth, especially the oceans, are without gravity anomalies. Hence, the gravity measurements have to be combined with the results of satellite observations to improve the knowledge about the earth's gravity field. Either given gravity anomalies are expanded into spherical harmonics and compared

[1] M. S. Molodenskii, V. F. Eremeev, and M. I. Yurkina, "Methods for Study of the External Gravitational Field and Figure of the Earth," Israel Program for Scientific Translations, Jerusalem, 1962.

[2] M. Hotine, "Mathematical Geodesy," *ESSA Monograph* 2, Government Printing Office, October, 1969.

TABLE 2h-4. NORMAL GRAVITY FROM THE EQUATOR TO THE POLE: COMPUTED FROM THE INTERNATIONAL GRAVITY FORMULA

[$\gamma = 978.0490(1 + 0.0052884 \sin^2 B - 0.0000059 \sin^2 2B)$ cm/sec². Unit 1 milligal]

B, deg	Gravity	Difference	B, deg	Gravity	Difference	B, deg	Gravity	Difference
0	978,049.00		31	979,416.53	78.78	61	982,001.46	77.55
1	978,050.57	1.57	32	979,496.80	80.27	62	982,077.35	75.89
2	978,055.27	4.70	33	979,578.46	81.66	63	982,151.49	74.14
3	978,063.10	7.83	34	979,661.40	82.94	64	982,223.77	72.28
4	978,074.06	10.96	35	979,745.54	84.14	65	982,294.12	70.35
5	978,088.12	14.06	36	979,830.77	85.23	66	982,362.45	68.33
6	978,105.26	17.14	37	979,916.98	86.21	67	982,428.67	66.22
7	978,125.48	20.22	38	980,004.08	87.10	68	982,492.70	64.03
8	978,148.74	23.26	39	980,091.94	87.86	69	982,554.46	61.76
9	978,175.02	26.28	40	980,180.48	88.54	70	982,613.88	59.42
10	978,204.29	29.27	41	980,269.57	88.09	71	982,670.89	57.01
11	978,236.50	32.21	42	980,359.12	89.55	72	982,725.41	54.52
12	978,271.63	35.13	43	980,449.01	89.89	73	982,777.37	51.96
13	978,309.63	38.00	44	980,539.14	90.13	74	982,826.72	49.35
14	978,350.44	40.81	45	980,629.39	90.25	75	982,873.39	46.67
15	978,394.04	43.60	46	980,719.65	90.26	76	982,917.33	43.94
16	978,440.35	46.31	47	980,809.82	90.17	77	982,958.47	41.14
17	978,489.33	48.98	48	980,899.78	89.96	78	982,996.77	38.30
18	978,540.92	51.59	49	980,989.42	89.64	79	983,032.19	35.42
19	978,595.05	54.13	50	981,078.64	89.22	80	983,064.67	32.48
20	978,651.66	56.61	51	981,167.33	88.69	81	983,094.19	29.52
21	978,710.68	59.02	52	981,255.37	88.04	82	983,120.69	26.50
22	978,772.05	61.37	53	981,342.67	87.30	83	983,144.16	23.47
23	978,835.68	63.63	54	981,429.10	86.43	84	983,164.55	20.39
24	978,901.49	65.81	55	981,514.58	85.48	85	983,181.85	17.30
25	978,969.42	67.93	56	981,598.99	84.41	86	983,196.03	14.18
26	979,039.38	69.96	57	981,682.23	83.24	87	983,207.08	11.05
27	979,111.28	71.90	58	981,764.19	81.96	88	983,214.99	7.91
28	979,185.03	73.75	59	981,844.79	80.60	89	983,219.73	4.74
29	979,260.55	75.52	60	981,923.91	79.12	90	983,221.31	1.58
30	979,337.75	77.20						

with the harmonic coefficients found by satellite observations, or gravity anomalies are computed, using the harmonic coefficients obtained from satellites, and compared with given gravity anomalies, in order to compute corrected harmonic coefficients. The geoid map of Fig. 2h-1 and the gravity anomalies for 5° by 5° surface elements of Tables 2h-5 and 2h-6 were obtained by such a combination. Combination methods, using instead of the expansion into spherical harmonics the solution of the geodetic boundary-value problem to express the earth's potential, are under investigation.[1,2]

[1] K. Arnold, An Attempt to Determine the Unknown Parts of the Earth's Gravity Field by Successive Satellite Passages, *Bull. Géod.* no. 87, p. 97, Paris, 1968.

