3m. Mobility Analogy

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The analogies that have been presented and considered in Sec. 3l have been formal ones owing to the similarity of the differential equations of electrical, mechanical and acoustical vibrating systems. For this reason these analogies have been termed the classical impedance analogies; they are, however, not the only ones possible of development for useful applications. For example, mechanical impedance has been defined by some authors—in addition to the ratio of force to velocity as developed in Sec. 3l—as the ratio of pressure to velocity, the ratio of force to displacement, and the ratio of pressure to displacement. During the past three decades the developments in the field of analogies have been reported in publications[1] by many investigators. In this connection a useful analogy, developed by Firestone and designated by him as the "mobility analogy," has been employed on a wide scale to solve problems in mechanical vibrating systems. In the mobility analogy mechanical mobility is defined as the complex ratio of velocity to force. Although the mobility analogy can be applied and used with all types of vibrating systems, its most direct and useful application is in the field of mechanical vibrating systems. Therefore, in order to make the subject of analogies complete in this handbook, it seems logical to include the mobility analogy. Accordingly, it is the purpose of this chapter to develop the mobility analogy, particularly as applied to mechanical rectilineal systems.[2]

3m-1. Mechanical Rectilineal Mobility. Mechanical rectilineal mobility is the inverse of mechanical rectilineal impedance. Mechanical rectilineal mobility z_I, in mechanical mhos, is defined as the complex ratio of linear velocity to linear force as follows:

$$z_I = \frac{v}{f_M} \tag{3m-1}$$

where v = velocity, cm/sec

f_M = force, dynes

It will be evident that a mechanical element in the mechanical mobility sense is analogous to the electric element if velocity difference across the mechanical element is analogous to the voltage difference across the electric element and if the force through the mechanical element is analogous to the electric current through the electric element.

[1] See the end of Section 3 for a list of references.

[2] The considerations in this section will be confined to mechanical rectilineal systems. The mobility analogy is equally applicable to mechanical rotational systems. In this connection mechanical rectilineal and mechanical rotational systems are not sufficiently different to warrant a separate treatment for the mechanical rotational system, particularly in view of the fact that fundamental aspects of the two systems have been considered from the classical impedance analogy viewpoint in this book.

Mechanical rectilineal mobility z_I, in mechanical mhos, is a complex quantity and may be written as follows:

$$z_I = r_I + jx_I \tag{3m-2}$$

where r_I = responsivity, mechanical mhos
x_I = excitability, mechanical mhos

3m-2. Responsivity (Mobility Resistance). In the mechanical rectilineal mobility system mechanical rectilineal responsivity (mobility resistance) r_I, in mechanical mhos, is defined as

$$r_I = \frac{v}{f_M} = \frac{1}{r_M} \tag{3m-3}$$

where v = velocity, cm/sec
f_M = force, dynes
r_M = mechanical impedance, mechanical ohms

3m-3. Mass (Mobility Capacitance). In the mechanical rectilineal mobility system the mass (mobility capacitance) m_I, in grams, is analogous to electric capacitance C_E.

The mechanical rectilineal excitability x_I of a mass (mobility capacitance), in mechanical mhos, is defined as

$$x_I - \frac{1}{\omega m_I} \tag{3m-4}$$

where $\omega = 2\pi f$
f = frequency, hertz

Equation (3m-4) shows that the mass (mobility capacitance) m_I in the mechanical rectilineal mobility system is analogous to electric capacitance C_E in the electric system.

Mass (mobility capacitance) m_I in the mechanical rectilineal mobility system may also be defined as follows:

$$f_M = m_I \frac{dv}{dt} \tag{3m-5}$$

$$v = \frac{1}{m_I} \int f_M \, dt \tag{3m-6}$$

In the electric system electric capacitance C_E may be defined as follows:

$$i = C_E \frac{de}{dt} \tag{3m-7}$$

where i = electric current, abamp
C_E = electric capacitance, abfarads
e = electromotive force, abvolts
t = time, sec

$$e = \frac{1}{C_E} \int i \, dt \tag{3m-8}$$

where i = current in abamperes.

