3i. Radiation of Sound

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3i-1. Introduction. Radiation of sound may take place in a number of ways, but basically, all sound generators cause an alternating pressure to be set up in the fluid medium within which the sound energy is established. The sound energy that is set up in a medium depends not only on the physical characteristics of the medium and the oscillatory volume displacement of the fluid set up by the vibrating source but also upon the size and shape of the generator. The acoustic power generated by any vibrating source can be expressed by

$$P = U^2 R_A \times 10^{-7} \qquad \text{watts} \tag{3i-1}$$

where U = rate of volume displacement of fluid, cc/sec

R_A = acoustic radiation resistance seen by source, acoustic ohms

If the rate of volume displacement is taken in peak cc/sec, Eq. (3i-1) will yield peak watts of power. If the volume displacement is taken in rms cc/sec, the power will be given in rms watts.

Of the many possible methods for generating sound, two types of generators will effectively serve to typify most of them. These basic generators are (1) pulsating sphere and (2) vibrating piston.

Each type of generator has its own acoustic impedance characteristic which depends on the dimensions of the source and on the frequency of vibration.

3i-2. Acoustic Impedance. *Pulsating Sphere.* The specific acoustic impedance of a pulsating sphere is given by

$$z = \frac{\rho c}{1 + [1/(\pi D/\lambda)]^2} + j\frac{\rho c/(\pi D/\lambda)}{1 + [1/(\pi D/\lambda)]^2} \qquad \text{acoustic ohms/cm}^2 \tag{3i-2}$$

where ρ = density of the medium, g/cc

c = velocity of sound in the medium, cm/sec

D = diameter of the sphere, cm

$\lambda = c/f$

f = frequency, Hz

It can be seen from inspection that at high frequencies, where D/λ becomes very large, the specific acoustic impedance becomes a pure resistance equal to ρc and the reactance term vanishes. At low frequencies, where D/λ is small, the specific acoustic impedance becomes

$$z = \rho c \left(\frac{\pi D}{\lambda}\right)^2 + j\rho c \frac{\pi D}{\lambda} \qquad \text{acoustic ohms/cm}^2 \tag{3i-3}$$

A plot of the specific acoustic resistance and reactance of a pulsating sphere as a function of D/λ is shown in Fig. 3i-1. To obtain the total acoustic radiation resistance

R_A of the sphere, it is necessary to divide the specific acoustic resistance by the total surface area of the sphere in cm². The value of R_A thus determined, when substituted in Eq. (3i-1), will give the actual acoustic watts being generated by the spherical source.

Vibrating Piston. The specific acoustic impedance of a circular piston set in an infinite rigid baffle and radiating sound from one of its surfaces is given by

$$z = \rho c \left[1 - \frac{J_1(2\pi D/\lambda)}{\pi D/\lambda} \right] + j\rho c \frac{K_1(2\pi D/\lambda)}{2(\pi D/\lambda)^2} \quad \text{acoustic ohms/cm}^2 \qquad \text{(3i-4)}$$

where D is the diameter of the piston in centimeters, J_1 and K_1 are Bessel functions, and the remaining symbols are defined under Eq. (3i-2).

