6d. Geometrical Optics and the Index of Refraction of Various Optical Glasses

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6d-1. Geometrical Optics. Geometrical optics ignores the wave nature of light. The concept of a light ray is introduced in order to make the optical calculations easier. If a light ray traverses the boundary between two homogeneous media, the deviation from the normal is determined by *Snell's Law*, which states: The refracted ray, the normal to the surface at the point of incidence, and the incoming ray are in one plane; they obey the relationship

$$n \sin \phi = n' \sin \phi' \tag{6d-1}$$

in which n is the refractive index of the first medium, n' the refractive index of the second medium, ϕ and ϕ' the angles between the normal and the ray before and after refraction. This is the fundamental relationship in geometrical optical calculations.

Since spherical surfaces are most common in optics, only these will be described. An optical system always possesses an axis of symmetry, so that all centers of curvature lie on this axis.

To investigate the behavior of a light ray traversing the boundary between two optical media with refractive indices n and n', we select a coordinate system with the Z axis along the axis of symmetry. The positive direction is the direction in which the light travels. The origin is selected in such a way that the XY plane goes through the point where the ray intersects the surface. The ray can then be defined by its direction cosines L, M and its intersection point x, y. We can derive the following relation, with the help of Snell's Law, written in matrix form:

$$\begin{bmatrix} n'L' \\ x' \end{bmatrix} = \begin{bmatrix} 1 & -(n' \cos \phi' - n \cos \phi)/r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} nL \\ x \end{bmatrix}$$

and

$$\begin{bmatrix} n'M' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & -(n' \cos \phi' - n \cos \phi)/r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} nM \\ y \end{bmatrix} \tag{6d-2}$$

in which r is the radius of curvature of the refracting surface (r is positive if the direction from the vertex to the center of curvature measured along the z axis is the direction in which the light travels) and in which primes refer to conditions after refraction.

If it is desired to follow the light ray through more refracting surfaces, we translate the coordinate system to the incidence point on the next surface without rotation. The relation governing this translation is given by

$$\begin{bmatrix} n'L'' \\ x'' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ d'/n' & 1 \end{bmatrix} \begin{bmatrix} n'L' \\ x' \end{bmatrix}$$

and

$$\begin{bmatrix} n'M'' \\ y'' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ d'/n' & 1 \end{bmatrix} \begin{bmatrix} n'M' \\ y' \end{bmatrix} \tag{6d-3}$$

in which d' is the distance between the two incidence points measured along the ray.

By multiplying the proper matrices for a complete system, we can find the relationship between the incoming ray and the outgoing ray. This relationship can always be written in the form

$$\begin{bmatrix} n'L' \\ x' \end{bmatrix} = \begin{bmatrix} B & -A \\ -D & C \end{bmatrix} \begin{bmatrix} nL \\ x \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} n'M' \\ y' \end{bmatrix} = \begin{bmatrix} B & -A \\ -D & C \end{bmatrix} \begin{bmatrix} nM \\ y \end{bmatrix} \tag{6d-4}$$

A is called the *power of the system*. The value of all matrices, calculated as determinants, is $+1$. This gives us a good check on the final calculations in the form of

$$BC - AD = +1 \tag{6d-5}$$

In general, the values of A, B, C, and D are different for different rays.

