2i. Seismological and Related Data

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2i-1. List of Symbols

V	velocity of longitudinal wave P
v	velocity of transverse wave S
P	symbol denoting longitudinal wave
S	symbol denoting transverse wave
k	bulk modulus or volume elasticity
μ	rigidity or shear modulus
ρ	density
τ	Poisson's ratio
A	ratio V/v
t	temperature in degrees centigrade, time
p	pressure in bars
h	depth in the earth
T	period of seismic disturbance
G	symbol denoting surface shear waves
R_a	symbol denoting Rayleigh waves
Δ	epicentral distance
SH	symbol denoting component of S wave in horizontal plane
SV	symbol denoting component of S wave in vertical plane
i	actual angle of incidence at a discontinuity
$\bar{i}$	apparent angle of incidence at a discontinuity
u	ratio of horizontal ground displacement to incident amplitude

2i-2. Fundamental Equations for Elastic Constants and Wave Velocities. In purely elastic, isotropic, homogeneous media the velocity V of longitudinal waves P, v of transverse waves S, the bulk modulus k, the rigidity μ, the density ρ, and Poisson's ratio σ are connected by the following equations:

$$V^2 = \frac{k + \frac{4}{3}\mu}{\rho} \qquad v^2 = \frac{\mu}{\rho} \tag{2i-1}$$

$$\sigma = \frac{\frac{1}{2}A^2 - 1}{A^2 - 1} \qquad A = \frac{V}{v} \tag{2i-2}$$

$$k = \rho(V^2 - \tfrac{4}{3}v^2) \qquad \mu = v^2\rho \tag{2i-3}$$

[1] Deceased.

2i-3. Elastic Constants and Wave Velocities in Rocks (Laboratory Experiments). In rocks the elastic constants and the wave velocities usually increase with increasing pressure p (Tables 2i-2 and 2i-3) and decrease with increasing temperature t and with porosity. Phase changes affect all elastic quantities. Many sedimentary rocks show significant anisotropy, with an axis of symmetry. Table 2i-4 gives an example of velocity differences for vertical and horizontal travel[1] and for shear polarization.

TABLE 2i-1. CORRESPONDING VALUES OF POISSON'S RATIO σ AND V/v

σ	0.00	0.10	0.20	0.22	0.24	0.25	0.26	0.28	0.30	0.40	0.50
V/v	1.414	1.500	1.633	1.670	1.710	1.732	1.756	1.809	1.871	2.449	∞

TABLE 2i-2. ELASTIC CONSTANTS AND WAVE VELOCITIES IN ROCKS AT ROOM TEMPERATURE†

	μ, 10^{11} dynes/cm^2		k, 10^{11} dynes/cm^2		σ	V, km/sec	v, km/sec
	1 atm	4,000 atm	1 atm	4,000 atm			
Dunite	$4\frac{3}{4}$–6	$6\frac{1}{2}$–7	?	12 ±	0.25–0.30	$7\frac{1}{2}$–$8\frac{1}{2}$	$4\frac{1}{4}$–$4\frac{3}{4}$
Gabbro	3–4	4–5	6 ±	$8\frac{3}{4}$ ±	0.2–0.3	5–7	$3\frac{1}{2}$–4
Granite	$1\frac{1}{2}$–$2\frac{1}{2}$	$3\frac{1}{4}$–$3\frac{1}{2}$	$2\frac{3}{4}$–$3\frac{1}{2}$	$5\frac{1}{4}$ ±	0.20–0.26	5–$6\frac{1}{4}$	2–$3\frac{1}{2}$
Obsidian glass	$2\frac{3}{4}$–3	?	$3\frac{1}{4}$ ±	$3\frac{3}{4}$–4	0.1–0.2?	5 ±	$3\frac{1}{2}$ ±
Ice	$\frac{1}{4}$–$\frac{1}{2}$	?	$\frac{3}{4}$–1	?	0.3–0.4	$3\frac{1}{4}$–$3\frac{3}{4}$	$1\frac{1}{2}$–2

† F. Birch, ed., Handbook of Physical Constants, *Geol. Soc. Am., Spec. Paper* 36 (1942); L. H. Adams, Elastic Properties of Materials of the Earth's Crust, in "Internal Constitution of the Earth," 2d ed., pp. 50–80, 1951. See also S. P. Clark, Jr., ed., Handbook of Physical Constants, rev. ed., *Geol. Soc. Am., Mem.* 97 (1966).

TABLE 2i-3. LONGITUDINAL VELOCITIES, KM/SEC, AT PRESSURES p AND TEMPERATURES t CORRESPONDING TO THE DEPTH h IN THE EARTH AFTER LABORATORY MEASUREMENT†

p, bars	t, °C	h, km	Dunite	San Marcos gabbro	Texas gray granite	Woodbury granite
260	45	1	7.55	6.70	5.90	5.90
1,300	135	5	7.50	6.90	6.02	6.15
2,600	225	10	7.22	6.96	6.02	6.14
3,900	290	15	....	6.95	6.01	6.04
6,700	400	25	....	6.80		

† D. S. Hughes and C. Maurette, Variation of Elastic Wave Velocities in Basic Igneous Rocks with Pressure and Temperature, *Geophysics* **22**, 23–31 (1957).

2i-4. Periods and Amplitudes of Seismic Waves. Seismological instrumentation has made great advances in fidelity of observation, geographic distribution of stations, and machine data reduction. Strain seismometers have uniform sensitivity from periods of many hours down to a few seconds. Tilt meters and gravimeters also

[1] J. E. White and R. L. Sengbush, Velocity Measurements in Near-surface Formations, *Geophysics* **18**, 54 (1963).

indicate earth motion down to "dc," i.e., periods much greater than the tidal period. A worldwide net of 125 stations has been established, recording three-component motion in 0.1-to-1-sec range and 10-to-100-sec range. A few array stations exist at which signals from dozens of seismometers in an array can be combined. This improved instrumentation gives an improved portrayal of earthquakes and more accurate knowledge of the structure of the earth.

Earthquakes create permanent displacements, which may be observed at great distances.[1] Great earthquakes excite the free oscillations of the earth to measurable amplitudes,[2] at periods of 3 to 54 min. Love waves and Rayleigh waves in the period range 10 to 100 sec are governed by velocity contrasts in the crust and mantle. Body waves display periods of 0.1 to 10 sec, depending on range, with shear waves tending to longer periods than compressional waves.

TABLE 2i-4. VELOCITIES IN SHALLOW SEDIMENTS, KM/SEC

	V vert.	V horiz.	v_{SV} vert.	v_{SH} horiz.
Chalk	2.6	3.0	1.1	1.2
Shale	1.8	2.4	0.4	0.6

Periods of natural microseisms (continuous motion from meteorological sources and ocean waves) range from a fraction of a second to a minute or more. The largest amplitudes of the most frequent types of microseisms (periods 4 to 10 sec) are a few microns at inland stations on rock and between 10 and 100 microns at stations near oceans during heavy storms.