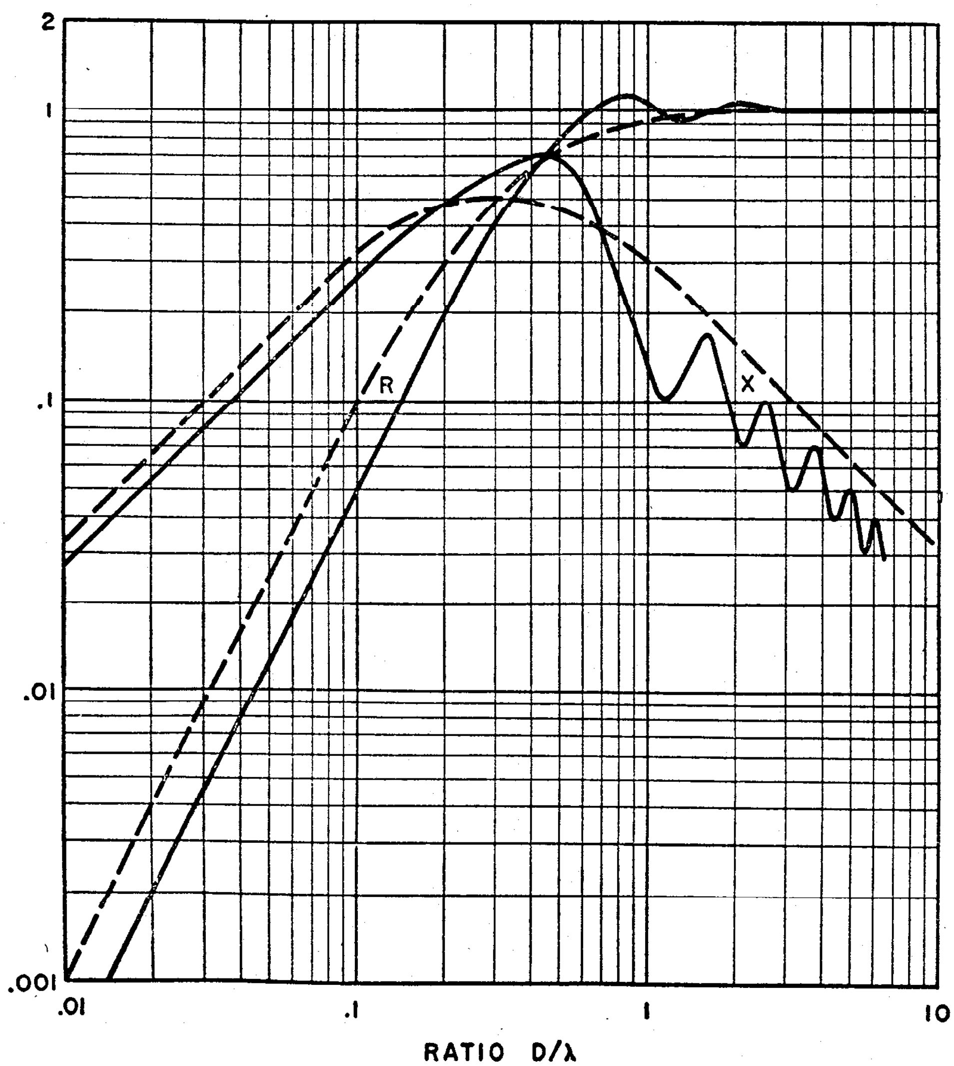

FIG. 3i-1. Specific acoustic resistance R and reactance X of a pulsating sphere (dashed curves) and a vibrating piston set in an infinite baffle (solid curves). To obtain magnitude of R or X multiply ordinates by ρc of the medium.

At high frequencies, where D/λ is large, Eq. (3i-4) reduces to a pure resistance equal to ρc. At low frequencies, where D/λ is small, the specific acoustic impedance for a piston set in an infinite baffle with one side radiating becomes

$$z = \frac{\rho c(\pi D/\lambda)^2}{2} + j\rho c \frac{8D}{3\lambda} \quad \text{acoustic ohms/cm}^2 \qquad \text{(3i-5)}$$

A plot of the specific acoustic resistance and reactance for a vibrating piston mounted in an infinite baffle is shown in Fig. 3i-1. To obtain the total acoustic radiation resistance of the piston, it is necessary to divide the specific resistance by the piston area in cm². The value of R_A so determined, when substituted in Eq. (3i-1), will give the actual acoustic watts being generated by a piston.

Summary of Radiation Impedance Characteristics. In Table 3i-1 are shown the magnitudes of the acoustic radiation resistance and reactance for a sphere and piston for both low-frequency (D/λ small) and high-frequency (D/λ large) operation.

TABLE 3i-1. TABULATED VALUES OF THE TOTAL ACOUSTIC RADIATION RESISTANCE AND REACTANCE OF A SPHERE AND PISTON IN ACOUSTIC OHMS

	$D/\lambda \ll 1$		$D/\lambda \gg 1$	
	R_A	X_A	R_A	X_A
Pulsating sphere	$\rho c \frac{\pi}{4\lambda^2}$	$\frac{\rho c}{\pi D\lambda}$	$\frac{\rho c}{A}$	0
Vibrating piston (in infinite baffle)	$\rho c \frac{\pi}{2\lambda^2}$	$\rho c \frac{8}{3\pi D\lambda}$	$\frac{\rho c}{A}$	0

ρ = density of the medium, g/cm³
c = velocity of sound in the medium, cm/sec
λ = wavelength of sound in the medium, cm
$\lambda = c/f$
f = frequency of the sound vibration, Hz
D = diameter of sphere or piston, cm
A = surface area of sphere or piston, cm²

3i-3. Directional Radiation of Sound. Whenever sound energy is generated from a source whose dimensions are small compared with the wavelength of the vibration in the medium, the intensity will be uniform in all angular directions and the generator is generally defined as a point source. When the dimensions of the vibrating surface are large compared with the wavelength, phase interferences will be experienced at different points in space due to the differences in time arrival of the vibrations originating from different portions of the surface, which results in a nonuniform directional radiation pattern. Practical use is made of this phenomenon when it is desired to produce special directional patterns by arranging the geometry and size of the vibrating surfaces of a sound generator to create the desired characteristic.