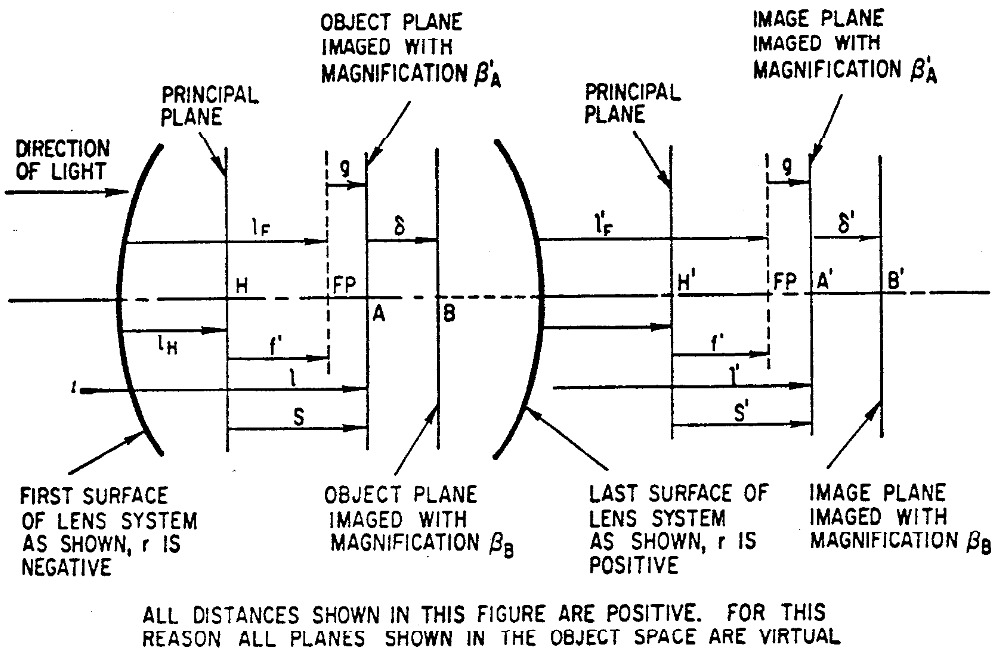

FIG. 6d-1. Significant planes in a lens system.

For image forming, as opposed to analyzing, optical systems require that all rays coming from a certain object point should go through its corresponding image point. To investigate this we select an object point at a distance l, measured along the ray, from the point of incidence with the first surface of the system and an image point on the same ray at a distance l' from the point where this ray leaves the last surface of the system. This is shown in Fig. 6d-1. With the help of matrices of the form (6d-3) we find for the transformation matrix

$$\begin{bmatrix} B + (l/n)A & -A \\ (l'/n')(l/n)A + (l'/n')B - (l/n)C - D & C - (l'/n')A \end{bmatrix} \tag{6d-6}$$

An examination of the matrix elements shows that, in general, it will be impossible to find an image point which satisfies these conditions. If, however, we define the image point of the ray as the point where this ray intersects, in the image space, the plane through the axis of the system and the object point, we shall have the condition

$$\frac{x'}{x} = \frac{y'}{y} = \beta' \tag{6d-7}$$

(β' is called the linear magnification), and we get the following relations:

$$\begin{bmatrix} n'L' \\ x' \end{bmatrix} = \begin{bmatrix} 1/\beta' & -A \\ 0 & \beta' \end{bmatrix} \begin{bmatrix} nL \\ x \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} n'M' \\ y' \end{bmatrix} = \begin{bmatrix} 1/\beta' & -A \\ 0 & \beta' \end{bmatrix} \begin{bmatrix} nM \\ y \end{bmatrix} \tag{6d-8}$$

and

$$\beta' = C - \frac{l'}{n'} A = \frac{1}{B + (l/n)A} \tag{6d-9}$$

$$\frac{l'}{n'} \frac{l}{n} A + \frac{l'}{n'} B - \left(\frac{l}{n}\right) C - D = 0 \tag{6d-10}$$

The task of the designer is to find a system for which Eqs. (6d-8) hold true for every object point lying in a given object plane. This, however, is impossible, and in practice a good lens will be a solution for which the deviations are within the tolerances allowed by the purpose for which the lens will be used. These deviations are called *aberrations*.

