6i. Glass, Polarizing, and Interference Filters

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This chapter briefly surveys methods of spectral filtering, by which we mean the technique of isolating a portion of the electromagnetic spectrum with filters which function in either reflection or transmission. Several survey articles [1,2] discuss these filters in detail. The important classes of filters which are discussed here are absorption filters, polarizers, mesh filters, interference devices, and polarization interference filters.

6i-1. Absorption Filters. There are many types of these filters, such as: (1) Glass-doped with impurities such as metal ions [3]. The commonly available filters [4,5] are useful in the spectral region from 0.25 to 2.5 μm. (2) Crystals, such as alkali halides or semiconductors. Spectral transmittance data are tabulated by several authors [6,7] and also by manufactures [8]. (3) Gelatin sheets impregnated with organic dyes [9] are inexpensive filters for the region 0.3 to 1.5 μm. (4) Gas cells and liquid solutions are often excellent filtering devices [1,10,11]. Infrared filters consisting of alkali halide powders dispersed in a matrix of polyethene [12,13] exhibit [illegible] bands in the spectral region from 20 to 200 μm. (6) Thin films of metals such as aluminum and indium [14] are used as bandpass filters in the spectral region below 0.1 μm, and the alkali metals [15] are effective at longer wavelengths. Absorption filters have several advantages: (1) They are relatively inexpensive, compared to the usual interference type of filter. (2) The spectral transmittance changes comparatively little as the incidence angle of the flux changes.

6i-2. Sheet Polarizers. Sheet polarizers have several advantages over the nicol prism and other early types of linear polarizers. They accept a wide cone of light (half angle of 30 to 45 deg, for example). They are thin, light, and rugged and are easily cut to any desired shape. Pieces many feet in length can be made. The cost is almost negligible compared with that of a nicol prism.

If a sheet polarizer is mounted perpendicular to a beam of 100 percent linearly polarized radiation, and if the polarizer is slowly turned in its own plane, the transmittance k varies between a maximum value k_1 and a minimum value k_2 according to the following law:

$$k = (k_1 - k_2)(\cos^2 \theta) + k_2 \tag{6i-1}$$

When such a polarizer is placed in a beam of unpolarized radiation, the transmittance is $\frac{1}{2}(k_1 + k_2)$. When two identical polarizers are mounted in the bean with their axes crossed, the transmittance is $k_1 k_2$.

The principal transmittance values k_1 and k_2 vary with wavelength, the variation being different for different types of polarizers. Table 6i-1 presents data for several

TABLE 6i-1. SPECTRAL PRINCIPAL TRANSMITTANCE OF SHEET POLARIZERS*

Wave-length, μm	HN-22 sheet		HN-32 sheet		HN-38 sheet		KN-36 sheet		HR sheet	
	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
0.375	.11	.000,005	.33	.001	.54	.02	.42	.002	.00	.00
0.40	.21	.000,01	.47	.003	.67	.04	.51	.001	.00	.00
0.45	.45	.000,003	.68	.000,5	.81	.02	.65	.000,3	.00	.00
0.50	.55	.000,002	.75	.000,05	.86	.005	.71	.000,05	.00	.00
0.55	.48	.000,002	.70	.000,02	.82	.000,7	.74	.000,04	.00	.00
0.60	.43	.000,002	.67	.000,02	.79	.000,3	.79	.000,03	.01	.00
0.65	.47	.000,002	.70	.000,02	.82	.000,3	.83	.000,08	.05	.00
0.7	.59	.000,003	.77	.000,03	.86	.000,7	.88	.02	.10	.00
1.0									.55	.05
1.5									.65	.00
2.0									.70	.00
2.5									.10	.02

* Data supplied by R. C. Jones, Polaroid Corporation, Cambridge, Mass. For each type of polarizer, the transmittance values near the ends of the useful range depend on the type of supporting sheet or lamination used. Also some variation from lot to lot must be expected.

well-known types, produced by Polaroid Corporation, Cambridge, Mass. H sheet, perhaps the most widely used sheet polarizer, is effective throughout the visual range; it is produced in three modifications having total luminous transmittance (for C.I.E. Illuminant C light) of 22 percent (Type HN-22), 32 percent (Type HN-32), and 38 percent (HN-38). Type HN-22 provides the best extinction, Type HN-38 provides the highest transmittance, and Type HN-32 represents a compromise that is preferred in many applications. K sheet, also useful throughout the visual range, is particularly intended for applications involving very high temperature. Its transmittance is 35 to 40 percent. HR sheet is effective in the infrared range from 0.7 to 2.2 μm.

Table 6i-2 presents data for a Polaroid Corporation ultraviolet light-polarizing filter HNP′B. The characteristics for wavelengths longer than 0.400 μm are the same as for HN-32 in Table 6i-1. In Fig. 6i-1 are curves showing a range of k values which can be achieved with this class of ultraviolet polarizer. Absorbing polarizers are also made by Polacoat, Inc., Blue Ash, Ohio.

