2o. Cavitation in Flowing Liquids

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2o-1. Introduction—Status of Available Data. Although the possibility of occurrence of cavitation in hydrodynamic systems was recognized as long ago as 1754 by Euler,[1] significant researches on the physical phenomena have been developed only during the first half of the present century. This has resulted from the growing importance of the effects of cavitation (both useful and detrimental) in such diverse fields as underwater propulsion and hydraulic machinery (loss of efficiency, damage to materials, noise), underwater signaling (background noise, absorption of acoustical power), hydroballistics (increased drag and instability of missiles), medicine (divers' bends, bullet wounds), and chemical processing (acceleration of reactions and mixing processes, industrial cleaning). Because of the complexities of the phenomena—hydrodynamical and physicochemical—in cavitated regions, research activity continues to emphasize understanding and description of events. Consequently, this section is restricted to brief descriptions of the various factors involved in the cavitation process and to the presentation of data which, while consistent within themselves, are intended primarily to illustrate the text. In all cases, reference should be made to the original source for guidance in judging the limits of accuracy and applicability of these data.

The discussion given here is concerned particularly with phenomena associated with flowing liquids and excludes cavitation produced by heat addition (boiling) and acoustical pressure waves as well as problems of pure liquids (e.g., ultimate tensile strength). Rather complete discussions of cavitation in flowing liquids (and about forms moving through stationary liquids) have been given by Ackeret[2] and Eisenberg,[3] and extensive bibliographies will be found in the papers of these authors and in a compilation by Raven et al.[4]

2o-2. Definitions and Nomenclature

$\sigma = \dfrac{P - p_v}{\frac{1}{2}\rho U^2}$	cavitation number
P	ambient pressure

[1] Leonhard Euler, Théorie plus complète des machines, qui sont mises en mouvement par la réaction de l'eau, *Historie de l'Academie Royale des Sciences et Belles Lettres*, Classe de Philosophie Experimentale, Mém. 10, pp. 227–295, 1754, Berlin, 1756.

[2] J. Ackeret, Kavitation (Hohlraumbildung), *Handbuch der Experimentalphysik* IV (1), 461–486 (Leipzig, 1932).

[3] Phillip Eisenberg, Kavitation, *Forschungshefte für Schiffstechnik* **3**, 111–124, 1953; **4**, 155–168 (1953); **5**, 201–212 (1954); On the Mechanism and Prevention of Cavitation, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 712, July, 1950; A Brief Survey of Progress on the Mechanics of Cavitation, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 842, June, 1953.

[4] F. A. Raven, A. M. Feiler, and Anna Jesperson, An Annotated Bibliography of Cavitation, *David Taylor Model Basin, U.S. Navy Dept. Rept.* R-81, December, 1947.

p_v	vapor pressure or actual pressure within a cavity
ρ	mass density of liquid
U	stream velocity
σ_i or K	cavitation number for inception of cavitation ("critical" cavitation number)
$Re = \dfrac{Ud}{\nu}$	Reynolds number
ν	kinematic viscosity
d	diameter of a body of revolution
d_m	maximum diameter of steady-state cavity
l	length of a steady-state cavity
R	radius of a transient cavity
h	altitude of a cone
$C_D = \dfrac{D}{\frac{1}{2}\rho U^2 A}$	drag coefficient
D	drag
A	area of body in plane normal to stream or cross-sectional area of circular cylinder
$C_D(\sigma)$	drag coefficient at cavitation number σ
α	total absolute air content
α_s	total saturation air content