In order to get an insight into the properties of an optical system, the *paraxial laws* are often used. To arrive at these laws we develop all the quantities involved in power series and use only the first term of every series. The quantities L and L' can now be replaced by α and α', being the angles between the axis and the ray. The origin of the coordinate system can now be chosen at the intersection point of the surface and the axis. The distances can all be measured along the axis instead of along the ray. Equations (6d-2) and (6d-3) now become

$$\begin{bmatrix} n'\alpha' \\ x' \end{bmatrix} = \begin{bmatrix} 1 & -(n' - n)/r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n\alpha \\ x \end{bmatrix} \tag{6d-11}$$

and

$$\begin{bmatrix} n'\alpha'' \\ x'' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ t'/n' & 1 \end{bmatrix} \begin{bmatrix} n'\alpha' \\ x' \end{bmatrix} \tag{6d-12}$$

Similar to Eqs. (6d-8), (6d-9), and (6d-10) we find for the paraxial relationship of object and image points that

$$\begin{bmatrix} n'\alpha' \\ x' \end{bmatrix} = \begin{bmatrix} 1/\beta'_p & -A_p \\ 0 & \beta'_p \end{bmatrix} \begin{bmatrix} n\alpha \\ x \end{bmatrix} \tag{6d-13}$$

$$\beta'_p = C_p - \frac{l'}{n'} A_p = \frac{1}{B_p + (l/n)A_p} \tag{6d-14}$$

$$\frac{l'}{n'} \frac{l}{n} A_p + \frac{l'}{n'} B_p - \frac{l}{n} C_p - D_p = 0 \tag{6d-15}$$

in which the subscript p denotes that we deal with paraxial quantities.

It is customary, however, to measure the distances l and l' not from the outer surfaces of the lens but from its *principal points*. The principal points are defined as the object and image points on the axis with a magnification of +1. These points follow from Eq. (6d-14):

$$\frac{l_H}{n} = \frac{1 - B_p}{A_p} \quad \text{and} \quad \frac{l'_H}{n'} = \frac{C_p - 1}{A_p} \tag{6d-16}$$

If we measure the object distance s and image distance s' from the principal points, it is easily shown that the following relationship holds:

$$\frac{n'}{s'} = \frac{n}{s} + A_p \tag{6d-17}$$

If we move the object point to infinity, we call its image the *focal point*. For the distance between this focal point and its corresponding principal point, which is called the *focal length* f', we find, with the help of Eq. (6d-17), that

$$s'_F = f' = \frac{n'}{A_p} \tag{6d-18}$$

By making $s' = \infty$, we find that

$$s_F = f = \frac{-n}{A_p} \tag{6d-19}$$

Table 6d-1 gives a selection of useful paraxial formulas. From this table we see that all the image distances can be calculated if we know the four quantities A_p,

TABLE 6d-1. PARAXIAL FORMULAS

$$\frac{l_H}{n} = \frac{1 - B_p}{A_p} \qquad \frac{l'_H}{n'} = \frac{C_p - 1}{A_p}$$

$$\frac{l_F}{n} = -\frac{B_p}{A_p} \qquad \frac{l'_F}{n'} = \frac{C_p}{A_p}$$

$$\frac{f}{n} = -\frac{1}{A_p} \qquad \frac{f'}{n'} = \frac{1}{A_p}$$

$$\frac{n'}{s'} = \frac{n}{s} + A_p$$

$$\beta' = \frac{ns'}{n's} = 1 - A_p\frac{s'}{n'} = \frac{1}{1 + A_p(s/n)} = -A_p\frac{g'}{n'} = \frac{1}{A_p(g/n)}$$

$$\beta'^2 = -\frac{g'n}{gn'} \qquad \frac{gg'}{nn'} = -\frac{1}{A_p^2}$$

$$\frac{\delta}{n} = -\frac{\beta'_B - \beta'_A}{\beta'_A\beta'_B A_p} \qquad \frac{\delta'}{n'} = \frac{\beta'_B - \beta'_A}{A_p}$$

$$\frac{\delta'}{n'} = \beta'_A\beta'_B\frac{\delta}{n}$$

B_p, C_p, and D_p of our lens system. These are readily found with the help of Eqs. (6d-11) and (6d-12). For example, we find for a lens in air with radii r_1 and r_2, refractive index n, and thickness t, the following:

$$\begin{bmatrix} B_p & -A_p \\ -D_p & C_p \end{bmatrix} = \begin{bmatrix} 1 & -(1-n)/r_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t/n & 1 \end{bmatrix} \begin{bmatrix} 1 & -(n-1)/r_1 \\ 0 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} 1 - (t/n)(1-n)/r_2 & -\{(n-1)/r_1 + (1-n)/r_2 - (t/n)[(n-1)(1-n)/r_1r_2]\} \\ t/n & 1 - (t/n)(n-1)/r_1 \end{bmatrix} \tag{6d-20}$$