TABLE 6i-2. SPECTRAL PRINCIPAL TRANSMITTANCE OF ULTRAVIOLET SHEET POLARIZER HNP′B (3.5)

λ	k_1	k_2	λ	k_1	k_2
(275)*	(0.250)	(0.0126)	340	0.602	0.0002
280	0.328	0.0110	350	0.568	0.0001
290	0.340	0.0040	360	0.550	0.0003
300	0.372	0.0017	370	0.568	0.0007
310	0.448	0.0009	380	0.604	0.0009
320	0.546	0.0006	390	0.644	0.0008
330	0.611	0.0003	400	0.688	0.0005

* This is the effective cutoff wavelength of the supporting plastic layer. The HNP′B foil itself transmits to about 250 nm. In this region the foil has much lower dichroism.

6i-3. Mesh Filters and Interference Devices. *Metal mesh filters* consist of an array of thin metal strips, rectangles, disks, apertures, etc., which are either unsupported or deposited on a thin plastic sheet. They have an effect similar to an iris in a microwave guide, with the exception that the mesh array functions in free space. They have been used principally in the spectral region from 100 μm to one millimeter

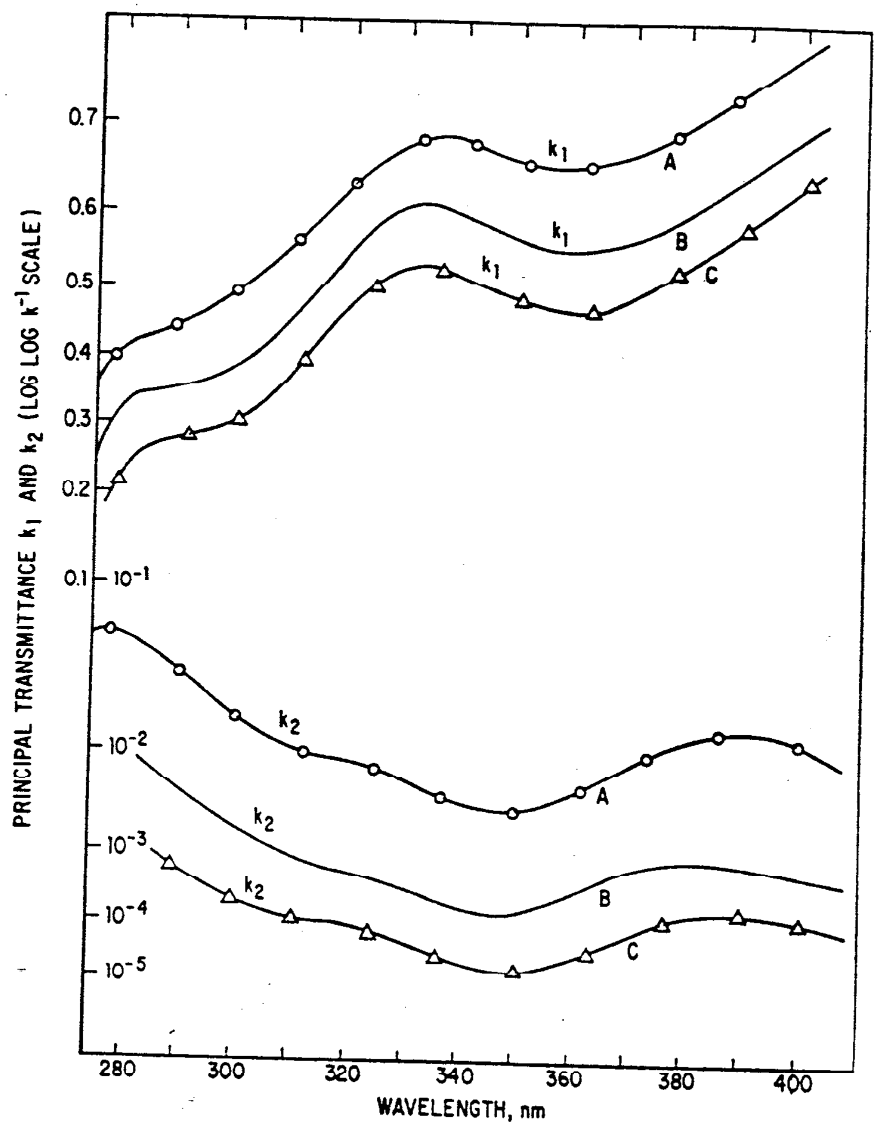

FIG. 6i-1. Polarized spectral transmittances of Polariod type HNP′B film. (Sample *B* represents standard concentration.)

as high-frequency pass filters [16], low-frequency pass filters [17], and bandpass filters [18].

Optical interference devices utilize the optical interference between reflecting surfaces and usually consist of a stack of thin films deposited on a suitable substrate. The term *multilayer* is often used generically to describe such coatings. They are used as mirrors and bandpass filters in the spectral region from the far infrared to 0.12 μm, which is the transmission limit of the lithium fluoride substrate. The wavelength at which they function is determined by the thickness of layers.

At the risk of oversimplification, such filters can be classed in two broad categories: (1) long-wave pass or short-wave pass filters, which consist of a periodic structure; (2) bandpass filters, which consist of a single optical cavity or several optical cavities. The terms "optical cavity" and "periodic structure" are explained below.