After great earthquakes, waves through the earth's interior may reach the surface at great distances with amplitudes of over 10 microns and periods of the order of 5 sec, while the largest surface waves may have ground amplitudes of 10 mm with periods of 20 sec. Much greater amplitudes occur near the source. In motion from not too close artificial explosions, longitudinal waves usually carry the largest amplitudes; even waves through the earth's core have been identified on such records.[3]

2i-5. Travel Times of Earthquake Waves. Examples of travel times are given in Table 2i-5. Surface waves traveling a few times around the earth have travel times of several hours. No dispersion has been established for waves through the earth's body except for waves through the transition zone from the liquid outer core to the probably solid inner core.[4] However, the prevailing increase in the velocity of longitudinal and transverse waves with depth results in a prevailing increase in wave velocity of surface waves as their length (depth of energy penetration) increases. Surface waves of first, second, and third modes have been observed. The group velocity of surface waves of first mode has a minimum[5] for periods of several seconds, depending on the crustal structure.

[1] C. J. Wideman and M. W. Major, Strain Steps Associated with Earthquakes, *Bull. Seis. Soc. Am.* **57**, 1429 (1967).

[2] L. E. Alsop, Spheroidal Free Periods of the Earth Observed at Eight Stations around the World, *Bull. Seis. Soc. Am.* **54**, 755 (1964).

[3] B. Gutenberg, Travel Times of Longitudinal Waves from Surface Foci, *Proc. Natl. Acad. Sci. U.S.* **39**, 849 (1953).

[4] B. Gutenberg, Wave Velocities in the Earth's Core, *Bull. Seis. Soc. Am.* **48**, 301–314 (1958).

[5] M. Ewing and F. Press, Crustal Structure and Surface-wave Dispersion, *Bull. Seis. Soc. Am.* **40**, 271–280 (1950); **42**, 315–325 (1952); **43**, 137–144 (1953). Surface Waves and Guided Waves, "Encyclopedia of Physics," vol. 47, pp. 119–139, Springer-Verlag, Berlin, 1956.

2i-6. Reflection and Refraction of Waves. If a longitudinal wave P or a transverse wave S arrives at a discontinuity, one P and one S wave are reflected and one of each type is refracted if the velocity ratio V_r/V_i of the reflected or refracted (r) and incident (i) wave permits.[1]

$$\sin i_r = \frac{V_r}{V_i} \sin i_i \tag{2i-4}$$

where i_i is the angle of incidence. Examples are given in Table 2i-6. Amplitudes of transverse waves (vibrations perpendicular to the ray) are frequently resolved into two components, SH in the horizontal plane, and SV (with a vertical component) perpendicular to SH. If an SH wave is incident, the reflected wave and the refracted wave (if it exists) are always of the SH type.

TABLE 2i-5. TRAVEL TIMES† (MIN:SEC) OF DIRECT LONGITUDINAL WAVES P AND TRANSVERSE WAVES S THROUGH THE EARTH STARTING AT DEPTH h, AND OF SURFACE SHEAR WAVES G AND RAYLEIGH WAVES R_a WITH PERIODS OF ABOUT 1 MIN (INDEPENDENT OF FOCAL DEPTH)
(Δ = epicentral distance, deg; P waves arriving at $\Delta > 100$ deg enter the earth's core)

Δ	h = 25 km		G, min	R_a, min	h = 300 km		h = 700 km	
	P	S			P	S	P	S
0	0:04	0:07	....	....	0:39	1:08	1:20	2:24
2	0:32	0:55	....	....	0:46	1:24	1:24	2:30
4	0:59	1:56	....	....	1:07	1:51	1:32	2:48
10	2:28	.....	4.1	4.5	2:17	4:03	2:20	4:12
20	4:34	8:16	8.3	9.0	4:15	7:39	3:55	7:02
40	7:36	13:42	16.5	17.9	7:11	12:52	6:44	12:01
70	11:12	20:20	28.9	31.4	10:44	19:21	10:11	18:20
100	13:46	25:14	41.3	44.8	13:15	24:23	12:37	23:14
120	18:54	28:00	49.5	53.8	18:19	27:09	17:38	26:01
150	19:46	.....	61.9	67.2	19:11	.....	18:31	
180	20:10	.....	74.2	80.6	19:35	.....	18:54	

† B. Gutenberg, Travel Times of Longitudinal Waves from Surface Foci, *Proc. Natl. Acad. Sci, U.S.* **39**, 849 (1953); H. Jeffreys and K. E. Bullen, "Seismological Tables," British Association for the Advancement of Science, 1940; B. Gutenberg, and C. F. Richter, On Seismic Waves, *Gerlands Beitr. Geophys.* **43**, 56–133 (1934); **54**, 94–136 (1939).

If a wave arrives at the earth's surface (actual angle of incidence i) a wave of the same type is reflected (angle i), and one of the other type may be reflected [Eq. (2i-4)] (see Table 2i-7). As a consequence of these three waves, the apparent angle of incidence $\bar{\imath}$ calculated from records of horizontal H and vertical V instruments ($\tan \bar{\imath} = H/V$) differs from i. In case of incident transverse waves the particles move in ellipses,[2] if $(V \sin i)/v > 1$. If an SH wave is incident, the reflected wave has the same amplitude as the incident wave, the ground displacement is twice the incident amplitude, and $\bar{\imath} = i$. For energy ratios of waves reflected and refracted at the boundary of the earth's core, see Table 2i-8. An SH wave incident upon the core is totally reflected.

[1] M. Ewing, W. S. Jardetzky, and F. Press, "Elastic Waves in Layered Media," pp. 74–93, McGraw-Hill Book Company, New York, 1957; B. Gutenberg, Energy Ratio of Reflected and Refracted Seismic Waves, *Bull. Seis. Soc. Am.* **34**, 85–102 (1944).

[2] B. Gutenberg, SV and SH, *Trans. Am. Geophys. Union* **33**, 573–584 (1952).

TABLE 2i-6. SQUARE ROOT OF ENERGY REFLECTED OR TRANSMITTED AT A DISCONTINUITY WITH DENSITY RATIO (UPPER LAYER TO LOWER) 1.103, CORRESPONDING VELOCITY RATIO 1.286 FOR P AND FOR S, POISSON'S RATIO 0.25 IN BOTH LAYERS

(Incident energy taken as unity. Based on Slichter-Gabriel.† 1– indicates values between 0.95 and 1.0. i = angle of incidence. P = longitudinal, SV = component of transverse wave in plane of ray)

i°	Refracted waves								Reflected waves							
	P from				SV from				P from				SV from			
	Above		Below		Above		Below		Above		Below		Above		Below	
	P	SV	P	SV	P	SV	P	SV	P	SV	P	SV	P	SV	P	SV
0	1–	0.0	1–	0.0	0.0	0.2	0.0	1–	0.2	0.0	0.2	0.0	1.0	0.0	0.0	0.2
15	1–	0.1	1–	0.1	0.1	0.1	0.1	1–	0.2	0.1	0.2	0.1	1–	0.1	0.1	0.1
30	1–	0.1	1–	0.1	...	0.2	0.2	1–	0.1	0.1	0.1	0.1	0.9	0.2	0.1	0.0
45	0.5	0.2	0.9	0.1	...	0.4	0.3	1–	0.2	0.0	0.1	0.1	0.9	0.3	...	0.2
60	...	0.3	0.9	0.2	...	...	...	1–	0.9	0.1	0.2	0.1	...	...	...	0.3
75	...	0.4	0.8	0.3	...	...	...	0.8	0.9	0.1	0.4	0.1	...	...	...	0.5
90	...	0.0	0.0	0.0	...	...	...	0.0	1.0	0.0	1.0	0.0	...	...	...	1.0

† B. Gutenberg, *Bull. Seis. Soc. Am.* **34**, 85 (1944).