2o-3. Inception of Cavitation. It is now generally agreed that cavitation originates with the growth of undissolved vapor or gas nuclei existing in the liquid or trapped on microscopic foreign particles. It is well known that the rupture forces of very clean and carefully degassed liquids are of the order of those predicted by kinetic theoretical formulations. Experimental evidence has also been obtained that water saturated with air, but denucleated by application of very high pressures, exhibits large tensile strength (of the order of several hundred atmospheres).[1] Thus the presence of nuclei is evidently necessary for the inception of cavitation at pressures of the order of vapor pressure. In supersaturated liquids, it is easy to account for the presence and stability of such nuclei, but in saturated and undersaturated liquids, the situation is not clear, and the presence of nuclei is usually accounted for on the basis that they are stabilized on suspended particles.[2] As a consequence, depending upon the size and number of these nuclei, cavitation may be expected to begin above as well as below the vapor pressure. The effect of total air content was shown in experiments of Crump[3] using a venturi nozzle having a diffuser of 5 deg included angle. Figure 2o-1 shows that in the undersaturated liquid it was possible to obtain tensions as the air content was reduced. Results in a nozzle with an abrupt expansion, however, show opposite trends in the pressures required for inception,[3] although here too tensions were obtained. Comparable results for sea water[4] are shown in Fig. 2o-2; since the water is supersaturated, thus presumably containing a large number of undissolved nuclei, bursts of cavitation are observed at pressures well above vapor pressure.

[1] Newton E. Harvey, W. D. McElroy, and A. H. Whiteley, On Cavity Formation in Water, *J. Appl. Phys.* **18**, 162–172 (February, 1947).

[2] Eisenberg, *loc. cit.;* P. S. Epstein and M. S. Plesset, On the Stability of Gas Bubbles in Liquid-Gas Solutions, *J. Chem. Phys.* **18** (11), 1505–1509 (November, 1950).

[3] S. F. Crump, Critical Pressures for the Inception of Cavitation in a Large-scale Numachi Nozzle as Influenced by the Air Content of the Water, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 770, July, 1951.

[4] S. F. Crump, Determination of Critical Pressures for the Inception of Cavitation in Fresh and Sea Water as Influenced by Air Content of the Water, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 575, October, 1949.

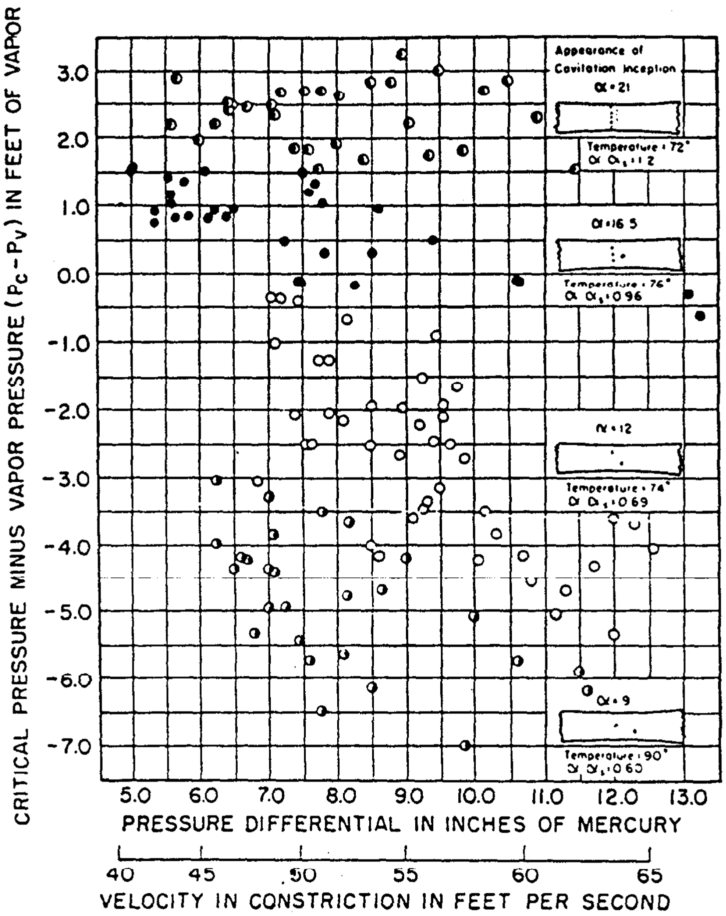

FIG. 2o-1. Cavitation inception in fresh water of varying air content. (*After Crump.*)

