

## 2r. Viscosity of Gases

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### Symbols

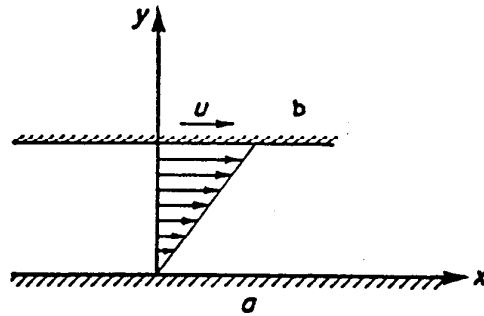
$a$	parameter in viscosity correlation, Eq. (2r-12)
$b$	virial coefficient, Eq. (2r-13)
$b^*$	reduced virial coefficient
$c$	virial coefficient, Eq. (2r-13)
$E$	potential energy of molecular binding
$e_{ij}$	strain tensor
$F_\mu$	reduced viscosity function, Eq. (2r-12)
$f_\mu^{(n)}$	$n$ th-order viscosity correction factor
$k$	Boltzmann's constant
$M$	molecular weight
$m$	mass of a molecule; parameter in Lennard-Jones ( $m - 6$ ) potential
$n$	number of molecules per unit volume
$p$	hydrostatic pressure
$p_s$	saturation pressure
$r_m$	molecular distance for maximum binding energy
$s$	viscosity stretching factor, Eq. (2r-12a)
$T$	absolute temperature
$T^*$	( $= kT/\epsilon$ ) reduced temperature
$t$	Celsius temperature
$t_{ij}$	stress tensor
$u$	velocity of fluid flow
$u_1, u_2, u_3$	fluid-flow velocity components
$\delta_{ij}$	Kronecker symbol
$\epsilon$	maximum binding energy between molecules
$\theta$	reduced temperature, Eq. (2r-12b)
$\lambda$	constant in constitutive equation
$\mu$	viscosity of fluid; constant in constitutive equation
$\mu_0$	viscosity at reference temperature $T_0$ or at zero density
$\mu_r$	( $= \mu/\mu_0$ ) viscosity ratio
$\nu$	kinematic viscosity
$\rho$	density
$\rho^*$	reference density
$\sigma$	molecular distance for vanishing potential
$\tau$	( $= 1/T$ ) inverse temperature
$\tau_0$	shear stress
$\Omega^{(2,1)*}$	reduced collision integral

TABLE 2t-1. ABSOLUTE VISCOSITY  $\mu$ : UNITS AND CONVERSION FACTORS

Units	centipoises	kg/m-sec	kp sec/m <sup>2</sup>	lbf-sec/ft <sup>2</sup>	lbm/ft-hr	lbm/ft-sec	micropoises	poises	slugs/ft-sec
centipoises.....	1.0	$1.0 \times 10^{-3}$	$1.01972 \times 10^{-4}$	$2.08854 \times 10^{-6}$	2.41909	$6.71969 \times 10^{-4}$	$1.0 \times 10^4$	$1.0 \times 10^{-2}$	$2.08854 \times 10^{-3}$
kg/m-sec.....	$1.0 \times 10^3$	1.0	$1.01972 \times 10^{-1}$	$2.08854 \times 10^{-2}$	$2.41909 \times 10^3$	$0.71969 \times 10^{-1}$	$1.0 \times 10^7$	$1.0 \times 10^1$	$2.08854 \times 10^{-1}$
kp sec/m <sup>2</sup> .....	$9.80665 \times 10^3$	$9.80665$	1.0	$2.04816 \times 10^{-1}$	$2.37232 \times 10^3$	$0.58976 \times 10^{-1}$	$9.80665 \times 10^7$	$9.80665 \times 10^1$	$2.04816 \times 10^{-1}$
lbf-sec/ft <sup>2</sup> .....	$4.78803 \times 10^4$	$4.78803 \times 10^1$	4.88243	1.0	$1.5826 \times 10^4$	$3.21740 \times 10^1$	$4.78803 \times 10^8$	$4.78803 \times 10^2$	1.0
lbm/ft-hr.....	$4.13379 \times 10^{-1}$	$4.13379 \times 10^{-4}$	$4.21559 \times 10^{-4}$	$8.63361 \times 10^{-6}$	1.0	$2.77778 \times 10^{-4}$	$4.13379 \times 10^3$	$4.13379 \times 10^{-3}$	$8.63361 \times 10^{-3}$
lbm/ft-sec.....	$1.48816 \times 10^3$	$1.48816$	$1.51750$	$3.10810 \times 10^{-2}$	$3.60000 \times 10^3$	1.0	$1.48816 \times 10^7$	$1.48816 \times 10^1$	$3.10810 \times 10^{-1}$
micropoises.....	$1.0 \times 10^{-4}$	$1.0 \times 10^{-7}$	$1.01972 \times 10^{-8}$	$2.08854 \times 10^{-9}$	$2.41909 \times 10^{-4}$	$6.71969 \times 10^{-8}$	1.0	$1.0 \times 10^{-6}$	$2.08854 \times 10^{-5}$
poises (g/cm-sec).....	$1.0 \times 10^5$	$1.0 \times 10^2$	$1.01972 \times 10^{-2}$	$2.08854 \times 10^{-3}$	$2.41909 \times 10^2$	$6.71969 \times 10^{-2}$	$1.0 \times 10^8$	1.0	$2.08854 \times 10^{-2}$
slugs/ft-sec.....	$4.78803 \times 10^4$	$4.78803 \times 10^1$	4.88243	1.0	$1.5826 \times 10^4$	$3.21740 \times 10^1$	$4.78803 \times 10^8$	$4.78803 \times 10^2$	1.0

Note. When the last significant digit is shown in boldface type, the conversion factor represents a conventional factor which is accurate by definition and involves no approximation.

2r-1. Definitions. The *viscosity* of a fluid is defined in relation to a *macroscopic* system which is assumed to possess the properties of a *continuum*. To obtain an *elementary definition* of viscosity (Fig. 2r-1) consider two infinite flat plates, *a* at rest and *b* moving at a constant velocity *u*, the space between them being filled with the fluid under consideration. In the resulting *shear flow* the velocity distribution is linear with a constant transverse gradient  $du/dy$ . It is assumed (*Newton's law of fluid friction*) that the shearing stress  $\tau_0$  at either wall is proportional to the velocity gradient



$$\tau_0 = \mu \frac{du}{dy} \quad (2r-1)$$

FIG. 2r-1. Illustration of Newton's law of fluid friction.

The coefficient of proportionality  $\mu$  is known as the *viscosity*, or more precisely, as the *dynamic* or *absolute viscosity* of the fluid. The various units of viscosity and their conversion factors are given in Table 2r-1.

The ratio

$$\nu = \frac{\mu}{\rho} \quad (2r-2)$$

is known as the *kinematic viscosity*; the respective units and conversion factors are given in Table 2r-2.