The term *periodic structure* refers to a group of two or more layers which is repeated many times. For example, the simplest type is a quarter-wave stack [19], which consists of a stack of layers of equal optical thickness and alternating between a low-index **L** and a high-index **H**. The stack is then of the design:

air **H L H L · · · H L H** *substrate*

where *air* is the incident medium. If such a stack were fabricated to reflect in the visible portion of the spectrum, then the **H** layer would typically consist of titanium dioxide (index 2.35) and **L** is silicon dioxide (index 1.55). Such multilayers are used

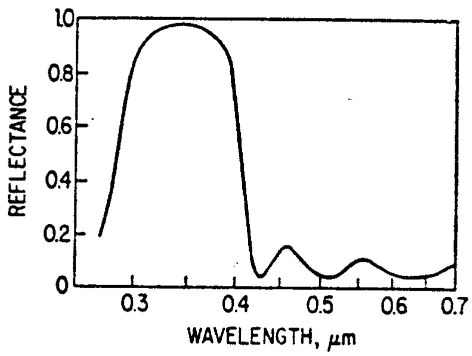

FIG. 6i-2. The measured spectral reflectance R of a multilayer mirror. Its transmittance is approximately $1 - R$. (*Courtesy of Schott Glaswork, Mainz.*)

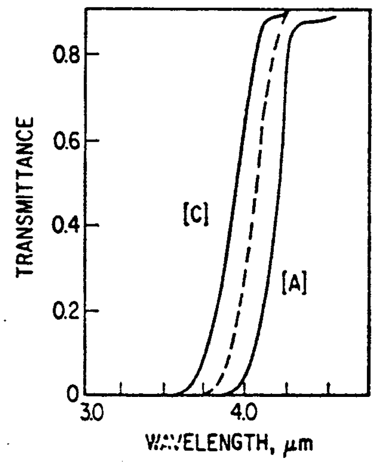

FIG. 6i-3. Measured spectral transmittance of a multilayer type of long-wave pass filter at normal incidence (curve *a*), at $\phi = 47$ deg incidence (dashed curve), and cooled to liquid nitrogen temperature at $\phi = 47$ deg (curve *c*). (*Courtesy of Optical Coating Laboratory, Inc.*)

because they have one or more useful *stop bands*—a region of the spectrum in which they are strongly reflecting and hence have a low transmittance. They have a substantial transmittance in the spectral region outside their stop bands. For example, Fig. 6i-2 depicts the spectral reflectance of a multilayer whose stop band extends from approximately 0.3 to 0.4 μm and which has a passband in the visible portion of the spectrum. Similar types of stacks with periodic structures are used as long-wave pass and short-wave pass filters, particularly in the infrared portion of the spectrum. As is true of all optical-interference devices, their transmittance changes with the angle of incidence, as shown in Fig. 6i-3. Since these periodic stacks are strongly reflecting in their stop bands, they are used as laser mirrors, reflection filters, and also as components of the optical-cavity type filters described in the next paragraph.

Filters with a relatively narrow spectral passband width are constructed by separating two reflectors with a "spacer" to form an *optical cavity*. More complex filters are produced by combining several optical cavities. There are several ways in which such optical-cavity filters can be fabricated. The reflectors can be composed of (1) semireflecting metal layers, such as silver or aluminum, or (2) a multilayer mirror, such as the quarter-wave stack described previously.

The optical cavity can be formed in several ways: (1) The reflectors are deposited on ultraflat fused-quartz plates and are held extremely parallel by a mechanical structure. Thus the cavity contains either a vacuum or some gas. This instrument is called the "Fabry-Perôt interferometer." (2) The cavity can contain a solid material, as when the reflectors are deposited on a solid slab of fused quartz or a sheet of mica. (3) The spacer is a thin solid film, which is deposited by evaporation in a vacuum when the filter is fabricated.

Before surveying the bandpass filters which can be fabricated with such optical cavities, we must define the filter's important attributes, which are: (1) The maximum transmittance T_{max} in its passband. (2) The wavelength λ_0 at which T_{max} occurs. (3) The spectral width of the passband, which is expressed in terms of its half width $\Delta\lambda_{\frac{1}{2}}$ at $0.5T_{max}$ or its deciwidth $\Delta\lambda_{\frac{1}{10}}$ at $0.1T_{max}$. (4) The resolution $Q = \lambda_0/\Delta\lambda_{\frac{1}{2}}$. (5) The off-band rejection, which is nebulously defined as the attenuation outside the passband. (6) The angle shift, which is the change of λ_0 at the angle of the incidence flux changes. (7) The angular field, which is related to the angle shift. The passband broadens when the filter is illuminated with convergent flux. The angular field is the maximum permissible cone angle of this flux. (8) The shift of λ_0 with temperature. (9) The order of interference m, which is the number of half wavelengths in the resonant cavity.

The Fabry-Perôt interferometer is described in many texts [20] and is principally used to measure with high resolution the spectral profile of lines in emission spectra. It can also be used as a narrow-band filter, especially if a regulator is used to maintain the parallelism of the reflectors [21]. It has the following properties:

1. It has a high resolution. A Q of 10^5 is easily attained, which corresponds to a bandwidth of 0.005 nm in the visible portion of the spectrum.