If the thickness of a single lens is small compared with its focal length, the thickness is often assumed to be zero in order to simplify the calculation. Such a lens is referred to as *thin*. For a thin lens we find from Eq. (6d-20) that

$$A_p = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad B_p = C_p = 1 \qquad D_p = 0 \tag{6d-21}$$

The principal points of such a thin lens are now located in the lens, according to Eqs. (6d-16).

All these formulas can be used for *mirrors* if the following sign rules are adopted: For the first mirror, n' is to be taken negative; i.e., for a mirror in air $n' = -1$. All refractive indices after the first reflection but before the second reflection are negative. After the second reflection the refractive indices are again positive until the third reflection, where they become negative, and so on. All distances remain positive when they are in the direction the light was traveling before it entered the system.

The deviations of actual optical systems from ideal optical systems are called *aberrations*. There are two kinds of aberrations:

1. Deviations of the rays from the paths given by the ideal laws as given by the paraxial equations, called *monochromatic aberrations*.

2. Changes in image position and size due to changes of refractive index with wavelength, called *chromatic aberrations*. These already exist in the paraxial approximation.

To find the aberrations for a given system we have to follow several rays step by step through the system. This is called *ray tracing*.

To get an insight into aberrations, the next term in the power series which gave us the paraxial law is often used. The deviations of the rays found in this way are called *third-order aberrations*.

In an aberration-free system β' and A, in Eqs. (6d-8), are constants. For every ray in such a system

$$n'L' = \left(\frac{1}{\beta'}\right) nL - Ax \tag{6d-22}$$

For a point on the axis of the system we can replace L and L' by $\sin U$ and $\sin U'$, where U and U' represent the angle between the optical axis and the ray before and after passing through the system. Now Eq. (6d-22) becomes

$$n' \sin U' = \frac{n \sin U}{\beta'} \tag{6d-23}$$

This is called *Abbe's sine condition*. Sometimes β' is replaced by x'/x, where x is a small distance perpendicular to the axis in the object plane and imaged as x' perpendicular to the axis in the image plane. The sine condition now reads

$$n'x' \sin U' = nx \sin U \tag{6d-24}$$

For a system in air without chromatic aberrations it is seen from Eq. (6d-13) that β' and A_p should be constants for light of different wavelengths, while for a thin lens this reduces to $A_p = \text{const}$. In general, however, using Eqs. (6d-21) we find after differentiation with respect to n that

$$\Delta A_p = \Delta \frac{1}{f} = \frac{\Delta n}{n-1}\frac{1}{f} = \frac{\Delta n}{n-1} A_p \tag{6d-25}$$

For calculations of chromatic aberrations the ν-value is often used. The ν-value is defined as

$$\nu = \frac{n_d - 1}{n_F - n_C} \tag{6d-26}$$

in which n_d is the refractive index of the glass at the d-line in a helium spectrum ($\lambda = 0.58756\ \mu$m); n_F is the refractive index of the glass at the F-line in a mercury spectrum ($\lambda = 0.48613\ \mu m$); n_C is the refractive index of the glass at the C-line in a ...ury spectrum ($\lambda = 0.65628\ \mu m$). Using the ν-value we find for Eq. (6d-26) that

$$\frac{\Delta A_p}{A_p} = -\frac{\Delta f'}{f'} = \frac{1}{\nu} \tag{6d-27}$$

The *illuminance of image* can be calculated for an optical system. Let dS be an element of area of the object. The flux dF into the element of solid angle $d\omega$, defined by a double cone of aperture α to $\alpha + d\alpha$, is

$$dF = B\ dS \cos\alpha\ d\omega$$

in which B is the *luminance* of the object. Now, from the way $d\omega$ is defined, it follows that

$$d\omega = 2\pi \sin\alpha\ d\alpha$$

so

$$dF = 2\pi B\ dS\ d\alpha \cos\alpha \sin\alpha$$

If the cone admitted by the optical system has an angle U, we can calculate the total light flux going into the system. We find that

$$F_U = \int_0^U 2\pi B\ dS\ d\alpha \cos\alpha \sin\alpha = \pi B\ dS \sin^2 U \tag{6d-28}$$