TABLE 2r-2. KINEMATIC VISCOSITY  $\nu$ ; UNITS AND CONVERSION FACTORS

Units	m <sup>2</sup> /sec	m <sup>2</sup> /hr	cm <sup>2</sup> /sec (stokes)	ft <sup>2</sup> /sec	ft <sup>2</sup> /hr
m <sup>2</sup> /sec.....	1	3,600	1 × 10 <sup>4</sup>	10.7639	3.875 × 10 <sup>4</sup>
m <sup>2</sup> /hr.....	277.8 × 10 <sup>-6</sup>	1	2.778	200.0 × 10 <sup>-6</sup>	10.7630
cm <sup>2</sup> /sec (stokes).....	1 × 10 <sup>-4</sup>	0.36	1	10.7639 × 10 <sup>-4</sup>	3.875
ft <sup>2</sup> /sec.....	0.092903	334.45	929.03	1	3,600
ft <sup>2</sup> /hr.....	25.806 × 10 <sup>-6</sup>	0.092903	0.25806	277.8 × 10 <sup>-6</sup>	1

From British Standard Code B.S. 1042: 1943 amended March, 1946. See Note to Table 2r-1.

In a general field of flow,  $u_1, u_2, u_3$  of a homogeneous Newtonian incompressible fluid, the *shearing stresses* are proportional to the respective *rates of change of strain* (Stokes' law). The symmetric *stress tensor*  $t_{ij}$  is assumed to be a linear function of the *rate of strain tensor*  $e_{ij}$ . Taking into account that in a fluid at rest the stress is an isotropic tensor, we put

$$t_{ij} = -p\delta_{ij} + \lambda\delta_{ij}e_{kk} + 2\mu e_{ij}$$

where  $\delta_{ij}$  is the Kronecker symbol ( $\delta = 1$  for  $i = j$  and  $\delta = 0$  for  $i \neq j$ ) and  $p$  is arbitrary. Since  $t_{ij} = 0$  for  $e_{ij} = 0$ , we have  $t_{ii} = -3p$  and  $3\lambda + 2\mu = 0$  (Stokes' hypothesis). Consequently

$$t_{ij} = -p\delta_{ij} - \frac{2}{3}\mu\delta_{ij}e_{kk} + 2\mu e_{ij} \quad (2r-3)$$

where now  $p$  denotes the hydrostatic pressure. The scalar  $\mu$  is defined as *the absolute viscosity of the fluid*.

The viscosity is assumed to be a function of the thermodynamic state of the fluid and independent of the velocity field. For a homogeneous fluid  $\mu$  is a function of *two properties*. It is customary to use either of the following two alternative representations:

$$\mu = \mu(p, T) \quad \text{or} \quad \mu = \mu(\rho, T) \quad (2r-4)$$

where  $T$  is the absolute temperature,  $p$  is the pressure, and  $\rho$  is the density of the fluid.

Numerical values of viscosity cannot be calculated with the aid of the equations of thermodynamics. They must be measured directly, the measurement being usually very difficult, particularly at higher pressures and temperatures. In principle, values of viscosity can be calculated by the methods of the kinetic theory of gases and statistical mechanics with quantum corrections where necessary.

In relation to a *microscopically* defined system the viscosity of a gas is assumed to be due to a transfer of momentum effected by molecules, their velocity being composed of the molecular (random) velocity and the macroscopic (ordered) velocity. In shear flow (Fig. 2r-2), the shearing stress acting on a small element of area  $aa$  is equal to the integral of the change in momentum effected by the particles moving across, both from above and from below it, the integral extending over all particles crossing.

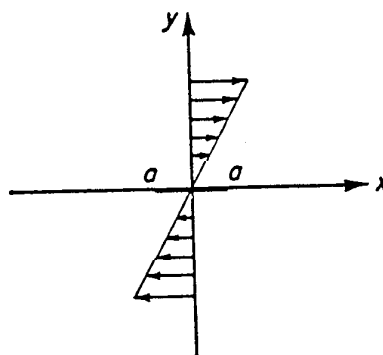


FIG. 2r-2. Kinetic interpretation of viscosity.

**2r-2. Variation of Viscosity with Temperature and Pressure.** The calculation of the viscosity of gases has so far met with only limited success, extensive experimental determinations still forming the basis for practical applications. The calculation of the viscosity of gases must make use of a *molecular model* for the gas, increasing refinements being possible.

On the simplest assumption of infinitely small, perfectly elastic molecules with zero fields of force (Maxwell) it is found that the absolute viscosity of a gas is independent of pressure and that it increases in proportion to  $T^{1/2}$ :

$$\begin{aligned} \mu &= K_1 T^{1/2} & \left( \frac{\partial \mu}{\partial p} \right)_T &= 0 \\ \nu &= K_2 T^{1/2} & p &= \text{const} \end{aligned} \quad (2r-5)$$

where  $K_1$  and  $K_2$  are empirical constants.

On the assumption of hard elastic spheres with a weak attraction force (Sutherland), it is found that

$$\mu = \frac{KT^{3/2}}{C + T} \quad \tau = \frac{1}{T} \quad (2r-6)$$

where  $K$  and  $C$  are empirical constants. Sutherland's equation (2r-6), as well as experimental results, show the increase with temperature to be *faster* than that in Maxwell's equation (2r-5).

The fact that the viscosity of a gas increases with temperature can be understood if it is realized that in gases the effects of molecular motion dominate over those due to intermolecular forces. In liquids cohesion forces are more important, and since the molecular bonds in a liquid are loosened as the temperature is increased, the absolute viscosity of a *liquid* decreases with temperature; that for a *gas* increases with

temperature. Sutherland's equation (2r-6) is inadequate for the correlation of experimental data over large temperature intervals.

In problems of compressible fluid flow it is customary to use the empirical relation

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^\omega \quad (2r-7)$$

where  $\mu_0$  is the value of  $\mu$  at a reference temperature  $T_0$  and  $\omega$  is an empirical constant ranging from 0.6 to 1.5. This correlation is less precise than those given later.

All preceding formulas relate to gases at low pressures (say atmospheric). Experimental results (which are still very scarce) show that the viscosity of gases at constant temperature *increases* with pressure, the increase being of the order of 20 to 40 per cent per 1,000 atm. For moderate pressure ranges it is possible to use a linear interpolation formula

$$\frac{\mu}{\mu_0} = 1 + k \quad (2r-8)$$

where  $\mu_0$  is the viscosity at temperature  $T$ , but at zero density, and  $k$  is an empirical constant.

More precisely, the viscosity of a gas increases as its density is increased. Since the viscosity of a gas consisting of molecules which exert no forces upon one another (Maxwell) is independent of density, this behavior is taken as evidence of the existence of intermolecular fields of forces. However, exceptions exist to this rule, notably steam and hydrocarbons, whose viscosity at constant temperature *decreases* with pressure, and therefore density, in certain ranges of states. In turn this is taken as evidence of the existence of some form of molecular association whose precise nature is not understood.