2. Since this Q is attained by using a high order of interference, there are undesired passbands adjacent to the main passband separated by a wavelength of approximately λ_0/m. For most applications, auxiliary "blocking filters" must be ganged in tandem with the FP to suppress these undesired passbands.

3. Its off-band rejection is rather poor, compared to that of a multicavity filter or a polarization-interference filter. The spectral shape of the passband is given to a good approximation by $T(\lambda) = T_{max}[1 + (\lambda - \lambda_0)^2 a]^{-1}$ where the constant a is related to $\Delta\lambda_{\frac{1}{2}}$. This line shape is called "Lorentzian" and has the property that the off-band transmittance decreases quite slowly. The ratio $\Delta\lambda_{\frac{1}{2}}/\Delta\lambda_{\frac{1}{10}}$ is independent of the Q.

4. As the angle of incidence ϕ increases, the angle θ of the flux in the cavity also increases, and the wavelength λ of the passband shifts to shorter wavelengths, as given by

$$2nd \cos \theta = \lambda m \tag{6i-2}$$

where n and d are respectively the refractive index and physical thickness of the spacer. Although this equation neglects any effects of the phase shift upon reflection from the mirrors, it is a good approximation for $m > 10$. For small angles Eq. (6i-2) is combined with Snell's law, and the fractional change in wavelength is expressed as

$$\frac{\lambda_0 - \lambda}{\lambda_0} \cong \frac{n_0^2\phi^2}{2n^2} \tag{6i-3}$$

where n_0 is the index of the incident medium in which ϕ is measured. As the fractional bandwidth of the filter is decreased, the flux must be more nearly perfectly collimated in order to maintain the fractional "angle shift" of the passband comparable to the Q^{-1} of the filter.

5. Although it is theoretically possible to attain a high resolution by using cavity reflectors with a reflectance very close to 100 percent, this is usually undesirable for

several reasons: First, the effect of any absorption in cavity spacer or in the reflectors is greatly enhanced, and this drastically reduces T_{max}. Second, the passband width does not decrease below a certain limit owing to a lack of planeness in the surface of the reflectors [22].

Single-cavity bandpass filters have also been constructed using a solid material for the cavity, as, for example, a slab of fused quartz [23] or a sheet of mica [24]. Compared with the conventional Fabry-Perôt, these have the advantages of greater mechanical and thermal stability. They also have a smaller angle shift, as can be seen from the effect of n^2 in the denominator of Eq. (6i-3). However, the T_{max} is smaller, because of the absorption of the spacer. A typical mica filter [24] has a T_{max} of 60 percent, $\Delta\lambda_{\frac{1}{2}}$ of 0.15 nm, and an order number of 150 at λ_0 of 600 nm. Multilayer mirrors are usually used for the reflectors in both the "air-spaced" and the "solid spacer" types of Fabry-Perôt filters.

The bandpass filters which are fabricated entirely of thin films have several distinct features: (1) Multiple-cavity filters [25] can be constructed, which have the advantage of superior off-band rejection. (2) Metal films can be used to advantage in the

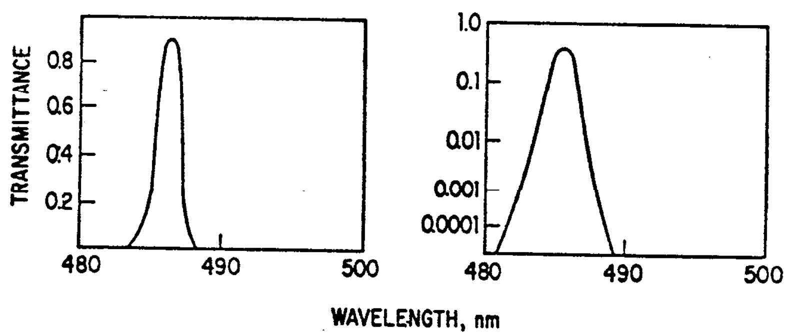

FIG. 6i-4. The measured spectral transmittance of a multiple cavity bandpass filter composed of thin films. (*Courtesy of Spectrolab, Inc.*)

FIG. 6i-5. The measured spectral transmittance (on linear and logarithmic scales) of a multilayer bandpass filter of the multiple-cavity type. (*Courtesy of Spectrolab, Inc.*)

filters. This provides another means of achieving a large off-band rejection. (3) The passbands can be located at wavelengths from the vacuum ultraviolet [26] to the infrared [2].

In the selection of such filters, the problem of blocking unwanted transmission bands should be considered. For example, Fig. 6i-4 shows the spectral transmittance of a single-cavity filter composed of dielectric layers. In addition to the passband in the green portion of the spectrum, there are broad regions of transmittance in both the blue and red. In most applications, auxiliary filters must be added in tandem with the multilayer filter to suppress these unwanted bands. Such a filter is said to be "completely blocked." Of course, the addition of these auxiliary filters reduces the T_{max}. Thus it is important to note whether a manufacturer is supplying the transmission curve of a "blocked" or an "unblocked" filter.

