5c. Electrical Standards

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5c-1. Fundamental Considerations. The standards in terms of which electrical quantities are evaluated are derived from absolute measurements which serve to establish the magnitudes of the electrical units in terms of the base mechanical units. The relations between the fundamental mechanical units and the electrical units derived from them are required to satisfy two conditions: (1) the electrical watt should equal the mechanical watt; and (2) in a rationalized system the unit of resistance must be such as to make the wave impedance of free space numerically equal to $\mu_v c$, where μ_v is the conventionally assigned value of the permeability of free space, and c is the velocity of the electromagnetic wave.[1] The first condition fixes the *product* of the volt and the ampere (the watt), while the second fixes their *quotient* (the ohm).

Two types of *absolute* measurement have been used in assigning numerical values to the basic electrical standards in terms of mechanical units of length and time. In one measurement the ohm is evaluated in terms of the mechanical units of length and time; in the second the ampere is measured in units of length, mass, and time.

Most of the *absolute-ohm* determinations in the past have involved an inductor (either self or mutual) constructed in such a way that its inductance can be computed from its measured dimensions, together with the conventionally assigned permeability of the space around it. This inductor is then supplied with a periodically varying current, and its reactance at the known frequency is, in effect, compared with the resistance of a standard resistor.[2] Now, there is no reason why absolute-ohm determinations should not be made in terms of a computable capacitance and a frequency. Indeed, such a determination offers decided advantages in that the electric field of a capacitor can be confined by shields and the capacitance value made completely independent of neighboring objects, whereas the magnetic field of a computable inductor cannot be so limited as to be completely free from proximity influences. However, only since the recent discovery of a new theorem in electrostatics[3] has it been possible to design and construct capacitors whose value can be computed with sufficient accuracy from simple dimensional measurements to make attractive an absolute-ohm determination in terms of capacitance. In such a determination, the permittivity of the dielectric medium in the capacitor must be used in computing its value, and this is obtained through permeability—an assigned unit in the absolute mksa system—and an experimental value for the speed of light. Thus, the essential limitation in such an absolute-ohm determination is the uncertainty in the value used for the speed of light in vacuum. Two ohm determinations in terms of capacitance have been reported.[4]

[1] F. B. Silsbee, *Instruments* **26**, 1522 (1953).
[2] Thomas et al., *J. Research NBS* **43**, 291 (1949); Rayner, *Metrologia* **3**, 12 (1967).
[3] The Thompson-Lampard theorem is discussed in Sec. 5c-5.
[4] Cutkosky, *J. Research NBS* **65A**, 147 (1961); Thompson, *Metrologia* **4**, 1 (1968).

In an *absolute-ampere* experiment, a pair of coils is so arranged that the force or torque between them when they carry a current can be measured accurately. The arrangement is called a *current balance*. The current, thus measured in absolute amperes, is passed through a resistor whose value is known in absolute ohms. The resulting voltage drop is opposed to the electromotive force of a standard cell, and its emf is determined in *absolute* volts.[1]

Values having been assigned to physical standards of resistance and voltage on the basis of absolute measurements, the values of the other electrical units can be derived from them, using appropriate relationships. Thus the *ohm* and *volt* become the base units of electrical measurement, and their physical embodiments in resistance coils and standard cells become the fundamental electrical standards.

5c-2. History of Electrical Standards. The British Association Ohm (1864), resulting from the work of a committee under the leadership of Maxwell, represented the first concerted attempt by a responsible organization to realize an electrical standard based on absolute measurements correlating a mechanical and electrical system of units. At that time the Daniell cell was commonly used as the standard of emf. Later the Clark cell (1872) and its modification by Lord Rayleigh (1884) were used. Still more recently (by international agreement in 1908) the cadmium cell invented by Weston (1891) has entirely replaced the Clark cell and is in use today as the standard of emf.

Although the assignment of values to electrical standards on the basis of an absolute system of units has been generally recognized as desirable since the initial proposal of the British Association, the difficulties encountered in absolute measurements led to rather large uncertainties in the values of the standards. This resulted in the adoption (1894) of an auxiliary set known as the "international" units, which were a "reasonable approximation" of the absolute units and which could, it was hoped, be experimentally reproduced with sufficient accuracy for measurement purposes. These units were defined by the resistance of a uniform column of mercury of specified length and mass and by the current required for the deposition of silver at a specified rate from a silver nitrate solution. The units defined in terms of the "mercury" ohm and the "silver" ampere could be established easily within a few hundredths of a percent, but presently there was need for greater accuracy in measurements. Fortunately the techniques needed in absolute measurement also improved, and it became possible to establish values of the electrical units within about 10 ppm by absolute methods.

Accordingly, on January 1, 1948, the *international* units based on the "mercury" ohm and "silver" ampere were abandoned, and *absolute* units were adopted by international agreement. These absolute units are defined identically with the meter-kilogram-second-ampere (mksa) units of the Systéme International (SI) formalized by international agreement in 1960.