It will be seen that Eqs. (3m-5) and (3m-6) in the mechanical rectilineal mobility system are analogous to Eqs. (3m-7) and (3m-8) in the electric system.

3m-4. Compliance (Mobility Inertia). In the mechanical rectilineal mobility system the compliance (mobility inertia) C_I, in centimeters per dyne, is analogous to electric inductance L.

The mechanical rectilineal excitability x_I of a compliance (mobility inertia), in mechanical mhos, is defined as

$$x_I = \omega C_I \tag{3m-9}$$

where $\omega = 2\pi f$
f = frequency, Hz

Equation (3m-9) shows that compliance (mobility inertia) C_I, in centimeters per dyne, is analogous to inductance.

Compliance (mobility inertia) C_I in the mechanical rectilineal mobility system may also be defined as

$$v = C_I \frac{df_M}{dt} \qquad \text{(3m-10)}$$

In the electric system inductance may be defined as

$$e = L \frac{di}{dt} \qquad \text{(3m-11)}$$

where L = inductance in abhenrys.

It will be seen that Eq. (3m-10) in the mechanical rectilineal mobility system is analogous to Eq. (3m-11) in the electric system.

3m-5. Representation of Electrical and Mechanical Rectilineal Mobility Elements. Electric elements have been defined in Sec. 3l. Elements in the mechanical rectilineal mobility system have been described in this section.

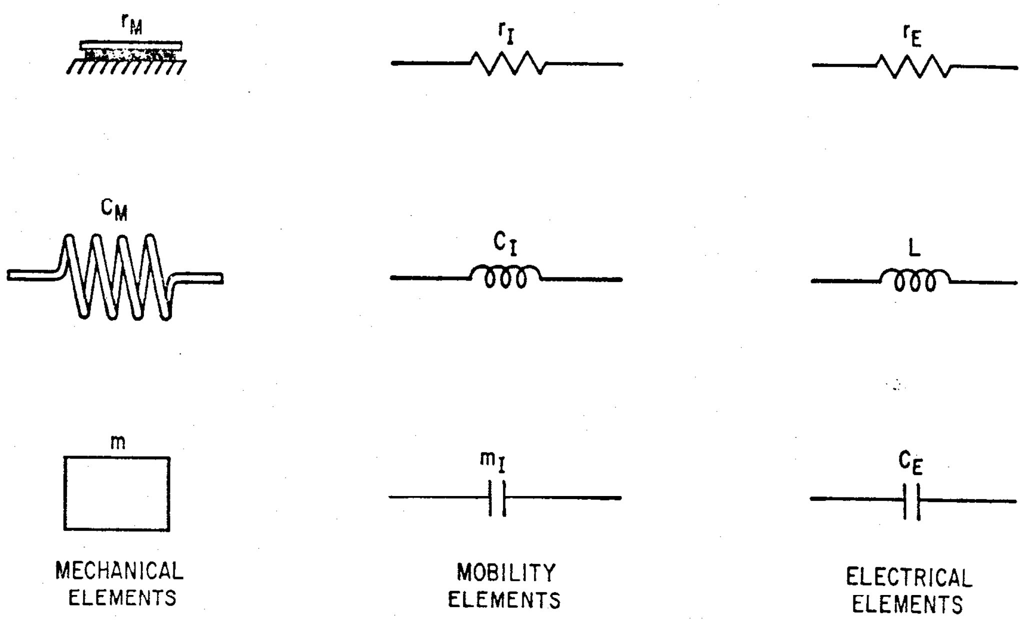

FIG. 3m-1. Graphical representation of the three basic elements in mechanical rectilineal, mobility, and electric systems.

r_M = mechanical rectilineal resistance r_I = responsivity r_E = electrical resistance
C_M = compliance C_I = mobility inertia L = inductance
m = mass m_I = mobility capacitance C_E = electric capacitance

(After Olson, "Solutions of Engineering Problems by Dynamical Analogies," D. Van Nostrand Co., Princeton, N.J., 1966.)

Figure 3m-1 illustrates schematically the mechanical elements and the analogous elements in the electric and mechanical rectilineal mobility systems.

