3g. Properties of Transducer Materials

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To determine the acoustic properties of gases, liquids, and solids and to utilize them in acoustic systems, it is necessary to generate the appropriate waves by means of transducer materials which convert electrical energy into mechanical energy and vice versa. For liquids and solids, the most common types of materials are piezoelectric crystals, ferroelectric materials of the barium titanate type, and magnetostrictive materials.

3g-1. Piezoelectric Crystals. The static relations for a piezoelectric quartz crystal producing a single longitudinal mode are for rationalized mks units

$$S_2 = s_{22}^E T_2 + d_{21} E_x \qquad D_x = d_{21} T_2 + \epsilon_1^T E_x \tag{3g-1}$$

where S_2 and T_2 are the longitudinal strain and stress, respectively, s_{22}^E the elastic compliance along the length measured at constant electric field, d_{21} the piezoelectric constant relating the strain with the applied field E_x, D_x the electric displacement, and ϵ_1^T the dielectric constant measured at constant stress. Equations of this type suffice to determine the static and low-frequency behavior of piezoelectric crystals. Using the first equation, one finds that the increase in length for no external stress and the external force for no increase in length are, respectively,

$$\Delta l = d_{21} \frac{Vl}{t} \qquad F = T_2 tw = -d_{21} \frac{Vw}{s_{22}^E} \tag{3g-2}$$

where V is the applied potential, l, w, and t are the length, width, and thickness of the crystal, and F is the force which is considered positive for an extensional stress. From the second equation one finds that the open-circuit voltage and the short-circuited charge for a given applied force are, respectively,

$$V = -\left(\frac{d_{21}}{\epsilon^T}\right) \frac{lF}{tw} \qquad Q = \int_0^l \int_0^w D_x \, dl \, dw = d_{21} \frac{Fl}{t} \tag{3g-3}$$

Another important criterion for transducer use is the electromechanical-coupling factor k whose square is defined as the ratio of the energy stored in mechanical form to the total input electrical energy. Using Eqs. (3g-1), this can be shown to be

$$k^2 = \frac{{d_{21}}^2}{s_{22}^E \epsilon^T} \tag{3g-4}$$

It is readily shown that the clamped dielectric constant ϵ^S, obtained by setting $S_2 = 0$, and the constant-displacement elastic compliance s^D, obtained by setting $D_x = 0$, are related to the constant-stress dielectric constant ϵ^T and the constant-field elastic compliance s_{22}^E by the equations

$$\frac{\epsilon_1^S}{\epsilon_1^T} = \frac{s_{22}^D}{s_{22}^E} = 1 - k^2 \tag{3g-5}$$

Equivalent circuits in which the properties of the crystal are expressed in terms of equivalent electrical elements are often useful (see Sec. 31). An equivalent circuit for a piezoelectric crystal for static conditions is shown by Fig. 3g-1*A*. In this network the compliance $C_1 = s_{22}{}^E l/wt$ represents the compliance of the crystal with the electrodes short-circuited, the capacitance C_0 is the capacitance of the clamped crystal, i.e., $C_0 = lw\epsilon_1{}^S/t$, while the transformer shown is a perfect transformer, i.e., a transformer having no loss between zero frequency and the highest frequency for which the piezoelectric effect is operative, having a turns ratio of φ to 1 where

$$\varphi = -d_{21}\frac{w}{s_{22}{}^E} \tag{3g-6}$$

The fact that this equivalent circuit presents the same information as Eq. (3g-1) is readily verified by substitution and integration over the area of the crystal.