A similar relation holds for an area dS' in the image space, where the total flux is given by

$$F_{U'} = \pi B'\ dS' \sin^2 U' \tag{6d-29}$$

In the case where dS' is the image of dS and the cone given by U' is the same cone as given by U after passing through the optical system with a transmittance τ, $F_{U'} = \tau F_U$ and the relation between B and B' becomes

$$B'\ dS' \sin^2 U' = \tau B\ dS \sin^2 U$$

If the optical system is aberration-free, we can use the sine condition and we find that

$$B' = \tau \left(\frac{n'}{n}\right)^2 B \tag{6d-30}$$

In applications where the total energy in the image is important, we can use these results. In other applications, however, the total energy is not so important as the *light flux density* or *illuminance* E. Here we find on the image side that

$$E' = \frac{F'}{dS'} = \pi B' \sin^2 U'$$

If the system is again aberration-free, we can use the other form of the sine condition and Eq. (6d-30) to arrive at

$$E' = \tau\pi B \frac{\sin^2 U}{G^2} \tag{6d-31}$$

It is also important to find the maximum value of $\sin U$ which is passed by an optical system. For a single thin lens, this is determined by the diameter of the lens.

In a multiple system the situation is more complicated. In order to find what cone is passed, for instance, by the nth surface, we calculate where a system consisting of the first $(n - 1)$ surfaces images the nth surface in the object space and with what magnification. If this surface has a radius ρ_n, then it will be imaged with a magnification $G_n\rho_n$. Now every ray going through a point in this image in the object space will go through the nth surface if this ray is passed by all the other surfaces. So if we image all surfaces and diaphragms in this way in the object space, we can determine which rays are passed by the whole system. The smallest of these images as seen from the axial point of the object is called the *entrance pupil*. The diameter of the entrance pupil determines U for the whole system. The image of the entrance pupil in the image space of the whole system is called the *exit pupil*. A ray through the center of the entrance pupil is called a *chief ray* or *principal ray*. It follows from the definition of an image that a chief ray after passing through the system goes through the center of the exit pupil.

A special case arises when the object is at infinity. The entrance pupil is determined in the same way. In this case we can write $\sin U = \rho/p$, in which ρ is the radius of the entrance pupil and p the object distance. For G we write p'/p and arrive at the illuminance in this case by

$$E' = \tau\pi B \left(\frac{\rho/p}{p'/p}\right)^2 = \tau\pi B \left(\frac{\rho}{p'}\right)^2 = \tau\pi B \left(\frac{\rho}{f'}\right)^2$$

If we now introduce the *f-number* N, defined by $N = f/2\rho$, we find that

$$E' = \frac{1}{4}\tau\pi B \left(\frac{1}{N}\right)^2 \tag{6d-32}$$

The *field of view* is another important property of an optical system. There are two things which can limit the field of view. Aberrations have in general a tendency to increase if the bundles travel more obliquely through the system. If the aberrations become so large that the image is no longer useful, this is a limiting factor. On the other hand, the bundles may become so oblique that they are no longer passed by the system. In order to find this limit, we consider a cone of rays all going through the center of the entrance pupil. This cone will be limited to an angle V by one of the other images in the object space of the surfaces and diaphragms. We call V the *field angle* and the limiting boundary the *field stop*. The chief rays with an angle V will come from certain points of the object and so define the size of the object which can be imaged usefully by the optical system. In general, the cones from the edge of the field will be smaller than the one in the center of the field. This effect is called *vignetting*.