In the 1948 reassignment of the electrical units, the International Committee rounded to 10 ppm the factors needed to convert from "mean international" units to "absolute" units. These mean international units were based on intercomparisons at the International Bureau (BIPM) of *ohm* and *volt* standards supplied by several countries. The absolute ohm and ampere determinations performed at various national laboratories were reduced to this common basis in establishing conversion factors.[2] Table 5c-1 may be used to compute the value in "1948 absolute" units of a quantity that is known in the "international" units previously used in the United States. The table takes account of the difference between the *mean* international

[1] Driscoll, *J. Research NBS* **60,** 287 (1958); Driscoll and Cutkosky, *ibid.* 297; Vigoureux, *Metrologia* **1,** 3 (1965).

[2] Curtis, *J. Research NBS* **33,** 235 (1944).

ohm and volt established by the intercomparison of national units at BIPM, and the international ohm and volt maintained at NBS and used in the United States. Corresponding tables based on the "international" units maintained by other countries would be slightly different because each country has maintained its own standards independently, and small differences developed over the years between the units of one country and those of another.

In October, 1968, the International Committee reviewed the results of recent absolute-ohm and ampere determinations with the following results. The unit of resistance maintained by BIPM appeared to be within 0.2 ppm of its intended value in absolute ohms, and no change was made in the assignment of the standards with which this unit is maintained. Also, on the basis of the 1967 international comparison of standards, BIPM reported that $\Omega_{NBS} = \Omega_{BIPM} - 0.19\ \mu\Omega$. Thus, it appeared that the unit of resistance maintained by NBS differs from its intended value by less than 0.4 ppm; and therefore the values of the standards with which the NBS ohm is maintained were not reassigned. On the other hand, from recent ampere determinations it was concluded that the unit of voltage maintained by BIPM was too large by 11 ppm[1]; and

TABLE 5c-1. UNITED STATES VALUES (1948)

1 international ohm	=	1.000495 absolute ohms
1 international volt	=	1.000330 absolute volts
1 international ampere	=	0.999835 absolute ampere
1 international coulomb	=	0.999835 absolute coulomb
1 international henry	=	1.000495 absolute henrys
1 international farad	=	0.999505 absolute farad
1 international watt	=	1.000165 absolute watts
1 international joule	=	1.000165 absolute joules

its assigned value was decreased by this amount, effective January 1, 1969. BIPM reported, as a result of the 1967 intercomparisons that $V_{NBS} = V_{BIPM} - 2.58\ \mu V$. Thus the NBS unit of voltage was brought to equivalence with that maintained by BIPM (on January 1, 1969) by increasing the numerical assignment of emf of the standard cells of the National reference group that maintains the NBS volt, by 8.4 ppm.

5c-3. Maintenance of the Electrical Units. The National Bureau of Standards is assigned the duty of establishing and maintaining the electrical units defined by an Act of Congress,[2] and used by science and technology in the United States. It also measures and reports values of electrical standards for other laboratories[3] in terms of the NBS units—the legal units of the United States.

The NBS *ohm* and *volt* are maintained by groups of wire-wound resistors and of standard cells whose group averages are assumed to remain constant with time. These groups constitute the *primary* electrical standards of the United States, and, in effect, all values of the various electrical quantities are derived from them. The constancy of these primary standards is checked at intervals by appropriate absolute-ohm and ampere determinations; and the constancy of their ratio is monitored by annual determinations of proton precession frequency in a magnetic field that can be related directly to the NBS ampere. The ratio of the NBS volt to ohm, thus determined, has not varied by as much as 1 ppm in the eight years (1968) during which this

[1] It should not be supposed that this entire amount represents a drift since 1948 in the emf of the standard cells with which the unit had been maintained; a considerable part of the discrepancy may be attributable to incorrect assignment in 1948, on the basis of ohm and ampere determinations then available, together with the Committee's decision to round the assignment to a part in 10^5.

[2] Public Law 617—81st Congress.

[3] This service is voluntary on the part of the organization requesting the test, as NBS has no police powers. Also, with some exceptions, a fee is charged covering the cost of the test.

test has been available. Monitoring the stability of the electrical units by relating them to an invariant atomic constant (proton gyromagnetic ratio) marks a significant step toward better maintenance.

Values of the NBS volt and ohm are also compared with the corresponding units of other countries every three years by intercomparison of standards at BIPM. Values of the NBS standards have been reassigned on occasion[1] in the conformity with international action, to bring the NBS units into closer agreement with their intended absolute values.

