2q. Liquid Jets[1]

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2q-1. Circular Jet. We first deal with the laminar flow due to a circular jet of viscous fluid issuing from a point orifice into a space filled with the same fluid.

Symbols

- J momentum crossing a plane normal to the axis of the jet per second
- u, v x, y components, respectively, of fluid velocity in the jet
- x distance parallel to the axis of the jet
- y distance perpendicular to the axis of the jet
- ρ fluid density
- ν kinematic viscosity of the fluid

The flow-velocity components in the jet are given by the following formulas due to Schlichting:[2]

$$u = \frac{3}{8\pi}\frac{K}{\nu x}\frac{1}{(1+\epsilon^2/4)^2} \qquad v = \frac{1}{4}\sqrt{\frac{3K}{\pi}}\,\frac{1}{x}\,\frac{\epsilon(1-\epsilon^2/4)}{(1+\epsilon^2/4)^2} \tag{2q-1}$$

where

$$\epsilon = \frac{1}{4}\sqrt{\frac{3K}{\pi}}\,\frac{1}{\nu}\,\frac{y}{x} \qquad K = \frac{J}{\rho} \tag{2q-2}$$

The formulas (2q-1) have been checked experimentally by Andrade and Tsien,[3] who found good agreement between the theory and experimental results for a jet of finite radius a at a distance of 8 jet diameters or more from the orifice, provided the x in (2q-1) is given by

$$x = x_o + 0.16u_o\frac{a^2}{\nu} \tag{2q-3}$$

where x_0 is the actual distance to the real orifice, and x may be interpreted as the distance to an effective point orifice upstream from the real one.

Figure 2q-1 shows a family of streamlines for a circular jet from a point orifice plotted from Eqs. (2q-1). (For reasons of clarity the figure is expanded in the y direction.) Typical velocity profiles (plots of u vs. y) are also given for two distances x from the orifice.

[1] For a general reference see H. Schlichting, "Boundary Layer Theory," 6th ed., translated by J. Kestin, McGraw-Hill Book Company, New York, 1968.
[2] H. Schlichting, *Z. angew. Math. Mech.* **13**, 260 (1933).
[3] E. N. da C. Andrade and L. C. Tsien, *Proc. Phys. Soc.* (*London*) **49**, 381 (1937).

2q-2. Plane Jet. Laminar flow due to a *plane* jet of viscous fluid issuing from a line orifice into a space filled with the same fluid is described by the following formulas:

$$u = 0.4543 \left(\frac{K^2}{\nu x}\right)^{\frac{1}{3}} \operatorname{sech}^2 \epsilon$$

$$v = 0.5503 \left(\frac{K\nu}{x^2}\right)^{\frac{1}{3}} (2\epsilon \operatorname{sech}^2 \epsilon - \tanh \epsilon) \qquad (2q\text{-}4)$$

where

$$\epsilon = 0.2751 \left(\frac{K}{\nu^2}\right)^{\frac{1}{3}} y x^{-\frac{2}{3}}$$

$$K = \frac{J}{\rho}$$

Here x is distance from the line source, measured parallel to the plane of symmetry

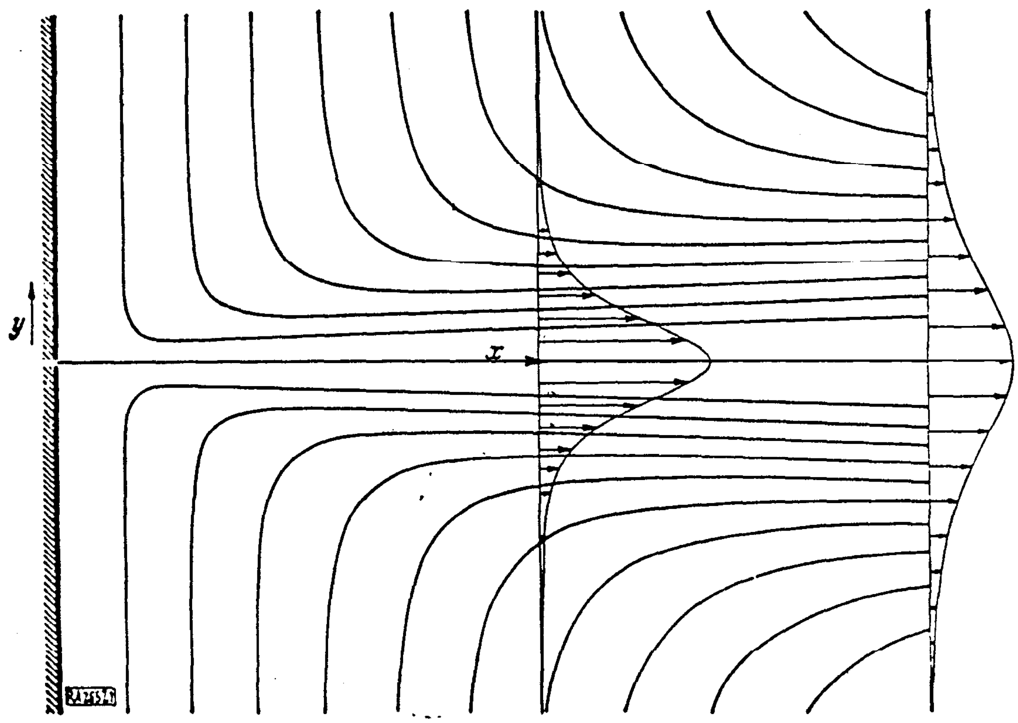

FIG. 2q-1. Streamlines for a circular jet from a point orifice.

of the jet, and y is measured normal to this plane; all other symbols have meanings analogous to those used in Eqs. (2q-1). This theoretical result due to Bickley[1] has been checked experimentally by Andrade[2] and found to be valid for jets from slits of finite width w, provided that x in Bickley's formula is given by

$$x = x_0 + \frac{0.65Kw}{v\nu} \qquad (2q\text{-}5)$$

[1] W. G. Bickley, *Phil. Mag.* **23**, 727 (1937).
[2] E. N. da C. Andrade, *Proc. Phys. Soc.* (*London*) **51**, 784 (1939).