6d-2. Index of Refraction of Various Glasses. Table 6d-2 includes the index of refraction for the wavelengths for the d, A', C, F, and h lines for each glass. For Schott glasses, the values of reciprocal dispersion ν are based on n_d; for B & L and EK glasses, they are based on n_D. To interpolate the index for any other wavelength, obtain the universal functions[1] a_i for the desired wavelength from Table 6d-3 and substitute in

$$n(\lambda) = a_1 n_{A'} + a_2 n_C + a_3 n_F + a_4 n_h \tag{6d-33}$$

The matrix C_{ij} given in Table 6d-4 can be used to compute the universal functions for other wavelengths in the range 0.4047 to 0.7682 μm.

$$a_j(\lambda) = C_{1j} + C_{2j}\lambda^2 + C_{3j}L + C_{4j}L^2 \tag{6d-34}$$

where λ is the wavelength in micrometers, and $L = 1/(\lambda^2 - 0.028)$.

[1] M. Herzberger, *Opt. Acta* **6**, 197 (1959).

TABLE 6d-2. A REPRESENTATIVE SELECTION OF AVAILABLE OPTICAL GLASSES

Manufacturer*	Type	ν	n_d, 0.5876 μm	$n_{A'}$, 0.7682 μm	n_C, 0.6563 μm	n_F, 0.4861 μm	n_h, 0.4047 μm
Schott	FKS01	81.61	1.48523	1.48135	1.48342	1.48936	1.49520
Schott	FK05	70.34	1.48749	1.48282	1.48534	1.49227	1.49893
Schott	PKS01	69.69	1.52054	1.51556	1.51824	1.52571	1.53295
Schott	FK6	67.28	1.44628	1.44188	1.44424	1.45088	1.45737
Schott	PSKS01	67.25	1.55753	1.55205	1.55498	1.56327	1.57137
B & L	BSC	67.0	1.49808	1.49316	1.49577	1.50320	1.51048
Schott	PSKS6	65.41	1.60310	1.59704	1.60028	1.60950	1.61857
B & L	BSC	63.5	1.51107	1.50578	1.50860	1.51665	1.52454
Schott	PSK1	62.88	1.54771	1.54198	1.54505	1.55376	1.56230
Schott	K11	61.59	1.50013	1.49479	1.49765	1.50577	1.51379
B & L	DBC	61.2	1.58811	1.58184	1.58513	1.59474	1.60424
B & L	C	60.5	1.51258	1.50708	1.50999	1.51846	1.52685
B & L	DBC	60.3	1.62011	1.61342	1.61696	1.62724	1.63748
Schott	BaK2	59.66	1.53996	1.53407	1.53720	1.54625	1.55528
B & L	C	58.6	1.52307	1.51729	1.52036	1.52929	1.53819
B & L	LBC	57.4	1.57497	1.56619	1.56956	1.57953	1.58951
B & L	DBC	57.2	1.61109	1.60423	1.60785	1.61853	1.62923
B & L	EDBC	57.2	1.65709	1.64972	1.65362	1.66510	1.67649
Schott	K10	56.46	1.50137	1.49560	1.49867	1.50755	1.51647
EK	110	56.15	....	1.68877	1.69313	1.70554	1.71786
B & L	DBC	55.5	1.63810	1.63074	1.63461	1.64611	1.65772
B & L	LaC	54.8	1.69111	1.68305	1.68730	1.69910	1.71248
Schott	LaK18	54.67	1.72875	1.72004	1.72469	1.73802	1.75126
B & L	CF	54.6	1.52568	1.51954	1.52277	1.53239	1.54215
B & L	EDBC	53.9	1.61711	1.60980	1.61368	1.62512	1.63670
Schott	BaLF5	53.61	1.54739	1.54086	1.54432	1.55453	1.56494
B & L	LBF	53.4	1.58809	1.58110	1.58479	1.59580	1.60697
Schott	LaK19	53.24	1.75496	1.74574	1.75065	1.76483	1.77902
Schott	KF6	52.16	1.51742	1.51105	1.51443	1.52435	1.53446