5c-4. Standards of Resistance. The primary standard of resistance in the United States is a group of ten 1-Ω resistors of special construction. The present group comprising the primary standard are of the Thomas[2] type, made in 1933. They were wound of No. 12 Awg manganin wire, vacuum-annealed at 500°C, and sealed in air in double-walled containers. The individual members of the group are intercompared annually. The maximum net change in any member of the group, with respect to the group average, has been a little over 2 ppm and the average about 1 ppm over the last 30 years (to 1968). They can be intercompared or can be compared with other similar standards to about 1 part in 10^7. By suitable comparisons and by series and parallel combinations, ratios can be established to a few parts in 10^7, and the range of resistance can be extended from the primary standards to higher and lower values. Secondary standards can be assigned values stepwise from the primary group to a maximum of 10^8 Ω and to a minimum of 10^{-5} Ω or less. Manganin (a Cu-Ni-Mn alloy) is generally used in resistance standards as sheet material for resistors of low value (below 1 Ω) and wire for resistors of higher value. Evanohm (a Ni-Cr-Al-Cu alloy) is coming into use for stable wire-wound resistors, particularly in the higher resistance range. In this application, its higher resistivity is of advantage. Both alloys have low-temperature coefficients of resistance (a few ppm/°C at most) when appropriately heat-treated, and both have low thermoelectric power against copper (2 to 3 μV/°C).

The stability of a resistance standard depends on its construction, the extent to which initial strains have been removed by annealing, freedom from strain in use, and protection of the resistance element against air and moisture. The construction of the Thomas-type standards in the primary group probably represents the best approach yet made to the ideal: complete elimination of initial strain by a high-temperature anneal in an inert atmosphere, practically strain-free mounting in use, a reasonably large ratio of volume to surface area, and protection by sealing from atmospheric effects. The stability of these standards is better by a factor of 10 or even 100× than that of the usual resistance standard. No general statement is possible concerning the stability of standards, except that those of higher value are usually less stable and that hermetic sealing provides better protection from atmospheric effects than other forms of enclosure.

For resistors having values of 10^9 Ω or more, wire-wound construction is not practical, and a film of resistive material on an insulating base is regularly used. Such resistors are inherently less stable than wire-wound units, and in many instances their values depend on impressed voltage, humidity, and other factors. Thus, accurately known stable standards are not available in the very high resistance region. The usual techniques of measurement can be applied, but with increasing difficulty, only up to about 10^{12} Ω, and higher values cannot be determined stepwise from the primary standard. Stability of selected hermetically sealed resistors up to 10^{14} Ω has been studied over a period of years by a rate-of-charge method.[3] Under specified measurement conditions, the best of this group of resistors appear to be drifting at rates that range from 0.5 percent per year for 10^{14}-Ω units to 0.1 percent per year for 10^9-Ω units.

[1] See Sec. 5c-2.
[2] Thomas, *J. Research NBS* **5,** 295 (1930); **36,** 107 (1946).
[3] Scott, *J. Research NBS* **50,** 1947 (1953).

5c-5. Standards of Electromotive Force. The *primary* standard consists of a group of 40 saturated cadmium (Weston) cells[1] which are maintained at a temperature of 28°C, held constant within 0.01°C. Of these cells, 33 are of the acid type, sulfuric acid being present in the electrolyte at a concentration of 0.03 to 0.05 N. The remainder of the group are neutral in the sense that no acid has been added. The presence of acid prevents hydrolysis of the mercurous sulfate in the cell and decreases the solvent action of the electrolyte on the glass container. Thus it contributes to the constancy of emf of the cell. However, the emf of an acid cell is lower than that of a neutral cell by an amount proportional to the concentration of the acid (30 μV for 0.05 N acid).[2] Of the cells which make up the primary standard, 7 have been in the group since 1906. Of the remainder, 7 made in 1932 and 26 made in 1949 were added in 1955. New cells are made periodically, employing carefully purified materials, and are used to supplement the primary group.[3] The cells of the primary standard are intercompared periodically, and the group average is used as the standard of emf. An international comparison made in 1967 indicated that, at that time, the United States standard differed by 2.6 ppm from the standard maintained by the International Bureau. The report stated that $V_{USA} = V_{BIPM} - 2.58\ \mu V$. On the basis of recent absolute measurements, it was decided that V_{BIPM} was in error, and that the emfs assigned to the BIPM reference bank of standard cells should be increased by 11 ppm; simultaneously the emfs assigned to the NBS reference bank were increased by 8.4 ppm to bring them into agreement with the international standard. These changes became effective January 1, 1969.

Secondary standards of emf may be cadmium cells of either the saturated or unsaturated type. Most modern cells, both saturated and unsaturated, are the "acid" type, containing sulfuric acid at a normality of about 0.05 N. Saturated cells maintain a more nearly constant emf over long periods of time than unsaturated cells, and are being used to an increasing extent as reference standards in many laboratories. The temperature coefficient of emf of a standard cell is the difference between rather large positive and negative coefficients of the positive and negative cell limbs, respectively. For a saturated cell this difference amounts to about $-50\ \mu V/°C$ at 28°C while the coefficients of the separate limbs are more than 300 μV/°C each. Thus it is not only essential that the cell temperature be held within close limits ($<0.02°C$ for assignment to 1 μV), but in addition temperature differences between the cell limbs must be held within much closer limits. The coefficients of the limbs of an unsaturated cell match much more closely (usually within 5 μV), but the requirement of small temperature difference between the cell limbs is just as strict as for a saturated cell. Temperature control for saturated cells can be maintained by a regulated oil bath or air bath. If oil is used, it should be clear, of medium viscosity, acid-free, and without appreciable vapor pressure. Air baths are used in a number of laboratories to maintain standard cells at a constant temperature. In one such construction,[4] the cells are within a thick-walled aluminum box which is enclosed by and thermally insulated from an outer aluminum box. The latter is thermally insulated to protect it against ambient temperature changes and is maintained at a constant temperature (within a few hundredths of a degree) somewhat above ambient (usually at a temperature between 28 and 37°C). Temperature fluctuations within the inner compartment are attenuated, and the cells are in a nearly constant temperature environment with very low gradients.