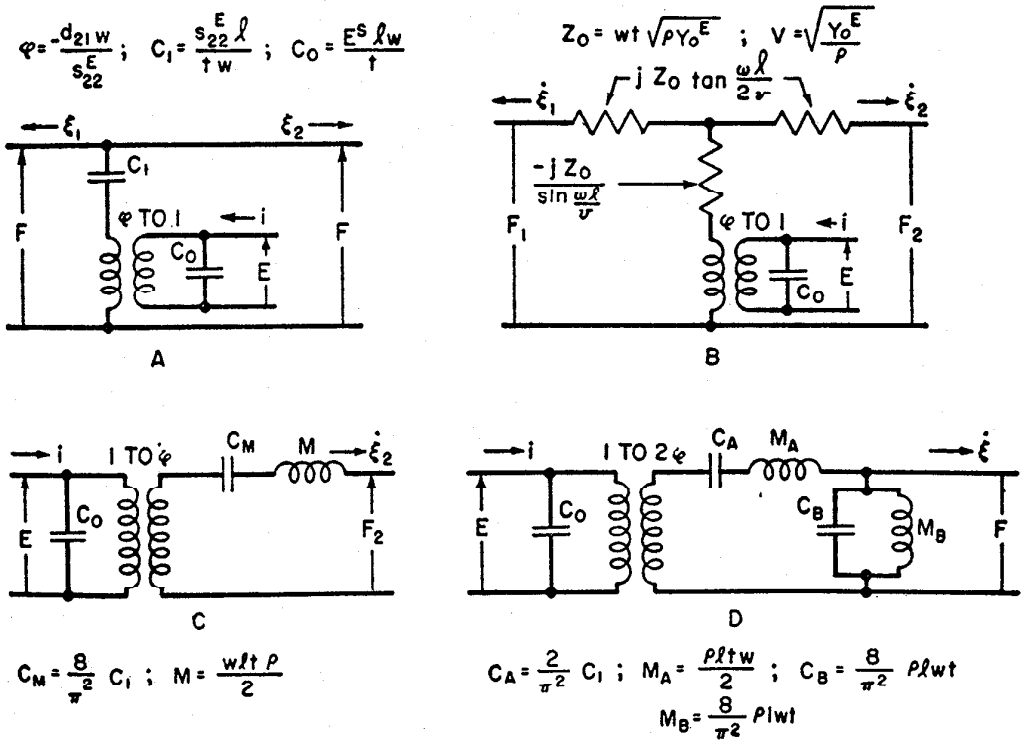


FIG. 3g-1. Equivalent circuit for a piezoelectric crystal for clamped and free conditions.

As an example of the use of such a network, one can calculate from it the efficiency of transformation of mechanical to electrical energy, or vice versa, under various conditions. Suppose that we clamp one end of the crystal and apply a force through the sending-end mechanical resistance R_M and receive the power generated into an electrical resistance R_E. Solving the network equations and obtaining the conditions for maximum power output, it is readily shown that the maximum power is obtained if

$$R_M = \frac{1}{\omega C_1 \sqrt{1 - k^2}} \qquad R_E = \frac{\sqrt{1 - k^2}}{\omega C_0} \tag{3g-7}$$

where $\omega = 2\pi$ times the frequency f. With these values the power in the termination is

$$P_0 = \frac{F^2 k^4}{4\varphi^2 R_E} \tag{3g-8}$$

The available power that can be obtained from a source having an open-circuit force F with an internal impedance R_M is maximum when $\varphi^2 R_E = R_M$. This power is then

$$i_2^2 R_E = \frac{F^2}{4\varphi^2 R_E} \tag{3g-9}$$

and hence the power-conversion efficiency is

$$P_E = k^4 \tag{3g-10}$$

Hence, unless the coupling is high, the efficiency of conversion by static means is low.

This efficiency can be improved by resonating the capacitance C_0 by an electric coil L_0 at the frequency of operation and can be further improved by mechanically resonating the static compliance of the crystal. The simplest way to analyze these circuits for their optimum conditions is to observe that, if the perfect transformer is moved to the end of the circuit, both equivalent sections are half sections of well-known filters. Equation (3g-11) gives the element values of the first filter resonated by an electrical coil, while Eq. (3g-14) gives the element values for the section tuned on both ends.

$$\begin{aligned} C_1' &= \frac{s_{22}^E l}{wt}\left(\frac{d_{21}w}{s_{22}^E}\right)^2 = \frac{lw}{t}\frac{d_{21}^2}{s_{22}^E} = \frac{f_1 + f_2}{2\pi f_1 f_2 Z_0'} = \frac{1}{2\pi f_1 Z_0} \\ C_0 &= \frac{\epsilon^S lw}{t} = \frac{f_1}{2\pi f_2(f_2 - f_1)Z_0'} = \frac{f_1}{2\pi(f_2^2 - f_1^2)Z_0} \\ L_0 &= \frac{(f_2 - f_1)Z_0'}{2\pi f_1 f_2} = \frac{(f_2^2 - f_1^2)Z_0}{2\pi f_1 f_2^2} \end{aligned} \tag{3g-11}$$

where f_1 is the lower cutoff, f_2 the upper cutoff, Z_0 the mid-shunt impedance occurring on the electrical side, and Z_0 the mid-series impedance occurring on the mechanical side. Solving for f_1, f_2, Z_0, and $Z_0'(\varphi^2)$, i.e., the actual mechanical resistance, we find

$$f_2 = \frac{1}{2\pi\sqrt{L_0 C_0}} \qquad f_1 = \frac{\sqrt{1-k^2}}{2\pi\sqrt{L_0 C_0}} \qquad Z_0 = R_E = \frac{1-k^2}{2\pi f_1 C_0}\cdot\frac{1+\sqrt{1-k^2}}{k^2}$$

$$R_M = \varphi^2 Z_0' = 2\pi f_1 (l s_{22}^E / tw) \tag{3g-12}$$

Hence, if there is no dissipation in the elements of the crystal, perfect power conversion can be obtained but only over a bandwidth of

$$\frac{f_2 - f_1}{f_2} = 1 - \sqrt{1-k^2} \tag{3g-13}$$

We consider next a wider bandpass filter having the element values

$$C_1' = \frac{lw}{t}\frac{d_{21}^2}{s_{22}^E} = \frac{f_2 - f_1}{2\pi f_1 f_2 Z_0} \qquad L_1' = \frac{\rho lt}{w}\left(\frac{s_{22}^E}{d_{21}}\right)^2 = \frac{Z_0}{2\pi(f_2 - f_1)}$$

$$C_0 = \frac{\epsilon^S lw}{t} = \frac{1}{2\pi(f_2 - f_1)Z_0} \qquad L_0 = \frac{(f_2 - f_1)Z_0}{2\pi f_1 f_2} \tag{3g-14}$$