In many instances, a transmitter is designed so that the sound is radiated in a relatively sharp beam so that the energy is concentrated only within a specific desired angular region. When such a directional structure is employed as a receiver, the transducer will be more capable of picking up weak signals from a specified direction than would be the case from a nondirectional transducer. The reason for this improvement is the reduced sensitivity of the directional receiver to random background noises that will be present in all directions from the source. The number of decibels by which the signal-to-noise ratio is improved by a directional receiver over a nondirectional receiver is known as the *directivity index* (directional gain) of the transducer. It will be defined more fully later. The following will show the directional radiation characteristics of several common structures.

Uniform Line Source. If a uniform long line is vibrating at uniform amplitude, the radiated sound intensity will be a maximum in a plane which is the perpendicular bisector of the line. At angles removed from the perpendicular bisector of the line, the intensity will fall off to a series of nulls and secondary maxima of diminishing amplitudes as the angle of incidence to the axis of the line deviates from the normal bisector of the line. For a line of length L vibrating uniformly over its entire length at a frequency corresponding to a wavelength of sound λ in the medium, the ratio of the sound pressure p_Θ produced at an angle Θ removed from the normal axis of maximum response to the sound pressure p_0 on the normal axis is given by

$$\frac{p_\Theta}{p_0} = \frac{\sin[(\pi L/\lambda)\sin\Theta]}{(\pi L/\lambda)\sin\Theta} \tag{3i-6}$$

If L is large compared with λ, the response as a function of Θ will go through a series of nulls and secondary maxima of successively diminishing amplitudes.

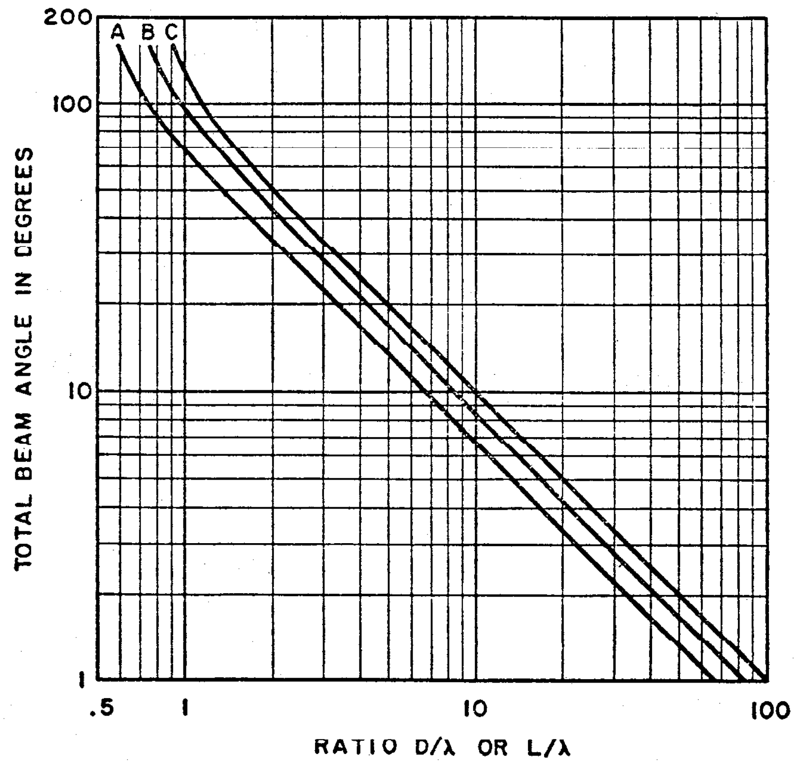

FIG. 3i-2. Total beam angle for a piston, ring, and line source as a function of size of source to wavelength of sound being radiated. *A*, thin ring of diameter *D*. *B*, uniform line of length *L*. *C*, piston of diameter *D*. *(Curves A and C from Massa, "Acoustic Design Charts," The Blakiston Division, McGraw-Hill Book Company, Inc., New York, 1942.)*