[1] Hamer, *NBS Monograph* 84 (1965).

[2] The initial small decrease of emf usually observed in a neutral cell during the first few months after it is made is believed to result from the formation of acid in the electrolyte.

[3] These supplementary groups are kept under the same conditions as the primary group and are regularly compared with them. Thus, if a cell in the primary group should fail, another cell with a known history of constancy could be used to replace it.

[4] Mueller and Stimson, *J. Research NBS* **13**, 699 (1933).

Recently, saturated cells have been developed in which the solid materials are mechanically held in place. These cells can be shipped with little danger of injury, whereas the older type of saturated cell must be hand-carried in transport. When these "transportable" cells are mounted in an air bath, it is advantageous to arrange the temperature control circuitry so that it can be operated alternatively from power lines or from a 12-V battery. Thus the bath temperature can be held constant during transport as well as in the laboratory; and the cells need not be subjected to temperature shock from which they would recovery slowly.

The international formula (adopted in 1908) relating the emf of a saturated cadmium cell to its temperature is

$$E_t = E_{20} - 40 \times 10^{-6}(t - 20) - 0.95 \times 10^{-6}(t - 20)^2 + 0.01 \times 10^{-6}(t - 20)^3$$

where E_t is the emf at temperature t, and E_{20} is the emf at 20°C. This formula is stated to apply to either neutral or acid cells and to hold within about 1 μV for temperatures between 0 and 40°C. However, it was developed for cells in which a 12 percent amalgam was used. As a 10 percent amalgam is now used almost universally, the formula of Vigoureux and Watts[1] is to be preferred. This may be stated, for cells with 10 percent amalgam, as

$$E_t = E_{20} - 39.39 \times 10^{-6}(t - 20) - 0.903 \times 10^{-6}(t - 20)^2 + 0.0066 \times 10^{-6}(t - 20)^3 - 0.00015 \times 10^{-6}(t - 20)^4$$

These formulas should be used with caution since, in manufacture, the concentration of the amalgam may vary slightly from its intended value. Thus for correction to better than 1 μV, the correction formula should not be used over an extended temperature range; i.e., where the highest accuracy is required, the cell emf should be assigned in terms of a reliable reference standard at a temperature within, say 1°C of the use temperature. Since cells are frequently maintained at 28°C, it is convenient to express the Vigoureux and Watts formula in terms of that temperature:

$$E_t = E_{28} - 52.90 \times 10^{-6}(t - 28) - 0.803 \times 10^{-6}(t - 28)^2 + 0.0018 \times 10^{-6}(t - 28)^3 - 0.00015 \times 10^{-6}(t - 28)^4$$

Unsaturated cells (becoming saturated at 4°C) are used extensively as working standards of emf. Their temperature coefficient of emf is less by a factor of 10 (usually $< -5\mu$V/°C at room temperature) than that of a saturated cell; and they can be shipped by express or parcel post, since the electrode material is held in place by porous plugs. The emf of an unsaturated cell decreases slowly with time; for cells of recent manufacture this decrease usually amounts to 20 to 30 ppm/year. Because of this change of emf with time, it is advisable to check them periodically against a stable standard, and to discard them when their emf has dropped below 1.0183 V.

Certain precautions should be observed in using standard cells.

1. They should be protected from large or sudden temperature changes, which are accompanied by a large temporary change in emf; recovery time is slow and depends on the magnitude and duration of the temperature shock. In the case of a 10°C shock it has been observed that recovery within 1 ppm may require more than two months.

2. Cells should not be exposed to nearby sources of heat that may produce temperature inequality between the limbs; this could produce a large change in emf, since the large positive and negative temperature coefficients of the individual limbs tend to cancel each other only if their temperatures are equal.

3. Saturated cells must not be exposed to temperatures above 43°C or below −8°C; a metastable change in the cadmium sulfate crystals takes place at 43.4°C, and at −8°C the two-phase liquid-solid amalgam becomes a one-phase solid amalgam.

[1] Vigoureux and Watts, *Proc. Phys. Soc.* (*London*) **45**, 172 (1933).

4. Cells should not be exposed to strong light, as the mercurous sulfate is photosensitive; some cells have a band of black paint on the positive limb to protect the mercurous sulfate.