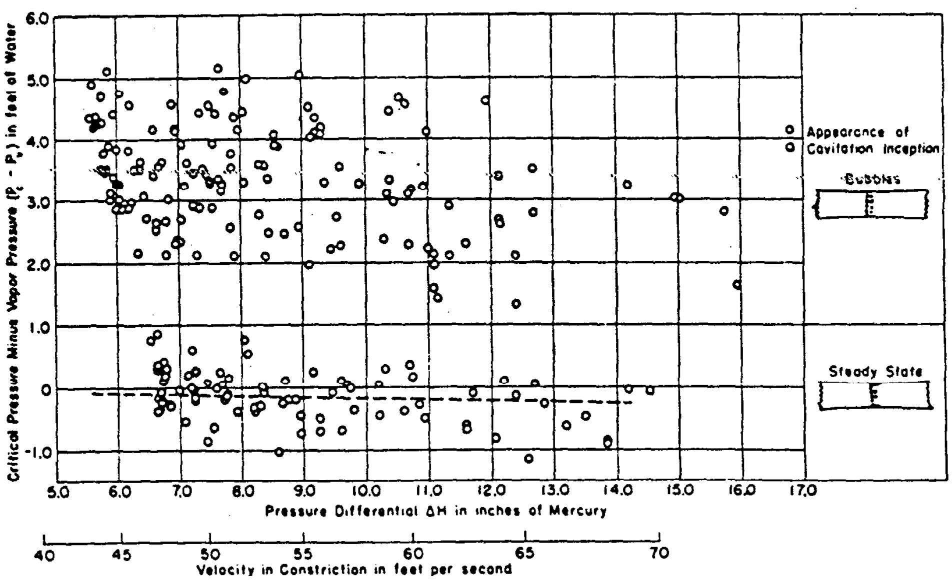

FIG. 2o-2. Critical pressure for inception of cavitation in sea water. (*After Crump.*)

It may be expected that a relation exists between the dissolved and entrained gas content, at least in an undisturbed liquid. Some evidence for this assumption exists in the measurements of Strasberg[1] on tap water with ultrasonically induced cavitation. Since, according to the analysis of Noltingk and Neppiras,[2] the time duration of the pressure for times of the order of milliseconds has very little influence on the inception pressure, and since this is also of the order of the time duration in hydraulic applica-

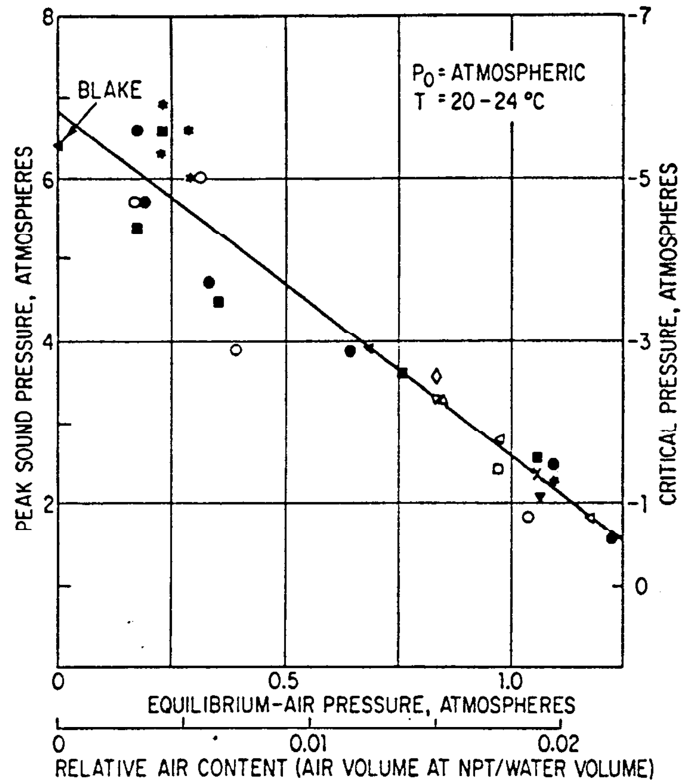

FIG. 2o-3. The effect of air content on the inception pressure for ultrasonic cavitation. (Each plotted point represents the average of 10 to 20 measurements. Each symbol represents a different sample of water.) (*After Strasberg.*)

tions, we can compare Strasberg's results as a basis for the effect of air content on cavitation inception. These are shown in Fig. 2o-3. Whether similar results would be obtained in flowing water using ultrasonic techniques is not known.