[2] Koch, K. R.: Alternate Representation of the Earth's Gravitational Field for Satellite Geodesy, *Boll. Geofisica teorica ed applicata* **10** (40) (1968).

TABLE 2h-5. 5° BY 5° MEAN GRAVITY ANOMALIES FROM A COMBINATION OF SATELLITE AND GRAVIMETRIC DATA REFERRED TO THE INTERNATIONAL GRAVITY FORMULA: EASTERN HEMISPHERE*

(Units milligals)

	0°						30°						60°						90°						120°						150°					180°
90°	6	7	2	3	3	3	3	3	2	3	3	3	−2	−3	2	2	3	3	4	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
	19	14	2	11	22	11	13	15	8	16	16	15	14	13	5	4	2	8	7	5	3	0	−2	−4	−5	−6	−5	−4	−2	0	3	6	8	11	14	11
	21	25	20	7	20	19	39	36	32	28	11	21	18	16	15	15	16	16	15	14	11	9	5	3	4	6	9	−8	18	22	24	25	25	21	−2	6
	10	17	27	18	12	9	29	20	17	17	4	7	12	8	4	2	−0	2	−12	−4	11	−4	0	−2	6	1	7	8	9	10	10	10	9	10	12	16
	55	67	32	19	8	7	12	22	19	7	3	6	13	1	3	−4	−10	−6	−2	−2	−2	−1	−5	5	8	11	12	14	15	13	10	8	11	18	23	24
	24	25	8	−2	−15	0	9	4	10	14	5	21	33	10	3	−5	−5	−10	−11	−9	−7	−6	−7	−9	−6	3	17	29	33	31	25	21	22	25	26	22
60°	5	18	10	−4	−6	3	1	12	11	18	17	20	13	3	4	2	−1	−1	−30	−35	−21	−20	−8	16	13	12	17	15	9	13	25	30	25	14	5	2
	−3	11	16	8	8	22	8	14	13	8	11	3	8	10	5	−13	−21	−2	−3	−27	−19	−15	−0	−0	−3	4	16	22	6	13	32	35	18	11	15	18
	9	23	26	31	18	16	16	5	10	−6	−6	−5	−9	−9	−15	−22	−18	−1	0	9	10	1	−5	1	8	7	5	8	6	16	6	10	3	4	10	6
	20	22	17	38	57	46	8	1	43	−11	4	1	−8	−28	−24	−11	−8	−0	−25	−26	23	−10	−8	−3	4	3	7	1	30	−1	−5	2	9	11	6	−3
	23	23	22	−5	−9	21	15	32	31	25	−5	−14	−25	−20	14	14	−10	−16	−1	18	−37	−15	−6	−11	2	13	17	35	19	17	−3	5	−16	−8	4	−6
	20	−14	18	2	−1	−35	−15	38	9	11	20	16	21	22	−45	25	5	62	39	39	16	−17	−26	−3	6	20	27	45	−25	3	−7	−9	−19	−13	−16	0
30°	−7	−24	−26	8	−7	−1	2	4	7	−2	−24	5	14	−3	1	−31	−49	−6	−58	−6	−14	−5	−7	1	1	10	−15	19	11	15	−7	−5	−8	−14	−10	8
	20	13	−2	−5	−12	−3	−3	13	19	−19	−24	17	−18	−3	4	6	−4	−8	−21	−17	−14	−11	1	3	13	5	7	1	3	−4	−3	1	−8	−11	−10	−10
	12	10	−15	−10	−3	−4	4	5	29	−11	−9	3	−12	−2	−20	−22	1	−30	−11	−8	−16	−10	7	13	41	9	−2	−0	3	−0	−0	−4	−5	−11	−2	8
	1	4	31	−6	4	7	9	10	6	−5	−6	−11	−15	−29	−31	−21	−8	−32	−16	−1	−5	−1	16	15	32	2	4	−6	−2	−22	−4	−11	−11	−2	−20	21
	16	7	11	−6	−14	3	3	13	−0	3	−51	−7	−11	−28	−45	−20	−8	−23	−30	3	3	7	−1	16	31	2	15	48	8	12	3	2	−6	0	−5	8