5. The internal resistance of an unsaturated cell is about 500 Ω in the high-resistance type and 100 Ω in the low-resistance type; the latter should be used with a deflection potentiometer. The resistance of a saturated cell may be as much as 750 Ω. Loss of sensitivity in potentiometer measurements, which is traced to the standard-cell circuit, may indicate the presence of a gas bubble in the negative limb of the cell. If the bubble cannot be removed by gentle tapping, the cell should be discarded. A convenient way of measuring cell resistance without injury to it is a potentiometer determination of the difference in its terminal voltage with and without a 1-MΩ resistor across its terminals; the voltage difference in microvolts is equal to the cell resistance in ohms. The resistor should be connected across the cell only long enough to make the measurement.

6. Current drawn from a cell which is used as an emf standard should be kept small and drawn for only a few seconds at most. Current should never exceed 100 μA. A cell which has been short-circuited may be assumed to be permanently damaged and should be discarded. Circuit arrangements which permit an alternating current through the cell should be avoided, as this results in a temporary change in emf. Also to be avoided are laboratory conditions that could produce moisture condensation on the cell walls, with resultant leakage current supplied by the cell.

7. Cells which have been exposed to temperature shock or roughly handled during shipment can suffer a substantial change in emf, with a recovery period that may be of several weeks' duration. Evidence is accumulating that, for transportable-type cells which have been held at constant temperature during shipment, the recovery period is quite short, and the cells may recover their normal emf within 1 to 2 days.

Zener diode networks are coming into use as reference voltage standards for potentiometer applications.[1] As of 1969 zener packages were commercially available which are adequate for 0.01 percent use, and which are stable within 0.001 percent over short periods.

5c-6. Capacitance Standards. Capacitors whose values can be computed accurately from measured dimensions are necessarily small—at most 100 pF (picofarads)[2] or so—and are limited to three-terminal types in which air, or another gas, or vacuum is the dielectric. A three-terminal design is required both to define precisely the geometry of the active electrode system and to permit the solid insulation that supports the electrode arrangement to be so located that it does not influence either the computed

[1] Eicke, *Trans. ISA* **3,** 93 (1964).

[2] A list of prefixes, to represent powers of 10, sponsored by the International Union of Physics and approved by the International Electrotechnical Commission (05-35-080 in *IEC Publ.* 80), has gained wide acceptance both abroad and in the United States. These prefixes and the corresponding powers of 10, adopted by the National Bureau of Standards, are given in the table below.

Prefix	Value	Prefix	Value
Tera	10^{12}	Deci	10^{-1}
Giga	10^{9}	Centi	10^{-2}
Mega	10^{6}	Milli	10^{-3}
Kilo	10^{3}	Micro	10^{-6}
Hecto	10^{2}	Nano	10^{-9}
Deka	10^{1}	Pico	10^{-12}

capacitance or the quadrature relation between current and voltage. Until recently such capacitors were usually built as a parallel-plate guard-ring arrangement, or as a system of coaxial guarded cylinders; and the accuracy of the assigned value was at best around 0.01 percent. A modification of the parallel-plate guard-ring design has been used to construct computable "guard-well" capacitors in the range below 1 pF (down to 0.001 pF). In this construction the working electrode is recessed behind the plane of the guard ring to take advantage of the concentration of field at an exposed sharp edge. The capacitance is a function of depth of the recess,[1] and can be as small as desired, while linear dimensions remain large enough for precise construction and measurement.

The situation as regards computable capacitors has changed completely since 1956, thanks to a new theorem in electrostatics.[2] The Thompson-Lampard theorem may be generalized as follows: "If four infinite cylindrical conductors of arbitrary sections are assembled with their generators parallel, to form a completely enclosed hollow cylinder in such a way that the internal cross-capacitances per unit length are equal, then in vacuum these cross-capacitances are $\ln 2/4\pi^2$ statfarads per cm." For small inequalities of cross-capacitances, it can be shown that the departure of their mean from the theoretical value is $0.087\,(\Delta C/C)^2$, where ΔC is their difference and C their mean. Thus, if the inequality is no more than 0.1 percent, the error in the computed value is less than 1 in 10^7. A practical realization of such a capacitor consists of four equal closely spaced round cylindrical rods with their axes parallel and at the corners of a square. Arranged as a three-terminal capacitor, the internal cross-capacitance amounts to 1 pF for approximately 50 cm length of the assembly and, if end effects are eliminated,[3] this capacitance can be computed as accurately as the length can be measured. In an early arrangement, made of an assembly of cylindrical gage rods, the uncertainty was stated as 2 ppm. In a later version of the Thompson-Lampard cross-capacitor, in which the active length is the distance between screen electrodes inserted in the central opening from both ends of the assembly,[4] the effective length is determined in a Fabry-Perot interferometer formed by optical flats on the opposed ends of the screen electrodes. Accuracy of length measurement and of capacitance assignment is an order of magnitude better than for the earlier gage-rod assembly. Bridges with transformer ratio arms[5] and high-resolution detectors[6] are now available with which capacitors can be intercompared with a precision approaching a part in 10^8 at the 1-pF level or can be stepped up by successive factors of 10 from this level without significant loss of precision. Thus the assignment of values to three-terminal capacitors in the range 1 to 10^6 pF is limited only by the stability of the unit. In addition to time dependence of stability, many capacitors have a value which is voltage-dependent. This voltage effect can vary within wide limits, depending on the construction of the unit. In some solid-dielectric capacitors, a change of several hundredths of a percent in value between low and rated voltage is observed; in others it can be substantially less than 1 ppm. In low-value air-dielectric capacitors whose dimensions are not altered by the electrostatic forces resulting from the applied voltage, the change in capacitance from low to rated voltage may be as small as a few parts in[7] 10^9. The best fixed-value solid-dielectric capacitors commercially available in 1969 showed drift rates that range upward from a few