Properties of the liquid such as viscosity and surface tension influence the growth of nuclei and, consequently, the inception pressures. In this connection, the presence of surface-active materials (detergents, etc.) affect inception pressures through alteration of surface tension.

Environmental factors which must be considered when attempting to predict inception include not only the average pressure and pressure-gradient conditions determined by the flow boundaries (such as bounding walls or a moving body) but also the magnitude and duration of pressure fluctuations in turbulent regions and

[1] M. Strasberg, The Influence of Air-filled Nuclei on Cavitation Inception, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 1078, May, 1957.

[2] B. E. Noltingk and E. A. Neppiras, Cavitation Produced by Ultrasonics, *Proc. Roy. Soc (London)*, ser. B, **63**, 674–685 (1950); 1032–1038 (1951).

boundary-layer effects including flow in zones of separation. An example of the effects of the boundary layer and, in particular, local separation is shown in Fig. 2o-4 from the work of Rouse and McNown.[1] In this figure are compared the minimum pressure coefficients with the cavitation numbers at which the pressure distribution first showed a change. This change is attributed to microscale cavitation in locally separated flows and served to define the critical cavitation number. Effect of model size on inception has been studied by Kermeen[2] and others.[3] While the mechanisms

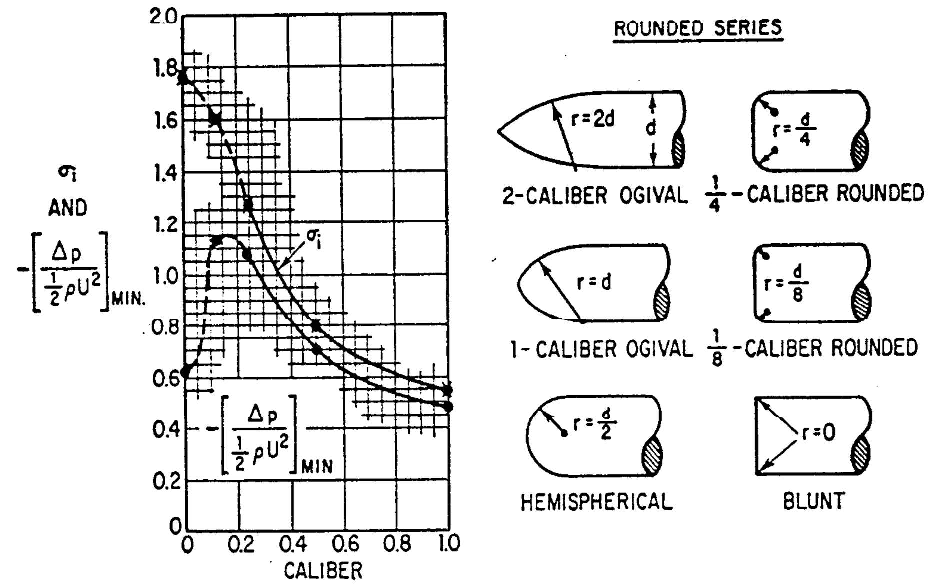

FIG. 2o-4. Critical cavitation number for first change in minimum pressure coefficient of bodies of revolution and minimum pressure coefficient vs. caliber of rounding. (*After Rouse and McNown.*)

are still only incompletely understood, trends are fairly well established and are consistent with the concept of nuclei and the role of the boundary layer.[4] An example of Kermeen's results is shown in Fig. 2o-5 wherein the average values of a large number of data are plotted for models of various diameters.