**2r-3. Variation of Viscosity with Temperature and Pressure According to Kinetic Theory.** There exists a rigorous kinetic theory of the equilibrium and transport properties of gases which is based on Boltzmann's equation. Thus, in particular, and in principle, the viscosity, thermal conductivity (see Sec. 4g) and virial coefficients of gases (see Sec. 4i) are calculated in a consistent and unified way. This theory is due to Chapman and Enskog (S. Chapman and T. G. Cowling, "Mathematical Theory of Non-uniform Gases," Cambridge University Press, New York, 1970; J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, "Molecular Theory of Gases and Liquids," John Wiley & Sons, Inc., New York, 1964.) The calculations are made on the basis of assumed semiempirical force potentials. For nonpolar gases the most widely used potentials have been the Lennard-Jones twelve-six potential and the modified Buckingham exp-six potential; that used for polar gases is the Stockmayer potential.

The viscosity at zero density is then calculated from the equation

$$\mu_0 = \frac{5 \sqrt{mkT} f_\mu^{(n)}(T^*)}{16\pi^{\frac{1}{2}} \sigma^2 \Omega^{(2,2)*}(T^*)} \quad (2r-9)$$

or, with the values of the universal constants substituted

$$\frac{\mu_0}{\text{micropoise}} = 26.694 \frac{\left(\frac{M}{\text{g/g-mole}} \cdot \frac{T}{\text{K}}\right)^{\frac{1}{2}} f_\mu^{(n)}(T^*)}{(\sigma^2/\text{\AA}^2) \Omega^{(2,2)*}(T^*)} \quad (2r-10)$$

Here  $\sigma$  is the molecular distance at which the potential vanishes,  $M$  is the molecular weight,  $k$  is Boltzmann's constant, and  $T^* = kT/\epsilon$  is a dimensionless temperature with  $\epsilon$  denoting the depth of the potential well. The collision integral  $\Omega^{(2,2)*}$  and the factor  $f_\mu^{(n)}$ , both of which are unique functions of the dimensionless temperature  $T^*$ , are given in terms of the intermolecular force potential and must be tabulated

for each one of them separately. Such tabulations for the more general  $m - 6$  potential can be found in "Tables of Collision Integrals for the  $(m - 6)$  Potential for Ten Values of  $m$ " by M. Klein and F. J. Smith (*Arnold Engineering Development Center Rept.*) AEDC-TR-68-92, May, 1968, Arnold Air Force Station, Tenn.), with  $m$  taking the values  $m = 9, 12, 15, 18, 21, 24, 30, 40, 50,$  and  $75$ . Tables for the exp-six potential can be found in "Transport Properties of Gases Obeying a Modified Buckingham (Exp-Six) Potential" by E. A. Mason [*J. Chem. Phys.* **22**, 169 (1954)]. The factor  $f_{\mu}^{(n)}$  with  $n = 1, 2, \dots$  represents successive approximations and it is usual to confine it to the third approximation,  $f_{\mu}^{(3)}$ , at most.

In principle, the form of and the constants in a potential can be determined by quantum mechanics from a knowledge of the structure of the molecule. However, the attendant mathematical difficulties preclude us from doing so, and potentials must be determined by fitting experimental data on a variety of properties to expressions like the one in Eq. (2r-9). The efforts to associate definite potentials and physically meaningful constants with even the simplest molecules have not yet met with complete success. One of the difficulties is connected with the fact that often several alternative potentials give equally good fits to a set of experimental data of a definite property of a gas, but none seems to reproduce all properties to within the experimental error. Thus, there exists no preferred or universal form of the potential, but, as a matter of experience, it can be stated that the viscosity of the simpler gases, except that of helium, is reproduced reasonably well by the potential family

$$E(r) = \frac{m\epsilon}{m-6} \left[ \frac{m}{6} \right]^{6(m-6)} \left[ \left( \frac{\sigma}{r} \right)^m - \left( \frac{\sigma}{r} \right)^6 \right] \quad (2r-11)$$

in which  $\sigma, \epsilon,$  and  $m$  are treated as adjustable constants. The viscosity of helium is best reproduced by the exp-six potential with  $r_m = 3.135 \text{ \AA}, \epsilon/k = 9.16 \text{ K},$  and  $\alpha = 12.4$  [E. A. Mason and W. E. Rice, *J. Chem. Phys.* **22**, 522, 843 (1954)]. Average, and to a certain extent preliminary, values of  $\sigma$  and  $\epsilon$  for the Lennard-Jones potential are listed in Table 2r-3. A better representation is obtained with the aid of the semiempirical formula

$$F_{\mu} = 1.0 - \frac{1.0}{\theta} + \frac{0.5}{\theta^2} \quad (2r-12)$$

where

$$F_{\mu} = \frac{s^2 \mu}{26.694 \sqrt{MT}} - a \quad (2r-12a)$$

and

$$\theta = gT \quad (2r-12b)$$

The optimum values of the constants  $a, g,$  and  $s$  are listed in Table 2r-4 for several gases.

Except for the neighborhood of the critical point, the effect of density (i.e., pressure) on the viscosity of gases, even up to pressures of the order of several hundred atmospheres, can be accounted for with the aid of the virial expansion

$$\mu(\rho, T) = \mu_0(T) + b(T)\rho + c(T)\rho^2 + \dots \quad (2r-13)$$

containing three or four terms. Kim and Ross [*J. Chem. Phys.* **42**, 263 (1965)] provided a theory for the virial  $b(T)$ . The diagram in Fig. 2r-3 represents the universally valid relation between

$$b^* = (T^*)^{-1} Q_{rel}(T^*) B_{00}(T^*) / \Omega^{(2,2)*}(T^*) \quad (2r-14)$$

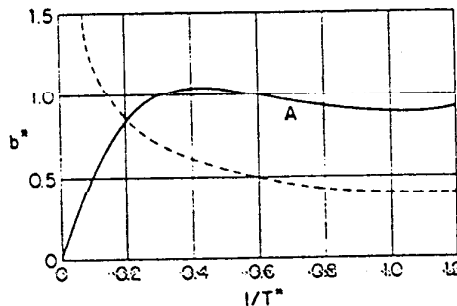


FIG. 2r-3. Dimensionless second virial coefficient for viscosity  $b^*$  as a function of reduced inverse temperature, according to Kim and Ross. [*J. Chem. Phys.* **42**, 263 (1965)].

and  $1/T^*$ . For the Lennard-Jones model, the expression reduces to

$$b^* = \frac{1}{15.20} \left( \frac{b}{\text{g/cm}^3} \right) \left( \frac{\text{\AA}}{\sigma} \right) \left( \frac{\text{K}}{\epsilon/k} \right)^{\frac{1}{2}} \left( \frac{M}{\text{g/g-mole}} \right)^{\frac{1}{2}} \quad (2r-15)$$