TABLE 2i-7. SQUARE ROOTS OF RATIO OF REFLECTED TO INCIDENT ENERGY a AT EARTH'S SURFACE AS FUNCTION OF ANGLE OF INCIDENCE i AND RATIO OF HORIZONTAL u AND VERTICAL w GROUND DISPLACEMENTS TO INCIDENT AMPLITUDE FOR CONTINUOUS SINUSOIDAL WAVES IF POISSON'S RATIO IS 0.25; $\bar{i}$ = APPARENT ANGLE OF INCIDENCE CALCULATED FROM OBSERVED HORIZONTAL AND VERTICAL COMPONENTS

(Elliptic motion of ground is indicated by *, and corresponding values for $\bar{i}$ are calculated on the assumption that the vertical and horizontal component reach their maximum simultaneously.† SV = component of transverse wave in plane of ray)

i	Longitudinal wave P incident					SV incident				
	a of P	a of SV	u	w	$\bar{i}$, deg	a of P	a of SV	u	w	$\bar{i}$, deg
0°	1.0	0.0	0.0	2.0	0	0.0	1.0	2.0	0.0	0
20	0.8	0.6	0.8	1.9	23	0.9	0.4	1.8	0.8	23
30	0.6	0.8	1.2	1.7	34	1.0	0.0	1.7	1.0	30
35.3	0.5	0.9	1.3	1.5	39	0.0	1.0	4.9	0.0	±0
40	0.4	0.9	1.4	1.4	44	...	1.0	0.7*	1.6*	−64*
45	0.3	0.9	1.5	1.3	48	...	1.0	0.0	1.4	±90
60	0.0	1.0	1.7	1.0	60	...	1.0	0.5*	1.1*	66*
80	0.1	1.0	1.3	0.5	69	...	1.0	0.3*	0.5*	59*
90	1.0	0.0	0.0	0.0	71	...	1.0	0.0*	0.0*	60*

† B. Gutenberg, SV and SH, *Trans. Am. Geophys. Union* **33**, 573–584 (1952).

2i-7. Wave Types and Their Symbols. The main discontinuities of the earth (Fig. 2i-1) are its surface, the "Mohorovičić discontinuity" (depth 10 ± km below the surface in the deeper parts of the major oceans, 30 ± km under the lower parts of continents, up to about 70 km under high mountain ranges, e.g. North Pamir[1]), and the boundary of the earth's core at a depth of 2,900 ± 10 km (radius r = 3,470 km). The transition from the outer to the inner core is probably gradual. At a distance of about 1,500 km from the earth's center, the velocity of longitudinal waves begins to increase more rapidly with depth than in the outer core but becomes approximately constant about 300 km deeper. This transition zone between the outer and the inner core may correspond to a transition from the liquid to the solid state.

TABLE 2i-8. SQUARE ROOTS OF ENERGY RATIOS FOR WAVES REFRACTED (REFR.) AND REFLECTED (REFL.) AT THE BOUNDARY OF THE EARTH'S CORE†

[Assumed at the core boundary: densities 5.4 (mantle), 10.1 (core); longitudinal velocities 13.7 and 8.0 km/sec, respectively; transverse velocity in the mantle 7.25 km/sec, 0 in core. i = angle of incidence of the arriving wave]

P incident in mantle				*P* incident in core				*SV* incident in mantle			
i	Refr. *P*	Refl. *P*	Refl. *S*	i	Refr. *P*	Refr. *S*	Refl. *P*	i	Refr. *P*	Refl. *P*	Refl. *S*
0	0.999	0.04	0.00	0	0.999	0.00	0.04	0	0.00	0.00	1.00
20	0.96	0.12	0.24	20	0.90	0.44	0.08	20	0.50	0.39	0.78
40	0.87	0.29	0.39	33½	0.79	0.62	0.00	30	0.61	0.47	0.64
60	0.79	0.42	0.44	35	0.83	0.55	0.10	31	0.58	0.49	0.65
80	0.84	0.20	0.51	35.7	0.00	0.00	1.00	32.0	0.00	0.00	1.00
83.8	0.85	0.00	0.52	37	.....	0.85	0.53	33	0.84	....	0.54
85	0.85	0.10	0.52	50	.....	0.92	0.40	40	0.92	....	0.40
89	0.60	0.71	0.36	80	.....	0.62	0.78	64	0.55	....	0.84
90	0.00	1.00	0.00	90	.....	0.00	1.00	65.0	....	....	1.00

† After S. Dana, The Partition of Energy among Seismic Waves Reflected and Refracted at the Earth's Core, *Bull. Seis. Soc. Am.* **34**, 189–197 (1944).

By international agreement longitudinal waves in the mantle are indicated by P (starting downward at the source) or p (starting upward), transverse waves by S or s, longitudinal waves through the outer core by K, through the inner core by I, and (hypothetical) transverse waves through the inner core by J (Fig. 2i-2). Some authors use $P' \equiv PKP$, $P'' \equiv PKIKP$. For a source below the surface, there is one reflection at the surface near the epicenter, another about halfway between source and station. The symbols for these waves are, respectively, pP and PP, sP and SP, pS and PS, sS and SS. Similarly, for twice-reflected waves pPP, PPP, etc., are used. Time differences $pP - P$, $sP - P$, $sS - S$, etc., give a good indication for the focal depth (Table 2i-9).[2] Among observed waves through the core reflected at the

[1] I. P. Kominskaya, G. G. Mikhota, and Yu. V. Tulina, Crustal Structure of the Pamir-Alai Zone from Seismic Depth-sounding Data, *Izvest., Geophys. Ser.*, trans. by Am. Geophys. Un., 1959, p. 673.

[2] B. Gutenberg and C. F. Richter, Materials for the Study of Deep-focus Earthquakes, *Bull. Seis. Sos. Am.* **26**, 341–390 (1936); see also H. Jeffreys and K. E. Bullen, "Seismological Tables," p. 24, British Association for the Advancement of Science, 1940.

surface of the earth are $pPKP$, $sPKP$, $P'P' \equiv PKPPKP$, $P'P'P'$, $P'P'P'P'$ (with a travel time of about $1\frac{1}{4}$ hr).