Compared with the single-cavity filter, the multiple-cavity filter has a passband shape which is more nearly rectangular, and it also has a superior off-band rejection. Figure 6i-5 depicts the spectral transmittance of a multicavity filter. Another method of obtaining an improved off-band rejection is to use filters which contain metal films [27]. The transmittance of such a filter is shown in Fig. 6i-6, and although its peak transmittance is rather small, its off-band transmittance is more than 40 db below T_{max}. Filters with a bandwidth $\Delta\lambda_{\frac{1}{2}}$ of a few angstroms have been fabricated [28] for use in the spectral region from 0.5 to 0.9 μm.

Reflection Filters. In certain regions of the spectrum, and especially in the ultraviolet, the reflection filter offers the simplest method of achieving a substantial T_{max} and a large off-band rejection. Figure 6i-7 depicts the spectral transmittance of an ultraviolet filter in which the incident flux reflects from four mirrors. The residual-ray type of reflection filter [2] has been used for many years to isolate portions of the infrared spectrum.

Choice of Filter. There are so many varied applications of bandpass filters that no single criterion can be used to judge their relative merit. Consider, for example, the selection of a filter in terms of its passband width. The half width is not a useful criterion of its performance if substantial off-band rejection is required. For many years solar physicists have relied upon the Lyot-Öhman filter described in Sec. 6i-4 to photograph absorption lines on the sun, because of its superior off-band rejection.

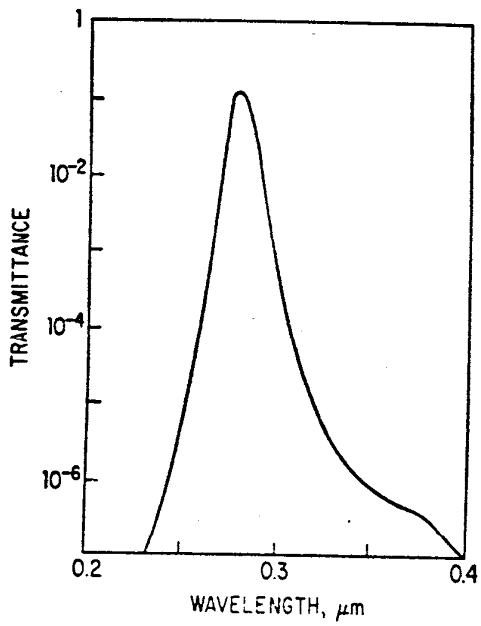

FIG. 6i-6. The measured spectral transmittance of an ultraviolet bandpass filter which contains metal films. (*Courtesy of Infrared Industries, Waltham, Mass.*)

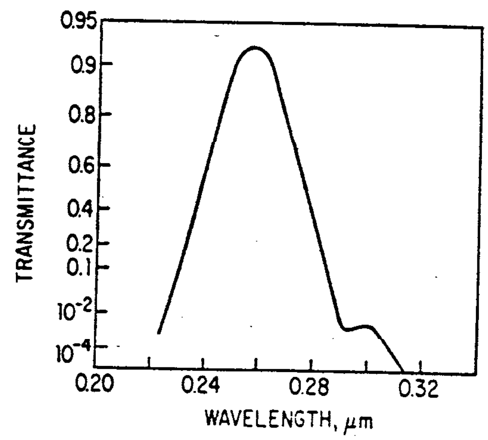

FIG. 6i-7. The measured spectral transmittance of a reflection filter. (*Courtesy of Schott Glaswork, Mainz.*)

The angular field is an attribute which must be related to the filter's physical size and to the optical system in which it is used. Suppose that a filter is combined with an objective lens which collects radiant flux from a distant source and focuses it upon a detector, as shown in Fig. 6i-8. Although a filter of small physical dimensions is required if it is placed at its focus, the highly convergent flux broadens its passband. Clearly the flux is least convergent at A in front of the objective lens, but this requires a filter of large diameter D_1. A smaller filter could be placed at B, although the angular subtense of the flux is magnified by the ratio D_1/D_2.

It is often necessary to compute the flux transmitted by the filter placed at the focus. If a uniform, collimated irradiance H_0 impinges upon the objective lens, then a flux $dF = H_0 2\pi y\, dy$ is contained in an annulus of width dy at a radius y from the center. Since $y = f \sin \phi$ for an aplanatic lens [29] of focal length f, the total flux is

$$2\pi H_0 f^2 \int_0^{\frac{1}{2}\phi_m} T(\phi) \cos \phi \sin \phi \, d\phi \qquad (6i\text{-}4)$$

where ϕ_m is the total cone angle. Several authors [30,31] have obtained approximate solutions to this equation, for specific types of filters.

Combinations of Filters. Often two or more filters are ganged in a tandem array to sharpen the passband or to increase the off-band rejection. Each filter has a reflectance R_i and transmittance T_i, and the tandem array has reflectance R and transmittance T. The largest attenuation is attained when the filters are either: (1) nonreflecting, $R_i = 0$, or (2) arranged so that the flux reflected at each surface leaves the optical system and is not collected by the detector. Only in this case is it true that T is the product: $T = T_1 T_2 \cdots T_i$.

The poorest attenuation is obtained when the multiply reflected beams are collected by the detector and all the filters are nonabsorbing, $R_i + T_i = 1.0$. In this case it can be shown [32] that $R/T = R_1/T_1 + R_2/T_2 + \cdots + R_i/T_i$.