Mechanical rectilineal resistance r_M in the mechanical rectilineal system is represented as sliding or viscous friction. Mechanical rectilineal responsivity (mobility resistance) r_I in the mechanical rectilineal mobility system is the reciprocal of mechanical rectilineal resistance r_M and is analogous to electrical resistance r_E.

Compliance C_M in the mechanical rectilineal system is represented as a spring. Compliance (mobility inertia) C_I in the mechanical rectilineal mobility system is analogous to inductance L in the electric system.

Mass m in the mechanical rectilineal system is represented as a mass or weight. Mass (mobility capacitance) m_I in the mechanical rectilineal mobility system is analogous to electric capacitance C_E in the electric system.

The electrical and the mechanical rectilineal quantities in the mobility system are shown in Table 3m-1. The units and the analogous elements and symbols also are shown in Table 3m-1.

3m-6. Mechanical Vibrating System Consisting of a Mass, Compliance, and Mechanical Resistance. The vibrating system[1] of one degree of freedom consisting of a mass, compliance, and mechanical resistance has been considered from the standpoint of the classical mechanical impedance analogy in Sec. 3l. It is the purpose of this section to consider the same mechanical vibrating system from the standpoint of the mechanical mobility analogy.[2]

TABLE 3m-1. CORRESPONDENCE BETWEEN ELECTRICAL AND MECHANICAL QUANTITIES IN THE MOBILITY SYSTEM

Electrical			Mechanical rectilineal mobility		
Quantity	Unit	Symbol	Quantity	Unit	Symbol
Electromotive force	Volts × 10^{-8}	e	Velocity	Centimeters per second	$\dot{x}$ or v
Charge or quantity	Coulombs × 10^{-1}	q	Impulse or momentum	Gram-centimeter per second	Q
Current	Amperes × 10^{-1}	i	Force	Dynes	f_M
Electrical impedance	Ohms × 10^9	z_E	Mechanical mobility	Mechanical mhos	z_I
Electrical resistance	Ohms × 10^9	r_E	Responsivity	Mechanical mhos	r_I
Electrical reactance	Ohms × 10^9	x_E	Excitability	Mechanical mhos	x_I
Inductance	Henrys × 10^9	L	Compliance or mobility inertia	Centimeters per dyne	C_I
Electrical capacitance	Farads × 10^9	C_E	Mass or mobility capacitance	Grams	m_I
Power	Joules per second	P_E	Power	Ergs per second	P_I

The mechanical system consisting of a mass, compliance, and mechanical resistance is shown in Fig. 3m-2*A*. The mechanical vibrating system may be rearranged to form the equivalent as shown in Fig. 3m-2*B*. From the mechanical vibrating system of Fig. 3m-2*B* it is a relatively simple matter to develop the mobility analogy of Fig. 3m-2*C*.

[1] The preceding paragraphs have been concerned with fundamental considerations. Therefore, the modifier rectilineal has been employed for the sake of accuracy. Since the remainder of this section will be concerned with applications of the mechanical rectilineal mobility, the modifier rectilineal will be dropped.

[2] In view of the fact that this section is concerned with mechanical systems, the modifier mechanical in relation to the mechanical mobility analogy is also superfluous and need not be used.