In the range where  $1/T^*$  exceeds 0.2 ( $T^* < 5$  approximately), the virial coefficient  $b$  is nearly a constant with  $b^* \approx 1$ . Consequently, Eq. (2r-14) can be simplified to

$$\mu(\rho, T) - \mu_0(0, T) = \frac{15.20\mu P \text{ cm}^3}{g} \left( \frac{\sigma}{\text{\AA}} \right) \left( \frac{\epsilon/k}{\text{K}} \right)^{\frac{1}{2}} \left( \frac{\text{g/g-mole}}{M} \right)^{\frac{1}{2}} + O(\rho^2) \quad (2r-16)$$

This form leads to an approximate equation for the excess viscosity  $\mu(\rho, T) - \mu_0(0, T)$  which has often been used for correlations. This form is

$$\mu(\rho, T) - \mu_0(0, T) \approx f(\rho) \quad (2r-17)$$

in which  $f(\rho)$  is a unique (empirical) function for each gas.

TABLE 2r-3. MOLECULAR-FORCE CONSTANTS FOR THE LENNARD-JONES (12-6) POTENTIAL

$$E(r) = 4\epsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right]$$

Gas	Symbol	$\epsilon/k, \text{K}$	$\sigma, \text{\AA}$	Ref.
Acetylene.....	$\text{C}_2\text{H}_2$	185	4.221	1
Air.....	—	{ 84.0 } { 117.5 }	{ 3.689 } { 3.512 }	1 2
Argon.....	Ar	{ 124.0 } { 152.8 }	{ 3.418 } { 3.292 }	1 2
Bromine.....	$\text{Br}_2$	520	4.268	1
Carbon dioxide.....	$\text{CO}_2$	261.1	3.705	2
Carbon monoxide.....	CO	110.3	3.590	3
Chlorine.....	$\text{Cl}_2$	257	4.40	1
Deuterium.....	$\text{D}_2$	39.3	2.948	1
Ethylene.....	$\text{C}_2\text{H}_4$	205	4.232	1
Helium.....	He	{ 10.22 } { 86.20 }	{ 2.576 } { 2.158 }	1 2
Hydrogen.....	$\text{H}_2$	38.0	2.915	1
Iodine.....	$\text{I}_2$	550	4.982	1
Krypton.....	Kr	206.4	3.522	2
Methane.....	$\text{CH}_4$	144	3.796	1
Neon.....	Ne	{ 35.7 } { 60.9 }	{ 2.789 } { 2.648 }	1 2
Nitric oxide.....	NO	119	3.470	1
Nitrogen.....	$\text{N}_2$	{ 91.5 } { 113.5 }	{ 3.681 } { 3.566 }	1 2
Oxygen.....	$\text{O}_2$	113	3.433	1
Propane.....	$\text{C}_3\text{H}_8$	254	5.061	1
Xenon.....	Xe	229	4.055	1

Note 1. Differences in the values in this table and the table in Sec. 4i are a measure of the uncertainties which still exist, as well as of the fact that the best fits to experimental values of virial coefficients and viscosity are obtained with slightly different values of the constants.

Note 2. In the case of helium the best form of potential function is that of the modified Buckingham exponential-six with parameters as quoted in the text. Consequently, the values of the parameters shown in the table may not be physically meaningful, especially in the case of those quoted from ref. 2.

#### References for Table 2r-3

- Hirschfelder, J. O., C. F. Curtiss, and R. B. Bird: "Molecular Theory of Gases and Liquids," Table 1-A, p. 1110, John Wiley & Sons, Inc., New York, corrected edition, 1964.
- DiPippo, R., and J. Kestin: Viscosity of Seven Gases up to 500°C and Its Statistical Interpretation, *Proc. 4th Symp. on Thermophys. Properties*, ASME, New York, 1968.
- Natl. Bur. Standards Circ. 564*, 1955.

TABLE 2r-4. PARAMETERS IN VISCOSITY CORRELATION, EQ. (2r-12)

Gas	Symbol	$a$	$g \times 10^3, (K)^{-1}$	$s, \text{Å}$	Temp. range, K
Air.....	.....	1.3034	6.0906	3.484	298-773
Argon.....	Ar	1.0300	7.5793	2.970	298-573
Butane.....	C <sub>4</sub> H <sub>10</sub>	0.91040	5.5145	4.730	311-511
Carbon dioxide.....	CO <sub>2</sub>	0.94147	5.3316	3.230	298-773
Ethane.....	C <sub>2</sub> H <sub>6</sub>	0.92669	6.2093	3.820	294-511
Ethylene.....	C <sub>2</sub> H <sub>4</sub>	0.71342	3.3598	2.235	303-368
Helium.....	He	1.5779	4.0302	2.250	298-673
Krypton.....	Kr	0.83447	8.4746	2.935	298-473
Methane.....	CH <sub>4</sub>	1.0532	5.2434	3.208	283-411
Neon.....	Ne	1.6602	6.6667	2.895	298-453
Nitrogen.....	N <sub>2</sub>	1.3127	6.2232	3.548	298-773

Unpublished correlation prepared by authors of this article.

**2r-4. Viscosity in the Neighborhood of the Critical Point.** Contrary to earlier views, it has now become accepted that the viscosity of a gas does not increase anomalously in the neighborhood of the critical point, even though the representation in the form of Eq. (2r-13) breaks down there. The viscosity in the neighborhood of the critical point has been measured (rather sketchily) for a very small number of substances only. A qualitative idea of the resulting behavior can be obtained from the diagram for CO<sub>2</sub>, given as Fig. 2r-4 [J. Kestin, J. H. Whitelaw and T. F. Zien, *Physica* 30, 161 (1964)].

**2r-5. Law of Corresponding States.** Attempts have also been made to correlate the viscosity of gases with the aid of the law of corresponding states. The most promising correlation [J. M. J. Coremans and J. J. M. Beenakker, *Physica* 26, 653 (1960)] makes use of molecular constants for the formation of reduced variables. The reference temperature is chosen as  $T^* = kT/\epsilon$ , the reference density being chosen as the fraction of volume occupied by the molecular core  $\rho^* = \frac{1}{3}\pi n(\frac{1}{2}\sigma)^3$  where  $n$  is the number density. The viscosity  $\mu$  is referred to  $\mu_0$  measured at zero density, so that  $\mu_r = \mu/\mu_0$  and

$$\mu_r = f(T^*, \rho^*) \tag{2r-18}$$

where  $f$  is an approximately universal function. It can be represented by the power series

$$\mu_r = 1 + \frac{0.55\rho^* + 0.96\rho^{*2} + 0.61\rho^{*3}}{T^{*0.59}} \tag{2r-19}$$

from which it is seen that the relative excess viscosity  $\mu_r - 1$  is a unique function of relative density  $\rho^*$  at constant relative temperature  $T^*$  according to Eq. (2r-17). Equation (2r-18) reproduces the experimental values for nonpolar or only slightly polar gases, with an error of the order of  $\pm 3$  per cent over a fairly large range of temperatures and densities. The error is negligible up to densities of approximately 200 amagat units, and the equation can be used up to about 500 amagat units.