[1] Snow, *NBS J. Research* **42**, 287 (1949).
[2] Lampard, *Proc. IEE* **104c**, 271 (1957); first announcement, *Nature*, **177**, 888 (1956).
[3] McGregor et al., *Trans. IRE* **7-I**, 253 (1958).
[4] Clothier, *Metrologia* **1**, 36 *LM17*, 232 (1965).
[5] Thompson, *Trans. IRE* **7-I**, 253 (1958); Cutkosky and Shields, *Trans. IRE* **9-I**, 243 (1960).
[6] Cutkosky, *IEEE Trans. IM*, 232 (1968).
[7] Shields, *J. Research NBS* **69C**, 265 (1965); Kusters and Petersens, *IEEE Trans.* **82**, (*Commun. & Electronics*) (1963).

ppm per year. A group of 10-pF reference-standard capacitors constructed at the National Bureau of Standards[1] have substantially less drift; over a three-year period their drift has been less than 1 in 10^7 per year. The voltage dependence of these capacitors is less than 1 in 10^8 up to 200 V.

Air capacitors (in both two- and three-terminal form up to 10^3 pF) are available as secondary standards. Adjustable standards in this range have a group of movable parallel plates that rotate with respect to a group of fixed interleaved parallel plates; the accuracy of adjustable air capacitors depends on the closeness with which the relative angular position of the fixed- and movable-plate systems can be set and reproduced, and on the quality of the bearings on which the electrode system rotates.

The phase-defect angles of air capacitors are always small, depending largely on the extent to which solid dielectric is present in the working field and to a much lesser extent on the presence of surface films[2] on the electrodes.

Solid-dielectric capacitors in which thin mica sheets are interleaved with metal foil are used as working standards up to 1 μF. The assembly is impregnated with wax to eliminate voids and air pockets, and is compressed through massive end plates to squeeze out excess wax. The quality and constancy of such a standard depend critically on the construction, being a function of the assembly pressure as well as the quality of the mica. In an alternative construction, mica sheets are coated with a thin layer of silver and stacked with interleaved metal foil. The assembly is not wax-impregnated but is sealed in a dry atmosphere. A general characteristic of mica capacitors is their increase in capacitance with increasing voltage. In the power and a-f range, this increase may be less than 10 ppm between low and rated voltage for the best wax-impregnated mica capacitors and has been observed to be in excess of 400 ppm in some silvered-mica capacitors. As the effect varies markedly from one unit to another, individual capacitors should be checked before they are used as working standards. Absorption and losses are always present in mica capacitors. The phase-defect angle of the best mica capacitors may amount to 1 to 2 minutes throughout the a-f range, and their capacitance value may be expected to remain constant within 0.01 and 0.02 percent over a period of many years. While mica is used in solid-dielectric capacitors of the best grade, polystyrene is used extensively in secondary standards. The phase-defect angle of such capacitors may be no larger than of mica capacitors of comparable size, but their temperature coefficient of capacitance is more than 4$\times$ as great.

5c-7. Inductance Standards. Self- and mutual inductors, whose values can be computed from measured dimensions, have been built at the National Bureau of Standards and at other national laboratories for use in absolute-ohm measurements. Computable self-inductors are single-layer solenoids wound on marble, porcelain, low-expansion glass, or fused silica forms. In some instances an accurate screw thread has been cut into the cylindrical form to control the spacing of the winding.[3] Computable mutual inductors have been built following a design of Campbell[4] or Wenner's modification of it.[5] In each of these designs the primary consists of single-layer helical windings on a marble or porcelain cylinder, the sections being spaced in such a way that a relatively large annular space is available around the central portion of the cylinder, within which the field is very small. The multilayer secondary winding is located in this space, and since the field is small, the exact location of the secondary becomes relatively less critical. Such mutual inductors can be computed as accurately as can the self-inductance standards. However, both types of inductor, apart from being

[1] Cutkosky and Lee, *J. Research NBS* **69C,** 173 (1965).
[2] Koops, *Philips Tech. Rev.* **5,** 300 (1940).
[3] Curtis, Moon, and Sparks, *J. Research NBS* **21,** 371 (1938).
[4] Campbell, *Proc. Roy Soc. (London)*, ser. A, **79,** 428 (1907).
[5] Thomas, et al., *J. Research NBS* **43,** 325 (1949).

very difficult and expensive to build and compute, have relatively low time constants and are not generally useful for work outside the special field (absolute measurements) for which they are designed.