Waves in the mantle with a reflection at the core surface permit accurate determination of the radius of the core. They are indicated by c, e.g., PcP, PcS, ScS; $pPcP$, $ScSScS$, etc., are in addition, reflected at the surface. All these waves usually have

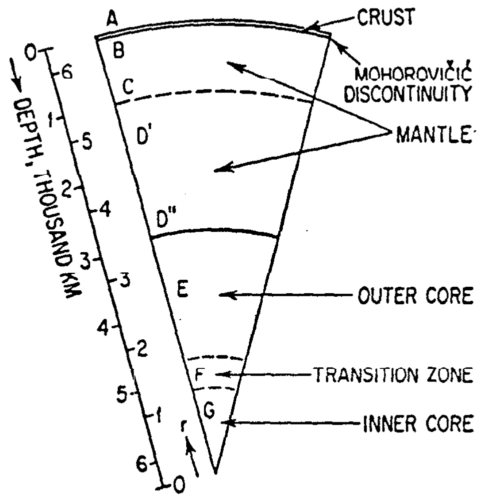

FIG. 2i-1. Main discontinuities of the earth. The letters A to G referring to the various regions in the earth have been suggested by Bullen and are used internationally. (*K. E. Bullen, Seismic Waves Transmission, vol.* 47, *p.* 104, *"Encyclopedia of Physics," Springer-Verlag, Berlin,* 1956.)

TABLE 2i-9. FOCAL DEPTH, KM, OF EARTHQUAKES FOR GIVEN TIME DIFFERENCES $pP - P$, $sP - P$, AND $sS - S$ FOR EPICENTRAL DISTANCES Δ OF 30, 80, AND 145 DEG

(* indicates that pP, sP, or sS, respectively, does not exist under given conditions)

Time diff., min:sec	Δ = 30 deg			Δ = 80 deg			Δ = 145 deg	
	$pP - P$	$sP - P$	$sS - S$	$pP - P$	$sP - P$	$sS - S$	$pP' - P'$	$sP' - P'$
0:20	100	60	50	75	55	40	70	55
0:40	205	120	100	160	105	85	150	105
1:00	310	195	165	250	165	140	235	160
1:30	*	295	270	395	255	220	375	250
2:00	*	415	425	565	350	300	525	345
2:30	*	535	*	755	460	390	690	440
3:00	*	*	*	?	575	485	?	540

periods of 1 to 4 sec. Waves reflected inside the core are indicated by $PKKP$, $SKKS$, etc. Their periods, too, are small ($PKKP$ waves with wavelengths $L < 10$ km have been observed), indicating a sharp boundary of the core. Waves refracted through the core (in addition to PKP) are PKS, SKP, SKS, etc. All observed travel times agree within a few seconds with those following from the velocities for P, K, and S (see Table 2i-11).

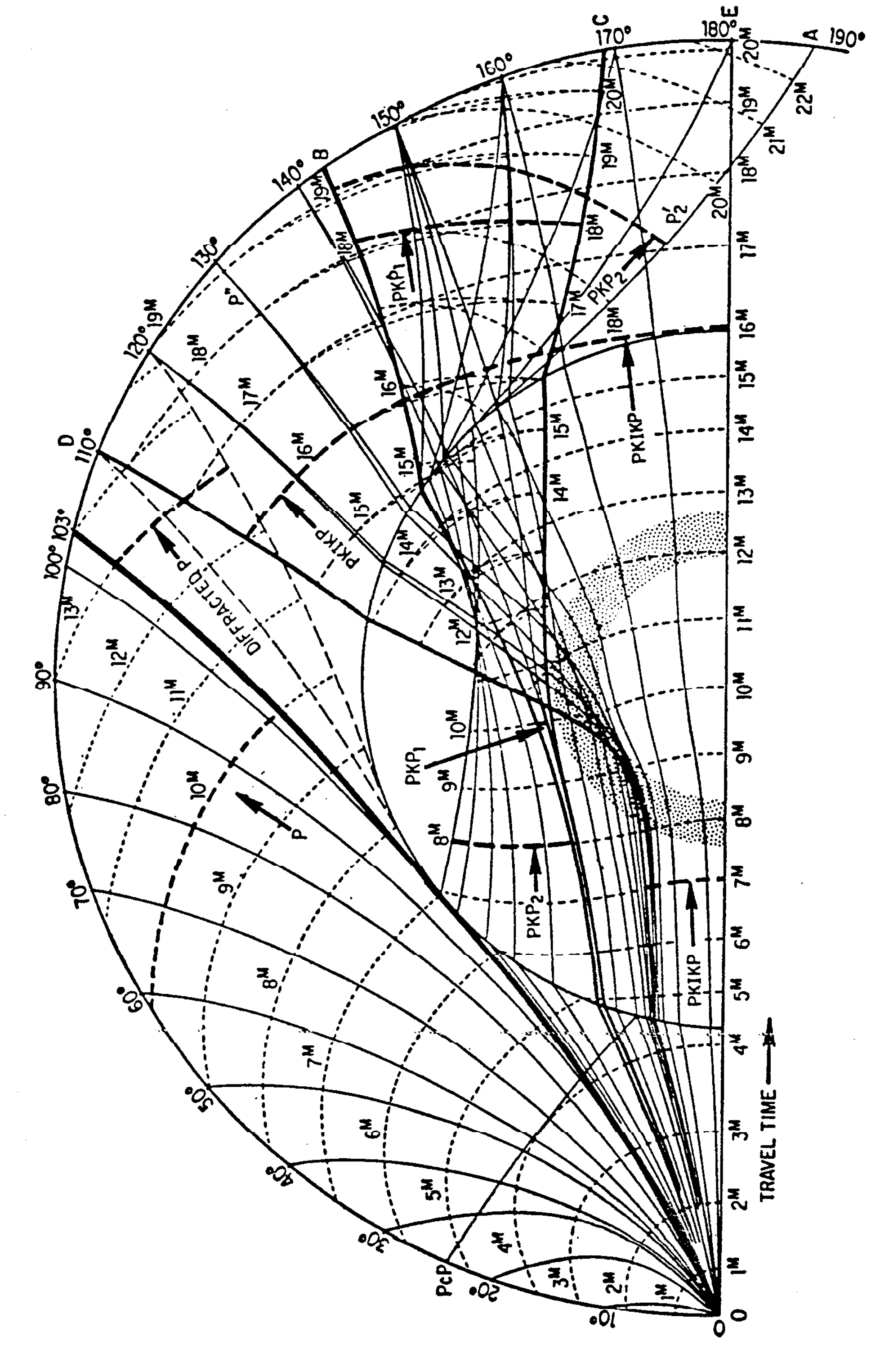

FIG. 2i-2. Paths and wavefronts (travel times) of longitudinal waves in the earth.