**2r-6. Mixtures of Gases.** The viscosity of a gaseous mixture cannot be deduced from the knowledge of its composition and of the viscosities of its components by macroscopic methods, and methods of statistical mechanics must be used. In any case it should be noted that the viscosity of a mixture is not equal to the weighted mean of the viscosity of its components, it being possible for the viscosity of a mixture to be higher than that of its components. For example, a mixture of argon ( $\mu_{Ar} = 222 \times 10^{-6}$  poise) and helium ( $\mu_{He} = 195 \times 10^{-6}$  poise) containing 40 per cent He



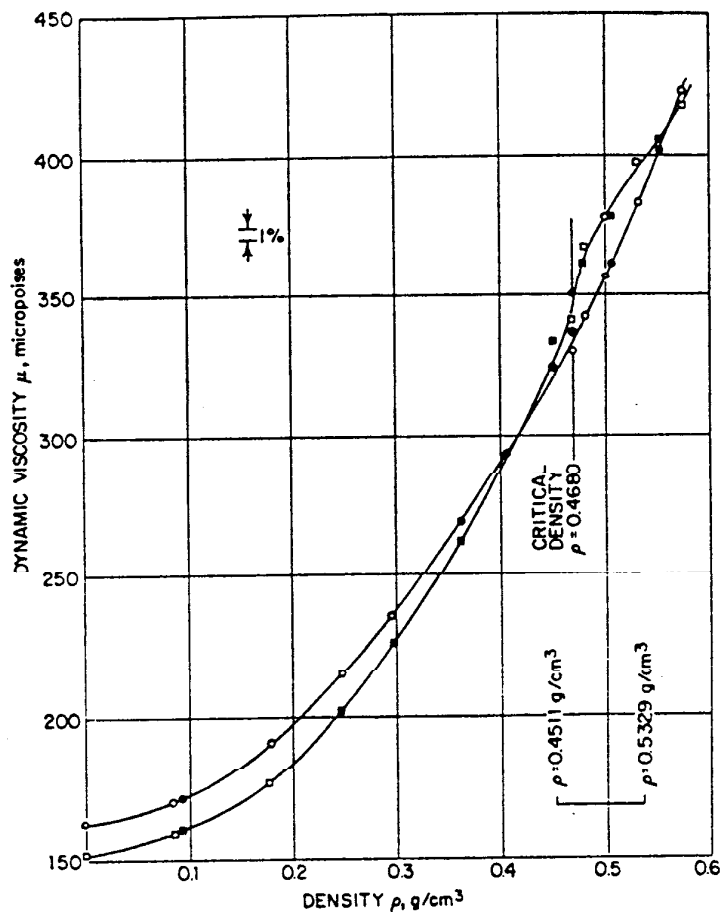


FIG. 2r-4. Viscosity of carbon dioxide as a function of density in the near-critical region according to Kestin, Whitelaw, and Zein [*Physica* **30**, 161 (1964)].

and 60 per cent Ar has a viscosity of  $\mu = 230 \times 10^{-6}$  poise. Thus for a given pressure and temperature, the viscosity of a mixture can pass through a maximum when plotted as a function of composition. Maxima are also exhibited by the binary mixtures  $\text{H}_2\text{-Xe}$ ,  $\text{He-Ne}$ ,  $\text{H}_2\text{-SO}_2$ ,  $\text{H}_2\text{-C}_3\text{H}_8$ ,  $\text{H}_2\text{-CO}_2$ ,  $\text{H}_2\text{-C}_3\text{H}_6$ ,  $\text{H}_2\text{-C}_2\text{H}_6$ ,  $\text{H}_2\text{-NO}$ ,  $\text{H}_2\text{-C}_2\text{H}_4$ ,  $\text{He-Ar}$ ,  $\text{H}_2\text{-NH}_3$ ,  $\text{H}_2\text{-CH}_4$ ,  $\text{NH}_3\text{-C}_2\text{H}_4$ ,  $\text{HCl-CO}_2$ ,  $\text{CH}_4\text{-NH}_3$ , and possibly many others.

Even in the case of binary mixtures, the relation among the viscosity of the mixture, the viscosities of the pure components, and the composition is quite complex. At present the quality of the statistical approximation obtained by the methods of statistical mechanics is somewhat uncertain, and it is necessary to refer the reader to the treatise by J. O. Hirschfelder et al. (see footnote to Table 2r-3) for further details. Table 2r-5 gives sources of data on the viscosity of gaseous mixtures.

TABLE 2r-5. REFERENCES TO DATA ON BINARY GASEOUS MIXTURES

Mixture	Pressure range, atm	Temperature range, °C	Reference
Air-H <sub>2</sub> O	1	25-75	1
He-Ar	1-50	20-30	2
He-Ne	1-35	20-30	3
He-Kr	1-25	20-30	4
He-H <sub>2</sub>	1-25	20-30	5
He-N <sub>2</sub>	1-25	20-30	4
He-O <sub>2</sub>	1-25	20-30	5
He-CO <sub>2</sub>	{ 1-25	20-30	6
	{ 1-70	20	9
Ne-Ar	1-35	20-30	3
Ne-N <sub>2</sub>	1-25	20-30	6
Ne-CO <sub>2</sub>	1-25	20-30	7
Ar-NH <sub>3</sub>	1-25	20-30	8
Ar-N <sub>2</sub>	1-25	20-30	6
Ar-CO <sub>2</sub>	1-25	20-30	4
Kr-CO <sub>2</sub>	1-25	20-30	5
N <sub>2</sub> -H <sub>2</sub>	1-25	20-30	5
N <sub>2</sub> -CO <sub>2</sub>	1-25	20-30	4
CO <sub>2</sub> -CH <sub>4</sub>	1-25	20-30	5
CH <sub>4</sub> -C <sub>4</sub> H <sub>10</sub>	1-25	20-30	5

References for Table 2r-5

1. Kestin, J., and J. H. Whitelaw: Measurement of the Viscosity of Dry and Humid Air, p. 301 in "Humidity and Moisture," vol. III, p. 301, Reinhold Book Corporation, New York, 1965.
2. Iwasaki, H., and J. Kestin: *Physica* **29**, 1345 (1963).
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6. DiPippo, R., J. Kestin, and K. Oguchi: *J. Chem. Phys.* **46**, 4758 (1967).
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9. Richardson, H. P., D. Cummins, and R. A. Guereca: Absolute Viscosity Determinations by Means of a Coiled-capillary Viscosimeter: Data for Helium, Carbon Dioxide Mixtures, *Proc. 4th Symp. Thermophys. Properties*, ASME, New York, 1968.