Self-inductance standards for laboratory work are usually multilayer coils of such shape that their inductance is maximum for a given size and length of wire.[1] Accurate computation of value from their measured dimensions is not possible, and their values are usually established from electrical measurements in terms of other inductors or a combination of resistance and capacitance. Laboratory mutual inductors also are usually designed to achieve a maximum time constant.

Higher inductance in a given volume or with a given amount of copper can be obtained if the winding is on a core of high-permeability material. Special ferromagnetic alloys are used for this purpose in sheet or strip form, or as a bonded granular or powder material. The gain in time constant is achieved at the expense of some nonlinearity in the inductor, since the core permeability is a function of current in the winding. Also, increased losses are to be expected from eddy currents and from hysteresis in the iron. By proper construction and the use of suitable core materials, these defects can be kept small, so that "iron-cored" inductance standards of moderate accuracy and stability are practicable.

Inductors wound as multilayer cylindrical coils of rectangular cross section, to achieve maximum time constant, set up an external field and, conversely are subject to pickup from stray fields in which they are placed. These effects are considerably reduced by dividing the coil into two equal sections wound in opposite directions so that the emfs induced in them by a changing external field tend to cancel. Such an arrangement is called *astatic*. A much greater degree of astaticism is attained when the coil is toroidal, with the winding uniformly distributed around the torus.

Adjustable standards of self- and mutual inductance are of two general kinds: the cross-coil type, in which the plane of a movable coil is turned to make various angles with the plane of the fixed coil; and the parallel-coil type, in which the plane of the movable coil is always parallel to the fixed coil. A familiar example of the cross-coil type is the Ayrton and Perry inductometer, in which the fixed and movable coils are zones of concentric spheres, with the movable coil pivoted on the common polar axis. Such an arrangement has serious faults. If the coils are nearly equal in radius to maximize their coupling, the rate of change of mutual inductance with angle is far from uniform; and the arrangement is not astatic. Probably the best example of the parallel-coil type is the Brooks inductometer with three pairs of link-shaped coils, designed to provide a uniform scale over most of its range. The coil dimensions are such that, at the maximum reading, the conditions for maximum time constant are approximately met. Also the system is arranged to be nearly astatic. The rotor, holding the movable pair of coils, turns on a shaft between pairs of fixed parallel coils. The coils are all connected in series when the instrument is to be used as a self-inductor; for use as a mutual inductor the circuits of the fixed and movable coils are separated.

All inductors are frequency-dependent as a result of distributed capacitance, eddy currents, and imperfect insulation between turns and layers of the winding. The effect of distributed capacitance is to increase both the effective resistance and inductance above their low-frequency values. At frequencies well below resonance the following formulas hold approximately:

$$R_{\text{eff}} = R_0(1 + 2\omega^2 L_0 C) \qquad \text{and} \qquad L_{\text{eff}} = L_0(1 + \omega^2 L_0 C)$$

where R_0 and L_0 are the values at zero frequency, and C is the equivalent capacitance considered to be connected across the terminals of the inductor. The effect of eddy

[1] Brooks, *J. Research NBS* **7**, 293 (1931).

currents is to increase the effective resistance and to decrease the effective inductance in accordance with the following formulas:

$$R_{\text{eff}} = R_0 + \frac{M^2\rho\omega^2}{\rho^2 + l^2\omega^2} \qquad \text{and} \qquad L_{\text{eff}} = L_0 - \frac{M^2 l\omega^2}{\rho^2 + l^2\omega^2}$$

where ρ and l are, respectively, the equivalent resistance and self-inductance of the eddy-current circuit, and M is its coupling with the inductor. The effect of imperfect insulation (equivalent to a shunt resistance across the terminals of the inductor) is to decrease the effective inductance. However, it may increase or decrease the effective resistance,[1] depending on conditions. If the leakage resistance ρ is very high compared with the coil resistance, the following formulas hold:

$$L_{\text{eff}} = L_0\left(1 - \frac{\omega^2 L_0^2}{\rho^2}\right) \qquad \text{and} \qquad R_{\text{eff}} = R_0\left(1 + \frac{\omega^2 L_0^2}{R_0\rho}\right)$$

It must be borne in mind that ρ is the a-c resistance of the insulation and hence may be itself a function of frequency.