2i-8. Equations Used in Calculating Travel Times and Velocities. If i = angle of incidence (between ray and vertical), r = radius vector measured from center of earth, V = velocity, and if quantities at the surface of the earth are indicated by the index 0, the ray equation in a sphere in which the velocity depends on r only is

$$\frac{r \sin i}{V} = \frac{r_0 \sin i_0}{V_0} = \text{const} \tag{2i-5}$$

The radius R of curvature of the ray is given by

$$R = \frac{V}{(dV/dr) \sin i} \tag{2i-6}$$

If $dV/dr = V/r$, and $i = 90$ deg, $R = r$. If V decreases with depth at a greater rate, no ray can have its deepest point in the respective layer, and the travel-time curve is interrupted. The angle of incidence i_0 at the surface at a given epicentral distance Δ in kilometers is found from

$$\sin i_0 = \frac{V_0}{\bar{V}_0} \tag{2i-7}$$

where $\bar{V} = d\Delta/dt$. The angular distance Θ of a ray section (or the whole ray) and the corresponding travel time t are given by

$$\Theta = \int_{r_1}^{r_2} \frac{\tan i}{r} dr \qquad t = \int_{r_1}^{r_2} \frac{dr}{V \cos i} \tag{2i-8}$$

The radius r_S to the deepest point of a ray arriving at the distance Δ in degrees and the corresponding velocity V_S are found from

$$\log r_S = \log r_0 - 0.0024127 \int_0^{\Delta} q \, d\Delta \tag{2i-9}$$

where $\cosh q = \bar{V}_\Delta / \bar{V}(\Delta)$.

$$V_S = \bar{V} \frac{r_S}{r_0} \tag{2i-10}$$

$\bar{V}_\Delta = \bar{V}$ at the distance Δ; $\bar{V}(\Delta)$ is variable as a function of Δ.

2i-9. Wave Velocity, Elastic Constants, and Pressure in the Earth. Equations (2i-9) and (2i-10) or other methods are used to calculate V and v as a function of r. Poisson's ratio follows from Eqs. (2i-2). If the density ρ is known as a function of depth, Eqs. (2i-3) give the bulk modulus k and the rigidity μ. The pressure p and gravity g are given by

$$g = \frac{4\pi K}{r^2} \int_0^r \rho r^2 \, dr = \frac{3g_0}{p_m r_0 r^2} \int_0^r \rho r^2 \, dr \qquad p = \int_r^{r_0} g \, dr \tag{2i-11}$$

K is the gravitational constant (6.673×10^{-8} cgs), ρ_m is the mean density of the earth (5.517 g/cm^3), r_0 is the radius of the earth (6,371 km), and g_0 is the gravity at the surface (981 gals).

In sediments V ranges from $1 \pm$ km/sec for sand to $7 \pm$ in well-cemented rocks. In the continents frequently "granitic rocks," $V = 6 \pm$ km/sec, are below the sediments. At a depth of $15 \pm$ km V and v seem to have minima (compare Table 2i-3), thus permitting the propagation of guided channel waves. Under the continents, there is at least one such channel. Waves propagated in these crustal channels are designated, for example, by *Lg*, *Li*, and *Rg*. Their periods are usually between 1 and 10 sec; their velocities between $3\frac{1}{4}$ and $3\frac{3}{4}$ km/sec.[1]

[1] B. Gutenberg, Low Velocity Layers in the Earth's Mantle, *Bull. Geol. Soc. Am.* **65**, 337–347 (1954); Channel Waves in the Earth's Crust, *Geophysics* **20**, 283–294 (1955). Summary in B. Gutenberg, "Physics of the Earth's Interior," pp. 39–41, Academic Press, Inc., New York, 1959.

In the next deeper layer in the continents V is usually $6\frac{1}{2}$ to 7 km/sec, which is, e.g., characteristic of gabbro and olivine-gabbro (selected data in Table 2i-10; details differ appreciably). In some regions indications of velocities of 7 to $7\frac{1}{2}$ km/sec have been found immediately above the Mohorovičić discontinuity. The velocity possibly decreases slightly with depth in this layer owing to the increase in temperature (compare Table 2i-3).

Below the Mohorovičić discontinuity (depths M_0 in Table 2i-10) the velocity of longitudinal as well as transverse waves decreases with depth to minima near depths of 80 and 140 km, respectively (Table 2i-11). This low-velocity layer makes determinations of velocities below the Mohorovičić discontinuity rather difficult. It is

TABLE 2i-10. VELOCITY V, KM/SEC, OF LONGITUDINAL WAVES AT SELECTED DEPTH INTERVALS h, KM, OBSERVED IN VARIOUS REGIONS, 1950–1958†
(SE = source of energy, AE = artificial explosions, EQ = earthquake, RB = rock burst. M_0 is the depth of the Mohorovičić discontinuity below sea level in km; V_M, v_M are reported longitudinal and transverse velocities, respectively, just below M_0. Corresponding values of Poisson's ratio are 0.23 to 0.27)

Region	SE	h	V	h	V	M_0	V_M	v_M
N.W. Germany	AE	6–15	5.9 ±	15–28	6.5 ±	28 ±	8.2	?
Black Forest	AE	1–21	6.0	21–30	6.55	31	8.2	4.8
Southern Alps	EQ	0–35	5.7 ±	35–45	6.6 ±	45 ±	8.0	4.4
Northern Italy	EQ	0–15	5.3 ±	15–30	6.5 ±	40 ±	8.2	4.5 ±
South Africa	RB	4–36?	6.2	?	6.8	34	8.2	4.7
New York	AE	0–35	6.3	?	?	35	8.1	4.7
Eastern U.S.	AE	0–5	6.0	5–15	6.5 ±	40 ±	8.1	?
Wisconsin	AE	$\frac{1}{2}$–3	4.5	3–40 ±	6.0–6.9	42 ±	8.2	?
So. California	AE	1 ±	5.8	4–12	6.1–6.7	32 ±	8.2	?
So. California	EQ	1–25	6.4	25–35	7.1	35 ±	8.1	4.55
Canadian Shield	RB	0–30	6.2	30–35	7.1	37	8.2	4.85
Japan	AE	1–23	6.1 ±	23–32	7.4	32 ±	8.2	4.7 ±
N.E. India	EQ	1–25	5.6	25–46	6.6	46	7.9	4.5
Central Asia	AE	1–20	5.7	20–50	6.2	50	8.0	?
W. Atlantic	AE	Water	.....	5–10 ±	6.7	10 ±	8.0	?
Pacific Basin	AE	Water	.....	5–11 ±	6.8 ±	11 ±	8.2	?

† B. Gutenberg, "Physics of the Earth's Interior," pp. 32–35, Academic Press, Inc., New York, 1959.

another locus for channel waves (Pa and Sa), especially if the energy source is in or near these layers.[1]

New data on body-wave travel times, surface-wave dispersion, periods of free oscillations, and high-pressure laboratory experiments are producing more precise values for the parameters which are listed in Table 2i-11.[2] Changes of only a few percent in velocity or density may govern the conclusions as to chemical composition versus depth. As precision improves, zonal variations in the mantle may emerge, for which Table 2i-11 could only represent averages for a given radius.