**2r-7. Tables of Viscosity.** The variation of the viscosity of several gases, all extrapolated to zero density (but accurate enough at atmospheric pressure), can be obtained from the correlation in Eq. (2r-12) and the data in Table 2r-4.

Table 2r-6 contains the best available data on the absolute viscosity  $\mu$  of gases at 20°C and atmospheric pressure together with the temperature increment  $(\Delta\mu)_T$  and the pressure increment  $(\Delta\mu)_p$  at that point. Table 2r-7 lists the same values for the kinematic viscosity  $\nu$ . The values have been carefully selected in each case, either mean values or preferred values having been chosen depending on the merits of the available experimental material. The estimated uncertainties are also based on a critical assessment of available data and are, to a certain extent, arbitrary. Experimental results for both high pressures and temperatures are, for all intents and purposes, nonexistent.

TABLE 2r-6. ABSOLUTE VISCOSITY  $\mu$  OF GASES IN MICROPOISES  
 ( $10^{-6}$  g/cm sec =  $10^{-6}$  dyne sec/cm<sup>2</sup>; at 20°C and 1 atm)

Gas	Symbol	$\mu$ , $\mu$ poises	Estimated uncertainty $\pm \Delta\mu$ , $\mu$ poises	Temp. increment $(\Delta T)$ , $\mu$ poises/°C	Pressure increment $(\Delta P)$ , $\mu$ poises/atm	Source
Acetylene.....	C <sub>2</sub> H <sub>2</sub>	93.5 (at 0°C)	.....	.....	.....	"International Critical Tables"
Air.....	.....	181.92	0.006	0.536	0.1224	Bearden, <i>Phys. Rev.</i> <b>56</b> , 1023 (1939)
Ammonia.....	NH <sub>3</sub>	97.4	3	0.425	.....	Wtd. mean of 2 values
Argon.....	Ar	222.86	0.1	0.704	0.1753	Ref. 1
Bromine.....	Br <sub>2</sub>	149.5	.....	0.500	.....	Ref. 2
iso-Butane.....	C <sub>4</sub> H <sub>10</sub>	74.8	.....	0.237	.....	Ishida, <i>Phys. Rev.</i> <b>21</b> (1923)
n-Butane.....	C <sub>4</sub> H <sub>10</sub>	84.8	.....	0.300	.....	Kuenen and Visser, <i>Amsterdam Acad. Sci.</i> <b>22</b> , 336 (1913)
Carbon dioxide.....	CO <sub>2</sub>	146.63	0.07	0.450	0.0046	Ref. 1
Carbon monoxide.....	CO	175.3	0.1	0.474	.....	Wtd. mean of 4 values
Chlorine.....	Cl <sub>2</sub>	133.0	.....	0.451	.....	Rankine, <i>Proc. Roy. Soc. (London)</i> , ser. A, <b>86</b> , 162 (1912)
Chloroform.....	CHCl <sub>3</sub>	100.0	.....	0.340	.....	Ref. 2
Cyanogen.....	C <sub>2</sub> N <sub>2</sub>	100.2	.....	0.360	.....	Ref. 2
Deuterium.....	D <sub>2</sub>	124.68	0.07	0.284	0.0082	Ref. 1
Deuteromethane.....	CD <sub>4</sub>	129.0	.....	0.580	.....	Ref. 2
Ethane.....	C <sub>2</sub> H <sub>6</sub>	91.0	0.8	0.277	.....	Wtd. mean of 2 values
Ethylene.....	C <sub>2</sub> H <sub>4</sub>	100.0	.....	0.320	.....	Van Cleave and Mauss, <i>Can. J. Research</i> <b>13B</b> , 140 (1935)
Helium.....	He	196.14	0.1	0.464	-0.0093	Ref. 1
Hydrogen.....	H <sub>2</sub>	88.73	0.05	0.200	0.0118	Ref. 1
Hydrogen bromide.....	HBr	184.3	.....	0.680	.....	Ref. 2
Hydrogen chloride.....	HCl	142.5	.....	0.500	.....	Ref. 2
Hydrogen deuteride.....	HD	111.8	0.3	.....	.....	Kestin and Nagashima, <i>Phys. Fluids</i> <b>7</b> , 730 (1964)

TABLE 2t-6. ABSOLUTE VISCOSITY  $\mu$  OF GASES IN MICROPOISES (Continued)

Gas	Symbol	$\mu$ , $\mu$ poises	Estimated uncertainty $\pm \Delta\mu$ , $\mu$ poises	Temp. increment $(\Delta T)_T$ , $\mu$ poises/ $^{\circ}$ C	Pressure increment $(\Delta p)_p$ , $\mu$ poises/atm	Source
Hydrogen iodide.....	HI	183.0	.....	0.640	.....	Ref. 2
Krypton.....	Kr	249.55	0.15	0.735	0.2816	Ref. 1
Mercury.....	Hg	450.0 (200 $^{\circ}$ C)	.....	.....	.....	Ref. 2
Methane.....	CH <sub>4</sub>	109.8	0.1	0.330	0.016	Kestin and Leidenfrost, "Thermodynamic Properties of Gases, Liquids, Solids," p. 321, ASME 1958
Methyl bromide.....	CH <sub>3</sub> Br	132.7	.....	0.460	.....	Ref. 2
Methyl chloride.....	CH <sub>3</sub> Cl	107.0	.....	0.425	.....	Breitenbach, <i>Ann. Phys.</i> <b>6</b> , 166 (1901)
Neon.....	Ne	313.81	0.15	0.697	0.0354	Ref. 1
Nitric oxide.....	NO	189.8	0.1	0.538	.....	Wtd. mean of 3 values
Nitrogen.....	N <sub>2</sub>	175.69	0.09	0.454	0.1234	Ref. 1
Nitrous oxide.....	N <sub>2</sub> O	145.6	.....	0.475	.....	Johnston and McCloskey, <i>J. Phys. Chem.</i> <b>44</b> , 1038 (1940)
Oxygen.....	O <sub>2</sub>	203.31	0.1	0.616	0.1205	Ref. 1
Propane.....	C <sub>3</sub> H <sub>8</sub>	80.0	.....	0.22	.....	Ref. 2
Sulfur dioxide.....	SO <sub>2</sub>	125.0	.....	0.400	.....	Ref. 2
Xenon.....	Xe	227.40	0.14	0.725	0.2624	Ref. 1

References

1. Kestin, J., and W. Leidenfrost: *Physica* **25**, 1033 (1959).
2. Golubev, I. F.: "Viaz'kost' gazov i zhozovykh smeseci," Moscow, 1959. This reference contains extensive data whose accuracy, however, it is difficult to assess.

