

2v. Shock Waves

ELI FREEDMAN

Ballistic Research Laboratories, Aberdeen Proving Ground

EDWARD F. GREENE

Brown University

2v-1. List of Symbols

u	flow velocity, measured in a coordinate system moving with the shock front
p	pressure
ρ	density
γ	ratio of heat capacities = C_P/C_V
H	enthalpy
E	internal energy
T	absolute temperature
S	entropy
R^*	gas constant per gram
c	local sound velocity
M_1	Mach number of incident shock = u_1/c_1
n	empirical constant in the Tait equation for liquids
$B(S)$	constant in the Tait equation for water
\mathbf{n}	unit vector normal to surface
\mathbf{u}	velocity vector
M_R	Mach number of reflected shock = u_{2R}/c_2

Subscripts 1, 2, and 3 on any quantity (e.g., u_1 , p_2 , ρ_3) mean that the quantity is measured in front of an incident shock, behind the incident shock, or behind a reflected shock, respectively.

Primed and double-primed quantities (e.g., p' , u'') are measured, respectively, on the two sides of a boundary between two media.

Subscript R on any quantity means that that quantity is measured in a coordinate system moving with a reflected shock.

2v-2. Introduction. Sound waves of infinitesimal amplitude in fluids always propagate without change of form (neglecting the effects of viscosity, thermal conductivity, and relaxation). For waves of finite amplitude this is no longer true. The denser regions move faster than the less dense, and hence the denser regions are always catching up with less dense ones in front of them, but since the velocity increases with density, the effect becomes more and more pronounced, the front of the wave becoming steeper and steeper until the density, temperature, and pressure changes across it are virtually discontinuous—a shock wave is formed. Mathematically, a shock wave is an actual discontinuity propagating with a velocity greater than the local sound velocity. Physically, although a shock transition is extremely abrupt (of the order of 10 mean free paths for a typical shock in a gas), it nevertheless is continuous, because of the action of dissipative forces. In what follows, attention will be focused exclusively on the regions behind or in front of the shock front. The relations that will be given are of general validity (except as noted) and are in any case independent of the actual course of events within the front itself.

It might be imagined that there could be a flow in which a shock moves from a dense region to a rarefied one. However, it can be shown from the energy-conservation law that steady-state flows of this type cannot exist in any fluid having an adiabat that is concave upward, the almost universally prevailing situation.

Another type of discontinuity occurring in gas flows is called a "contact discontinuity." It differs from a shock in that there is no mass flow across it, as there is in the case of a shock. Contact discontinuities cannot occur in steady-state flows and will not be further considered.

2v-3. Steady-state One-dimensional Flow. General Relations. Consider a shock propagating steadily in a fluid. Relative to a coordinate system moving with the shock, the equations of steady compressible flow are

$$u \frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} = 0 \quad (2v-1a)$$

$$u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \quad (2v-1b)$$

Equation (2v-1a) leads to

$$\rho_2 u_2 = \rho_1 u_1 \quad (2v-2)$$

From Eqs. (2v-1) and (2v-2) we have

$$\rho_2 u_2^2 + p_2 = \rho_1 u_1^2 + p_1 \quad (2v-3)$$

Also, from (2v-1b),

$$\frac{1}{2} u^2 + \int \frac{dp}{\rho} = \text{const} \quad (2v-4a)$$

From the energy-conservation equation, it can be shown that

$$\frac{1}{2} u_2^2 + H_2 = \frac{1}{2} u_1^2 + H_1 \quad (2v-4b)$$

These equations lead at once to the *Rankine-Hugoniot relations*:

$$E_2 - E_1 = \Delta E = \frac{1}{2} (p_2 + p_1) \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \quad (2v-5a)$$

$$H_2 - H_1 = \Delta H = \frac{1}{2} (p_2 - p_1) \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \quad (2v-5b)$$

and
$$u_1 = \frac{1}{\rho_1} \left(\frac{p_2 - p_1}{1/\rho_1 - 1/\rho_2} \right)^{\frac{1}{2}} \quad (2v-5c)$$

Equations (2v-5a), (2v-5b), and (2v-5c) are based solely upon hydrodynamics and

thermodynamics and are valid for all fluids. Further progress can now be made only when they are supplemented by an equation of state for the fluid.