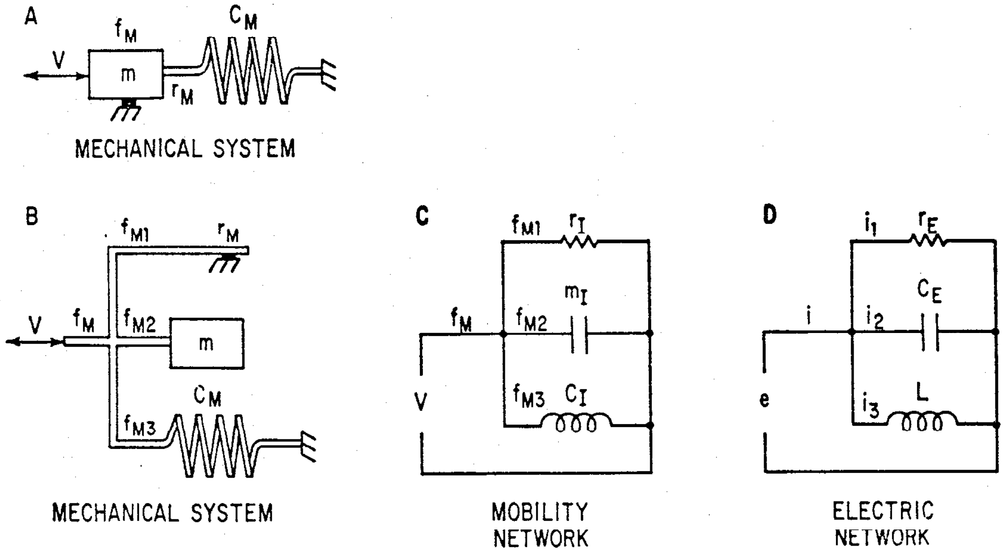

FIG. 3m-2. A mechanical vibrating system consisting of a mass, compliance, and mechanical resistance. *A*. Mechanical system. *B*. Mechanical system equivalent to the mechanical system of *A*. *C*. Mobility network of the mechanical system. *D*. Electric network analog of the mobility system. (*After Olson, "Solution of Engineering Problems by Dynamical Analogies," D. Van Nostrand Company, Princeton, N.J.*, 1966.)

The sum of the forces through the three branches of the mobility network[1] of Fig. 3m-2*C* is

$$f_M = f_{M1} + f_{M2} + f_{M3} \tag{3m-12}$$

where

$$f_{M1} = \frac{v}{r_I} \tag{3m-13}$$

$$f_{M2} = m_I \frac{dv}{dt} \tag{3m-14}$$

$$f_{M3} = \frac{1}{C_I} \int v\, dt \tag{3m-15}$$

From the sum of Eqs. (3m-13) to (3m-15) the differential equation of the mobility network of Fig. 3m-2*C* is

$$f_M = m_I \frac{dv}{dt} + \frac{v}{r_I} + \frac{1}{C_I} \int v\, dt \tag{3m-16}$$

The sum of the electric currents of the electric network of Fig. 3m-2*D* is

$$i = i_1 + i_2 + i_3 \tag{3m-17}$$

where

$$i_1 = \frac{e}{r_E} \tag{3m-18}$$

$$i_2 = C_E \frac{de}{dt} \tag{3m-19}$$

$$i_3 = \frac{1}{L} \int e\, dt \tag{3m-20}$$

[1] In establishing analogies between electric and mechanical systems the elements in the electric network have been labeled r_E, L, and C_E. However, in using analogies in actual practice, the conventional procedure is to label the elements in the analogous electric network as r_M, m, and C_M for the classical mechanical rectilineal system and as r_I, C_I, and m_I for the mobility mechanical rectilineal system. This procedure will be followed in this section in labeling the elements of the analogous electric network. It is literally accurate to label the network with the caption "Analogous electric network of the mechanical rectilineal system" (or, of the mobility mechanical rectilineal system). For the sake of brevity, these networks will be labeled "mechanical network" and "mobility network." Where there is only one path, "circuit" will be used instead of "network."

From the sum of Eqs. (3m-18) to (3m-20) the differential equation of the electric network of Fig. 3m-2*D* is

$$i = C_E \frac{de}{dt} + \frac{e}{r_E} + \frac{1}{L} \int e\, dt \qquad (3m\text{-}21)$$

Comparing the variables and coefficients of the mobility and electric networks in the differential equations (3m-16) and (3m-21) establishes the analogous variables and quantities in the two systems as given in Table 3m-1.

The classical mechanical impedance analogy of the mechanical system of Fig. 3m-2 has been considered in Sec. 3l and will not be repeated here.