[1] Summary in B. Gutenberg, "Physics of the Earth's Interior," pp. 86, 87, Academic Press, Inc., New York, 1959.

[2] Francis Birch, Density and Composition of Mantle and Core, *J. Geophys. Research* **69**, 4377–4388 (1964); S. P. Clark, Jr., and A. E. Ringwood, Density Distribution and Constitution of the Mantle, *Revs. Geophys.* **2**, 35–88 (1964); Frank Press, Density Distribution in Earth, *Science* **160**, 1218–1221 (1968).

2i-10. Intensity, Magnitude, and Energy of Earthquakes and Related Quantities. The "intensity" of an earthquake refers to the effects of shaking at a given point. In the United States the modified Mercalli scale[1] (I to XII) is used; a few greatly condensed examples follow.

II. Felt by few persons at rest.

IV. Felt outdoors by few; some sleepers awakened; dishes, windows disturbed.

V. Some dishes, windows broken; unstable objects overturned; pendulum clocks may stop.

VI. Felt by all; some fallen plaster or damaged chimneys.

VII. Considerable damage in poorly built structures.

IX. Buildings shifted off foundations; ground cracked.

XI. Few structures remain standing; rails bent.

TABLE 2i-11. WAVE VELOCITIES V (LONGITUDINAL) AND v (TRANSVERSE), KM/SEC [Poisson's ratio σ, Eqs. (2i-2); density ρ, g/cm^3; bulk modulus k and rigidity μ, both in 10^{12} dynes/cm^2, Eqs. (2i-3); gravitational acceleration g, cm/sec^2; and pressure p, million atm, in the earth as function of depth, km†]

Depth	V	v	σ	ρ	k	μ	g	p
Mantle:								
50	8.0	4.55	0.26	3.3	1.3	0.65	985	0.014
100	7.8	4.4	0.27	3.5	1.3	0.65	987	0.03
150	7.9	4.4	0.28	3.6	1.3	0.64	989	0.05
200	8.1	4.4	0.29	3.7	1.3	0.68	990	0.06
250	8.3	4.5	0.29	3.8	1.4	0.71	990	0.08
300	8.5	4.7	0.29	3.9	1.6	0.8	990	0.10
500	9.5	5.2	0.28	4.0	2	1.1	990	0.18
1,000	11.5	6.4	0.28	4.6	$3\frac{1}{2}$	1.9	990	0.39
1,500	12.2	6.7	0.28	5	$4\frac{1}{2}$	2.3	980	0.6
2,000	12.8	6.9	0.29	$5\frac{1}{4}$	5	2.6	980	0.9
2,900	13.7	7.3	0.30	$5\frac{3}{4}$	$6\frac{1}{2}$	3	1000	1.3
Outer core:								
2,900	8.0	$\ll 1$	0.5	$9\frac{1}{2}\pm$	$6\pm$	0?	1000	1.3
4,000	9.4	$\ll 1$	0.5	$11\pm$	$10\pm$	0?	$800\pm$	$2\frac{1}{4}\pm$
5,000	10.0	$\ll 1$	0.5	$12\pm$	$12\pm$	0?	$600\pm$	$3\frac{1}{4}\pm$
Inner core:								
5,400	11.1	?	0.4?	13?	$15\pm$	2?	$500\pm$	$3\frac{1}{2}\pm$
6,370	11.2	?	0.4?	13?	$16\pm$	2?	0	$3\frac{3}{4}\pm$

† Based on B. Gutenberg, "Physics of the Earth's Interior," Academic Press, Inc., New York, 1959. The probable errors of most quantities increase with depth.

The observed intensity depends on the depth of focus, the ground, the type of building, the density of population, etc. The intensity is useful for engineers but not for studies of seismicity, for which the earthquake magnitude is used. There are various scales. Magnitude M originally was defined[2] for southern California as the common logarithm of the maximum trace amplitude a or b in microns with which a seismograph of period 0.8 sec, magnification 2,800, damping 65:1 would record the shock at a distance of 100 km. Tables[3] permit the determination of M. In addition, for

[1] H. O. Wood and F. Neumann, Modified Mercalli Intensity Scale of 1931, *Bull. Seis. Soc. Am.* **21**, 277–283 (1931).

[2] C. F. Richter, An Instrumental Earthquake Magnitude Scale, *Bull. Seis. Soc. Am.* **25**, 1–32 (1935).

[3] M. Båth, Earthquake Seismology, *Earth-Sci. Revs.* **1**, 60–86 (1966).

$\Delta > 15°$, M_S is found from ground amplitudes b (in microns) of surface waves with periods of 20 sec in shallow earthquakes. The magnitude M is based on amplitudes a of P, PP, and S waves in shocks (focal depth h) recorded at the epicentral distance Δ:

$$M_S = \log b + F(\Delta) \qquad M = \log a - \log T + f(\Delta,h) \tag{2i-12}$$
$$M = M_S - 0.37(M_S - 6.74) \qquad \text{(approximately)}$$

For $F(\Delta)$ and $f(\Delta,h)$, see Table 2i-12; small station corrections are to be added. The amplitudes b of surface waves of length L decrease with increasing focal depth h

TABLE 2i-12. VALUES OF $f(\Delta,h)$ IN EQ. (2i-12) FOR VERTICAL COMPONENTS Z OF P AND PP, HORIZONTAL COMPONENT SH OF S, AND $F(\Delta)$ FOR HORIZONTAL COMPONENT OF MAXIMUM (MAX)
(h = focal depth; Δ = epicentral distance, deg*)

Δ	h = 25 km				h = 300 km			h = 600 km		
	PZ	PPZ	SH	Max	PZ	PPZ	SH	PZ	PPZ	SH
20	6.0	...	5.8	4.0	6.1	...	5.8	6.4	...	5.9
30	6.6	6.7	6.3	4.3	6.3	6.4	6.1	6.4	6.3	6.0
50	6.7	6.7	6.6	4.6	6.1	6.6	6.7	6.3	6.5	6.4
80	6.7	6.9	6.7	5.0	6.6	6.9	6.4	6.2	6.8	6.5
100	7.4	7.2	7.4	5.1	7.2	6.8	6.7	7.2	7.0	6.7
160	...	6.9	...	5.4	...	6.6	...	...	6.7	

* B. Gutenberg, Amplitudes of Surface Waves and Magnitudes of Shallow Earthquakes, *Bull. Seis. Soc. Am.* **35**, 3–12 (1945); Magnitude Determination for Deep-focus Earthquakes, *Bull. Seis. Soc. Am.* **35**, 117–130 (1945). B. Gutenberg and C. F. Richter, Magnitude and Energy of Earthquakes. *Ann. Geofis. Rome*, **9**, 1–15 (1956).