Special Cases. THE IDEAL GAS

$$p = \rho R^* T$$

From Eqs. (2v-5a), (2v-5b), and (2v-5c) and the equation of state it can be shown that

$$\frac{p_2}{p_1} = \frac{\rho_2(\gamma + 1) - \rho_1(\gamma - 1)}{\rho_1(\gamma + 1) - \rho_2(\gamma - 1)} \quad (2v-6a)$$

$$\frac{\rho_2}{\rho_1} = \frac{p_2(\gamma + 1) \div p_1(\gamma - 1)}{p_1(\gamma + 1) \div p_2(\gamma - 1)} \quad (2v-6b)$$

and
$$\frac{T_2}{T_1} = \frac{p_2 \rho_1}{p_1 \rho_2} \quad (2v-6c)$$

In terms of the Mach number of the incident shock M_1 ,

$$\frac{p_2}{p_1} = \frac{2M_1^2\gamma - \gamma + 1}{\gamma + 1} \quad (2v-7a)$$

and
$$\frac{\rho_2}{\rho_1} = \frac{M_1^2(\gamma + 1)}{M_1^2(\gamma - 1) + 2} \quad (2v-7b)$$

LIQUIDS. An often-used equation of state for liquids, especially water, is the Tait equation. A convenient form of it is

$$p = B(S) \left[\left(\frac{\rho(T,p)}{\rho(T,0)} \right)^n - 1 \right] \quad (2v-8a)$$

Approximately
$$B = \frac{\rho_0 c_1^2}{n} \quad (2v-8b)$$

It is a good approximation in liquids to assume that the initial and final states are connected by an adiabatic compression. With this assumption,

$$u_1 = c_1 \left(1 + \frac{n+1}{4c} \sigma \right) \quad (2v-9a)$$

where
$$\sigma = \frac{2c_1}{n-1} \left[\left(\frac{p_2}{p_1} \right)^{(n-1)/2} - 1 \right] \quad (2v-9b)$$

Systems Subject to Chemical Reaction. The Rankine-Hugoniot relation, Eq. (2v-5a), is plotted in the $(p, 1/\rho)$ plane in Fig. 2v-1 with an adiabat for comparison. This relation is of course valid when the system reacts chemically, if the chemical energy is included in ΔE . In this case the point (p_2, ρ_2) does not lie on the Rankine-Hugoniot curve, but either above or below it, depending on whether the chemical reaction is endothermic or exothermic. An especially interesting case, detonation, occurs when there is enough chemical energy alone to sustain the shock wave. Since the wave velocity is measured by the slope of the line through (p_1, ρ_1) which intersects the Rankine-Hugoniot curve [see Eqs. (2v-5)], there are usually an infinite number of possible velocities. However, in a steady-state detonation the lowest possible velocity, which corresponds to a line through (p_1, ρ_1) just tangent to the Rankine-Hugoniot curve, is the one that occurs. This is the *Chapman-Jouguet condition*:

$$u_{1\text{detonation}} = \frac{1}{\rho_1} \left(\frac{p_2 - p_1}{1/\rho_1 - 1/\rho_2} \right)^{\frac{1}{2}} \quad (2v-10)$$

which provides the extra relation needed so that the detonation velocity can be calculated from Eqs. (2v-5).

When mechanical as well as chemical energy is available, the velocity increases from the Chapman-Jouguet value as (p_2, ρ_2) moves upward along the Rankine-Hugoniot curve. There is no common physical process corresponding to the value of (p_2, ρ_2) below the Chapman-Jouguet value. The other branch of the Rankine-Hugoniot curve for which $p_2 < p_1$ and $\rho_2 < \rho_1$ corresponds to a deflagration and is a subsonic process.