Solving for the bandwidth and the impedances

$$\frac{f_2 - f_1}{f_m} = \frac{k}{\sqrt{1-k^2}} \qquad f_m = \sqrt{f_1 f_2} = \frac{1}{2\pi\sqrt{L_0 C_0}} = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

$$Z_0 = R_E = \frac{\sqrt{1-k^2}}{2\pi f_m C_0 k} \qquad R_M = \varphi^2 Z_0 = \frac{k}{\sqrt{1-k^2}}\frac{1}{2\pi f_m s_{22}^E}\frac{wt}{l} \tag{3g-15}$$

This filter section can efficiently transform mechanical into electrical energy and vice versa with a loss determined only by the dissipation in the elements of the crystal.

The simplest method for mechanically resonating the crystal is to use it near its natural mechanical resonance. An exact equivalent circuit for a vibrating crystal is shown by Fig. 3g-1*B*. Near the first resonant frequency, the equivalent circuit for a clamped quarter-wave crystal is shown by Fig. 3g-1*C* while the equivalent circuit for a half-wave crystal is shown by Fig. 3g-1*D*. When the half-wave crystal resonated by a shunt coil is applied to converting electrical into mechanical energy, the same formulas given in Eqs. (3g-14) and (3g-15) are applicable except that $k^2/(1 - k^2)$ is replaced by $(8/\pi^2)[k^2/(1 - k^2)]$. By using the complete representation of Fig. 3g-1*B* the effect can be calculated by using various backing plates on the radiation from the front surface.

The general form of Eq. (3g-1) holds for any single mode whether it is longitudinal or transverse as long as the appropriate constants are used. For longitudinal thickness modes when the radiating surface is a number of wavelengths in diameter, s_{22}^E is replaced by $1/c_{11}^E$ and d_{21} by e_{21}/c_{11}^E, the appropriate thickness piezoelectric constant. For a thickness shear mode, the appropriate shear stiffness (c_{44}, c_{55}, or c_{66})

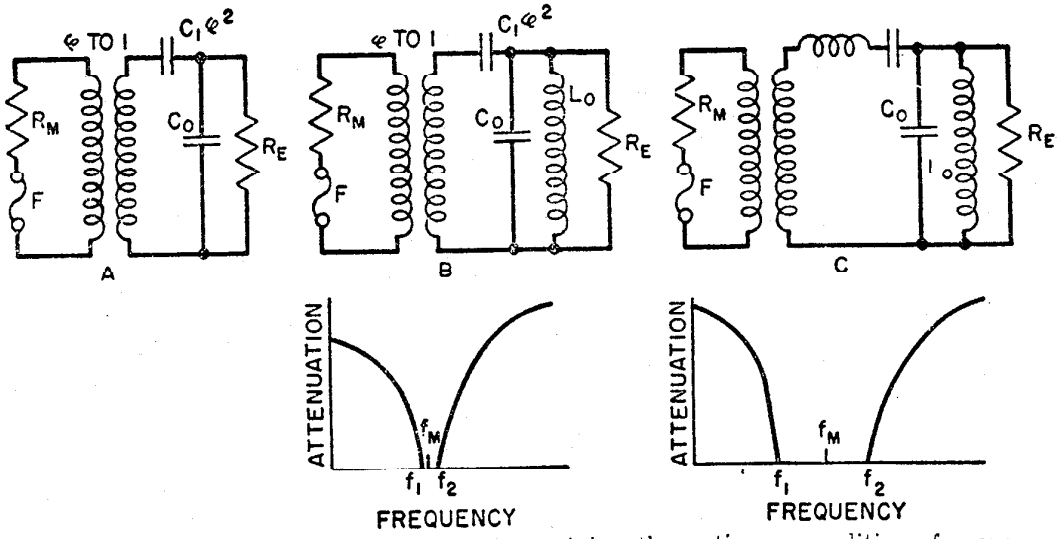


FIG. 3g-2. Use of equivalent circuit in determining the optimum conditions for energy transmission.

replaces $1/s_{22}$ and the appropriate shear piezoelectric constant replaces d_{21}. Table 3g-1 lists the constants in mks units for a number of standard crystal cuts.

3g-2. Electrostrictive and Magnetostrictive Materials. Other types of materials that have been used in transducers are ferroelectric crystals and ceramics of the barium titanate type and ferromagnetic crystals, polycrystals, and sintered materials of the ferrite type. All these materials have changes in lengths proportional to squares and even powers of the polarization and to obtain a linear response they have to be polarized. These polarized materials have relations between stresses, strains, electric and magnetic fields, and electric displacement and magnetic flux similar to those for a piezoelectric crystal shown by Eq. (3g-1) and hence these materials can be said to have "equivalent" constants which depend not only on the material but also on the degree of poling and in some cases on aging effects. The dielectric and permeability constants are those associated with the polarized medium as are also the elastic constants.