TABLE 2i-13. INTENSITY I AT THE EPICENTER, CORRESPONDING MAXIMUM ACCELERATION α, CM/SEC^2, MEAN RADIUS r_p OF AREA OF PERCEPTIBILITY, KM, FOR A GIVEN MAGNITUDE M IN AVERAGE SHOCKS IN SOUTHERN CALIFORNIA (h = 16± KM)
(Values for I, α, r are based on empirical equations†)

M	2.2	3	4	5	6	7	8	$8\frac{1}{2}$
I	1.5	2.8	4.5	6.2	7.8	9.5	11.2	12.0
α	1	3	10	36	130	460	1,670	3,160
r_p	0	25	55	110	200	390	740	1,000

† B. Gutenberg and C. F. Richter, Earthquake Magnitude, Intensity, Energy, and Acceleration, *Bull. Seis. Soc. Am.* **32**, 163–191 (1942).

corresponding to a factor $e^{-qh/L}$, where q (about 2) depends on crustal structure. The average relationship of intensity to magnitude in California earthquakes is given in Table 2i-13.

The energy E corresponding to the magnitude M found from body waves is given to a first approximation[1] by

$$\log E = 12.24 + 1.44M \tag{2i-13}$$

[1] M. Bâth, Earthquake Seismology, *Earth-Sci. Revs.* **1**, 69–86 (1966).

2i-11. Seismicity of the Earth. Earthquakes are divided into shallow shocks ($h \leq 60$ km), intermediate ($60 < h \leq 300$), and deep ($h > 300$, maximum $720 \pm$ km). Most shocks occur in narrow belts (Table 2i-14).[1] Deep and intermediate shocks are limited to the circumpacific belt and the trans-Asiatic (Alpide) belt.

For the magnitude of the largest observed shock and the relative frequency of earthquakes in various depth intervals, see Table 2i-15, which also shows examples of regional differences.

2i-12. Energy E of Earthquakes. Most calculations of E depend on Eq. (2i-13). This empirical formula is based on many observations, but is subject to adjustment.

TABLE 2i-14. NUMBER OF SHALLOW, INTERMEDIATE, AND DEEP-FOCUS EARTHQUAKES, % OF ALL EARTHQUAKES IN THE GIVEN DEPTH RANGE, AND CORRESPONDING ENERGY RELEASE (*a*) IN THE MAJOR UNITS OF THE EARTH AND (*b*) IN SELECTED AREAS
(Averages 1904–1957)

Region	Number, %			Energy, %		
	Shallow	Inter-med.	Deep	Shallow	Inter-med.	Deep
(*a*) Circumpacific belt	82	91	100	75	89	100
Trans-Asiatic belt	10	9	<1	23	11	0.3
Atlantic and Indian Oceans	5	0	0	1	0	0
All others	3	0	0	1	0	0
Total	100	100	100	100	100	100
(*b*) Pacific region, Alaska to U.S.	2	0	0	2	0	0
North and Central America, West Coast	12	10	0	12	8	0
South America, western part	10	19	6	15	9	19
Kermadec-Tonga Is.	3	3	41	4	5	25
New Hebrides and Solomon Is.	12	20	4	7	18	3
Marianas Is.	2	6	6	1	8	3
Japan-Kamchatka	15	16	35	19	22	44
Philippine Is.	5	3	4	6	2	3
Celebes-Sunda Is.	8	11	4	6	15	3
Hindu Kush	0	5	0	0	6	0
Asia Minor to Italy	2	2	0	1	4	0
Total	71	95	100	73	97	100

Shocks of magnitudes over 7 account for most of the total energy release (Table 2i-16). For annual extreme and average energy release, 1904 to 1957, see Table 2i-17. The annual energy release in shallow earthquakes decreased appreciably about 1907. While the average for 1897 to 1906 was about 20×10^{24} ergs, it was only 6×10^{24} ergs for 1907 to 1956 with no appreciable fluctuations in 10-year periods. As a consequence of the relatively short period of about 60 years for which data are available, averages for the annual energy release change noticeably with the period used for the calculation. The annual energy loss by heat flow through the earth's surface is of the order of 10^{28} ergs.

[1] B. Gutenberg and C. F. Richter, "Seismicity of the Earth," 2d ed., Princeton University Press, Princeton, N.J., 1954.

TABLE 2i-15. (a) MAGNITUDE m OF GREATEST KNOWN SHOCK (1905–1957) IN DEPTH INTERVALS d, CENTERING AT h; (b) PERCENTAGE OF SHOCKS FOR THE WHOLE EARTH; (c) CORRESPONDING FREQUENCY FOR SELECTED PARTS OF THE CIRCUMPACIFIC BELT

d, km h, km	60 30	60 90	100 175	100 275	100 375	100 475	100 575	50 650	50 700
(a) Largest observed m	8.2	8.0	8.0	7.8	7.8	7.5	7.5	7.5	6.9
(b) Number of shocks, %	72	12	7	2	2	2	2	1	$\frac{1}{3}$
(c) Mexico, Central America, %	73	20	6	1	0	0	0	0	0
Andes, %	36	30	20	5	0	0	4	4	0
New Zealand–Samoa, %	30	10	10	6	7	6	25	5	$\frac{1}{2}$
New Hebrides–New Guinea, %	43	30	20	4	3	1	0	0	0
Japan–Manchuria, %	36	16	11	6	15	9	6	$\frac{1}{4}$	0
Sunda Arc, %	30	26	20	1	4	1	10	2	5

TABLE 2i-16. AVERAGE ANNUAL ENERGY RELEASE IN ALL EARTHQUAKES WITH $M_S \leq M^*$

(Units 10^{23} ergs. Ratios of figures are good approximations; absolute values may be incorrect by factor 100)

M^*	6	7	8
Shallow shocks	0.2	1	5
Intermediate shocks	?	0.2	0.6
Deep shocks	?	0.05	0.1

TABLE 2i-17. MAXIMUM, MINIMUM, AND AVERAGE ANNUAL ENERGY RELEASE IN EARTHQUAKES 1904–1957

(Units 10^{23} ergs. Accuracy as in Table 2i-16)

	Max	Year	Min	Year	Average
Shallow shocks	340†	1906	9	1954	70
Intermediate shocks	100	1911	1	1935	16
Deep shocks	35	1906	0.2±	Several	3
All shocks	390	1906	12	1930	90

† Possibly 500 in 1897.