2o-4. Transient (Bubble) Cavities. These are small individual bubbles which grow, sometimes oscillate, and eventually collapse and disappear. Of particular interest here are the pressures produced in the vicinity of such cavities when they collapse. From studies of damage and acoustic radiation produced by such cavities, it is known that pressures of the order of thousands of atmospheres are developed. However, since the maximum pressure rise is confined to durations of the order of a microsecond, definitive measurements have not yet been achieved. The motion of such cavities depends not only upon the ambient pressure conditions but also upon the amount of permanent gas in the bubble and the condensation rates of the vapor as well as the properties of the liquid—compressibility, viscosity, surface tension.

[1] Hunter Rouse and John S. McNown, Cavitation and Pressure Distribution; Head Forms at Zero Angle of Yaw, *State Univ. Iowa Studies Eng. Bull.* 32, 1948.
[2] R. W. Kermeen, Some Observations of Cavitation on Hemispherical Head Models, *Calif. Inst. Technol. Hydrodynamics Lab. Rept.* E-35.1, June, 1952.
[3] Blaine R. Parkin, Scale Effects in Cavitating Flow, *Calif. Inst. Technol. Hydrodynamics Lab. Rept.* 21–8, July 31, 1952.
[4] Eisenberg, *loc. cit.;* Parkin, *loc. cit.*

Except for surface tension, all these factors tend to decrease the rate of collapse; in addition, distortion from spherical shape caused by pressure gradients or bubble-wall instability tends to result in reduced collapse rates and thus reduced pressures.

Plesset,[1] employing Rayleigh's[2] theoretical formulation for collapse of a spherical cavity in incompressible inviscid fluid but including effect of surface tension and comparing with the experimental results of Knapp and Hollander,[3] has shown that, in the region from maximum radius down to about one-quarter the maximum radius, the motion can be predicted with fair accuracy as long as the bubble is approximately

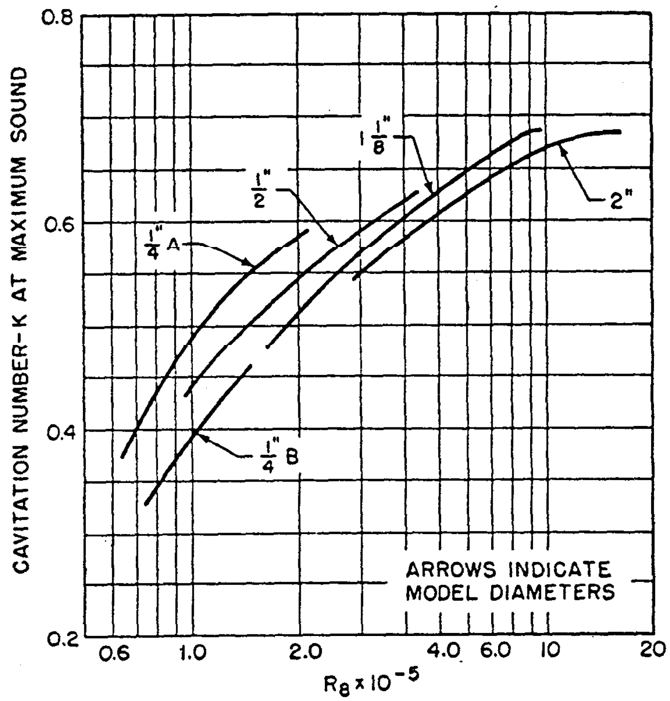

FIG. 20-5. Cavitation number K for incipient cavitation (as defined by value at which noise disappears) as a function of Reynolds number for bodies with hemispherical heads and cylindrical middle bodies. (The $\frac{1}{4}$-in. A model was more accurately constructed than the $\frac{1}{4}$-in. B model.) (*After Kermeen.*)

spherical. This idealized theory, which predicts that the bubble-wall velocity is of the order of $R^{-\frac{3}{2}}$ as $R \rightarrow 0$ (and that the maximum pressure is infinite), is, of course, inadequate for the final stages of collapse where the effects mentioned above become important. For example, a further approximation carried out by Gilmore[4] shows that the effect of compressibility of the liquid is to reduce the wall velocity to the order of $R^{-\frac{1}{2}}$.