To obtain these equivalent piezoelectric and piezomagnetic constants, one can start with the more fundamental potential equations which have the same form for either electrostrictive or magnetostrictive materials. For polycrystalline or sintered materials, these potential equations can be written in the form

TABLE 3g-1. PROPERTIES OF PIEZOELECTRIC CRYSTALS IN MKS UNITS

Crystal and cut	Mode	Elastic constant, 10^{-11} m²/newton	Piezoelectric constant d, 10^{-12} coulomb/newton	Dielectric capacitivity ϵ, 10^{-11} farad/m	Electromechanical coupling k	Open-circuit voltage $g = d/\epsilon$, volt-meters/newton	Force factor d/s, newtons/volt-meter	Density, 10^3 kg/m³
Quartz X cut, length Y	L.L.	$s_{22}^E = 1.27$	$d_{21} = 2.25$	4.06	0.099	0.055	0.177	2.65
X cut	T.L.	$\frac{1}{c_{11}^E} = 1.16$	$\frac{e_{11}}{c_{11}^E} = -2.04$	4.06	0.093	0.050	0.175	2.65
Y cut	T.S.	$\frac{1}{c_{66}^E} = 2.57$	$\frac{e_{26}}{c_{66}^E} = +4.4$	4.06	0.137	0.108	0.171	2.65
Rochelle salt, 45-deg X cut	L.L.	$s_{22}'^E = 6.7$	$\frac{d_{14}}{2} = 435$	444.0	0.78	0.098	6.5	1.77
45-deg Y cut	L.L.	$s_{11}'^E = 9.89$	$\frac{d_{25}}{2} = -28.4$	9.85	0.288	0.29	0.287	1.77
ADP, 45-deg Z cut	L.L.	$s_{11}^{E'} = 5.3$	$\frac{d_{36}}{2} = 24.6$	13.8	0.29	0.178	0.465	1.804
KDP, 45-deg Z cut	L.L.	$s_{11}^{E'} = 4.85$	$\frac{d_{36}}{2} = 10.7$	19.6	0.12	0.058	0.22	2.31
EDT, Y cut, length X	L.L.	$s_{11}^E = 3.88$	$d_{21} = 11.3$	7.4	0.215	0.152	0.29	1.538
DKT, 45-deg Z cut	L.L.	$s_{11}^{E'} = 4.25$	$d_{31}' = -12.2$	5.8	0.245	0.21	0.287	1.988
L.H., Y cut	T.L.	$\frac{1}{c_{22}^E} = 2$	$\frac{e_{22}}{c_{22}^E} = 15$	9.15	0.35	0.165	0.75	2.06
L.H., hydrostatic	H.		$d_{21} + d_{22} + d_{23} = 13$	9.15		0.143		
Tourmaline, Z cut	T.L.	$\frac{1}{c_{33}^E} = 0.61$	$\frac{e_{33}}{c_{33}^E} = -1.84$	6.65	0.092	0.0275	0.3	3.1
Tourmaline, hydrostatic	H.		$d_{31} + d_{33} = -2.16$	6.65		0.0325		

Abbreviations: L.L. = length longitudinal; T.L. = thickness longitudinal; T.S. = thickness shear; ADP = ammonium dihydrogen phosphate; KDP = potassium dihydrogen phosphate; EDT = ethylene diamine tartrate; L.H. = lithium sulfate monohydrate.

$$= -\tfrac{1}{2}[s_{11}^D(T_1^2 + T_2^2 + T_3^2) + 2s_{12}^D(T_1T_2 + T_1T_3 + T_2T_3) + 2(s_{11}^D - s_{12}^D)(T_4^2 + T_5^2 + T_6^2)] - \{Q_{11}(D_1^2T_1 + D_2^2T_2 + D_3^2T_3) + Q_{12}[T_1(D_2^2 + D_3^2) + T_2(D_1^2 + D_3^2) + T_3(D_1^2 + D_2^2)] + 2(Q_{11} - Q_{12})(T_4D_2D_3 + T_5D_1D_3 + T_6D_1D_2)\} + \tfrac{1}{2}\beta_{11}^T(D_1^2 + D_2^2 + D_3^2) + K_{11}^T(D_1^4 + D_2^4 + D_3^4) + K_{12}^T(D_1^2D_2^2 + D_1^2D_3^2 + D_2^2D_3^2) + K_{111}^T(D_1^6 + D_2^6 + D_3^6) + K_{112}^T[D_1^4(D_2^2 + D_3^2) + D_2^4(D_1^2 + D_3^2) + D_3^4(D_1^2 + D_2^2)] + K_{123}^TD_1^2D_2^2D_3^2 \quad \text{(3g-16)}\dagger$$

here T_1, T_2, T_3 are the three extensional stresses, T_4, T_5, T_6 the three shearing resses, D_1, D_2, D_3 the three components of the electrical displacement for ferroectric materials or the three components of the magnetic flux B for ferromagnetic aterials, the s constants are the compliance constants for an isotropic material easured at constant electric or magnetic displacement, the Q's are the electrorictive or magnetostrictive constants, β_{11}^T the inverse of the initial dielectric constant · permeability measured at constant stress, and the K^T's are constants determining ıe total energy stored for higher polarizations. The static equations can be obtained r differentiation of G according to the relations

$$S_i = -\frac{\partial G}{\partial T_i} \qquad E_m = \frac{\partial G}{\partial D_m} \quad \text{(3g-17)}$$