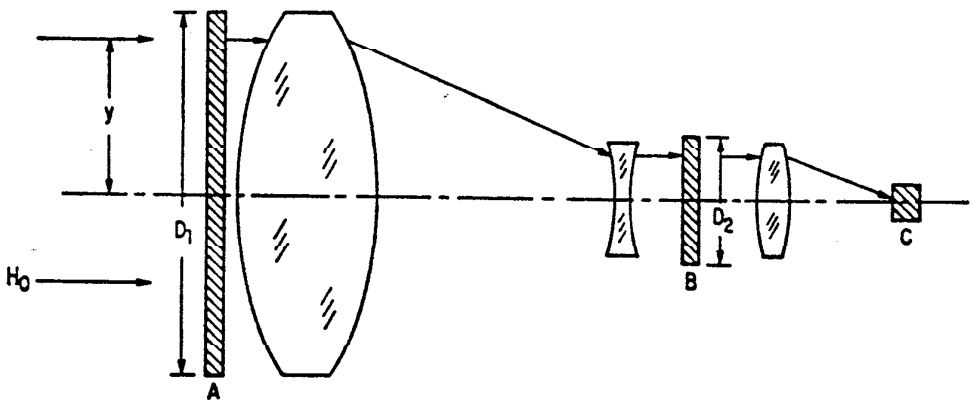

FIG. 6i-8. Showing the various positions at which a filter may be placed in a flux-collecting optical system.

If a single nonabsorbing filter transmits 1 percent, a tandem array of two identical filters transmits 0.503 percent. In the more general case of tandem array of semireflecting, absorbing filters, we must measure the reflectance from both sides of each filter and use more complex equations [32] to compute the transmittance of the array.

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6i-4. Birefringent Filters. Two types of birefringent polarizing filters with bandwidths of 0.12 to 100 Å have been successfully made and are widely used in solar research. They are the Lyot-Öhman [1,2] filter and the Solc [3] (pronounced "Scholtz") filter. Both depend on the interference of polarized light, and have spectral transmission curves with sharp regularly spaced primary maxima at successive orders of interference, separated by series of secondary maxima.

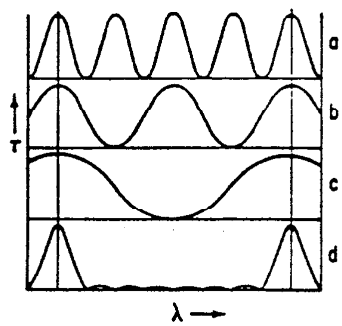

FIG. 6i-9. Spectral transmission of the single *b* elements of a three-element Lyot-Ohman filter (*a*, *b*, *c*) and the assembled filter *d*.

The Lyot-Öhman (LO) filter consists of alternating polarizers (usually Polaroid film) and plane-parallel birefringent plates cut with optic axes parallel to the surfaces. The first and last elements are polarizers. In the simplest configuration, the polarizers are parallel, and the birefringent elements are set with optic axes either alternating between plus and minus 45 deg to the plane of polarization or all parallel to either angle. The thicknesses of the birefringent elements (*b* elements) increase in powers of 2, but may be arranged in any order.

The light entering a given *b* element is divided into ordinary and extraordinary beams which traverse the element at different velocities and are combined by the following polarizer to interfere. The transmission of a single *b* element between a pair of polarizers is

$$\tau = \cos^2 \pi\gamma \tag{6i-5}$$

where $\gamma = (d/\lambda)\ (\epsilon - \omega)$, ϵ and ω being the extraordinary and ordinary indices of refraction, and d the thickness of the b element. Since the thickness of the ith element is $d_i = 2^{i-1}d_1$, the transmission of the whole filter is

$$\tau = \cos^2 \pi\gamma_1 \cos^2 \pi\gamma_2 \cdots \cos^2 \pi\gamma_n = \cos^2 \pi\gamma_1 \cos^2 2\lambda\gamma_1 \cdots \cos^2 2^{n-1}\pi\gamma_1 \tag{6i-6}$$

where n is the total number of b elements. Figure 6i-9 shows the spectral transmission curves for the individual b elements of a three-element filter and the curve of their product, i.e., the transmission curve for the assembled filter. All but two of the transmission peaks of the thickest element in Fig. 6i-9*a* coincide with transmission minima of one of the thinner elements and are thereby suppressed. Lyot [1] showed that Eq. (6i-6) is identical with the expression for the spectral transmission of a diffraction grating of 2^n rulings:

$$\tau = \frac{\sin 2^n\gamma_1}{2^n \sin \gamma_1} \tag{6i-7}$$

The bandwidth at half intensity, $\delta\lambda$, is very nearly the half width between the transmission peak at γ_1 = integer, and the first zero at γ_n = integer + $\frac{1}{2}$, given approximately by

$$\delta\lambda = \frac{\sigma\lambda}{2\gamma_n} = \frac{\sigma\lambda}{2^n\gamma_1} \tag{6i-8}$$

The band separation, $\Delta\lambda$, is the interval between successive transmission peaks for the thinnest b element, approximately:

$$\Delta\lambda = \frac{\sigma\lambda}{\gamma_1} \tag{6i-9}$$

The mysterious factor σ allows for the effect of dispersion in $\epsilon - \omega$. It is about 0.9, and varies by a few percent for different crystals and wavelength regions. The finesse is obviously

$$F = \frac{\Delta\lambda}{\delta\lambda} = 2^n \tag{6i-10}$$

A most important property of any narrow-band filter is its suppression ratio, $S = T_m/T_p$, where T_m is the integrated transmission over the whole principal band between the adjacent zeros (of width $2\delta\lambda$), and T_p is the integrated parasitic transmission between successive principal bands. For the LO filter, $S = 0.11$.