	−20	1	4	−14	−20	3	−3	−1	−34	−26	−28	−19	−21	−31	−42	−47	−50	−36	−16	9	14	18	19	27	28	19	52	3	15	10	3	9	−1	4	2	3
0°	−11	−1	−17	−27	−38	7	−4	22	−29	−22	−12	−16	−23	−30	−42	−49	−50	−51	−14	−1	22	32	29	28	4	11	25	16	17	33	13	22	9	10	9	−7
	−10	−28	−8	−12	−6	−3	−2	0	−31	−22	1	−6	−15	−10	−9	−30	−28	−11	−13	2	−3	12	18	11	−2	−18	−12	15	26	2	−20	21	20	−25	−1	7
	−14	−25	−7	9	−12	−10	1	−22	−21	2	−10	−14	−15	−4	−8	−25	−9	2	−11	5	8	2	−9	−14	−7	9	28	20	17	15	11	23	37	9	12	10
	−1	3	−2	−0	−4	−0	4	16	−14	7	−8	12	9	−4	−13	−12	−34	−25	−25	−34	−10	−14	−4	11	9	21	6	5	27	11	53	8	4	21	17	19
	7	8	8	17	−7	5	−10	−5	4	13	−3	11	25	35	19	−4	−6	−5	−3	−34	−28	−17	2	12	−6	−16	−14	0	12	24	10	5	36	23	22	21
	14	−1	3	23	8	26	17	1	3	−4	−16	1	12	13	31	3	−1	−4	−9	−11	−18	−32	4	10	4	−12	−24	−9	−0	2	20	6	11	21	18	11
−30°	8	13	14	12	30	18	13	2	5	9	11	17	2	15	32	7	−2	−3	−1	−25	−27	−32	−31	−3	−45	−25	−15	4	1	22	16	9	25	14	17	22
	5	7	10	13	12	13	21	7	2	3	9	14	14	20	13	3	−7	5	−6	−12	−24	−42	−15	−12	−22	−25	−14	1	8	22	21	0	2	9	27	14
	4	10	11	9	15	17	19	19	15	20	18	8	16	23	10	19	17	15	11	−0	−4	−6	−10	−12	−12	−9	−10	−14	9	27	8	−8	−6	−11	−3	3
	8	7	8	11	11	9	16	23	21	21	18	16	29	26	35	30	21	18	17	12	4	−4	−9	−11	−11	−3	4	−0	−1	6	2	−8	−4	7	−2	10
	25	16	14	22	24	20	21	25	24	23	23	20	19	28	32	30	20	11	11	13	9	4	3	3	−2	−1	5	9	9	7	−0	3	0	−2	13	2
	11	8	7	9	13	15	17	19	29	21	21	19	18	21	33	33	23	13	12	13	11	7	6	5	4	4	5	4	1	1	2	1	−4	−12	−16	−16
−60°	3	5	6	8	12	16	19	17	14	13	11	8	2	−1	1	5	6	6	5	5	6	6	4	−1	−4	−4	−1	−3	−6	−6	0	6	4	−2	−5	−3
	−6	−8	−10	−12	−12	−10	−5	36	15	29	38	37	28	15	10	14	23	21	13	28	26	25	21	23	20	19	19	9	−2	8	2	−3	−7	−12	−16	−18
	50	58	62	60	54	45	37	33	32	34	34	32	25	16	11	9	11	12	23	14	11	10	−37	7	6	5	4	−52	−11	−17	−23	−24	3	3	−16	−7
	15	14	11	7	3	−1	−4	−5	−4	−2	2	6	9	12	14	15	37	21	17	17	−15	1	11	7	4	0	−4	−35	−37	−35	−29	21	−36	−9	−13	−13
	−4	−6	−6	−5	−3	0	5	10	16	22	28	32	27	22	36	31	31	27	23	18	14	11	8	7	−25	−27	−19	−16	8	8	7	6	3	−0	−21	−16
	−8	5	5	5	5	5	6	6	6	6	6	6	6	6	5	5	1	4	−8	1	1	−10	5	6	7	9	10	−9	−11	−8	6	−7	−4	0	11	20
−90°																																				

* R. H. Rapp, Comparison of Two Methods for the Combination of Satellite and Gravimetric Data, *Ohio State Univ. Dept. Geod. Sci. Rept.* 113, p. 26, 1968.