Since linear equations are obtained only if a permanent polarization P_0 is introıced, we assume that

$$D_3 = P_0 + D_3^* \quad \text{(3g-18)}$$

here D_3^* is a small variable component superposed on P_0. Also, D_1 and D_2 are small that their squares and higher powers can be neglected compared with P_0. Introıcing these into (3g-16) and differentiating, we have

$$\begin{aligned}
S_1 &= s_{11}^DT_1 + s_{12}^D(T_2 + T_1) + Q_{12}(P_0^2 + 2P_0D_3^*)\\
S_2 &= s_{11}^DT_2 + s_{12}^D(T_1 + T_3) + Q_{12}(P_0^2 + 2P_0D_3^*)\\
S_3 &= s_{11}^DT_3 + s_{12}^D(T_1 + T_2) + Q_{11}(P_0^2 + 2P_0D_3^*)\\
S_4 &= 2(s_{11}^D - s_{12}^D)T_4 + 2(Q_{11} - Q_{12})P_0D_2\\
S_5 &= 2(s_{11}^D - s_{12}^D)T_5 + 2(Q_{11} - Q_{12})P_0D_1\\
S_6 &= 2(s_{11}^D - s_{12}^D)T_6\\
E_1 &= -2(Q_{11} - Q_{12})P_0T_5 + D_1(\beta_{11}^T + 2K_{12}^TP_0^2 + 2K_{112}^TP_0^4)\\
E_2 &= -2(Q_{11} - Q_{12})P_0T_4 + D_2(\beta_{11}^T + 2K_{12}^TP_0^2 + 2K_{112}^TP_0^4)\\
E_3 &= -2Q_{11}P_0T_3 - 2Q_{12}P_0(T_1 + T_2) + D_3^*(\beta_{11}^T + 12K_{11}^TP_0^2 + 30K_{111}^TP_0^4)
\end{aligned} \quad \text{(3g-19)}$$

It is obvious that the variable components of Eq. (3g-19) follow the same rule as for piezoelectric crystal. There are three longitudinal modes and a shearing mode. he length longitudinal mode has the following constants:

$$\text{L. mode} \quad s_{11}^E = s_{11}^D\left[1 + \frac{4Q_{12}^2P_0^2}{\beta_{33}^T(P_0)s_{11}^D}\right] \qquad d_{31} = \frac{2Q_{12}P_0}{\beta_{33}^T(P_0)}$$

$$\epsilon_{33}^T(P_0) = \frac{1}{\beta_{33}(P_0)} \quad \text{(3g-20)}$$

here $\beta_{33}^T(P_0) = (\beta_{11}^T + 12K_{11}^TP_0^2 + 30K_{111}^TP_0^4)$ is the dielectric impermeability the ceramic when it has a permanent polarization P_0

$$\text{T. bar} \quad s_{11}^E = s_{11}^D\left[1 + \frac{4Q_{11}^2P_0^2}{\beta_{33}^T(P_0)s_{11}^D}\right] \qquad d_{33} = \frac{2Q_{11}P_0}{\beta_{33}^T(P_0)}$$

$$\epsilon_{33}^T(P_0) = \frac{1}{\beta_{33}^T(P_0)} \quad \text{(3g-21)}$$

† If higher-order terms than those considered here are used, second-order electrostrictive d magnetostrictive terms and the change in elastic constants with polarization can be ken care of. For example, see W. P. Mason, *Phys. Rev.* **82** (5), 715–723 (June 1, 1951).

These formulas hold for a bar which is long in the direction of vibration compared with the cross-sectional dimensions. When a plate is used which is a number of wavelengths across, the sidewise motions S_1 and S_2 are zero and the constants are

L.T. plate $\qquad \frac{1}{c_{11}^E} \qquad d'_{33} \qquad \epsilon'_{33}(P_0)$ (3g-22)

where

$$\frac{1}{c_{11}^E} = \frac{1}{c_{11}^P} + d'^2_{33}\epsilon'_{33}(P_0) \qquad d'_{33} = 2P_0\left(Q_{11} - \frac{2s_{12}^D}{s_{11}^D + s_{12}^D}\,Q_{12}\right)\epsilon'_{33}(P_0)$$

$$\epsilon_{33}^{T'}(P_0) = \frac{1}{\beta_{33}^T(P_0) + [4Q_{12}^2P_0^2/(s_{11}^D + s_{12}^D)]} \qquad \text{and} \qquad c_{11}^D = \frac{s_{11}^D + s_{12}^D}{(s_{11}^D - s_{12}^D)(s_{11}^D + 2s_{12}^D)}$$

The thickness shear mode has the fundamental constants $2(s_{11}^E - s_{12}^E)$; d_{14}; $\epsilon_{11}^T(P_0)$,