[1] M. S. Plesset, The Dynamics of Cavitation Bubbles, *J. Appl. Mech.* **16**, 277–282 (September, 1949).

[2] Lord Rayleigh, On the Pressure Developed in a Liquid during the Collapse of a Spherical Cavity, *Phil. Mag.* **34**, 94–98 (1917).

[3] R. T. Knapp and A. Hollander, Laboratory Investigations of the Mechanism of Cavitation, *Trans. Am. Soc. Mech. Engr.* **70** (5), 419–435 (July, 1948).

[4] Forrest R. Gilmore, The Growth or Collapse of a Spherical Bubble in a Viscous Compressible Liquid, *Calif. Inst. Technol. Hydrodynamics Lab. Rept.* 26–4, Apr. 1, 1952.

2o-5. Steady-state Cavities. Such cavities (also referred to as "fixed" and "sheet") are large stationary cavities observed behind blunt obstacles and on hydrofoil profiles with relatively sharp leading edges. While such cavities are, especially at low cavitation numbers, usually filled only with vapor phase and other gas, they are often observed to contain a mixture of individual bubbles and liquid phase. The surface usually oscillates, and often parts or the entire cavity is observed to grow and collapse; the average envelope, however, behaves essentially as the boundary of a time-independent flow.[1]

Reliable measurements of cavity shape have been made up to now only for axisymmetric cavities. Data for the principal dimensions of cavities formed behind truncated forms with the apex upstream (disks, cones, hemispheres, semiellipsoids, ogives) have been reported by Reichardt[2] and Eisenberg and Pond.[3] Such measurements for cavities about bodies of revolution composed of cylindrical middle bodies and various head shapes have been reported by Rouse and McNown.[4] Reichardt's data are particularly of interest, since they extend to the lowest cavitation numbers yet attained (as low as 0.013).

For the truncated forms for which the leading edge of the cavity is essentially fixed at the trailing edge of the form (cones, disks), measurements of the principal dimensions can be represented within the experimental error by formulas given by Reichardt.[2] The ratio of maximum cavity diameter to diameter of disk or base of cone is

$$\frac{d_m}{d} = \sqrt{C_D(0)\,\frac{1+\sigma}{\sigma f}} \tag{2o-1}$$

where

$$f = 1 - 0.132\sigma^{\frac{1}{2}} \tag{2o-2}$$

and values of $C_D(0)$ are given in Table 2o-1. The ratio of maximum cavity diameter to cavity length is

$$\frac{d_m}{l} = \frac{0.066 + 1.70\sigma}{\sigma + 0.008} \tag{2o-3}$$

2o-6. Drag in Cavitating Flow. Available data indicate that, for the truncated bodies discussed above, the drag coefficient is a linear function of the cavitation number. Available data may be represented by[5]

$$C_D(\sigma) = C_D(0)(1 + \beta\sigma) \tag{2o-4}$$

where the value of β is given in Table 2o-1. This formula can also be used to represent available data for a circular cylinder with its axis normal to the flow. The value of $C_D(0)$ for the disk is the average of the extrapolated values of Reichardt[2] and Eisenberg and Pond.[3] The results for the cones are from Reichardt; the results for the hemisphere, semiellipsoid, and ogive are from Eisenberg and Pond. In each of these cases, the value of $C_D(0)$ is extrapolated from the experimental data from which the values of β were also obtained. The value of $C_D(0)$ for the circular cylinder is from a computation of Brodetsky.[6] The value of $\beta = 0.73$ for the circular cylinder

[1] Eisenberg, *loc. cit.*

[2] H. Reichardt, The Laws of Cavitation Bubbles at Axially Symmetrical Bodies in a Flow, *Ministry Aircraft Prod., Rept. Translations* 766, Aug. 15, 1946 (distributed in the United States by the Office of Naval Research, Washington, D.C.).