TABLE 2r-7. KINEMATIC VISCOSITY  $\nu$  OF GASES  
 ( $10^{-3}$  cm<sup>2</sup>/sec; at 20°C and 1 atm)

Gas	Symbol	$\nu$ , $10^{-3}$ cm <sup>2</sup> /sec	Estimated uncertainty $\pm \Delta \nu$ , $10^{-3}$ cm <sup>2</sup> /sec	Temp. increment $(\Delta \nu)_T$ , $10^{-3}$ cm <sup>2</sup> /sec(°C)	Pressure increment $(\Delta \nu)_P$ , $10^{-3}$ cm <sup>2</sup> /sec(atm)
Acetylene.....	C <sub>2</sub> H <sub>2</sub>	80.6 (at 0°C)	0.08	0.960	-150.9
Air.....	.....	151.1	4	1.07	
Ammonia.....	NH <sub>3</sub>	138	0.06	0.882	-134.1
Argon.....	Ar	134.3	.....	0.152	
Bromine.....	Br <sub>2</sub>	22.50	.....	0.204	
iso-Butane.....	C <sub>4</sub> H <sub>10</sub>	31.0	.....	0.244	
n-Butane.....	C <sub>4</sub> H <sub>10</sub>	35.1	.....	0.516	-80
Carbon dioxide.....	CO <sub>2</sub>	89.09	0.04	0.921	
Carbon monoxide.....	CO	159.6	0.09	0.307	
Chlorine.....	Cl <sub>2</sub>	45.11	.....	0.137	
Chloroform.....	CHCl <sub>3</sub>	29.16	.....	0.325	
Cyanogen.....	C <sub>2</sub> N <sub>2</sub>	45.35	.....	4.24	-740
Deuterium.....	D <sub>2</sub>	744.2	0.4	1.22	
Deuteromethane.....	CD <sub>4</sub>	154.7	.....	0.471	
Ethane.....	C <sub>2</sub> H <sub>6</sub>	72.9	0.6	0.997	
Ethylene.....	C <sub>2</sub> H <sub>4</sub>	85.84	.....	6.81	-1,200
Helium.....	He	1,179	0.6	0.389	
Hydrogen bromide.....	HBr	54.79	.....	0.651	
Hydrogen chloride.....	HCl	93.99	.....	6.01	-1,060
Hydrogen deuteride.....	HD	889.4	2.4	0.237	-72.20
Hydrogen.....	H <sub>2</sub>	1,059	0.6	0.460	
Hydrogen iodide.....	HI	34.42	.....	1.06	
Krypton.....	Kr	72.44	0.044		
Mercury.....	Hg	87.12 (at 200°C)			
Methane.....	CH <sub>4</sub>	164.8	0.2		-160

TABLE 2F-7. KINEMATIC VISCOSITY  $\nu$  OF GASES (Continued)

Gas	Symbol	$\nu$ , $10^{-3}$ cm <sup>2</sup> /sec	Estimated uncertainty $\pm \Delta \nu$ , $10^{-3}$ cm <sup>2</sup> /sec	Temp. increment $(\Delta \nu) \tau$ , $10^{-3}$ cm <sup>2</sup> /sec)(°C)	Pressure increment $(\Delta \nu) p$ , $10^{-3}$ cm <sup>2</sup> /sec)(atm)
Methyl bromide.....	CH <sub>3</sub> Br	33.64	.....	0.232	
Methyl chloride.....	CH <sub>3</sub> Cl	50.97	.....	0.376	
Neon.....	Ne	374.1	0.18	2.11	-374
Nitric oxide.....	NO	152.1	0.08	0.950	
Nitrogen.....	N <sub>2</sub>	150.9	0.08	0.905	-150.8
Nitrous oxide.....	N <sub>2</sub> O	79.57	.....	0.531	
Oxygen.....	O <sub>2</sub>	152.8	0.08	0.984	-152.6
Propane.....	C <sub>3</sub> H <sub>8</sub>	43.7	.....	0.269	
Sulfur dioxide.....	SO <sub>2</sub>	46.94	.....	0.310	
Xenon.....	Xe	42.02	0.026	0.278	-42.38

TABLE 2-8. VISCOSITY OF COMPRESSED WATER AND SUPERHEATED STEAM (MICROPOISES)  
Of each pair of figures the upper represents the adopted value and the lower the tolerance ( $\pm$ )

Pres- sure, bars	Temperature, °C																	
	0	50	100	150	200	250	300	350	375	400	425	450	475	500	550	600	650	700
1	17,500 400	5,440 140	121.1 1.2	141.5 1.4	161.8 1.6	182.2 1.8	202.5 2.0	223 7	233 7	243 7	253 8	264 8	274 8	284 8	304 9	325 10	345 10	365 11
5	17,500 400	5,440 140	2,790 70	1,810 50	1,60.2 1.6	181.4 1.8	202.3 2.0	234 9	234 9	244 10	254 10	264 11	274 11	284 11	305 12	325 13	345 14	366 15
10	17,500 400	5,440 140	2,790 70	1,810 50	158.5 1.6	180.6 1.8	202.2 2.0	234 9	234 9	244 10	255 10	265 11	275 11	285 11	305 12	326 13	346 14	366 15
26	17,500 400	5,440 140	2,800 70	1,820 50	1,340 30	177.8 1.8	201.6 2.0	236 9	236 9	246 10	256 10	266 11	276 11	287 12	307 12	327 13	347 14	367 15
50	17,500 400	5,450 140	2,800 70	1,820 50	1,350 30	1,070 30	200.6 2.0	240 10	240 10	250 10	259 10	269 11	279 11	289 12	309 12	329 13	349 14	369 15
75	17,500 400	5,450 140	2,800 70	1,830 50	1,350 30	1,080 30	199.2 2.0	244 10	244 10	253 10	263 10	273 11	282 11	292 12	312 12	332 13	352 14	372 15
100	17,500 400	5,450 140	2,810 70	1,830 50	1,360 30	1,080 30	905 23	249 10	249 10	258 10	267 11	276 11	286 11	295 12	315 13	334 13	354 14	374 15
125	17,500 400	5,460 140	2,810 70	1,840 50	1,360 30	1,090 30	911 23	254 10	254 10	263 10	271 11	280 11	289 12	299 12	318 13	337 14	357 14	376 15
150	17,400 400	5,460 140	2,820 70	1,840 50	1,370 30	1,100 30	917 23	262 11	262 11	269 11	276 11	285 11	294 12	302 12	321 13	340 14	359 14	379 15
175	17,400 400	5,460 140	2,820 70	1,850 50	1,380 30	1,100 30	924 23	273 11	273 11	276 11	282 11	290 12	298 12	307 12	324 13	343 14	362 14	381 15
200	17,400 400	5,460 140	2,830 70	1,860 50	1,380 40	1,110 30	930 23	291 12	291 12	286 11	289 12	296 12	303 12	311 12	328 13	346 14	365 15	384 15