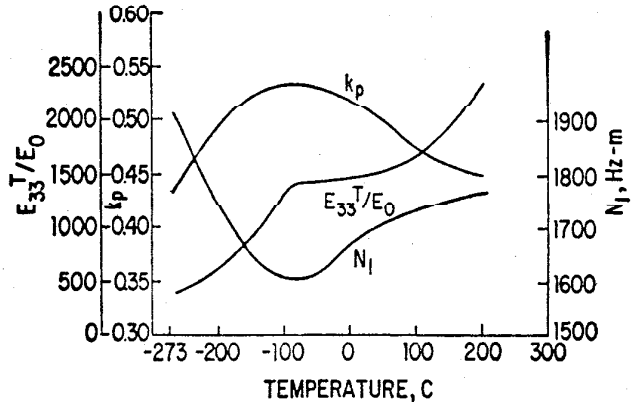


FIG. 3g-3. Temperature variation of k_p, permittivity, and bar frequency constant for prestabilized PZT-4.

i.e., the dielectric constant perpendicular to the poling direction, where

$$d_{14} = \frac{2(Q_{11} - Q_{12})P_0}{\epsilon_{11}^T(P_0)} \qquad 2(s_{11}^E - s_{12}^E) = 2(s_{11}^D - s_{12}^D) + \frac{4(Q_{11} - Q_{12})^2 P_0^2}{\beta_{11}^T(P_0)}$$

$$\epsilon_{11}^T(P_0) = \frac{1}{(\beta_{11}^T + 2K_{12}^T P_0^2 + 2K_{112}^T P_0^4)} \qquad \text{(3g-23)}$$

Two other modes have been used in electrostrictive and magnetostrictive materials,

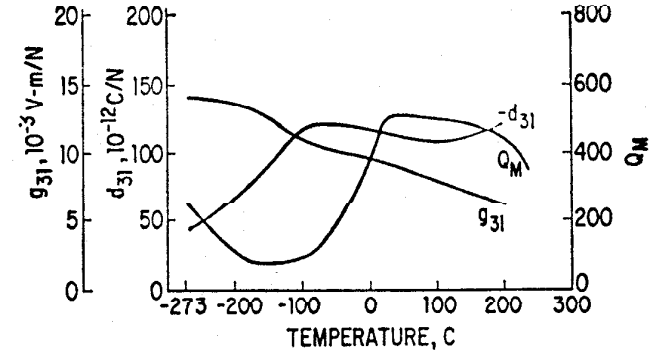


FIG. 3g-4. Temperature variation of g_{31}, d_{31}, and mechanical Q for prestabilized PZT-4.

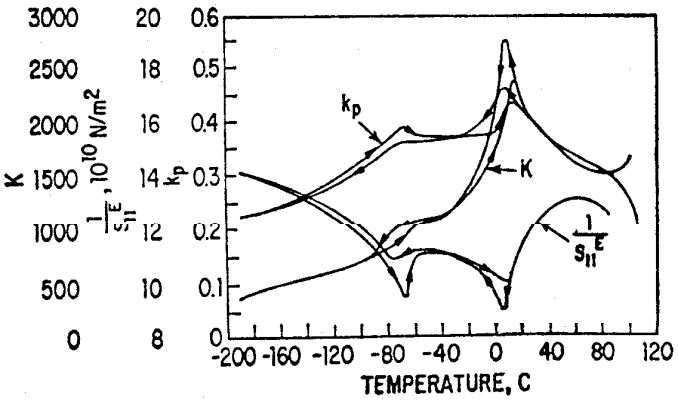


FIG. 3g-5. Temperature variation of k_p, permittivity K, and Young's modulus ($1/s_{11}^E$) for $BaTiO_3$ ceramic.

TABLE 3g-2. PROPERTIES OF FERROELECTRIC CERAMICS AT 25°C

Material	Young's modulus, 10^{11} newtons/m²	Piezoelectric constant d_{31}, 10^{-11} coulomb/newton	Piezoelectric constant d_{33}, 10^{-11} coulomb/newton	Dielectric capacitivity ϵ_{33}^T, 10^{-11} farad/m	Electromechanical coupling factors		Open-circuit voltage $g = d_{31}/\epsilon_{33}^T$, volt-meters/newton	Force factor $d_{33}Y_0^E$, newtons/volt-meter
					k_{31}	k_{33}		
Commercial $BaTiO_3$ ceramics	1.18	−5.6	16	1,250	0.17	0.45	0.0106	13.5
97% $BaTiO_3$, 3% $CaTiO_3$	1.22	−5.3	13.5	1,230	0.17	0.43	0.0111	11.0
96% $BaTiO_3$, 4% $PbTiO_3$	1.14	−3.8	10.5	880	0.14	0.39	0.012	9.2
90% $BaTiO_3$, 4% $PbTiO_3$, 6% $CaTiO_3$	1.24	−4.0	11.5	710	0.167	0.48	0.016	9.3
84% $BaTiO_3$, 8% $PbTiO_3$, 8% $CaTiO_3$	1.31	−2.7	8.0	530	0.124	0.4	0.015	6.1
80% $BaTiO_3$, 12% $PbTiO_3$, 8% $CaTiO_3$	1.28	−2.0	6.0	400	0.113	0.34	0.015	4.7
PZT #4*	0.815	−9.7	23.5	875	0.28	0.63	0.0268	19.2
PZT #5*	0.675	−14.0	32.0	1,200	0.32	0.70	0.0266	21.6
PZT #6*	0.865	−7.8	19.1	860	0.25	0.60	0.022	16.5
NbO_3 (K 50%; Na 50%)†	1.02	−3.2	8.0	235	0.226	0.52	0.034	8.15
$Pb(NbO_3)_2$‡	0.29	−3.3	9.0	240	0.115	0.31	0.037	2.6