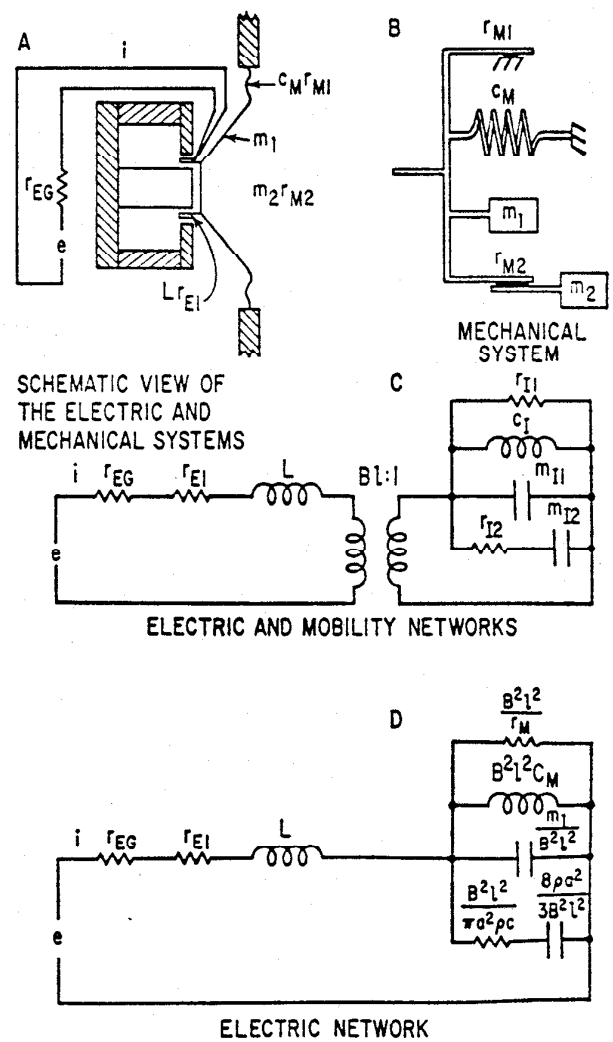

FIG. 3m-3. Cross-sectional view, the mechanical system, the electric and mobility networks, and the electric network of a direct radiator dynamic loudspeaker. In the electric and mechanical networks: e, the electromotive force of the electric generator. r_{EG}, the electrical resistance of the electric generator. L, the inductance of the voice coil. R_{E1}, the electrical resistance of the voice coil. m_1, the mass of the cone. C_M, and r_{M1}, the compliance and mechanical resistance of the suspension. m_2 and r_{M2}, the mass and mechanical resistance of the air load. m_I, the mobility capacitance of the cone. C_I and r_{I1}, the mobility inertia and responsivity of the suspension. m_{I2} and r_{I2}, the mobility capacitance and responsivity of the air load. B, the flux density in the air gap. l, the length of the voice coil conductor. a, the radius of the cone. ρ, the density of air. (*After Olson, "Solutions of Engineering Problems by Dynamical Analogies," D. Van Nostrand Company, Princeton, N.J.,* 1966.)

3m-7. Direct Radiator Loudspeaker. The direct radiator dynamic loudspeaker shown in Fig. 3m-3 is almost universally used for radio, phonograph, television, and other small-scale sound reproduction.

The electric and mechanical systems of the complete loudspeaker are shown in Fig. 3m-3A. The mechanical vibrating system consisting of the voice coil, cone, suspension, and air load is presented in Fig. 3m-3B.

The mass m_1 of the cone and voice coil, and the compliance C_M and mechanical resistance of the suspension system, can be obtained from measurements of the vibrating system.

The mechanical system of the air load—namely, the mechanical resistance r_{M2} and mass m_2 of the air load upon the front of the cone—is depicted in Fig. 3m-4A and