VISCOSITY OF GASES

225	17,400 400	5,460 140	2,830 70	1,860 50	1,390 40	1,120 30	936 23	747 30	491 20	299 12	298 12	302 12	309 12	316 13	332 13	350 14	368 15	386 15
250	17,400 400	5,470 140	2,840 70	1,870 50	1,390 40	1,120 30	943 24	760 30	597 24	321 13	309 12	310 12	315 13	321 13	336 13	353 14	371 15	389 16
275	17,400 400	5,470 140	2,840 70	1,870 50	1,400 40	1,130 30	949 24	772 31	633 25	367 15	324 13	320 13	322 13	327 13	341 14	357 14	374 15	392 16
300	17,400 400	5,470 140	2,850 70	1,880 50	1,400 40	1,130 30	955 24	785 31	657 26	458 18	345 14	331 13	330 13	334 13	346 14	361 14	377 15	395 16
350	17,300 400	5,480 140	2,860 70	1,890 50	1,420 40	1,150 30	968 24	805 32	693 28	573 23	416 17	363 14	351 14	349 14	357 14	369 15	385 15	401 16
400	17,300 700	5,480 200	2,870 120	1,900 80	1,430 60	1,160 50	981 39	825 33	721 29	628 25	503 20	411 16	379 15	369 15	379 15	392 16	408 16	415 17
450	17,300 700	5,490 220	2,880 120	1,910 80	1,440 60	1,170 50	993 40	837 33	743 30	664 27	565 23	468 19	415 17	393 16	383 15	389 16	401 16	415 17
500	17,200 700	5,490 220	2,890 120	1,920 80	1,450 60	1,180 50	1,010 40	850 34	762 30	693 28	609 24	521 21	456 18	421 17	400 16	401 16	423 17	423 17
550	17,200 700	5,500 220	2,900 120	1,930 80	1,460 60	1,200 50	1,020 40	860 34	780 31	716 26	643 26	564 23	497 20	453 18	418 17	414 16	420 17	431 17
600	17,200 700	5,500 220	2,910 120	1,940 80	1,480 60	1,210 50	1,030 40	870 35	795 32	736 26	670 27	600 24	534 21	485 19	439 18	428 17	430 17	439 18
650	17,200 700	5,510 220	2,920 120	1,960 80	1,490 60	1,220 50	1,040 40	882 35	809 32	754 30	698 28	629 25	567 23	516 21	460 18	442 18	441 18	448 18
700	17,100 700	5,510 220	2,930 120	1,970 80	1,500 60	1,230 50	1,060 40	895 36	822 33	770 31	713 28	654 26	596 24	545 22	482 19	458 18	453 18	458 18
750	17,100 700	5,520 220	2,940 120	1,980 80	1,510 60	1,240 50	1,070 40	905 36	835 33	784 31	732 29	676 27	621 25	572 23	504 20	474 19	466 19	468 19
800	17,100 700	5,520 220	2,950 120	1,990 80	1,520 60	1,260 50	1,080 40	915 37	846 34	798 32	748 30	695 28	644 26	596 24	526 21	491 20	478 19	478 19

Note 1. The entry shown for 0°C and 1 bar relates to a metastable liquid state. The stable state is here solid.  
 Note 2. The values and the tolerances in the region of the critical point do not take into account the possibility of an anomalous behavior of the viscosity in the immediate neighborhood of the critical point.



2r-8. Steam. The dynamic and kinematic viscosity of steam has been settled (subject to future amendment) by international agreement ["Supplementary Release on Transport Properties of the Sixth International Conference on the Properties of Steam," New York, 1963; obtainable from the Secretariat of the International Conference on the Properties of Steam, ASME, United Engineering Center, New York. See also E. Schmidt, "VDI-Wasserdampftafeln" (VDI-Steam Tables), 7th ed., Springer Verlag, 1968]. According to this internationally recognized correlation, the viscosity of steam and water can be represented empirically by the following equations, depending on the range of states under consideration:

Superheated steam at 1 bar pressure in temperature range  $100^{\circ}\text{C} < t < 700^{\circ}\text{C}$ :

$$\frac{\mu_1}{\text{micropoise}} = 80.4 + 0.407 \frac{t}{^{\circ}\text{C}} \quad (2r-20)$$

$\pm 1\%, \quad 100 < \frac{t}{^{\circ}\text{C}} < 300$   
 Tolerance for  $\mu_1$ :  
 $\pm 3\%, \quad 300 < \frac{t}{^{\circ}\text{C}} < 700$

Superheated steam from 1 bar pressure to saturation in temperature range  $100^{\circ}\text{C} < t < 300^{\circ}\text{C}$ . (range of anomalous behavior where the viscosity decreases with density along an isotherm):

$$\frac{\mu - \mu_1}{\text{micropoise}} = - \frac{\rho}{\text{g/cm}^3} \left[ 1858 - 5.90 \frac{t}{^{\circ}\text{C}} \right] \quad (2r-21)$$

Tolerance for  $\mu$ :  $\pm 1\%$

Supercritical steam from 1 to 800 bars pressure in temperature range  $375^{\circ}\text{C} < t < 700^{\circ}\text{C}$ :

$$\frac{\mu - \mu_1}{\text{micropoise}} = 353.0 \frac{\rho}{\text{g/cm}^3} + 676.5 \left( \frac{\rho}{\text{g/cm}^3} \right)^2 + 102.1 \left( \frac{\rho}{\text{g/cm}^3} \right)^3 \quad (2r-22)$$

Tolerance for  $\mu$ :  $\pm 4\%$

Liquid water along saturation line in temperature range  $0^{\circ}\text{C} < t < 300^{\circ}\text{C}$ :

$$\frac{\mu}{\text{micropoise}} = 241.4 \times 10^{247.8/(T/\text{K} - 140)} \quad (2r-23)$$

Tolerance for  $\mu$ :  $\pm 2.5\%$

Liquid water from saturation pressure to 800 bars in temperature range  $0^{\circ}\text{C} < t < 300^{\circ}\text{C}$ :

$$\frac{\mu}{\text{micropoise}} = \left( 1 + \frac{p - p_s}{10^6 \text{ bars}} \phi \right) \times 241.4 \times 10^{247.8/(T/\text{K} - 140)} \quad (2r-24)$$

where

$$\phi = 1.0467 \left( \frac{T}{\text{K}} - 305 \right)$$

$\pm 2.5\%, \quad 1 < \frac{p}{\text{bar}} < 350$   
 Tolerance for  $\mu$ :  
 $\pm 4.0\%, \quad 350 < \frac{p}{\text{bar}} < 800$

The International Skeleton Table, reproduced as Table 2r-8, gives values of the viscosity of steam and water at agreed grid points, together with their tolerances (uncertainties). The dynamic viscosity  $\mu$  of steam exhibits anomalous behavior below about  $270^{\circ}\text{C}$  in that an increase in density along an isotherm causes the viscosity to decrease.