* Data from Clevite Brush Company.
† L. Egerton, *J. Am. Ceram. Soc.* **42,** 438 (1959).
‡ G. Goodman, *J. Am. Ceram. Soc.* **36,** 368 (1953).

the radial mode and the torsional mode. The first is driven by polarizing the disk perpendicular to the major surface and involves the same fundamental constants as the length longitudinal mode of Eq. (3g-20). It has been shown[1] that the effective coupling and the resonant frequency of such disks are given by the equations

$$k^2 = \frac{2}{1-\sigma}\frac{4Q_{11}^2P_0^2\epsilon^T(P_0)}{s_{11}^E} \qquad f_R = \frac{2.03}{2\pi a}\sqrt{\frac{1}{s_{11}^E\rho(1-\sigma^2)}} \tag{3g-24}$$

where σ is Poisson's ratio, which is approximately 0.3 for barium titanate ceramics. The torsional mode is generated in electrostrictive and magnetostrictive materials when the alternating displacement is at right angles to the polarization. This is easily accomplished for a magnetostrictive material by polarizing a cylinder radially by one set of windings and driving the cylinder by a set of windings coaxial with the cylinder. In an electrostrictive material, a torsional vibration can be obtained by inducing a permanent polarization in different directions on two sides of the cylinder and driving the cylinder by a set of two electrodes with the two gaps between them coming in the region of greatest permanent polarization. The fundamental elastic constant is the shear constant ($s_{44}^E = s_{55}^E$) while the fundamental piezoelectric constant is the shear piezoelectric constant d_{15} or the similar magnetostrictive constants.

Table 3g-2 gives some typical constants for a number of barium titanate compositions with lead and calcium titanate additions. A number of new ceramics, particularly lead zirconate titanate (trade name PZT), sodium potassium niobate, and lead metaniobate, have recently appeared. These have higher Curie temperatures than barium titanate combinations but lower values of electrical and mechanical Q's. The stored electrical polarization in lead zirconate titanate (nearly 30 microcoulombs/cm^2) is higher than in any other ceramic and such materials are especially useful for producing a high current when depolarized by a mechanical shock (E.E.T. transducers). Figures 3g-3, 4, and 5 show how the fundamental constants vary with temperature over a wide temperature range for the most used ceramic PZT-4, and for the original $BaTiO_3$ ceramic. Table 3g-3 gives some typical constants for a number of magnetostrictive materials.

3g-3. Equivalent Circuits for Magnetostrictive Transducers. The energy equation (3g-16) is the same for magnetostrictive and electrostrictive materials, provided the electric field and displacement are replaced by the magnetic field H and the magnetic flux density B. Hence the equivalent circuit of Fig. 3g-1 also applies to a magnetostrictive material, provided we replace E and i by $\int_0^l H_i\,dl = U$, the magnetomotive force, and $\dot{B}S = \dot{\Phi}$, where S is the cross-sectional area, Φ the total flux through the magnetostrictive transducer, and $\dot{\Phi}$ the time rate of change of this flux. Hence all the fundamental quantities and coupling factors can be expressed in terms of the analogous quantities as shown by Table 3g-3. These hold for materials having a closed magnetic circuit such as a ring or a rod with closing magnetic circuit having a reluctance small compared with that for the rod. If this is not true, demagnetizing factors and additional reluctance values have to be taken account of and the value of Φ is the average value determined by all these factors.

In a transducer, however, it is not U and $\dot{\Phi}$ that we deal with, but rather the input voltage and current. These quantities are related by equations of the type

$$E = N\frac{d\Phi}{dt} \qquad U = Ni \tag{3g-25}$$

where N is the number of turns and the voltage, current, flux, and magnetomotive forces are directed as shown by Fig. 3g-6. These are the equations of a gyrator, shown

[1] W. P. Mason, "Piezoelectric Crystals and Their Application to Ultrasonics," chap. XII, D. Van Nostrand Company, Inc., Princeton, N.J., 1950.