Table 2h-6. 5° by 5° Mean Gravity Anomalies from a Combination of Satellite and Gravimetric Data Referred to the International Gravity Formula: Western Hemisphere*

(Units milligals)

Longitude 180°–210°

Latitude						
90°–60°	3	−0	−3	−3	6	0
	14	11	6	7	10	7
	16	−6	11	9	6	3
	20	24	6	9	11	−3
	22	19	3	−1	18	23
	19	8	7	19	24	16
60°–30°	7	14	18	42	29	29
	−18	−20	3	−16	−3	16
	8	6	13	15	9	12
	−7	−11	−8	−2	3	1
	−6	−9	−13	−12	−6	−6
	−6	−7	−37	−11	7	2
30°–0°	−13	−1	8	9	2	0
	−5	2	11	36	26	3
	−7	−4	−8	−3	2	−7
	17	−12	−1	1	9	−16
	−23	−8	−2	22	21	−12
	5	−7	−7	21	1	−8
0°–−30°	−1	−7	−5	−8	−0	−2
	−2	−6	−7	7	12	10
	11	2	−6	6	3	14
	7	4	−1	5	9	13
	−10	−16	0	5	5	4
	10	9	4	3	−3	−2
−30°–−60°	−20	4	8	11	4	−2
	−2	4	9	9	7	7
	9	9	7	8	10	8
	3	−0	0	7	8	4
	−7	−8	−6	−4	−6	−3
	−12	−10	−10	−10	−7	−2
−60°–−90°	−1	−2	−6	−7	−6	−4
	−18	−18	−17	−15	−14	−15
	−13	−19	−24	−28	−31	−32
	−18	−18	−22	−28	−12	−15
	−19	−18	−15	−19	−14	−12
	−9	20	20	20	−11	20

Longitude 210°–240°

Latitude						
90°–60°	6	4	8	10	13	15
	14	12	12	11	9	11
	2	5	9	16	22	13
	7	2	9	15	5	6
	24	26	10	−1	−6	1
	42	59	17	5	−10	−13
60°–30°	12	10	4	9	1	−0
	8	6	−1	−7	−36	3
	0	−5	−9	−5	−8	15
	−3	−14	−15	−5	−6	22
	−5	−13	−18	−25	−21	−8
	0	−8	−5	−12	−15	−20
30°–0°	10	10	−7	−21	−25	−26
	−7	−8	−12	−16	−18	−19
	−2	−14	−14	−29	−19	−23
	−3	−2	−5	−8	−18	10
	−14	−10	−5	−4	−11	−12
	−8	−6	−2	−2	−3	−7
0°–−30°	−6	−8	−6	−5	−1	−4
	−1	−1	−7	−5	−7	−4
	−2	−3	−13	−3	−3	2
	6	8	−3	−5	−8	−1
	−2	−8	−6	−1	3	0
	−0	−6	−4	−6	−2	−4
−30°–−60°	−3	−6	−3	−3	−1	−5
	7	−1	−2	−2	−1	−2
	6	3	4	0	−5	−1
	4	2	1	1	−1	1
	3	4	2	3	5	6
	1	−1	−1	3	8	10
−60°–−90°	−6	−7	−6	−2	0	1
	−16	−16	−13	−10	−8	−7
	−20	−24	−16	−8	−1	4
	−8	−22	−16	−5	13	5
	−10	−7	−5	−19	−2	−22
	21	20	20	20	20	−6

Longitude 240°–270°

Latitude						
90°–60°	14	12	10	9	8	13
	34	35	18	21	21	26
	27	33	33	33	29	31
	25	9	15	10	8	17
	−9	2	−10	3	2	1
	−13	−11	4	4	0	−3
60°–30°	2	3	−9	0	−11	−4
	27	6	11	19	7	3
	3	22	18	22	11	11
	14	24	27	13	4	−3
	6	5	22	−1	−6	−3
	−12	−10	7	−0	−1	2
30°–0°	−15	−19	2	26	−4	2
	−21	−11	−12	17	−4	7
	−15	−10	−5	1	23	5
	−22	2	−3	−4	0	4
	−14	−1	−12	−1	0	1
	−7	−3	−6	−2	−3	6
0°–−30°	−6	−8	−5	2	−5	2
	−5	−4	−3	−4	1	−5
	−4	2	1	0	2	−2
	−3	2	−7	2	1	−0
	−4	1	−5	4	−2	8
	−1	5	1	8	−4	6
−30°–−60°	−3	7	3	8	6	4
	−1	7	5	5	8	10
	3	3	5	8	9	12
	6	8	8	6	5	9
	8	8	5	5	10	13
	11	9	6	5	9	11
−60°–−90°	2	4	6	8	9	11
	−6	−3	1	3	2	−1
	7	8	10	11	−6	27
	1	−9	−18	−11	−12	13
	−2	−23	−1	17	17	13
	20	20	20	20	20	−23