5c-8. Frequency Standards. The reciprocal relationship between frequency and time interval makes it desirable to examine first the definition of the *second*. Previous to 1956 the second was defined in terms of the earth's rotation—the second was 1/86,400 part of a mean solar day. As frequency standards improved, this definition became unacceptable because of small variations in the earth's rotational motion. Because the earth's orbital frequency around the sun is less subject to perturbations than its rotational frequency, the second was redefined in 1956 by international agreement, as the fraction 1/31,566,925.9747 of the tropical year 1900, beginning at 12 hr ephemeral time. Obviously, the precise determination of a frequency in terms of such a "second" requires a lengthy and involved set of observations, and, as atomic frequency standards improved greatly during the decade that followed, it became possible to measure frequencies in terms of these standards quickly and far more precisely. Thus, by international agreement in 1967 the *second* was redefined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

The National Bureau of Standards maintains standard frequency services from four transmitting stations: WWVH in Hawaii; WWV, WWVB, and WWVL in Colorado.[2] WWVH broadcasts on frequencies of 2.5, 5, 10, and 15 MHz; WWV on 2.5, 5, 10, 15, 20 and 25 MHz; WWVB on 60 kHz; and WWVL on 20 kHz. Frequencies transmitted by WWV (derived from cesium-controlled oscillators) are held stable to better than 2 parts in 10^{11} at all times, with deviations much less than 1 part in 10^{11} from day to day; frequencies at WWVH are derived from quartz oscillators, and frequency adjustments do not exceed 5 parts in 10^{10}. Both these stations broadcast standard audio frequencies of 440 and 600 Hz on each carrier frequency. WWVB and WWVL frequencies (cesium-controlled) are normally stable to better than 2 parts in 10^{11}, with day-to-day variations less than 1 part in 10^{11}. WWVB frequency has no offset; the others are all intentionally offset (-300 parts in 10^{10} in 1968) from standard frequency to reduce the departure of their broadcast time signals from UT2 astronomical time. Changes in the propagation medium (causing Doppler effect, diurnal shift, etc.) result in fluctuations of the order of a part in 10^7 in the HF carrier frequencies as received; the effects of the propagating medium on the received frequencies are much less at LF and VLF, and the full transmitted accuracy may be obtained with appropriate receiving techniques.

[1] Campbell and Childs "Measurement of Inductance, Capacitance, and Frequency" p. 191, D. Van Nostrand Company, Inc., Princeton, N.J., 1953.

[2] *NBS Spec. Publ.* 236.

Quartz crystals are used in vast numbers to control the frequencies of oscillators through much of the spectrum, in both measurement and communication applications. Their constancy depends on the closeness with which their temperature and pressure are controlled.

Tuning forks may be used as laboratory standard at power and audio frequencies. A precision fork, operating at constant temperature, may have a frequency that is stable to 10 ppm and, when corrected for barometric pressure, to 1 ppm. A battery-driven fork without temperature control may have a temperature coefficient less than −0.015 percent/°C and a voltage coefficient less than 0.01 percent/V. It should provide a frequency known to better than 0.1 percent under any specified laboratory condition.

The frequency of 60-Hz power in most localities affords a convenient reference point. However, even where power is supplied from a network that includes generating stations over an area of many hundreds of square miles, the frequency is not continuously held at precisely 60 Hz. It may depart by as much as 0.1 or 0.2 Hz, occasionally even more. Also, the frequency can be corrected only very slowly because of the large inertia of the system, perhaps as much as half an hour being required. The average frequency will be very close to 60 Hz over an extended time period, and synchronous clocks will usually keep time within a few seconds. However, a commercial power source cannot be employed reliably as a frequency standard to much better than 1 percent.

5c-9. Deflecting Instruments. Instruments used for the measurement of current, voltage, or power are made in a number of accuracy classes. The best grades, called "laboratory standards," may be in the $\frac{1}{10}$ (or $\frac{1}{20}$) percent class, meaning that, over the useful part of the scale, no marked point is in error by more than $\frac{1}{10}$ (or $\frac{1}{20}$) percent of the full-scale value. These are large instruments which must be carefully leveled to ensure good performance. Smaller portable instruments are made in 0.2, 0.5, and $\frac{3}{4}$ percent accuracy classes. The class of the instrument is generally stated in the maker's catalog. Switchboard instruments are generally in the 1 percent class, and panel instruments in the 1, 2, or even 5 percent class. Direct-current ammeters and voltmeters are almost universally permanent-magnet moving-coil instruments, whereas the construction of a-c instruments depends on the intended application. Moving-iron or electrodynamic instruments are used at power frequencies and, if suitably compensated, in the lower a-f range. Thermocouple ammeters are useful from low frequencies up to many megahertz, whereas thermocouple voltmeters are generally applicable only at power and audio frequencies unless they have special multipliers designed for high-frequency operation. Electrostatic voltmeters have no frequency limitation other than that imposed by low impedance at very high frequencies, and many electronic voltmeters are designed to operate from power frequencies up to many megahertz without serious error. Depending on operating principle and construction, deflecting instruments are subject to errors of various types: temperature, magnetic field, frequency, waveform, spring hysteresis, use in other than the intended position, and others.[1]

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[1] Standard C-39 of the USA Standards Institute contains performance specifications for deflecting instruments of various types and accuracy classes. Sections of C-39 also deal with electronic and digital instruments. A text on electrical instruments or measurements should be consulted for complete details on instrument performance.

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