TABLE 3g-3. MAGNETOSTRICTIVE PROPERTIES OF METALS AND FERRITES

Data from C. M. Van der Burgt, *Phillips Research Repts.* **8**, 91–132, 1953

Material	$d_{33} \times 10^9$ webers/newton	$d_{14} \times 10^9$ webers/newton	Rev. per. long. $\mu^T(P_1) \times 10^4$ henrys/m	$Y_0{}^H = \frac{1}{s^H} \times 10^{-11}$ newtons/m²	k_{33}	Rev. per shear $\mu^T(P_0) \times 10^4$ henrys/m	Shear stiffness $G^E \times 10^{-11}$ newton/m²	Torsional coupling k_T	Energy stored $\frac{1}{2}(d_{33}{}^2/\mu^T) \times 10^{12}$ joules-m/newton²	Density, kg/m³ $\times 10^{-3}$
99.9 nickel	−5.3	.....	2.84	2.0	0.14	.....	....	.....	0.05	8.9
50 Co; 0.5 Cr; 49.5 Fe	12.3	.....	8.3	2.2	0.20	.....	....	.....	0.09	8.2
35 Co; 0.5 Cr; 64.5 Fe	13.4	.....	19.2	2.1	0.14	.....	....	.....	0.047	8.1
NiO (15%); ZnO (35%); Fe_2O_3 (50%)	−11.1	−28.5	190	1.8	0.034	139	0.68	0.063	0.003	5.06
NiO (18%); ZnO (32%); Fe_2O_3 (50%)	−16.0	−39.5	77.5	1.62	0.073	74	0.62	0.115	0.0165	4.9
NiO (25%); ZnO (25%); Fe_2O_3 (50%)	−9.8	−20.3	22.0	1.53	0.082	20	0.59	0.110	0.022	4.85
NiO (32%); ZnO (18%); Fe_2O_3 (50%)	−8.7	−15.8	13.4	1.5	0.093	13.2	0.58	0.105	0.0282	4.85
NiO (40%); ZnO (10%); Fe_2O_3 (50%)	−5.9	−13.0	5.5	1.37	0.112	5.35	0.54	0.13	0.0315	4.76
NiO (50%); Fe_2O_3 (50%)	−4.4	.....	2.8	0.93	0.08	2.4	0.36	0.09	0.0344	4.20

Data from R. M. Bozorth, E. A. Nesbit, and H. J. Williams

Material	Flux density B, webers/m²	Long. rev. per $\mu^T(P_0) \times 10^4$ henrys/m	Young's modulus $Y_0{}^A \times 10^{-11}$ newtons/m²	Longitudinal coupling k_{33}	$d_{33} \times 10^9$ webers/newton	Energy stored $\frac{1}{2}(d_{33}{}^2/\mu^T) \times 10^{12}$ joules-m/newton²	Density, kg/m³ $\times 10^{-3}$
99.9% nickel	0.4	0.93	2.1	0.232	−5.0	0.127	8.9
	0.5	0.515	...	0.208	−3.26	0.103	
	0.55	0.317	...	0.177	−2.18	0.075	
45% Ni, 55% Fe, i.e., 45% Permalloy	0.722	8.94	1.6	0.154	11.5	0.074	8.17
	0.965	7.35	...	0.179	12.2	0.101	
	1.2	4.45	...	0.178	9.4	0.099	
	1.4	1.97	...	0.15	5.3	0.071	
2V Permindur, 2% V, 50% Co, 48% Fe	1.5	3.54	2.3	0.238	9.35	0.123	8.3
	1.6	2.61	...	0.222	7.5	0.108	
	1.8	2.23	...	0.202	6.3	0.089	
	2.0	1.14	...	0.18	4.0	0.07	

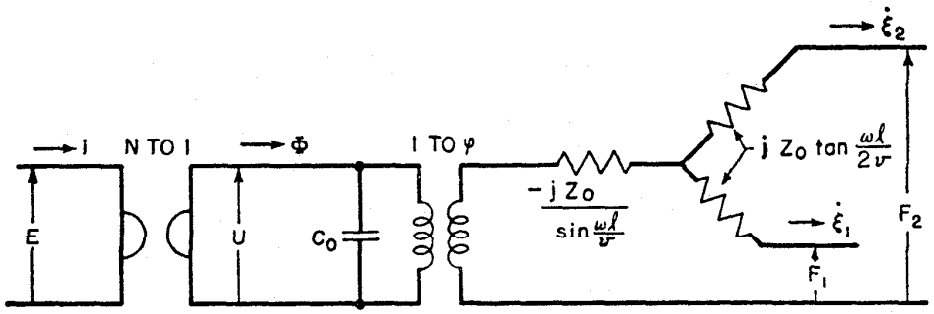
by the symbol of Fig. 3g-6, which does not satisfy the reciprocity relationship. If we call Z_M the magnetic impedance defined by

$$Z_M = \frac{U}{d\Phi/dt} \tag{3g-26}$$

it is evident that the electrical impedance at the terminals of the transducer is equal to

$$Z_E = \frac{E}{i} = \frac{N^2}{Z_M} \tag{3g-27}$$

Hence the effect of the gyrator coupling is to invert all the elements of the equivalent



$$C_0 = \frac{\mu^S l}{S} \; ; \; Z_0 = S\sqrt{\rho \gamma_0^H} \; ; \; v = \sqrt{\frac{\gamma_0^H}{\rho}} \; ; \; \varphi = \frac{d_{33}\gamma_0^H S}{l}$$

FIG. 3g-6. Equivalent circuit of a magnetostrictive rod.

circuit. Hence one should determine the element values of Fig. 3g-6 for the appropriate terminating conditions and then invert the values in accordance with Eq. (3g-27) to determine the elements of a magnetostrictive transducer. The values given in Fig. 3g-6 are for a longitudinally vibrating rod where S is the cross-sectional area and l the length. μ^S is the average value of the permeability in the equations for the reluctance R

$$R = \frac{l}{\mu^S S} \tag{3g-28}$$

where μ^S is for the constant stress condition.