Longitude 270°–300°

Latitude						
90°–60°	16	12	4	13	2	2
	28	16	16	27	34	6
	31	32	29	13	10	39
	3	12	−5	−2	0	−3
	2	5	9	4	1	2
	−7	−10	−5	−7	−7	−2
60°–30°	1	3	−33	−22	−5	11
	−10	−15	−9	−6	17	0
	−8	−5	−10	−11	−6	3
	−5	−2	−7	10	10	−15
	1	−0	6	−16	−15	−10
	−0	11	−6	−31	−18	−2
30°–0°	12	23	−27	−28	−24	−18
	22	−2	−5	−58	−27	−23
	15	−7	−20	4	−49	−16
	18	22	−11	−28	−36	−18
	12	17	34	12	18	−8
	9	5	31	24	23	11
0°–−30°	9	3	11	24	19	4
	−9	−6	13	3	−7	2
	2	0	−8	26	18	13
	−3	−6	−20	−5	44	21
	−12	21	−12	−2	48	−8
	−10	13	−4	21	27	16
−30°–−60°	−2	9	5	2	10	15
	9	4	5	13	10	9
	14	0	7	11	7	3
	12	3	0	28	5	14
	7	1	29	6	2	6
	8	5	6	7	9	2
−60°–−90°	12	9	6	7	14	25
	8	−2	5	17	31	21
	20	12	15	40	40	32
	16	11	15	22	13	−10
	7	−9	−6	−4	18	−10
	−9	19	18	18	13	7

Longitude 300°–330°

Latitude						
90°–60°	2	3	2	2	2	1
	37	38	39	38	37	34
	33	15	−14	−11	−15	22
	1	4	22	20	24	27
	4	−17	11	46	23	36
	−4	−17	−27	39	26	28
60°–30°	−2	−2	11	21	24	25
	−22	20	11	12	22	30
	11	17	33	18	39	13
	−20	0	17	19	22	23
	−8	−7	−3	12	23	35
	−9	−7	−2	17	19	23
30°–0°	−28	−19	11	24	−1	−10
	−28	−13	3	−4	−11	−13
	−27	−26	−16	−17	−20	−9
	−50	−29	−34	−13	−14	−17
	−17	−22	−37	−29	−8	4
	−1	−4	−11	−19	−7	4
0°–−30°	−9	−6	−5	−15	−7	−1
	−7	−10	−36	−34	1	−19
	5	−11	4	−9	−11	−10
	7	−2	−5	10	−11	−13
	−7	−18	−7	−11	−16	−11
	10	8	−2	−21	−23	−8
−30°–−60°	11	−0	16	−9	−1	11
	16	3	1	4	−5	−3
	6	−4	−3	8	3	−1
	−1	−5	−4	0	0	−2
	−1	15	15	16	16	12
	0	1	8	10	18	20
−60°–−90°	30	25	16	12	15	18
	30	17	4	−3	−1	1
	21	9	−1	−6	−6	−4
	8	3	0	−14	−13	2
	−22	−9	−14	7	7	7
	6	−14	−19	13	−27	11