EK	210	51.18	....	1.72483	1.72978	1.74417	1.75877
B & L	CF	51.0	1.52408	1.51759	1.52100	1.53127	1.54185
B & L	LBF	51.0	1.56210	1.55518	1.55879	1.56982	1.58115
B & L	EDBC	50.9	1.65714	1.64894	1.65323	1.66614	1.67927
Schott	LaK17	50.48	1.78847	1.77841	1.78375	1.79937	1.81514
Schott	SSK7	50.36	1.61847	1.61069	1.61479	1.62707	1.63972
Schott	BaF5	49.25	1.60729	1.59949	1.60359	1.61592	1.62871
B & L	LaF	48.0	1.70012	1.69098	1.69576	1.71033	1.72536
B & L	LaF	47.5	1.72013	1.71063	1.71561	1.73077	1.74645
Schott	LaF21	47.37	1.78831	1.77767	1.78330	1.79994	1.81694
B & L	ELF	47.3	1.54110	1.53386	1.53768	1.54912	1.56102
B & L	BF	47.2	1.67008	1.66123	1.66585	1.68004	1.69472
Schott	BaF8	47.04	1.62374	1.61541	1.61980	1.63306	1.64692
Schott	LaSF1	46.76	1.80279	1.79184	1.79763	1.81480	1.83239
EK	310	46.42	....	1.73491	1.74033	1.75638	1.77301
B & L	BF	46.0	1.58391	1.57598	1.58013	1.59282	1.60615
B & L	ELF	45.5	1.55860	1.55086	1.55495	1.56722	1.58010
EK	430	44.69	....	1.76582	1.77164	1.78902	1.80706
B & L	BF	43.6	1.60542	1.59682	1.60130	1.61518	1.62987
B & L	LF	42.5	1.57262	1.56425	1.56861	1.58208	1.59637
Schott	LFS1	42.19	1.54765	1.53959	1.54382	1.55680	1.57091
B & L	LaF	42.0	1.72016	1.70957	1.71508	1.73220	1.74965
Schott	BaSF6	41.88	1.66755	1.65766	1.66284	1.67878	1.69579
EK	450	41.80	....	1.79180	1.79814	1.81738	1.83767
Schott	LaSF5	41.00	1.88069	1.86722	1.87430	1.89578	1.91825
Schott	BaSFS6	40.99	1.70181	1.69108	1.69672	1.71384	1.73200
Schott	BaSF10	39.31	1.65016	1.63999	1.64529	1.66183	1.67968
B & L	DF	38.0	1.60514	1.59538	1.60045	1.61638	1.63358
Schott	BaSFS2	37.99	1.72340	1.71160	1.71779	1.73683	1.75729
Schott	LaF13	37.83	1.77551	1.76283	1.76946	1.78996	1.81201
Schott	FS1	37.06	1.58407	1.57440	1.57945	1.59521	1.61274
B & L	DBF	36.6	1.65715	1.64616	1.65189	1.66984	1.68943
B & L	DF	36.2	1.62114	1.61066	1.61610	1.63325	1.65197
Schott	BaSF14	34.95	1.69968	1.68749	1.69384	1.71386	1.73599
B & L	EDF	33.8	1.64916	1.63754	1.64355	1.66275	1.68397
B & L	EDF	32.2	1.67269	1.66012	1.66665	1.68751	1.71084
Schott	BaSFS7	32.15	1.73627	1.72243	1.72961	1.75251	1.77817
Schott	SFS08	31.30	1.61339	1.60174	1.60777	1.62737	1.65071
B & L	LaF	31.0	1.86725	1.85036	1.85910	1.88706	1.91874
B & L	EDF	29.3	1.72022	1.70555	1.71309	1.73766	1.76542
Schott	LaF9	28.39	1.79504	1.77827	1.78695	1.81495	1.84675
B & L	EDF	27.8	1.75084	1.73473	1.74303	1.77005	1.80089
Schott	SFS3	26.10	1.78470	1.76688	1.77607	1.80613	1.84085
Schott	SFS09	23.83	1.84666	1.82578	1.83651	1.87204	1.91363
Schott	SFS06	20.36	1.95250	1.92545	1.93928	1.98606	2.04280

* Schott Jenaer Glaswerk Schott & Gen.
B & L Bausch & Lomb.
EK Eastman Kodak.