[3] Eisenberg, *loc. cit.*; Phillip Eisenberg and Hartley L. Pond, Water Tunnel Investigations of Steady State Cavities, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 668, October. 1948.

[4] *Loc. cit.*

[5] Eisenberg, *loc. cit.*

[6] S. Brodetsky, Discontinuous Fluid Motion Past Circular and Elliptic Cylinders, *Proc. Roy. Soc. (London)*, ser. A, **102** (A718), 542–553 (February, 1923).

is given by Birkhoff[1] based on experiments of Martyrer. The other values of β for the circular cylinder are based on Kanstantinov's[2] experiments, which show differences depending on Reynolds number (based on cylinder diameter). It should be noted that Kanstantinov's results are for constant Reynolds number, whereas in Martyrer's tests the Reynolds number varied as the cavitation number was varied. There may be a question, however, as to the accuracy of Kanstantinov's results, since the forces were found by integrating pressure distributions rather than by direct measurement.

TABLE 2o-1. DATA FOR DRAG COEFFICIENT IN EQ. (2o-4)

Model	$C_D(0)$	Range of σ	β	Reynolds No.
Disk, $h/d = 0$	0.80	0.038–0.56	1.0	$2.6\text{–}7.9 \times 10^5$*
Cones:				
$h/d = \frac{1}{4}$	0.63	0.033–0.125	1.0	
$h/d = \frac{1}{2}$	0.5	0.032–0.118	1.0	
$h/d = 1$	0.32	0.026–0.069	1.0	
$h/d = 2$	0.15	0.013–0.086	1.0	
Hemisphere	0.241	0.168–0.38	2.024	$3\text{–}8.3 \times 10^5$
2:1 semiellipsoid and 2 caliber ogive	0.114	0.133–0.394	3.65	$\approx 3\text{–}9 \times 10^5$
Circular cylinder	≈ 0.55	....	0.81	2.72×10^5
			0.68	1.75×10^5
			0.73	$2\text{–}6 \times 10^5$

* Phillip Eisenberg and Hartley L. Pond, Water Tunnel Investigations of Steady State Cavities, *David Taylor Model Basin, U.S. Navy Dept. Rept.* 668, October, 1948.

2o-7. Nonstationary Cavities and Other Topics. A third type of flow which may be defined as part of a general classification of cavitating flows is the "nonstationary" (or "unsteady") cavity. This is a cavity resembling steady-state cavities but varying in time as in the air-water entry of an air-dropped missile or as in the motion of an initially submerged but accelerating body. Although all three are free-boundary flows, in the transient cavity, the pressure at the boundary varies with time; in the steady-state cavity, the boundaries are free streamlines; and, in the third, the boundaries are such that the material lines are not necessarily free streamlines. The nomenclature used here was chosen to provide a consistent representation for both the physical phenomena and the corresponding mathematical descriptions. Further discussions of nonstationary cavities and references will be found in Eisenberg[3] and Birkhoff.[3]

For problems of lift in cavitating flows, especially supercavitating hydrofoils, see Tulin.[4] Information on such subjects as cavitation damage and measurement of air content will be found in Eisenberg.[5]

[1] Garrett Birkhoff, "Hydrodynamics," chap. 2, Princeton University Press, Princeton, N.J., 1950.

[2] W. A. Kanstantinov, Influence of the Reynolds Number on the Separation (Cavitation) Flow, *David Taylor Model Basin, U.S. Navy Dept. Translation* 233, November, 1950.

[3] *Loc. cit.*

[4] Marshall P. Tulin, Supercavitating Flows, part 2, sec. 12, "Handbook of Fluid Dynamics," V. L. Streeter, ed., McGraw-Hill Book Company, New York, 1961.

[5] Phillip Eisenberg, Mechanics of Cavitation, part 1, sec. 12, "Handbook of Fluid Dynamics," V. L. Streeter, ed., McGraw-Hill Book Company, New York, 1961.