Longitude 330°–0°

Latitude						
90°–60°	0	−1	1	1	0	1
	30	24	16	12	16	13
	19	48	40	17	31	15
	23	22	24	23	24	8
	34	31	37	47	28	19
	20	49	31	49	40	42
60°–30°	30	27	10	14	23	21
	16	20	23	33	24	17
	15	11	5	−5	−5	13
	25	14	13	3	14	24
	36	16	−2	11	30	19
	16	−5	14	−4	43	52
30°–0°	−10	−7	8	24	10	−2
	−15	−8	−1	4	−7	−11
	−4	14	2	14	−3	11
	−12	−10	15	21	5	6
	1	3	5	16	29	18
	6	2	−4	0	7	19
0°–−30°	−6	−8	−4	5	10	1
	−7	−10	−4	2	0	−2
	−13	−12	−5	−4	−4	−4
	−9	−12	−4	−0	4	−7
	−10	−6	−4	5	0	−9
	−13	−5	−1	6	7	−2
−30°–−60°	0	0	6	4	0	16
	−5	−3	−4	5	3	9
	−2	−3	4	8	8	5
	−2	2	9	12	11	9
	9	10	14	18	23	28
	16	13	15	16	15	13
−60°–−90°	17	14	12	10	6	3
	1	−3	−6	−8	−7	−6
	−1	2	6	11	25	38
	4	7	9	11	13	14
	6	5	4	2	0	−2
	10	9	8	−22	7	−26

* Rapp, *op. cit.*, p. 27.

2h-7. Geodetic Reference System: 1967. In 1967 the General Assembly of the International Union of Geodesy and Geophysics recommended replacing the International Ellipsoid and the International Gravity Formula with the Geodetic Reference System 1967 defined by[1]

$$a = 6\,378\,160 \text{ m}$$
$$GM = 398\,603 \text{ km}^3/\text{sec}^2$$
$$J_2 = 10\,827 \times 10^{-7}$$

with $J_2 = -\sqrt{5}\,\bar{C}_{20}$. This set of parameters is identical with the parameters adopted by the International Astronomical Union in 1964 as part of a system of new

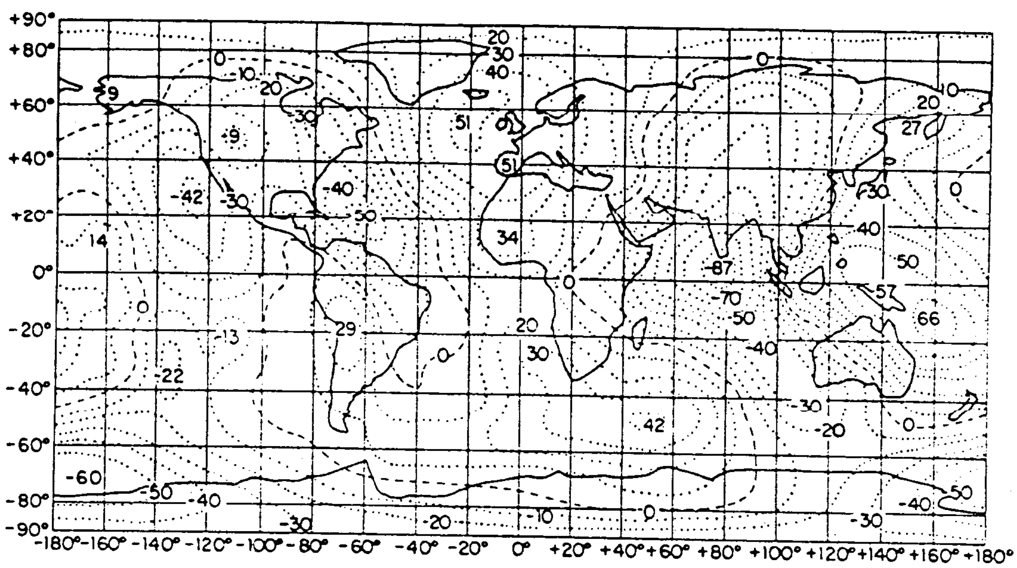

FIG. 2h-1. Geoid obtained by combining satellite and gravimetric data. Units: meters. (*W. Kohnlein, Smithsonian Astrophysical Observatory Special Report* 264, *p.* 57, 1967.)

astronomical constants. The values for a, GM, and J_2, together with the value for the earth's rotational velocity, define an equipotential ellipsoid of revolution completely, so that the shape of the ellipsoid and its external gravity field are determined by the four constants. Only preliminary numerical values for the shape of the ellipsoid and the gravity formula of the Geodetic Reference System 1967 have been published until now.[2,3]

[1] *Bull. Geod.* no. 86, p. 367, Paris, 1967.
[2] A. H. Cook, The Polar Flattening and Gravity Formula in the Geodetic Reference System 1967, *Geophys. J.* **15**, p. 431, Oxford, 1968.
[3] H. Moritz, "The Geodetic Reference System 1967," *Allgem. Vermess.*, p. 2, Karlsruhe, 1968.