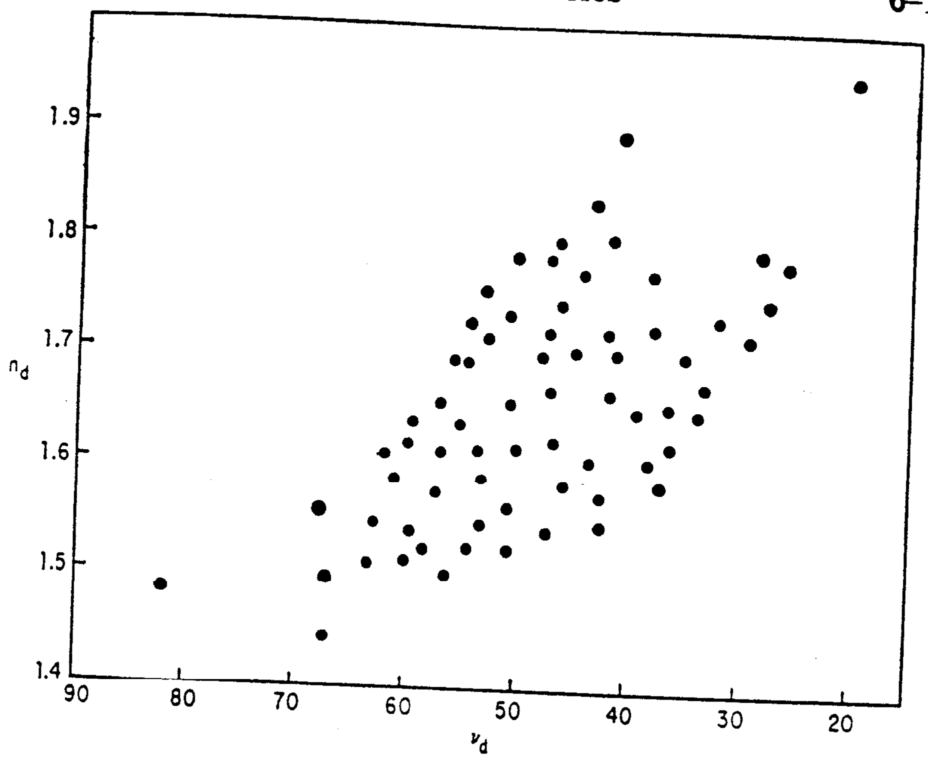

FIG. 6d-2. Graphical representation of n_d vs. ν_d of the glasses shown in Table 6d-2.

TABLE 6d-3. UNIVERSAL FUNCTIONS FOR USE IN EQ. (6d-33)

Line	λ, μm	a_1	a_2	a_3	a_4
A'	0.76820	+1.000000	0.000000	0.000000	0.000000
C	0.65630	0.000000	1.000000	0.000000	0.000000
D	0.58930	−0.219082	0.951088	0.317290	−0.049296
d	0.58760	−0.220644	0.943152	0.328101	−0.050609
e	0.54610	−0.198539	0.652493	0.619332	−0.073287
F	0.48610	0.000000	0.000000	1.000000	0.000000
g	0.43580	0.146293	−0.362709	0.835623	0.380793
G'	0.43410	0.145947	−0.360970	0.810864	0.404160
h	0.40470	0.000000	0.000000	0.000000	1.000000

TABLE 6d-4. MATRIX OF COEFFICIENTS FOR DETERMINING UNIVERSAL FUNCTIONS

j	C_{1j}	C_{2j}	C_{3j}	C_{4j}
1	−10.181350	18.313606	−9.815670	2.683413
2	13.140533	−19.603381	8.696877	−2.234029
3	2.192521	−4.346029	3.108244	−0.954736
4	−0.149673	0.311663	−0.267335	0.105345