2i-13. Aftershocks and Earthquake Sequences. Investigations by Benioff[1] show that elastic strain-rebound increments in series of earthquake aftershocks follow two types of functions:

$$(1)\ S_1 = A + B \log t \qquad (2)\ S_2 = C - De^{-\sqrt{t}} \qquad \text{(2i-14)}$$

[1] H. Benioff, Earthquakes and Rock Creep, *Bull. Seis. Soc. Am.* **41**, 31–62 (1951).

where t is time from a selected zero point and A, B, C, D are constants of the process. (1) was given previously by Griggs for compressional recoverable creep strain, (2) by Michelson for shearing creep recovery. For series of earthquakes in certain areas and for all earthquakes in certain depth ranges Benioff[1] has found strain-rebound characteristics of forms similar to Eqs. (2i-14). Yearly strain rebound in all deep shocks shows a decrease between at least 1905 and 1950 following Eq. (1), whereas most great shallow shocks have occurred in five active periods. The units of the Pacific belt have different patterns of activity.[2]

2i-14. Nonelastic Properties of the Earth's Interior. Earthquake-generated surface waves, body waves, and free oscillations yield data on energy losses in the earth for periods of one to a few thousand seconds. Field and laboratory measurements on crustal rocks cover periods of a few seconds to fractions of a microsecond. For a given rock, losses can be expressed in terms of a specific dissipation constant $1/Q$ which is found to be independent of frequency.[3] This result strongly suggests a nonlinear stress-strain relation for the rock, even at infinitesimal values of strain. A hysteresis loop, in which energy expended per cycle is independent of the rate of traversal, would yield $1/Q$ independent of frequency. Since the nonlinear terms are small, one may assume that superposition applies and hence that stresses and strains are expressible as Fourier transforms. A modified Hooke's law can then be applied, in which stresses are proportional to strains through complex constants which have a one-to-one relationship with any elastic constants one would use for an elastic solid of a given geometry and wave type.[4] For example, shear stress $s(t)$ in a plane shear wave is related to strain $\epsilon(t)$ thus:

$$\begin{aligned} s(t) &= (1/2\pi) \int_{-\infty}^{\infty} S(\omega) \exp (i\omega t)\, d\omega \\ \epsilon(t) &= (1/2\pi) \int_{-\infty}^{\infty} E(\omega) \exp (i\omega t)\, d\omega \\ S(\omega) &= (\mu + i \operatorname{sgn}\omega\, \mu^*) E(\omega) \end{aligned} \tag{2i-15}$$

This leads to a wave equation with a complex propagation constant $(a_S + i\omega/v)$. A plane wave along the x axis at angular frequency ω_0 is

$$\epsilon(t) = E_0 \exp (-a_S x) \cos \left(\omega_0 t - \frac{\omega_0 x}{v}\right) \tag{2i-16}$$

With the loss parameter μ^* much smaller than the shear modulus μ,

$$\frac{1}{Q_S} = \frac{\mu^*}{\mu} = \frac{2a_S v}{\omega} \tag{2i-17}$$

With the additional loss parameter k^* introduced through the complex incompressibility $(k + i \operatorname{sgn}\omega\, k^*)$, the corresponding quantities for a plane compressional wave are

$$\frac{1}{Q_P} = \frac{k^* + 4\mu^*/3}{k + 4\mu/3} = \frac{V_P 2a}{\omega} \tag{2i-18}$$

The two loss parameters Q_S and Q_P, applied to an isotropic solid, are sufficient to specify attenuation of Rayleigh waves, sharpness of resonance for free vibrations, etc.

[1] H. Benioff, Global Strain Accumulation and Release as Revealed by Great Earthquakes, *Bull. Geol. Soc. Am.* **62**, 331–338 (1951).

[2] H. Benioff, Orogenesis and Deep Crustal Structure—Additional Evidence from Seismology, *Bull. Geol. Soc. Am.* **65**, 385–400 (1954).

[3] S. P. Clark, Jr., "Handbook of Physical Constants," *Geol. Soc. Am. Mem.* **97**, 178 pp. (1966); L. Knopoff, Q, *Revs. Geophys.* **2**, 625–660 (1964).

[4] J. E. White, "Seismic Waves: Radiation, Transmission, and Attenuation," p. 94, McGraw-Hill Book Company, New York, 1965.

Since most data are not sufficiently complete and accurate to determine both, it makes some sense to attribute one parameter ($1/Q$) to a given solid. Some ranges of $1/Q$ for common rock types appear in Table 2i-18.

On the assumption that Q is independent of frequency and varies with depth, attenuation versus period for surface waves and sharpness of resonance of free oscillations can yield the dependence of Q on depth.[1] Shear waves reflected from the core also give Q versus depth in the mantle.[2] Average Q for the mantle above 600 km is about 200; for the rest of the mantle, about 2,000. There is an indication of a minimum Q near the top of the mantle, probably correlating with the minimum in velocities (see Table 2i-11). Anderson et al.[3] suggest Q values of 200 to 600 for the crust, 50 to 110 for the minimum, increasing to 70 to 190 at 120 km, and continuing to increase with depth.

Slow adjustments of the crust indicate that the stress-strain relation for the crust and mantle must permit creep under steady load. The core is best described as a

TABLE 2i-18. RANGES OF SPECIFIC DISSIPATION CONSTANT $1/Q$ FOR SEVERAL COMMON ROCKS

Rock type	$1/Q$
Granite	0.002–0.02
Limestone	0.002–0.03
Sandstone	0.004–0.05
Shale	0.015–0.1

viscous fluid. A relation between shear stress s and shear strain ϵ which incorporates both possibilities is

$$\frac{s}{\mu} + \frac{1}{\eta} \int s \, dt = \epsilon + \lambda \frac{d\epsilon}{dt} \tag{2i-19}$$

In terms of transformed stress and strain, the complex shear modulus can be included to yield an equation which reduces to Eq. 2i-15 for intermediate frequencies:

$$\frac{S}{\mu + i \operatorname{sgn}\omega \, \mu^*} + \frac{S}{i\omega\eta} = E + i\omega\lambda E \tag{2i-20}$$

The rate of rise of Fennoscandia toward equilibrium following the melting of the Pleistocene ice masses (maximum thickness about $2\frac{1}{2}$ km) has been used by many authors to estimate the viscosity of the mantle. McConnell[4] has analyzed the detailed information on the shape of the upwarping of the area covered by the Fennoscandian ice sheet to deduce variation of viscosity with depth. His best model consists of an elastic layer overlying a layered elastoviscous mantle. His values of η are: 2×10^{21} poises from 120 to 400 km, 10^{22} poises from 400 to 800 km, increasing to almost 10^{23} poises at 1,500 km.

Where the product $\mu\lambda = \nu$ defines the fluid viscosity ν, data on torsional free oscillations and shear reflections *ScS* indicate that the viscosity of the core lies between 0.35×10^{11} and 4.7×10^{11} poises.[5]

[1] Don L. Anderson and C. B. Archambeau, The Anelasticity of the Earth, *J. Geophys. Research* **69**, 2071–2084 (1964).

[2] Robert L. Kovach and Don L. Anderson, Attenuation of Shear Waves in the Upper and Lower Mantle, *Bull. Seis. Soc. Am.* **54**, 1855–1864 (1964).

[3] Don L. Anderson, Ari Ben-Menahem, and C. B. Archambeau, Attenuation of Seismic Energy in the Upper Mantle, *J. Geophys. Research* **70**, 1441–1448 (1965).

[4] Robert K. McConnell, Jr., Isostatic Adjustment in a Layered Earth, *J. Geophys. Research* **70**, 5171–5188 (1965).

[5] R. Sato and A. F. Espinosa, Dissipation Factor of the Torsional Mode $_0T_2$, *J. Geophys. Research* **72**, 1761–1767 (1967).