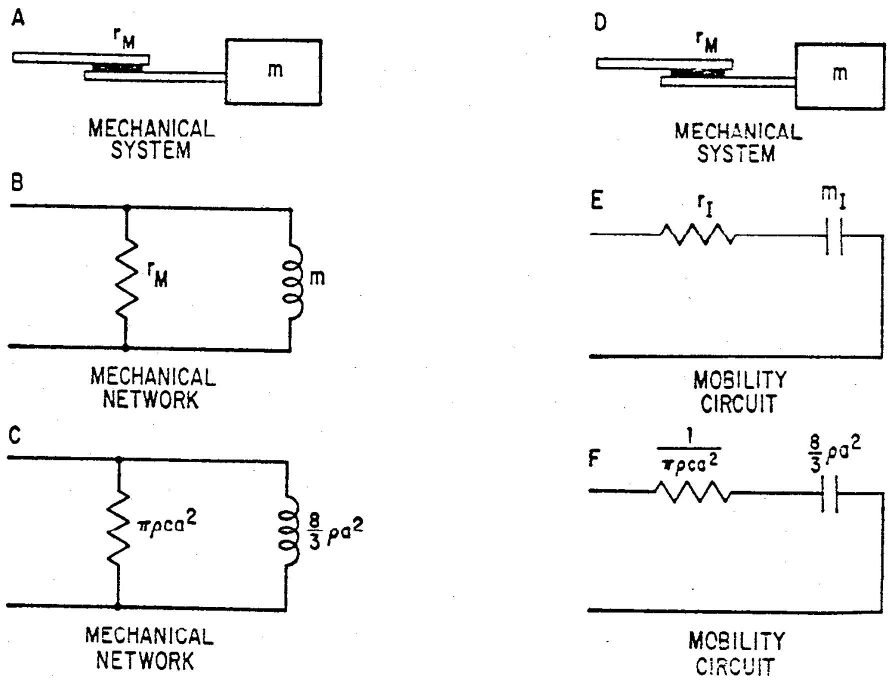

FIG. 3m-4. Air load upon a loudspeaker cone. *A.* Mechanical system: m, the mass of the air load. r_M, the mechanical resistance of the air load. *B.* Mechanical network of the air load upon a loudspeaker cone. *C.* Mechanical network of the air load upon a loudspeaker cone: a, the radius of the cone. ρ, the density of air. c, the velocity of sound. *D.* Mechanical system same as *A.* *E.* Mobility circuit of the air load upon a loudspeaker cone: m_I, the mobility capacitance of the air load. r_I, the responsivity of the air load. *F.* Mobility circuit of the air load upon a loudspeaker cone: a, the radius of the cone. ρ, the density of air. c, the velocity of sound. (*After Olson, "Solution of Engineering Problems by Dynamical Analogies." D. Van Nostrand Company, Princeton, N.J.*, 1966.)

3m-4*D*. The mechanical network of the air load upon the front of the cone is shown in Fig. 3m-4*B*. The constants of the mechanical resistance and mass of the air load upon the front of the cone are shown in the mechanical network of Fig. 3m-4*C*. The mobility circuit of the air load upon the front of the cone appears in Fig. 3m-4*E*. The constants of the responsivity and compliance are given in the mobility circuit of Fig. 3m-4*F*.

The electric and mobility networks with the ideal transformer connecting the electric and mobility sections are shown in Fig. 3m-3.

In Fig. 3m-3*D* the ideal transformer has been eliminated, and the entire vibrating system reduced to an electric network. The electrical impedance due to the mechanical system is given by Eq. (3l-26) as follows:

$$z_{EM} = \frac{(Bl)^2}{z_M} \qquad (3m\text{-}22)$$

where z_{EM} = electrical impedance due to the mechanical system, abohms
z_M = mechanical impedance of the mechanical system, mechanical ohms
B = flux density in the air gap, gauss
l = length of the voice coil conductor, cm

Since $1/z_M = z_I$, Eq. (3m-22) may be written as

$$z_{EM} = (Bl)^2 z_I \tag{3m-23}$$

where z_I = mobility in mechanical mhos.

By means of Eq. (3m-23) it is possible to convert the combined electric and mobility networks to the electric network, as shown in Fig. 3m-3.

The process employing the mobility analysis of this section may be compared with the classical impedance analysis of Sec. 3l.

References

1. Olson, H. F.: "Dynamical Analogies" 2d ed., D. Van Nostrand Company, Inc., Princeton, N.J., 1958.
2. Olson, H. F.: "Solution of Engineering Problems by Dynamical Analogies," Van Nostrand Reinhold Co., New York, N.Y., 1968.