If $\epsilon - \omega$ is known in the neighborhood of a desired passband, Eqs. (6i-2), (6i-4), and (6i-5) are sufficient to calculate the γ_1 and n required for a filter of any desired bandwidth and band separation. Normally one provides for $\Delta\lambda$ sufficiently large to allow unwanted passbands to be eliminated with glass filters or an interference filter. In making the calculation, one must allow for the dependence of γ on temperature. The net effect is a shift in the wavelength of the passband by $\Delta\lambda_T = k\,\Delta T$. For quartz and calcite, $k = -0.66$ and -0.42 Å per °C, respectively. It is necessary, therefore, to control the temperature of the filter to keep $\Delta\lambda_T$ within an acceptable tolerance (usually about 0.2 bandwidth). However, the temperature dependence provides a convenient fine tuning of sufficient range to correct for small deviations from an assumed $\epsilon - \omega$.

This is the basic Lyot-Öhman filter. It has limitations. The wavelength of the passband for light traversing it at an angle to the instrumental axis is shifted. Let ϕ be the angle in air between the light ray and the axis, and θ the azimuth of the incident plane measured from the crystal optic axis of the first b element. Then

$$\Delta\lambda(\phi,\theta) = \pm\lambda\sigma\frac{\phi^2}{2\omega^2}(\cos^2\theta - \sin^2\theta) \tag{6i-11}$$

The plus and minus signs apply, respectively, for positive and negative crystals. Let $\Delta\lambda(\phi)$ be the maximum acceptable shift. The corresponding ϕ at $\theta = 0$ or $\pi/2$ is

$$\phi = \left[2\Delta\lambda(\phi)\frac{\omega^2}{\sigma\lambda}\right]^{\frac{1}{2}} \approx 2.2\left[\frac{\Delta\lambda(\phi)}{\lambda}\right]^{\frac{1}{2}} \tag{6i-12}$$

This is a fairly stringent restriction. For example, in a filter for $\lambda = 6{,}563$ ($H\alpha$) with $\delta\lambda = 0.5$ Å, $\Delta\lambda(\phi) = 0.1$ Å is a reasonable tolerance. Then $\phi = 9 \times 10^{-3}$ radian.

Lyot invented a wide field elaboration of the simple filter in which each b element is made of two equal pieces of half the calculated thickness. The two are rotated 90 deg with respect to each other, and a 90-deg polarization rotator (usually a $\lambda/2$ plate) is mounted between them. Then for a given light ray, θ in the first half element becomes $\theta + \pi/2$ in the second half element, and by Eq. (6i-11), the $\Delta\lambda(\phi,\theta)$'s compensate. Since Eq. (6i-11) neglects higher-order terms, the compensation is not perfect, but Lyot's device does increase the radius of the useful field by factors of 26 and 6 for b elements of quartz and calcite.

Another problem is the loss of light by absorption in the polarizers of filters with large finesse. A Polaroid film usually transmits about 80 percent of the desired light, which means an optical density of $0.091n + 0.39$ for the filter as a whole. In some uses the loss of 95 percent of the light is serious. To alleviate the pain, Evans [2] devised a "split element" filter. Here half the b elements are cut into two equal halves and crossed as in Lyot's wide-field filter. The remaining elements are inserted between these equal halves with axes at 45 deg, and a unit of two birefringent elements can then be placed between successive polarizers. This reduces the optical density to $0.091n/2 + 0.39$, a very considerable improvement when n approaches 8 or 10. A better solution is to use more transparent polarizers like Rochon prisms, which may be no more expensive than the split-element construction. The prism polarizers are the only presently practical approach at wavelengths less than about 4,200 Å, where the absorption of Polaroid film begins to become excessive.

Beyond the limited slow tuning by temperature variation, each transmission band of a birefringent filter has a fixed central wavelength. It is feasible, however, to tune to any desired wavelength by adding phase shifters to the b elements. The condition for a transmission band at a given wavelength, λ_1, is that γ be integral at λ_1 in all b elements. If each b element has a phase shifter which adds $\Delta\gamma$, adjustable from 0 to 1, this condition can be satisfied.

The elegant approach is a filter with b elements of adjustable thickness. Each element is a pair of wedges, one of which slides with respect to the other to vary the total thickness in the optical path. The $\Delta\gamma$ is then a function of the wedge position and wavelength, but at every wavelength the required $\Delta\gamma$ can be achieved. Hence the variable thickness tuning works at all wavelengths for which the polarizers are effective. So far, the mechanical problems of control in filters of 8 or 10 elements have prevented use of this method. However, one of the modern small control computers could deal with these problems quite easily.