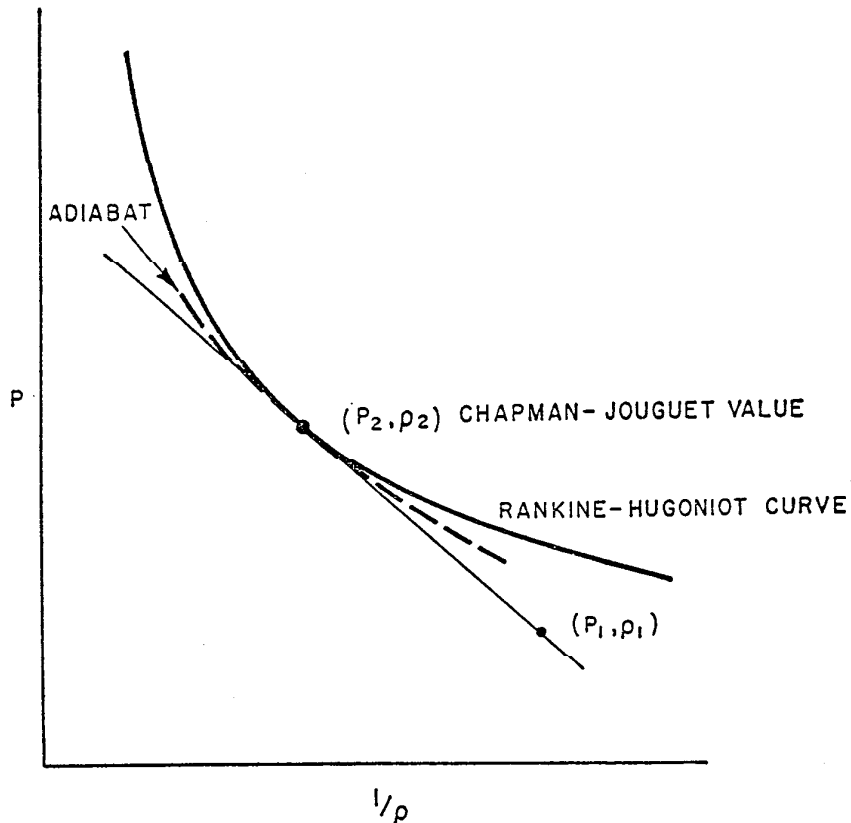


FIG. 2v-1. Plot of Rankine-Hugoniot relation.

2v-4. Reflection and Refraction at a Rigid Wall. At a rigid boundary, in addition to the previous Eqs. (2v-5) there must be added the condition

$$\mathbf{u} \cdot \mathbf{n} = 0 \quad (2v-11)$$

Normal Incidence. The use of (2v-11) along with (2v-5) for a perfect gas leads to

$$\frac{p_3}{p_2} = \frac{(3\gamma - 1)(p_2/p_1) - \gamma + 1}{(\gamma - 1)(p_2/p_1) + \gamma + 1} \quad (2v-12a)$$

and
$$-u_{3R} = c_1 \frac{2(p_2/p_1)(\gamma - 1) + 2}{\{2\gamma[(\gamma + 1)(p_2/p_1) + \gamma - 1]\}^{1/2}} \quad (2v-12b)$$

which is the velocity of the reflected shock relative to the reflecting surface.

Oblique Incidence. In this case a second condition may be imposed: The incident and reflected waves should intersect at the surface. This condition cannot always be satisfied; when it is one speaks of *regular reflection*. Regular reflection always occurs at a sufficiently small angle of incidence (i.e., the angle between the normal to the surface and the normal to the shock front). The two boundary conditions then completely determine the direction and strength of the reflected shock.

There exists a critical angle of incidence above which regular reflection cannot occur. The point of intersection of the incident and reflected shocks rises above the surface and is joined to it by a third shock, called the Mach stem. This case is called "Mach reflection." Experimentally it is found that Mach reflection sets in at angles smaller than those predicted by theory.

2v-5. Reflection and Refraction at a Nonrigid Wall. There are now two boundary conditions that must be satisfied:

$$\mathbf{u}' \cdot \mathbf{n}' = \mathbf{u}'' \cdot \mathbf{n}'' \quad (2v-13a)$$

and

$$p' = p'' \quad (2v-13b)$$

Normal Incidence. In order to satisfy both (2v-13a) and (2v-13b) it is necessary that there be a transmitted and a reflected wave. The transmitted wave is always a shock, but the reflected wave may be either a shock or a rarefaction wave, depending on the properties of the two media and, in some cases, on the strength of the incident shock.