Circular Piston in Infinite Baffle. The directional radiation pattern from a large circular piston vibrating at constant amplitude and phase and set into an infinite rigid baffle may be obtained from the expression

$$\frac{p_\Theta}{p_0} = \frac{2J_1[(\pi D/\lambda)\sin\Theta]}{(\pi D/\lambda)\sin\Theta} \tag{3i-7}$$

where p_Θ = sound pressure at an angle Θ from the normal axis of the piston
p_0 = sound pressure on normal axis of piston
D = diameter of piston
λ = wavelength of sound
J_1 = Bessel function of order 1

From this equation, it can be seen that, as D/λ increases, the beam width becomes smaller and the sound pressure goes through a series of nulls and secondary maxima as Θ progressively departs from the normal axis to the piston.

Thin Circular Ring. The directional radiation pattern from a large narrow circular ring of diameter D vibrating at constant amplitude and fitted into an infinite plane baffle may be obtained from the expression

$$\frac{p_\Theta}{p_0} = J_0\left(\frac{\pi D}{\lambda}\sin\Theta\right) \tag{3i-8}$$

where J_0 = Bessel function of order zero and all other symbols are defined under Eq. (3i-7).

Beam Width for Line, Piston, and Ring. From Eqs. (3i-6), (3i-7), and (3i-8), the total beam width has been computed for the radiation from each of the three types of sound generators. The total beam width is here defined as the angle 2Θ at which the pressure p_Θ is reduced 10 dB in magnitude from the maximum on axis reponse p_0. By setting p_Θ/p_0 equal to -10 dB or 0.316 in magnitude in these equations, the three curves plotted in Fig. 3i-2 were computed.

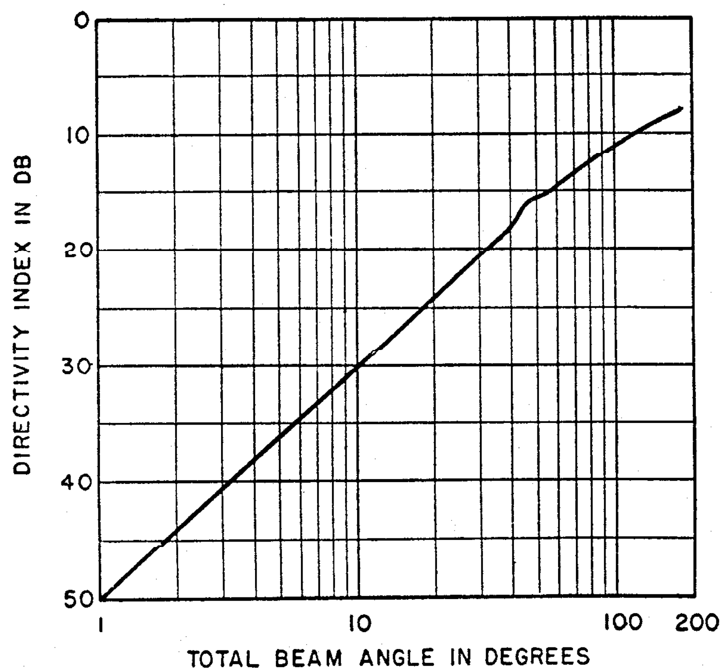

FIG. 3i-3. Directivity index of a piston or ring as a function of total beam angle where beam angle is defined as the included angle of the main beam between the 10-decibel-down points in the directional response. (*Computed from Massa, "Acoustic Design Charts," The Blakiston Division, McGraw-Hill Book Company, Inc., New York, 1942.*)

3i-4. Directivity Index. It has already been mentioned that a directional transducer has an advantage over a nondirectional structure whenever it is desired to send or receive signals from a particular localized direction only. The fact that the directional transducer is less sensitive to sounds coming from random undesired directions makes it possible for it to detect weaker signals than would be possible with a nondirectional unit. The measure of this improvement in decibels corresponds to the *directivity index* of the transducer, which is 10 times the logarithm (to the base 10) of the ratio of intensity of the response along the axis of maximum sensitivity to the average intensity of the response over the entire spherical region surrounding the transducer. See Sec. 3a for a more detailed definition.

The directivity index of a transducer is expressed in decibels, and a plot of the directivity index as a function of beam width for a piston or ring is shown in Fig. 3i-3.