The second approach to filter tuning is relatively simple mechanically, but is effective only over a limited spectral range. Before entering the following polarizer, the light emerging from a b element is elliptically polarized. As λ varies and γ goes through a range of 1, the elliptical figure goes through the cycle from vertical linear to right circular, horizontal linear, left circular, and back to vertical linear. If now we add a $\lambda/4$ plate with its axis at 45 deg to the axis of the b element, elliptical polarization is converted to plane polarization rotated at an angle $\Psi = \frac{1}{2}\gamma$ to the axis of the $\lambda/4$ plate. If now we rotate the following polarizer, we can adjust it to transmit any $\Delta\gamma$, regardless of the wavelength. In a simple filter the Ψ's are simply proportional to the thicknesses of the preceding b elements, which progress in powers of two. The wavelength limitation is imposed by the fact that a simple $\lambda/4$ plate is $\lambda/4$ at only one wavelength. Light leaks develop in the wavelength intervals between principal bands as we depart from that wavelength. However, the leaks are tolerable for most purposes over a range of several hundred angstroms.

The tuning range of the rotating polarizer filter can be greatly extended by the use of achromatic $\lambda/4$ plates, which can now be constructed by known principles [4]. One such filter with a 0.25 Å passband at λ6,563, tunable from 4,200 to 7,000 Å, is presently under construction.

All commercially available LO filters have b elements of quartz with $\epsilon - \omega \sim 0.009$, or calcite with $\epsilon - \omega \sim 0.18$, or both. The use of other materials has been confined to a few experiments. A typical example has an aperture of 30 mm, a 0.5 Å passband at 6,563, with high-order calcite elements of wide field construction. The two thickest elements are tunable over a range of ± 2 Å by rotating polarizers. The total length is about 16 cm.

The Solc filter [3] consists of a pile of N plane-parallel birefringent plates placed between two polarizers. The plates are identical in thickness, and are cut with the optic axes parallel to the surfaces. Two basic arrangements give identical filtering characteristics. The folded filter has crossed polarizers, and the optic axes of the plates are set successively at angles of plus or minus $\pi/4N$ to the electric vector of light from the first polarizer. The fan filter has parallel polarizers, and the orientations of successive b elements increase monotonically, $\pi/4N$, $3\pi/4N$, $5\pi/4N$, etc. The action of the pile on polarized light is not readily apparent, but can be understood qualitatively if one thinks of the pile as a device for producing $N + 1$ different pathlengths, among which the light is distributed somewhat unevenly. The resulting transmission curve resembles that of a grating of $N + 1$ rulings, but has some significant differences. The transmission is

$$\tau = \frac{\sin N\chi}{\sin \chi} \cos \chi \tan \frac{\pi}{2N} \qquad \text{(6i-13)}$$

where χ is defined by

$$\cos \chi = \cos \pi\gamma \cos \frac{\pi}{2N}$$

and γ is the retardation of a single plate.

The separation of successive transmission bands is approximately

$$\Delta\lambda = \sigma \frac{\lambda}{\gamma} \qquad \text{(6i-14)}$$

The bandwidth approaches

$$\delta\lambda = \frac{\sqrt{3}}{2} \frac{\sigma\lambda}{N\gamma} \qquad \text{(6i-15)}$$

as N increases. Equation (6i-15) is accurate within one percent when $N \geq 5$. The Solc and LO filters have the same band separation and bandwidth if $\gamma = \gamma_1$ and $N = \sqrt{3} \times 2^{n-1}$. In its basic form, however, the Solc filter suppresses parasitic light much less effectively than the Lyot filter. S (Solc) increases with N. It is 0.22 for $N = 16$ and approaches an upper limit of 0.27. However, Solc [5] showed that S can be reduced to <0.05 by simply altering the orientations of the plates slightly at the cost of some increase in $\delta\lambda$ (which can be compensated by increasing N about 20 percent). This is the only practical elaboration of the Solc filter so far proposed.

The effect of inclination of the light rays to the instrumental axis is about the same as that for the simple LO filter. No one has devised an analytic expression for $\Delta\lambda(\phi,\theta)$, but Beckers and Dunn [6] calculated it numerically. The field could be enlarged by using plates of Lyot's compound form, but in a filter of large finesse this would be very expensive.

Similarly, wavelength tuning is possible only by varying the thickness of all N individual plates equally.

The great virtue of the Solc filter is its transparency. It has only two polarizers. Against this advantage we must consider a very expensive form of construction. There are more optical elements to be worked than for an LO filter, and the tolerances in thickness are very much smaller. The thickness tolerances for all b elements in an LO filter are about 3 μm for quartz and 0.15 μm for calcite. Beckers and Dunn found that the tolerances in a Solc filter are smaller by a factor of the order of $1/N$. They note, however, that Solc has made a successful filter with $N = 80$, which speaks well for the skill of his opticians.

On the whole, it is probably far less expensive to build an LO filter with prism polarizers than a Solc filter of equivalent performance. If a wide angular field is required, or wavelength tuning, the Solc filter is not a realistic competitor.

References for Sec. 6i-4

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