TABLE 2v-1. SOME PROPERTIES OF SHOCKS IN IDEAL GASES

M_1	Monatomic				Diatomic			
	p_2/p_1	ρ_2/ρ_1	T_2/T_1	M_R	p_2/p_1	ρ_2/ρ_1	T_2/T_1	M_R
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
1.5	2.562	1.714	1.495	1.397	2.458	1.862	1.320	1.426
2	4.750	2.286	2.078	1.648	4.500	2.667	1.688	1.732
2.5	7.562	2.703	2.798	1.808	7.125	3.333	2.138	1.949
3	11.00	3.000	3.667	1.915	10.33	3.857	2.679	2.104
4	19.75	3.308	5.803	2.041	18.50	4.571	4.047	2.297
5	31.00	3.571	8.680	2.104	29.00	5.000	5.800	2.408
6	44.75	3.692	12.12	2.142				
8	79.75	3.821	20.87	2.182				
10	124.8	3.884	32.12	2.201				
15	281.0	3.947	71.19	2.220				
20	499.8	3.970	125.9	2.227				

Oblique Incidence. There can occur either regular reflection or Mach reflection, of which the first case has been well investigated. It is shown that there is always a transmitted wave (i.e., total reflection of a shock wave cannot occur). If the second medium has a high acoustic impedance, the observed phenomena are similar to those found at a rigid surface; if the second medium has a low acoustic impedance, the observed phenomena are similar to those found at a free surface.

Free Surface (for Liquids Only). The condition (2v-13b) here becomes $p' = 0$. For a sufficiently small angle of incidence there is always a reflected rarefaction wave intersecting the incident shock at the surface. At some critical angle of incidence, determined by the strength of the incident shock as well as the properties of the liquid, this picture no longer applies. The phenomena in this case have not yet been intensively investigated.

Table 2v-1 lists some important properties of shock waves in ideal monatomic and diatomic gases. The following values have been used for γ , the ratio of heat capacities: For the monatomic gas, $\gamma = \frac{5}{3}$; for the diatomic gas, $\gamma = \frac{7}{5}$. For both gases, the possibility of electronic excitation has been neglected. In addition, for the diatomic gas, the possibilities of dissociation and the activation of the vibrational

heat capacity have been neglected. Since the latter assumption becomes increasingly unrealistic at high temperatures, this part of the table has not been extended beyond $M_1 = 5$.

References

General

- Courant, R., and K. O. Friedrichs: "Supersonic Flow and Shock Waves," Interscience Publishers, Inc., New York, 1948. Comprehensive, thorough treatment of entire subject, emphasizing mathematical aspects; 198 references.
- Greene, E. F., and J. P. Toennies: "Chemische Reaktionen in Stosswellen," Steinkopf Verlag, Darmstadt, 1959. Over-all survey, emphasizing applications to chemistry.
- Penney, W. G., and H. H. M. Pike: *Repts. Progr. in Phys.* **13**, 46-82 (1950). Over-all survey of problems and results, emphasizing physical aspects; 40 references.

Special

- Bleakney, W., and A. H. Taub: *Revs. Modern Phys.* **21**, 584-605 (1949).
- Cole, R. H.: "Underwater Explosions," Princeton University Press, Princeton, N.J., 1946. Thorough treatment of propagation of shocks in water.
- Fletcher, C. H., A. H. Taub, and W. Bleakney: *Revs. Modern Phys.* **23**, 271-286 (1951). Mach reflection considered theoretically and experimentally.
- Hirschfelder, J. O., C. F. Curtiss, and R. B. Bird: "Molecular Theory of Gases and Liquids," John Wiley & Sons, Inc., New York, 1954. Chapter 11 applies the rigorous kinetic theory of gases to detonations and shocks.
- Lewis, Bernard, and Guenther von Elbe: "Combustion, Flames and Explosions of Gases," Academic Press, Inc., New York, 1951. Chapter XI treats detonation waves in gases.
- Polachek, H., and R. J. Seeger: *Phys. Rev.* **84**, 922-929 (1951). Refraction at a gaseous interface.
- Taub, A. H.: *Phys. Rev.* **72**, 51-60 (1947). Reflection and refraction of plane shocks. See also "Waves of Finite Amplitude," Sec. 3c-6 of this book.